TAIPEI TECH

國立臺北科技大學

High-Frequency Electronic Circuits

Lecture 3

Impedance Matching Networks

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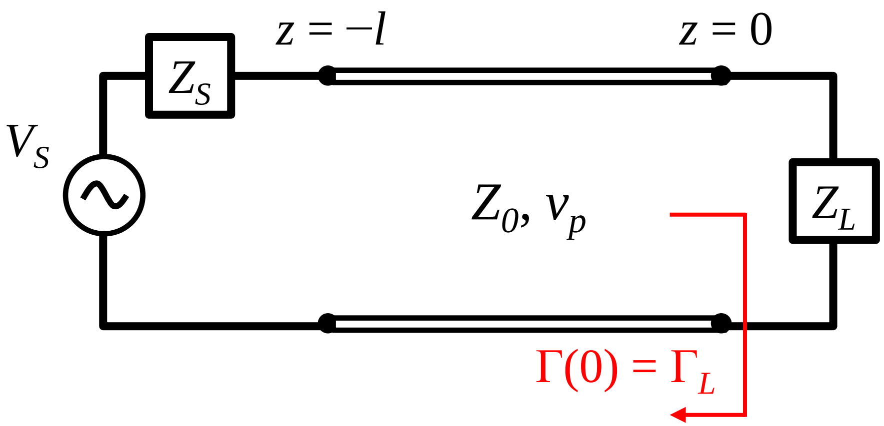
Contents

Lecture 3:
Impedance Matching Networks

Contents

3.1 Objective of Impedance Matching

Effect of Reflected Voltage Waves $V^{-}e^{j\beta z}$



Reflection coefficient

$$\Gamma(z) \triangleq \frac{V^{-}e^{j\beta z}}{V^{+}e^{-j\beta z}} = \Gamma_L e^{2j\beta z}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The Voltage wave along the line: $V(z) = V^{+}e^{-j\beta z} + V^{-}e^{j\beta z}$

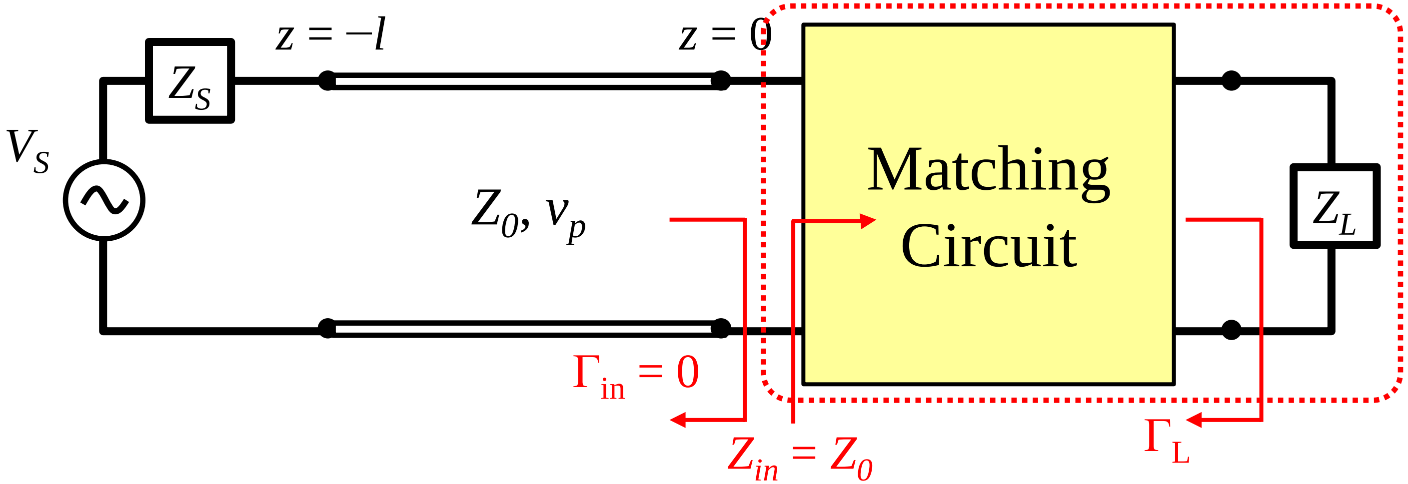
The current wave along the line: $I(z) = \frac{1}{Z_0}\left(V^{+}e^{-j\beta z} - V^{-}e^{j\beta z}\right)$

