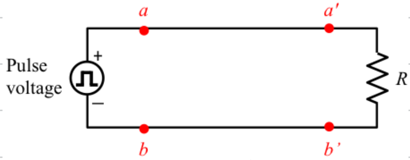
§ Transmission Line Theorey.

2.1 Why Does Frequency Matter

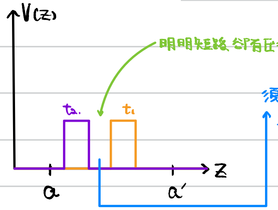
· Assumption in Circuit and High Frequency.



· In Electric Circuit, we usually consider there's no voltage rise or drop alone the line (short circuit)

· In Reality, ① the transmission line is so long that cause "propagation"
② high frequence cause short wavelength, short line can have "voltage rise or drop"

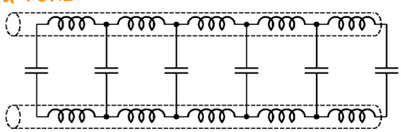
⇒ graphical expression.



明明短路 卻有压升压降.

須修正若考慮時間延遲問題.
↑ 具有 component 在此 short.

☆ revise



當傳輸頻率↑, 則以此傳送.

电感 $L=\dfrac{\phi\ (\text{磁通量密度})}{I\ (\text{电流})}\ (H/m)$

傳輸線本身.

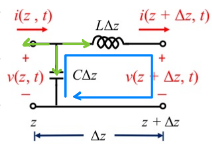
电容 $C=\dfrac{\rho_s\ (\text{电通量密度})}{V\ (\text{电压})}\ (F/m)$

傳輸線與地 or 其他

皆與導線長度有关.
distributed. 建構式 (非真实, 有其他因次)
lump. (真实, 無多餘物件本身外因次)

· Modeling Equivalent of Short Section

consider very short section transmission line Δz



① KVL: $-v(z,t)+(L\Delta z)\cdot\dfrac{\partial i(z+\Delta z,t)}{\partial t}+v(z+\Delta z,t)=0$

② KCL: $-i(z,t)+(C\Delta z)\cdot\dfrac{\partial v(z+\Delta z,t)}{\partial t}+i(z+\Delta z,t)=0.$

from ① ⇒ $\dfrac{v(z+\Delta z,t)-v(z,t)}{\Delta z}=-L\dfrac{\partial i(z+\Delta z,t)}{\partial t}.$

from ② ⇒ $\dfrac{i(z+\Delta z,t)-i(z,t)}{\Delta z}=-C\dfrac{\partial v(z+\Delta z,t)}{\partial t}$

皆取 $\lim\limits_{\Delta z\to 0}$ →
$\boxed{\lim\limits_{\Delta z\to 0}\dfrac{v(z+\Delta z,t)-v(z,t)}{\Delta z}}=\boxed{\lim\limits_{\Delta z\to 0}-L\dfrac{\partial i(z+\Delta z,t)}{\partial t}}$ ⇒ $\boxed{\dfrac{\partial v(z,t)}{\partial z}=-L\dfrac{\partial i(z,t)}{\partial t}}$

$\boxed{\lim\limits_{\Delta z\to 0}\dfrac{i(z+\Delta z,t)-i(z,t)}{\Delta z}}=\boxed{\lim\limits_{\Delta z\to 0}-C\dfrac{\partial v(z+\Delta z,t)}{\partial t}}$ ⇒ $\boxed{\dfrac{\partial i(z,t)}{\partial z}=-C\dfrac{\partial v(z,t)}{\partial t}}$

微分定義. Δz 消失. Telegrapher's Equation

$\dfrac{\partial v(z,t)}{\partial z}=-L\dfrac{\partial i(z,t)}{\partial t}$ <2邊同取 $\frac{\partial}{\partial z}$> ⇒ $\dfrac{\partial^2 v(z,t)}{\partial z^2}=-L\dfrac{\partial^2 i(z,t)}{\partial z\partial t}$

$\dfrac{\partial i(z,t)}{\partial z}=-C\dfrac{\partial v(z,t)}{\partial t}$ <2邊同取 $\frac{\partial}{\partial t}$> ⇒ $\dfrac{\partial^2 i(z,t)}{\partial z\partial t}=-C\dfrac{\partial^2 v(z,t)}{\partial t^2}$

} 合併得. $\boxed{\dfrac{\partial^2 v(z,t)}{\partial z^2}-LC\dfrac{\partial^2 v(z,t)}{\partial t^2}=0.}$

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} 合併得. $\boxed{\dfrac{\partial^2 i(z,t)}{\partial z^2}-LC\dfrac{\partial^2 i(z,t)}{\partial t^2}=0.}$

Wave Equation.

⇒ $\dfrac{\partial^2 f(z,t)}{\partial z^2}-\dfrac{1}{v_p^2}\dfrac{\partial^2 f(z,t)}{\partial t^2}=0.$

具有通解形式
$f(z,t)=f^i\left(t-\dfrac{z}{v_p}\right)+f^r\left(t+\dfrac{z}{v_p}\right)$

$\therefore v(z,t)=\boxed{V^i}\left(t-\dfrac{z}{v_p}\right)+\boxed{V^r}\left(t+\dfrac{z}{v_p}\right)$ $\quad \dfrac{z}{v_p}=\dfrac{\text{位移}}{\text{速度}}$ = 時間 ⇒ 其实為 time delay.