When $Z_0 \neq Z_L$:

- It creates reflected voltage waves: $V^{-}e^{j\beta z}$
- The reflected voltage waves may damage the source circuitry
- It wastes transmitting power:

$$P_{av}(z) = \frac{\left|V^{+}\right|^2}{2Z_0}\left(1 - \left|\Gamma(z)\right|^2\right)$$

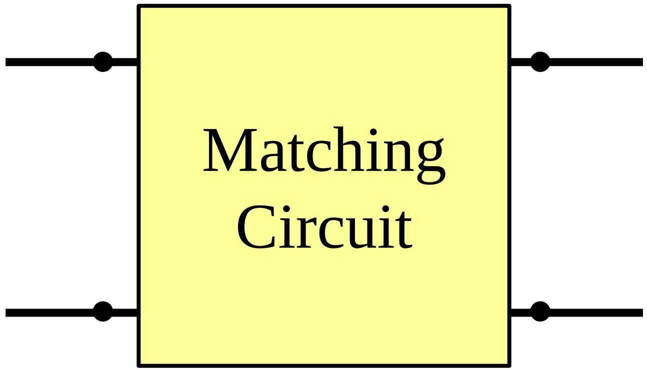
How to Eliminate the Reflected Voltage Waves?



If we purposely insert a matching circuit…

- With a well-design circuit, we make $Z_{in} = Z_0$
- Therefore, $|\Gamma_{in}| = 0$, and no reflected voltage wave goes back to the source
- Although Γ_L still exists, the matching circuit acts like a buffer; It stores the transmitting power coming from V_S, waiting for a right moment to push power to the load

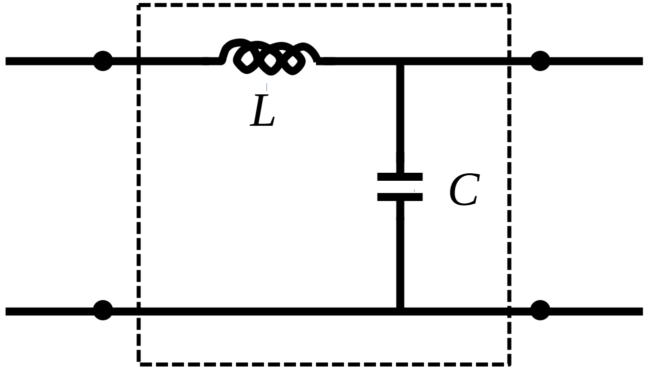
How Does a Matching Circuit Looks Like?



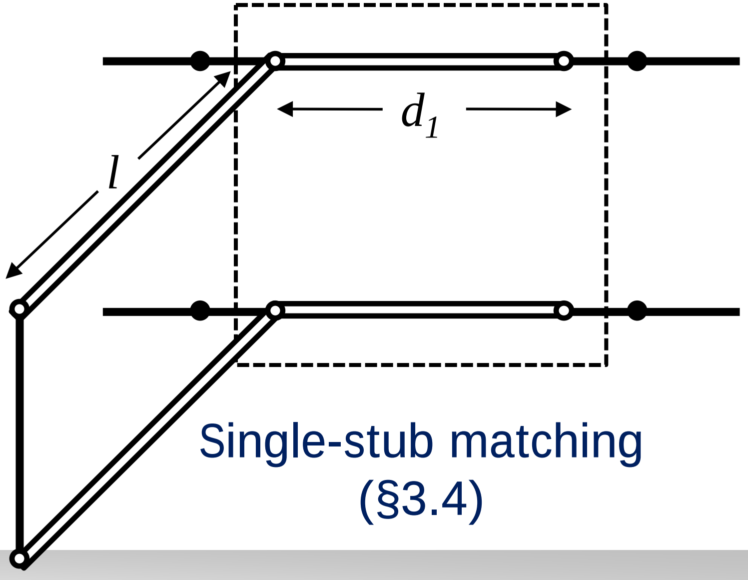
A matching circuit should comprise **lossless** components:

- Lumped Inductors
- Lumped capacitors
- Transmission-line stubs

Example:

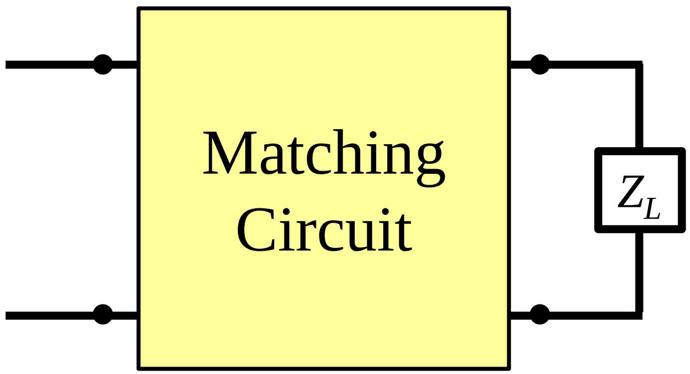


Matching with L-section circuit (§3.3)



Single-stub matching (§3.4)

Classification of Matching Circuit

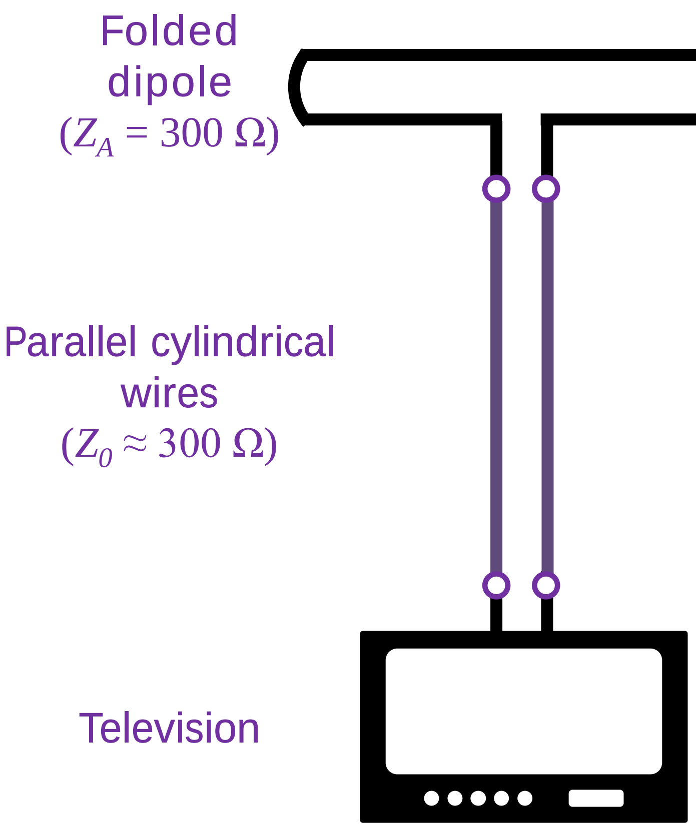


- There are ∞ possible solutions for the matching circuit
- Factors for selecting matching circuit:
  - Complexity
  - Bandwidth
  - Implementation
  - Adjustability
  - Size

The techniques of impedance matching can be classified into:

1. If Z_L is purely real: Quarter-wavelength transformer (§3.2)
2. If Z_L is complex: L-section lumped matching circuit (§3.3)
   Series connection of single-stub tuner (§3.4)
   Parallel connection of single-stub tuner (§3.4)
   Double-stub tuner (§3.4)
   Quarter-wavelength transformer with shunt stub (§3.2)
3. If impedance matching should be done broad-band: (§3.5)