$i(z,t)=\boxed{i^i}\left(t-\dfrac{z}{v_p}\right)+\boxed{i^r}\left(t+\dfrac{z}{v_p}\right)$

incident ⇒ 入射波. reflect ⇒ 反射波.

$v_p=\dfrac{1}{\sqrt{LC}}$ (phase velocity)

∴代入 $\dfrac{\partial v}{\partial z}=-L\dfrac{\partial i}{\partial t}$ $\displaystyle\int\frac{-1}{v_p}\left[v^i\left(t-\frac{z}{v_p}\right)-v^r\left(t+\frac{z}{v_p}\right)\right]dt=\int -L\frac{\partial i}{\partial t}\,dt$

$i=\dfrac{1}{Lv_p}\left[v^i\left(t-\dfrac{z}{v_p}\right)-v^r\left(t+\dfrac{z}{v_p}\right)\right]$

為了將 V, i 聯关係 $i=\dfrac{V}{Z_0}$

def $Z_0=Lv_p=\sqrt{\dfrac{L}{C}}$
並非絕对阻值, 僅代表电路特性, 無任何耗能意義

2.2 Frequency-Domain Analysis.

• Getting Complete Signal Expression.

$v(z,t) = v^i\left(t - \frac{z}{v_p}\right) + v^r\left(t + \frac{z}{v_p}\right)$

voltage source $\Rightarrow v^i, v^r$ related to source impedance, load impedance, voltage source.
$\Rightarrow$ by using sinusoidal wave, we can have phasor expression.

$v_s(z,t) = \underset{\text{amplitude}}{A} \cos\left[\omega\left(t - \frac{z}{v_p}\right) + \phi\right]$

ω ↳ angular frequency $\omega = 2\pi f$.

from definition of phase velocity. $v_p = \frac{\Delta z}{\Delta t} = \frac{\lambda}{T} = \lambda f = \left(\frac{\lambda}{2\pi}\right)\cdot(2\pi f)$ ω.

β, wave number 波數. $\left(\frac{2\pi}{\lambda}\right)$.

$\therefore v_p = \lambda f = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

$\xrightarrow{\text{apply in phasor transform}}$ $V(z) = V\angle -\beta z + \phi$.

apply telegraph equation in frequency domain. $\Rightarrow \begin{cases} \frac{\partial v(z,t)}{\partial z} = -L\frac{\partial i(z,t)}{\partial t} \\ \frac{\partial i(z,t)}{\partial z} = -C\frac{\partial v(z,t)}{\partial t} \end{cases} = \begin{cases} \frac{dV(z)}{dz} = -j\omega L I(z) \\ \frac{dI(z)}{dz} = -j\omega C V(z). \end{cases}$

$(v(z,t) = Re\{\vec{V}(z)e^{j\omega t}\} \Rightarrow \frac{dV}{dt} = j\omega\, Re\{\vec{V}e^{j\omega t}\})$

在頻域(phasor domain)微分等於乘以 $j\omega$。

$\frac{d^2V}{dz^2} = -j\omega L \frac{dI}{dz}.$ 代入. $\frac{d^2V}{dz^2} = -\omega^2 LCV. \Rightarrow \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ $\therefore \beta = \omega\sqrt{LC}$

令 $V = e^{\alpha z}$. $\therefore \alpha^2 + \beta^2 = 0 \Rightarrow \alpha = \pm j\beta$. $\therefore V(z) = V^+e^{-j\beta z} + V^-e^{j\beta z}$, and I is vice versa.

inverse phasor transform $|V^+|\cos(\omega t - \beta z + \phi)$ $\quad -\beta z \to -\frac{z}{v_p}$.

knowing V^+ means incident wave. V^- means reflect wave.

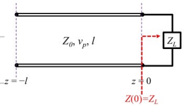
$\Rightarrow$ define "reflection coefficient" $\Gamma(z) = \frac{V^-e^{j\beta z}}{V^+e^{-j\beta z}} = \left(\frac{V^-}{V^+}\right)e^{2j\beta z}$. <special case: at $z=0$. $\Gamma(0) = \frac{V^-}{V^+}$.

substitute in. $V(z) = V^+e^{-j\beta z} + V^-e^{j\beta z} = V^+e^{j\beta z}(1+\Gamma(z))$

$I(z) = \frac{1}{Z_0}(V^+e^{-j\beta z} - V^-e^{j\beta z}) = \frac{1}{Z_0}V^+e^{-j\beta z}(1-\Gamma(z))$

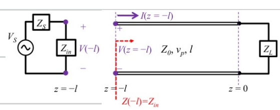
$\Rightarrow$ input impedance $Z(z) = \frac{V(z)}{I(z)} = Z_0\frac{1+\Gamma(z)}{1-\Gamma(z)}$

transfer to. $\frac{Z(z)}{Z_0} = \frac{1+\Gamma(z)}{1-\Gamma(z)} \Rightarrow (Z(z)+Z_0)\Gamma(z) = Z(z) - Z_0 \Rightarrow \Gamma(z) = \frac{Z(z)-Z_0}{Z(z)+Z_0}$



place lump component Z_L at $z=0$.

$\Gamma(z=0) = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow \Gamma(z) = \Gamma(0)e^{2j\beta z}$ < $\Gamma(0) = \Gamma_L$ >

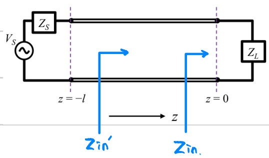


$V(-l) = \frac{Z_{in}}{Z_{in}+Z_S}V_s. \Rightarrow V(z) = V^+e^{-j\beta z}[1+\Gamma(z)]$

$V^+ = V(-l)e^{j\beta z}[1+\Gamma(z)]^{-1}$

Apply all the coefficient $V(z) = V_s\frac{1}{1+\Gamma(-l)}\frac{Z_{in}}{Z_{in}+Z_s}e^{-j\beta l}(1+\Gamma(z))$

• Impedance Transformation.



$$Z(z) = Z_0 \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

$$= Z_0 \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0} e^{2j\beta z}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0} e^{2j\beta z}} = Z_0 \frac{(Z_L+Z_0)(\cos\beta z - j\sin\beta z) + (Z_L - Z_0)(\cos\beta z + j\sin\beta z)}{(Z_L+Z_0)(\cos\beta z - j\sin\beta z) - (Z_L - Z_0)(\cos\beta z + j\sin\beta z)}$$

$$= Z_0 \frac{2Z_L\cos\beta z - 2jZ_0\sin\beta z}{-2jZ_L\sin\beta z + 2Z_0\cos\beta z} = Z_0 \frac{Z_L - jZ_0\tan\beta z}{Z_0 - jZ_L\tan\beta z}.$$

• Average Power along Lossless TL.

若為 rms phasor 則不須乘 $\frac{1}{2}$

$$P_{av} = \frac{1}{2}\mathrm{Re}\{V(z) I^*(z)\}$$

令 $\Gamma(z) = u + jv$.

$$= \frac{1}{2}\mathrm{Re}\left\{V^+ e^{-j\beta z}[1+\Gamma(z)] \frac{V^{+*} e^{j\beta z}}{Z_0}[1-\Gamma(z)]^*\right\}$$

$1+u+jv \quad 1-u-jv$

$$(1+\Gamma(z))(1-\Gamma(z))^* = (1-u^2) - j(1+u)v + j(1-u)v - v^2$$
$$= (1-u^2) - j2v - v^2$$

$$= \frac{|V^+|^2}{2Z_0}\mathrm{Re}\{[1+\Gamma(z)][1-\Gamma(z)]^*\} = \frac{|V^+|^2}{2Z_0}\mathrm{Re}\{(1-(u^2+v^2)) - j2v\} = \frac{|V^+|^2}{2Z_0}[1-(u^2+v^2)] = \frac{|V^+|^2}{2Z_0}[1-|\Gamma(z)|^2]$$

reflect power

incident power.

• Return Loss & Insertion Loss.

Return Loss.

power reflect.

$$RL = -10\log\left(\frac{P^r}{P^i}\right)$$

power transmitt to load.

$$= -10\log\left(\frac{V^-}{V^+}\right)^2 = -20\log|\Gamma(z)| = -20\log|\Gamma_L|$$

Insertion Loss

$$IL = -10\log\left(\frac{P^t}{P^i}\right) = -10\log\left(\frac{P^i - P^r}{P^i}\right) = -10\log(1-|\Gamma_L|^2)$$

• Special Case of Input Impedance

1. $Z_L = 0$. <short>.

$$RL = -20\log|\Gamma_L| = -20\log\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = 0\,dB.$$

2. $Z_L = \infty$ <open>

$$RL = -20\log|\Gamma_L| = -20\log\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = 0\,dB.$$

2者相當於全其輸入完全未傳送到輸出.

3. $Z_L = Z_0 \longrightarrow Z(z) = Z_0\frac{1+\Gamma(z)}{1-\Gamma(z)} = Z_0$. $(\Gamma(z) = \Gamma_L e^{2j\beta z} = 0)$

$RL = -20\log|\Gamma_L| = \infty$ 輸入完全到輸出.