An Example of Matching Circuit



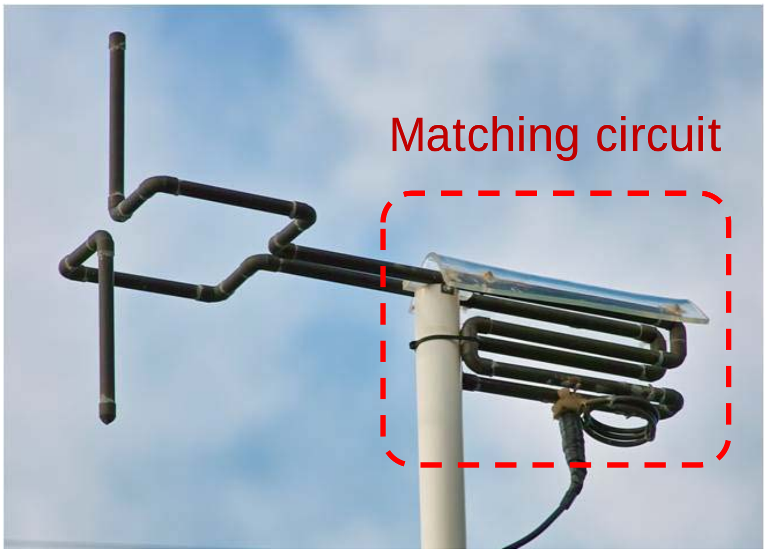
TV antennas: folded dipoles

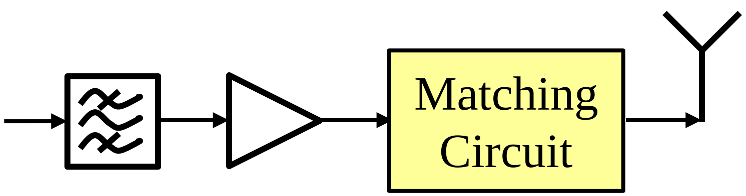
- The folded dipole receives wireless signal and pass it to the parallel cylindrical wires
- However, the parallel cylindrical wires tend to receive noises since the emanation of fields
- If coaxial cables ($Z_0 = 50$ or $75\ \Omega$) are used instead of parallel lines, we need to add a matching circuit

[Online]. Available: http://tuxmadrid.blogspot.tw/2011/05/small-tv-aerial-uda-yagi.html

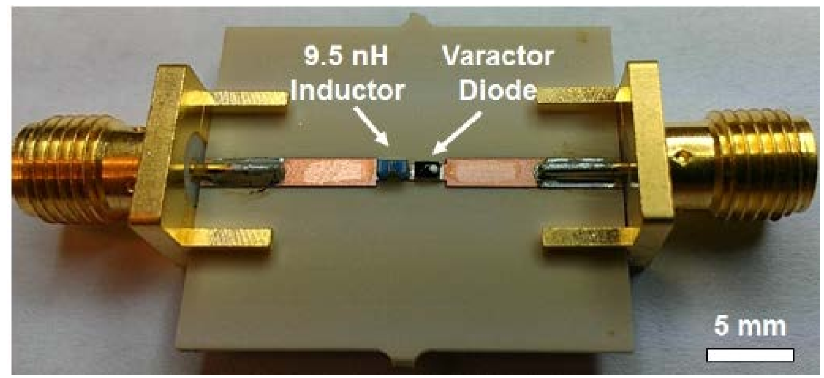
More Examples of Matching Circuits

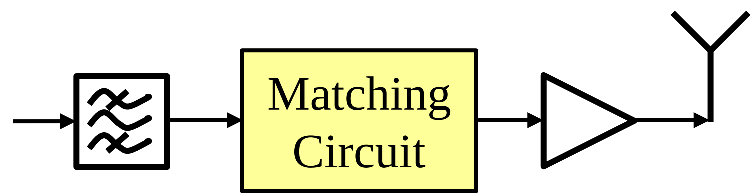
Matching circuit for antenna applications:





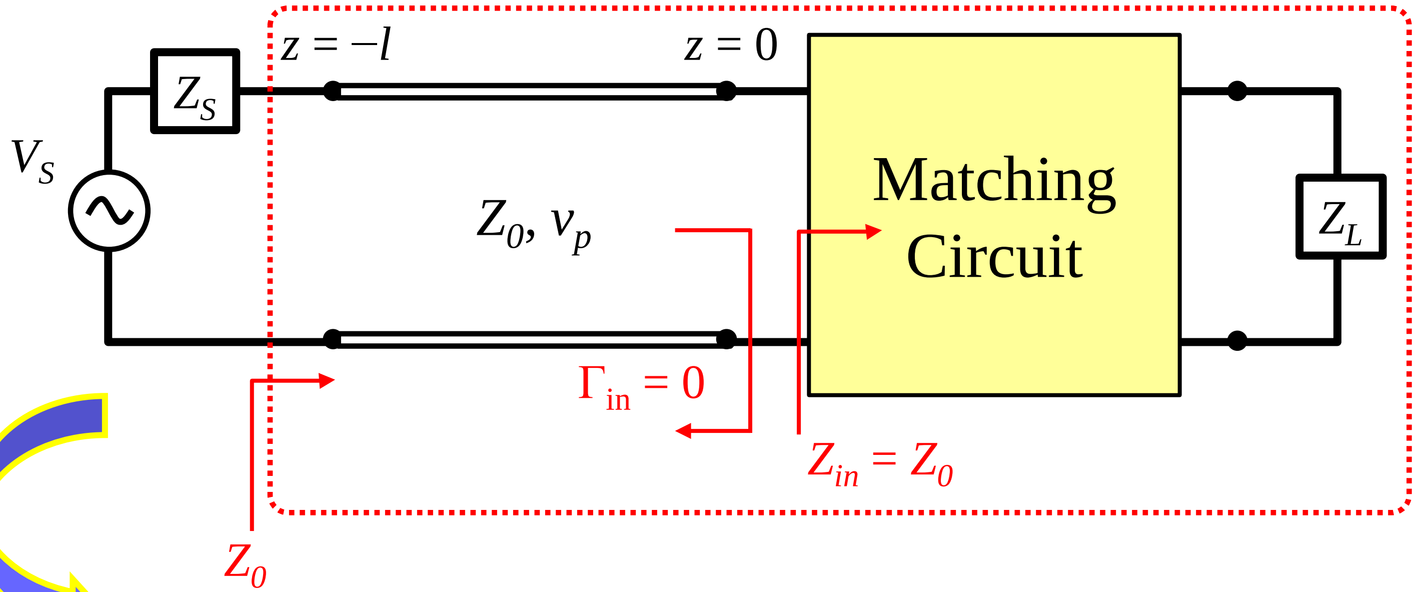
Matching circuit for power amplifiers:

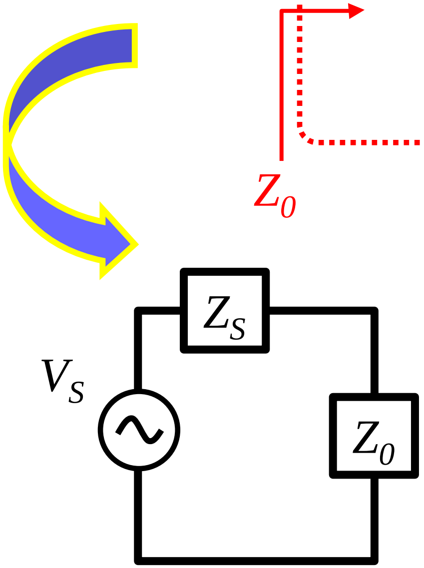




The WA7X Beacon: The Cycloid Dipole. [Online]. Available: http://www.wa7x.com/cycloid_info.html
W. N. Allen and D. Peroulis, “Wideband diode-based reconfigurable matching network operating at 36 dBm input power,” *IEEE AP-S/URSI International Symposium*, Chicago, Illinois, Jul. 2012