4. $z = \frac{\lambda}{2} \longrightarrow Z(z) = Z_0\frac{Z_L - Z_0 j\tan\beta z}{Z_0 - Z_L j\tan\beta z} = Z_L$. $(\tan\beta z = \tan\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = 0.)$

5. $z = \frac{\lambda}{4} \longrightarrow Z(z) = Z_0\frac{Z_L - Z_0 j\tan\beta z}{Z_0 - Z_L j\tan\beta z} = \frac{Z_0^2}{Z_L}$ $(\tan\beta z = \tan\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \infty)$

• Voltage Standing Wave Ratio

1. Voltage Standing Wave Phenomenon.

from $V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z} = V^+ e^{-j\beta z}[1+\Gamma(z)] = V^+ e^{-j\beta z}[1+\frac{Z_L - Z_0}{Z_L + Z_0} e^{2j\beta z}]$

at short circuit $\Gamma_L = 0$. $V(z) = V^+ e^{-j\beta z}[1 - e^{2j\beta z}]$ having perfect standing wave because of $V_{min} = 0$.

open circuit $\Gamma_L = \infty$ $V(z) = V^+ e^{-j\beta z}[1 + e^{2j\beta z}]$ having perfect standing wave because of $V_{min} = 0$.

→ Conclusion: only have perfect standing wave when. $1 + \Gamma(z) = 0$. for particular z.
and else there having similar phenomenon.

2. Extreme Value of $V(z)$.

from conclusion above. we can knew when extreme value exists. then.

$$|V_{max}| = |V^+|(1 + |\Gamma_L|)$$

$$|V_{min}| = |V^+|(1 - |\Gamma_L|)$$

3. Voltage Standing Wave Ratio.

to define measure of "mismatching line" define VSWR.

$$VSWR = \frac{|V(z)|_{max}}{|V(z)|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

→ for matching load : VSWR = 1.
for open or short: VSWR = ∞

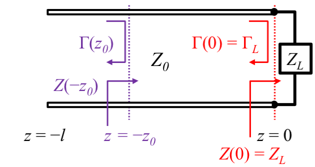
4. Some Useful Number

VSWR = 2:1
10 dB threshold
VSWR = 3:1
6 dB threshold.
VSWR = 1.5:1
14 dB threshold.
VSWR = 2.5:1
7.4 dB threshold.

VSWR	$\lvert\Gamma(z)\rvert$	R.L.	$1 - \lvert\Gamma(z)\rvert^2$
	50%	3dB	50%.
3.	25%	6dB	75%.
	20%	7dB.	80%.
2 ≒ 1.96	10%	10dB	90%.
	5%	13dB.	95%.
	1%	20dB	99%.

2.8 Smith Chart

· Properties of Γ plane and Z plane



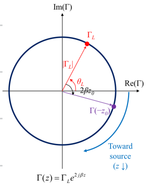
from $Z(z)=Z_0\dfrac{Z_L-jZ_0\tan\beta z}{Z_0-jZ_L\tan\beta z}$ $\longrightarrow$ $Z(z)$阻抗會隨位置不同而改變大小.

⇒無明顯規則可依循.

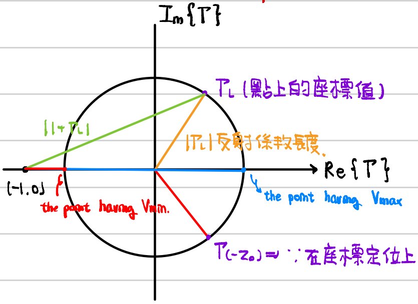
from $\Gamma(z)=\Gamma_L e^{2j\beta z}$. $\longrightarrow$ 在定 Z_L 的狀況下,其反射係數僅跟位置有关.

⇒大小固定,僅為反射參數変換.

可用「Γ plane」来实行



• Γ plane to make impedance

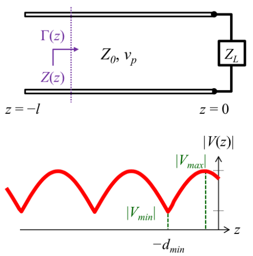


Γ_L (點上的座標值)

(以 Z_L 為原點)

$\Gamma(-z_0)$ ⇒ ∵在座標定位上都是用"負號" ⇒ ∴在旋轉時皆用「順轉」

• Slotted Line Measurements.



First $|V_{min}|$ occurs at $z=-d_{min}$

1. 在 $-d_{min}$ 時有最小的Γ.

$\Gamma(z)=\Gamma_L e^{2j\beta z}$

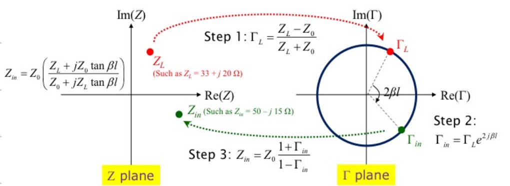
$\Rightarrow \Gamma_L e^{2j\frac{2\pi}{\lambda}(-d_{min})}=-\Gamma_L$

$\therefore\ d_{min}=\dfrac{\lambda}{4}$

2. $VSWR=\dfrac{1+|\Gamma_L|}{1-|\Gamma_L|}\Rightarrow|\Gamma_L|=\dfrac{VSWR-1}{VSWR+1}$ 得到 $\Gamma_L=|\Gamma_L|e^{j\theta_L}$

3. 最後, $Z_L=Z_0\dfrac{1+\Gamma_L}{1-\Gamma_L}$

· Concept of Smith Chart



∵ $\Gamma_L=\dfrac{Z_L-Z_0}{Z_L+Z_0}$ 會有2個未知量 ⇒ ∴會先進行標準化

才可讓 Γ_L 不會受到 Z_0 的影响. ⇒ $z_n=\dfrac{Z_L}{Z_0}$.