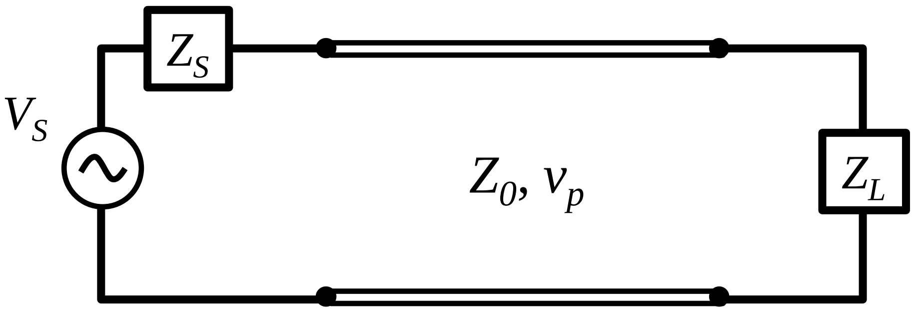
Is All the Power Delivered Into The Load?





- When the transmission line connects to a matched load, the impedance on any point z is Z_0
- Not all the power generated by source transmits into the load
- How to achieve maximum power transfer? $Z_0 = Z_S^*$!

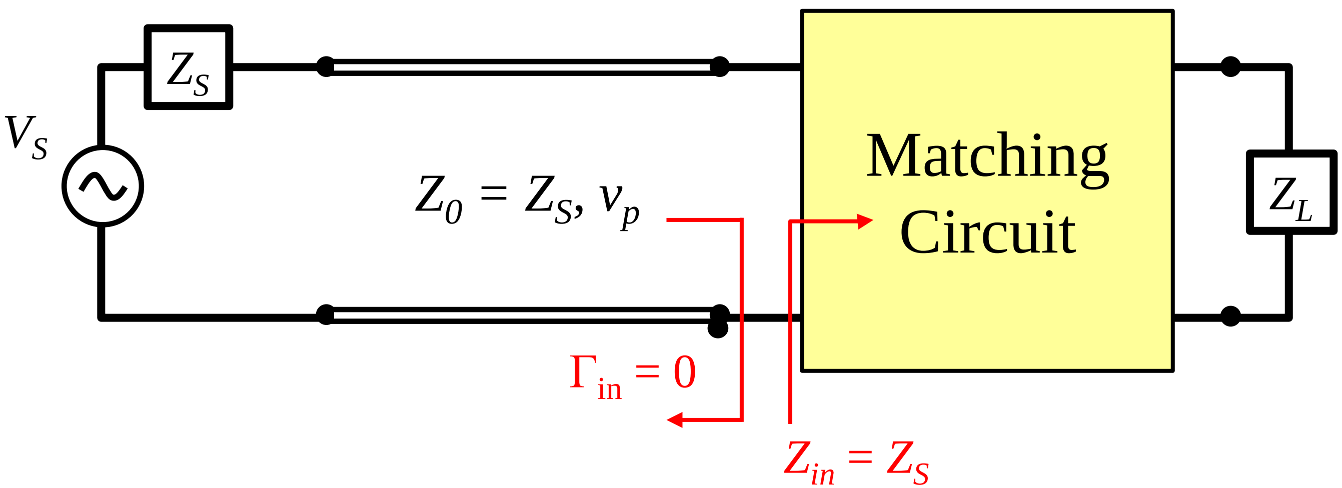
The Best Situation of Power Transfer



A practical scenario:

- V_S, Z_S (assuming purely real) and Z_L are given
- Engineers must choose a proper Z_0 and design a matching circuit so that maximum power deliver to the load

How to achieve maximum power transfer:



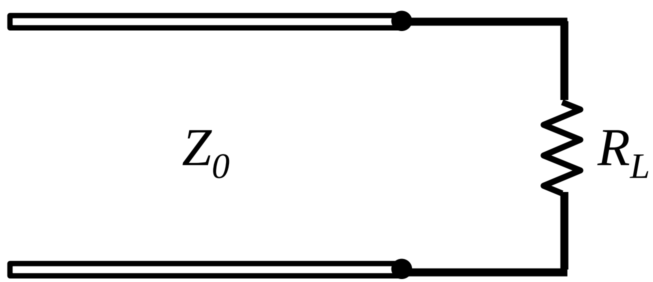
- Choosing $Z_0 = Z_S$
- Design a matching circuit that $Z_{in} = Z_S$
- Power efficiency: 50 %
- Maximum power transfers to the load

Contents

3.2 Quarter-Wavelength Transformers

Objective of $\lambda_g/4$ Transformers

Operational scenario:

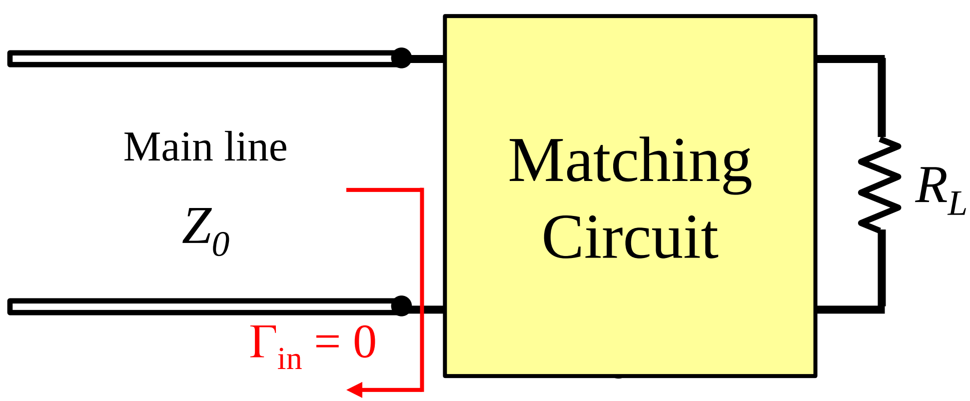


- Z_0 and R_L are both real and given
- If $Z_0 \neq R_L$, it incurs a reflected voltage wave:

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

- We'd like to match the load to the main line

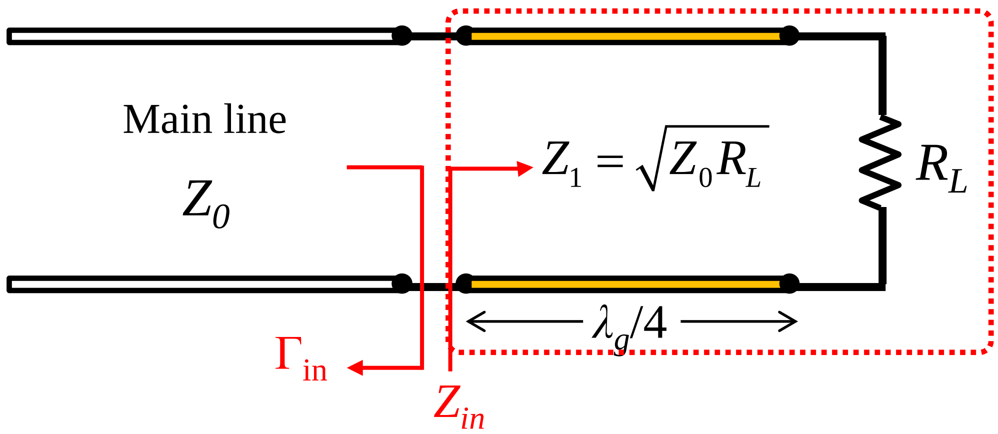
To make $\Gamma_{\text{in}} = 0$:



- Solution: by using a lossless piece of transmission line of characteristic impedance Z_1 and length $\lambda_g/4$, where

$$Z_1 = \sqrt{Z_0 R_L}$$

Why Does It Work?