• Normalized Impedance on Γ Plane

由上式可知。$z_n = \frac{1+\Gamma}{1-\Gamma}$. 令 $z_n = r + jx$. $\Gamma = u + jv$.

$$\therefore r + jx = \frac{1+\Gamma}{1-\Gamma} = \frac{1+u+jv}{1-u-jv} = \frac{(1+u+jv)(1-u+jv)}{(1-u-jv)(1-u+jv)} = \frac{1-u^2-v^2+j2v}{(1-u)^2+v^2}$$

得出 $r = \frac{1-u^2-v^2}{(1-u)^2+v^2}$ $\quad x = \frac{2v}{(1-u)^2+v^2}$

1. 求 r 的关係:

$$r(1-u)^2 + rv^2 = 1-u^2-v^2$$

$$(1+r)v^2 = (1-r) + 2ur - (1+r)u^2$$

$$v^2 = \frac{1-r}{1+r} + \frac{2ur}{1+r} - u^2$$

$$v^2 = \frac{1-r^2}{(1+r)^2} + \frac{2ur}{1+r} - u^2$$

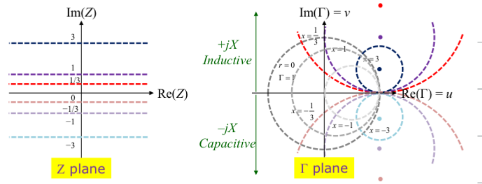
$$\left(\frac{1}{r+1}\right)^2 = \left(u - \frac{r}{r+1}\right)^2 + v^2$$

2. 求 x 的关係.

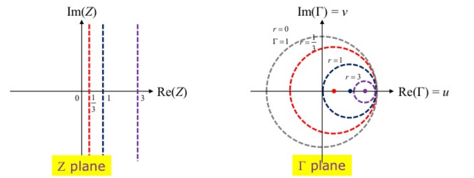
$$(1-u)^2 + v^2 = \frac{2v}{x}$$

$$(1-u)^2 + \left(v^2 - \frac{2v}{x} + \frac{1}{x^2}\right) = \frac{1}{x^2}$$

$$(1-u)^2 + \left(v - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

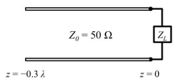


Called "constant x circle" or "constant reactance circle"



Called "constant r circle" or "constant resistance circle"

• Basic Procedure of Smith Chart



- A load impedance of $130 + j90\ \Omega$ terminates a $50\ \Omega$ transmission line that is 0.3λ long
  1. Find the reflection coefficient at the load
  2. Find the reflection coefficient at the input to the line
  3. Find the VSWR on the line
  4. Find the return loss
  5. Find the impedance seen at the input to the line
  6. Find the admittance seen at the input to the line

1. 先正規化 $z_n = \frac{Z_L}{Z_0}$. 並找对应 r 圓. x 圓
2. 將2圓交點列出. 並以圓點為圓心找出交點長度.
3. 將長度対外圓的半徑做比例比較. 即為 Γ.
4. 若要找 $Z(z)$ 可直接用 Smith Chart 繞圓. 以 $2\cdot\frac{2\pi}{\lambda}\cdot z$ 繞.

• Connecting Z_L with Series.

1. Inductor:

$\because Z = j\omega L \Rightarrow \therefore$ 对於 $z_n = r + jx$ 來說. 其 "x" 的成份变大. → 观察其频率可知 f 大會往 x 大方向走.

$\therefore$ 其結果為定 r 圓 . 朝 x 大方向走

若只看頻率其頻率不會使 x 從正到負 or 負到正.

2. Capacitor.

$\because Z = -j\frac{1}{\omega C} \Rightarrow \therefore$ 对於 $z_n = r + jx$ 來說. 其 "x" 的成份变小. → 观察其频率可知 f 大會往 x 小方向走.

$\therefore$ 其結果為定 r 圓 朝 x 小方向走.

3. Resistor.

串联电阻會使 $z_n = r + jx$ 变大.
其結果為定 x 圓朝 r 大方向走. → 與頻率無关.

• Connecting Z_L with Parallel.

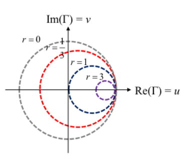
⇒ If we using parallel. the impedance would be complicated because of $Z_{in} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

↳ changing to "admittance" form then $Y_{in} = Y_1 + Y_2$. ⇒ methods to change Z_{in} to Y_{in} is $Z(\frac{\lambda}{4}) = Z_0 \frac{Z_0}{Z_L}$

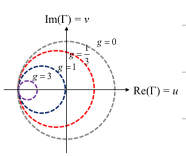
⇒ $\frac{Z_L}{Z_0} = \frac{Z_0}{Z(\frac{\lambda}{4})}$ gets $z_n \big|_{z=\frac{\lambda}{4}} = Y_L$. 实際上為將Z(z)旋轉180°

another approach to get is rotating axis in 180°

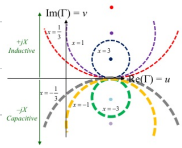
Z chart



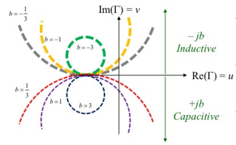
Y chart



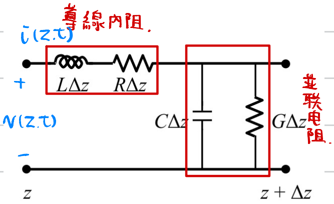
Z chart



Y chart



2.4 Lossy Transmission Lines



Why does loss exist

R: imperfect conductors (非良導体.)

理想狀況下. $\sigma \to \infty$ 对於導体而言.其導电率$=\infty$ (S/m)

目前狀況.石墨烯最佳.

G: imperfect dielectric (非良絕緣体)

理想狀況下. $\sigma \to 0$.

R.G. distributed element (Ω/m)

at low frequency. $Z_L = j\omega L \longrightarrow 0$.

$Y_C = j\omega C \longrightarrow 0$.

$G(\omega) \propto \sigma(\omega)$. and $\sigma(\omega) \propto \omega \longrightarrow 0$.

dielectric conductivity.

Revise KVL.

$$-v(z,t) + (L\Delta z)\frac{\partial i(z,t)}{\partial t} + (R\Delta z)\, i(z,t) + v(z+\Delta z, t) = 0.$$

$\Rightarrow$ phasor expression $\quad \frac{dV}{dz} = -(R+j\omega L)I.$

Revise KCL

$$-i(z,t) + (C\Delta z)\frac{\partial v(z+\Delta z,t)}{\partial t} + (G\Delta z)\, v(z+\Delta z,t) + i(z+\Delta z,t) = 0$$

$\Rightarrow$ phasor expression $\quad \frac{dI}{dz} = -(G+j\omega C)V$

$\Rightarrow$ getting $\frac{d^2V}{dz^2} - \gamma^2 V = 0$. $\quad \gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta.$

case 1: Lossless.

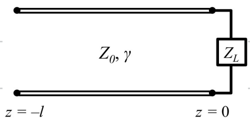
$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = j\omega\sqrt{LC} \Rightarrow \alpha = 0,\ \beta = \omega\sqrt{LC}$$

case 2: Low loss, high frequency. $(R \ll \omega L,\ G \ll \omega C)$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \underbrace{\sqrt{\left(\frac{R}{j\omega L}+1\right)\left(\frac{G}{j\omega C}+1\right)}}_{\text{二項式展开.}}\left(\sqrt{j\omega L \times j\omega C}\right)$$

$$\approx j\omega\sqrt{LC}\sqrt{1+\frac{R}{j\omega L}+\frac{G}{j\omega C}} \approx \underbrace{j\omega\sqrt{LC}}_{\beta}\left[1+\underbrace{\frac{1}{2}\left(\frac{R}{j\omega L}+\frac{G}{j\omega C}\right)}_{\alpha.}\right]$$

$\Rightarrow$ getting $\alpha = \underbrace{\frac{1}{2}R\sqrt{\frac{C}{L}}}_{\text{conductor loss}} + \underbrace{\frac{1}{2}G\sqrt{\frac{L}{C}}}_{\text{dielectric loss.}}$. $\beta = \omega\sqrt{LC}$. $Z_0 = \sqrt{\frac{L}{C}}$ 與lossless相同.



In time domain expression.

$$v(z,t) = \mathrm{Re}\left\{V(z)\, e^{j\omega t}\right\}$$

$$= \underbrace{|V^+|e^{-\alpha z}\cos(\omega t - \beta z + \phi)}_{\text{Incident Wave}} + \underbrace{|V^-|e^{\alpha z}\cos(\omega t + \beta z + \phi)}_{\text{Reflect Wave}}$$

★ two waves contains an attenuation of coefficient $e^{-\alpha z}$.

Comparement

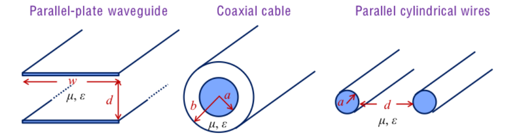
Lossless lines		Lossy lines
$\beta = \omega\sqrt{LC}$	Propagation constant	$\gamma = \alpha + j\beta$ α : attenuation, $\beta = \omega\sqrt{LC}$
$Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$	Input impedance	$Z_{in} = Z_0\left(\frac{Z_L + Z_0\tanh\gamma l}{Z_0 + Z_L\tanh\gamma l}\right)$
$P_{av}(z) = \frac{\lvert V^+\rvert^2}{2Z_0}\left(1-\lvert\Gamma(z)\rvert^2\right)$	Time-average power flow along the line	$P_{av}(z) = \frac{\lvert V^+\rvert^2}{2Z_0}\left(1-\lvert\Gamma(z)\rvert^2\right)e^{-2\alpha z}$

$\longrightarrow$ 隨長度有不同長度的旋轉與縮減

$\longrightarrow$ 有一丁 $e^{-2\alpha z}$ 縮減

attenuation of line (insertion loss) $= -10\log e^{-2\alpha l} = \alpha l \times 8.69$ dB.

Three-Dimensional Structures



$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon'(1 - j\tan\delta) = \varepsilon' + j(\sigma/\omega)$ is the complex permittivity

ε′(能量集中), ε″(能量外擴) $\varepsilon = \varepsilon'(1 - j\tan\delta)$

loss tangent $\tan\delta = \dfrac{\sigma}{\omega\varepsilon}$

Parallel-plate waveguide	Coaxial cable	Parallel cylindrical wires
$C = \varepsilon' \dfrac{w}{d}$	$C = \dfrac{2\pi\varepsilon'}{\ln(b/a)}$	$C = \dfrac{\pi\varepsilon'}{\cosh^{-1}(d/2a)}$
$L = \mu \dfrac{d}{w}$	$L = \dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$	$L = \dfrac{\mu}{\pi}\cosh^{-1}(d/2a)$
$Z_0 \approx \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{\mu}{\varepsilon'}}\dfrac{d}{w}$	$Z_0 \approx \sqrt{\dfrac{L}{C}} = \dfrac{1}{2\pi}\sqrt{\dfrac{\mu}{\varepsilon'}}\ln\dfrac{b}{a}$	$Z_0 \approx \sqrt{\dfrac{L}{C}} = \dfrac{1}{\pi}\sqrt{\dfrac{\mu}{\varepsilon'}}\cosh^{-1}(d/2a)$
$R = \dfrac{2R_S}{w}$	$R = \dfrac{R_S}{2\pi}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$	$R = \dfrac{R_S}{\pi a}$
$\boxed{G = \omega\varepsilon'' \dfrac{w}{d}}$	$\boxed{G = \omega\varepsilon'' \dfrac{2\pi}{\ln\frac{b}{a}}}$	$\boxed{G = \omega\varepsilon'' \dfrac{\pi}{\cosh^{-1}(d/2a)}}$

R_s (surface resistance) $= \dfrac{1}{\sigma\delta_s}$ $\delta_s = \sqrt{\dfrac{2}{\omega\mu\sigma}}$

$R_s = \dfrac{1}{\sigma\delta_s}$

肌膚深动. 电荷一般僅通过表面

但不可能有理想導体. 电流会進入到金屬

造成肌膚深动.

G在頻率愈高 其必須要考慮.

EM Properties in some Dielectrics.

Phase Velocity. $v_p = \sqrt{\dfrac{1}{\varepsilon\mu}} = \sqrt{\dfrac{1}{\varepsilon_0\varepsilon_r\mu_0}} = \dfrac{c}{\sqrt{\varepsilon_r}}$

⇒ wavelength in material. $\lambda_g = \dfrac{v_p}{f} = \dfrac{\lambda_0}{\sqrt{\varepsilon_r}}$.

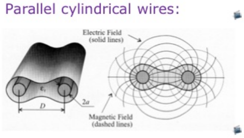
the larger dielectric constant. smaller associated wavelength.

Material	Dielectric constant (ε_r)
Air	1.0054
FR4 (玻璃纖維板)	4.4
Epoxy (環氧樹脂)	3.9
Polyimide (聚亞醯胺)	4.5
Polytetrafluoroethylene (聚四氟乙烯)	2.1

Material	Dielectric constant (ε_r)
Polystyrene (聚苯乙烯)	2.5 – 2.6
Polypropylene (聚丙烯)	2.25
Ceramic (陶瓷)	3 – 7
Duroid RO 5880	2.2
Duroid RO 4003	3.55

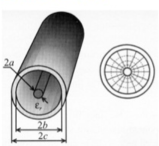
Practical Issues of 3-D TLs

Parallel cylindrical wires:



- A bad example of transmitting RF voltage and current waves
  - Electric and magnetic field lines emanating from the conductors extend to infinity and thus tend to influence electronic equipment in the vicinity of the lines
  - Radiation loss tends to be very high
- Applications: connecting TV set to receiving antenna, 50-60 Hz power line, and local telephone connection

Coaxial cable:



- Used for almost all cases of RF systems or measurement equipment (up to 18 GHz)
- The outer conductor is grounded, so the radiation loss and field interference are minimized
- Coaxial connectors: a special structure of coaxial cable

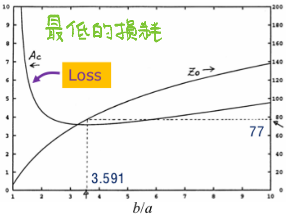
(特徵阻抗: 跟内外半徑有关)

RG or M17	Characteristic impedance (Ω)	Dielectric Type	Loss at 1 GHz (dB/100 ft)	Capacitance (pF/ft)
RG 8A/U	50	PE	9.0	29.5
RG 213A/U	50	Foam PE	9.0	30.8
RG 188A/U	50	Solid TFE	30.0	29.4
RG 179B/U	75	Solid TFE	25.0	19.5
RG 141A/U	50	Solid TFE	13.0	29.4
M17/90	93	Air space PE		13.5
M17/56	95	PE		17.0
M17/6	75	PE		20.6
M17/95	95	Solid TFE		15.4

Coaxial Cable Data Sheet

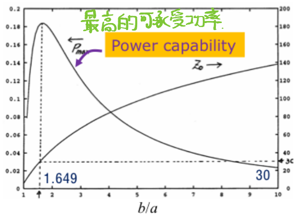
Reasons for using 50-Ω Coaxial Lines

For the aspect of loss:



- Different geometric parameters result in different Z_0 and different α
- For an air-filled coaxial line, loss has the minimum value at $Z_0 = 77\ \Omega$

For the aspect of breakdown:

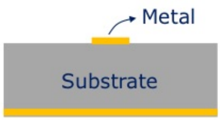


- Different geometric parameters result in different power capacity (that is, the time-average power at breakdown)
- For an air-filled coaxial line, power capability has the maximum value at $Z_0 = 30\ \Omega$

➡ To compromise between $77\ \Omega$ and $30\ \Omega$, we choose $Z_0 = 50\ \Omega$

- **Type-N connector**: The outer diameter of the female end is 0.625 in. (passband: 11-18 GHz)
- **TNC connector**: This connector is similar to the very common BNC connector (passband: up to 1 GHz)
- **APC-7 connector**: This connector is sexless (passband: up to 18 GHz)
- **SMA connector**: The outer dimension of the female end is 0.25 in. This connector can be used up to 18-25 GHz and is the most popular coaxial connector for RF and microwave measurement applications

Planar Structures.



Microstrip line

Slotline

Coplanar waveguide (CPW)

Coplanar strip (CPS)

- Z_0, α, β depend on geometric characteristics, dielectric properties ε_r.
  carefully determined geometric characteristics a number Z_0 is obtained.

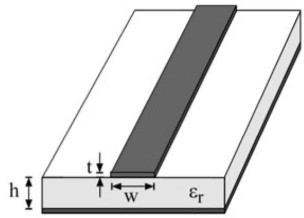
good for microwave integrated circuit applications

⇒ problem of Field Leakage.

PCB transmission line:

- They have rather high radiation loss (in comparison with 3-D TLs)
- They are prone to "crosstalk" (interfere) between neighboring conductor systems
- The severity of field leakage depends on the relative dielectric constants
- Substrates having high dielectric constants can minimize field leakage and cross coupling

Microstrip Lines



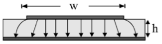
EM characteristics in MSL.

phase velocity in MSL : $v_p = \dfrac{c}{\sqrt{\varepsilon_{eff}}}$

wavelength in MSL : $\lambda = \dfrac{v_p}{f} = \dfrac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$

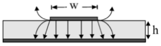
- Effective dielectric constant: the dielectric constant of a homogeneous medium that replaces the air and dielectric regions
- For very wide lines (w/h >> 1):

$$\varepsilon_{eff} = \varepsilon_r$$



- For very narrow lines (w/h << 1):

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2}$$

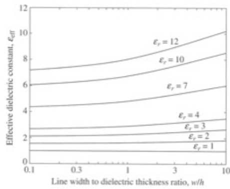


Analysis:

1. If the line width is narrow ($w/h \le 1$)

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(8\frac{h}{w} + \frac{w}{4h}\right)$$

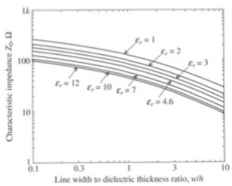
$$\text{where } \varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\left(1 + 12\frac{h}{w}\right)^{-0.5} + 0.04\left(1 - \frac{w}{h}\right)^2\right]$$



2. If the line width is wide ($w/h \ge 1$)

$$Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{eff}}\left(1.393 + \frac{w}{h} + 0.667\ln\left(\frac{w}{h} + 1.444\right)\right)}$$

$$\text{where } \varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{w}\right)^{-0.5}$$



Synthesis or design:

1. If the line width is narrow ($w/h < 2$)

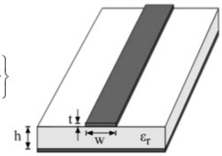
$$\frac{w}{h} = \frac{8e^A}{e^{2A} - 2}$$

$$\text{where } A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

2. If the line width is wide ($w/h \ge 2$)

$$\frac{w}{h} = \frac{2}{\pi}\left\{B - 1 - \ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B - 1)\right] + 0.39 - \frac{0.61}{\varepsilon_r}\right\}$$

$$\text{where } B = \frac{377\pi}{2Z_0\sqrt{\varepsilon_r}}$$



80 . 5 nH.

$z_n = 1.6 + j\,\frac{2\pi \cdot 2G \cdot 5\,nH}{Z_0}$

$= 1.6 + j\,0.4\pi$

$-j1.7 = -j\,\frac{Z_0}{\omega C}$

$y_n = 0.38 - j\,0.3.$

$\rightarrow -j\,0.19 = -j\,\frac{1}{\omega L Y_0}$

$C = \frac{50}{4\pi G \times 1.7}$

$L = \frac{50}{4\pi G}$

$= 20.94\,nH$ (병)