- Keep in mind that our objective is to have $Z_{in} = Z_0$
- According to

$$Z_{in} = Z_1 \left(\frac{R_L + jZ_1 \tan \beta l}{Z_1 + jR_L \tan \beta l} \right)$$

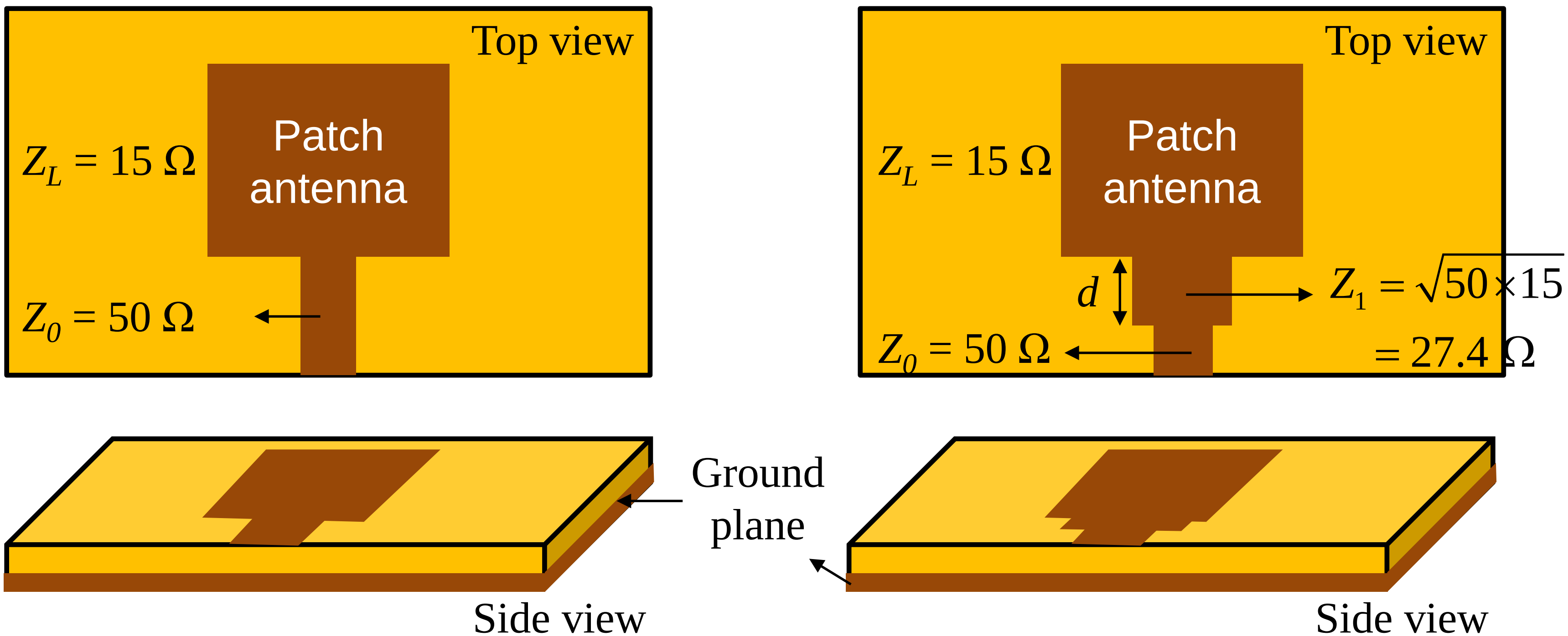
Now $l = \lambda/4$, so $\beta l = \pi/2$ and $\tan\beta l \to \infty$; therefore, $Z_{in} = \dfrac{Z_1^2}{R_L}$

- When $Z_1 = \sqrt{Z_0 R_L}$, clearly, $Z_{in} = Z_0$ and $\Gamma_{\text{in}} = 0$

How to Implement It by Microstrip Lines?

Example: To match a patch antenna by microstrip lines

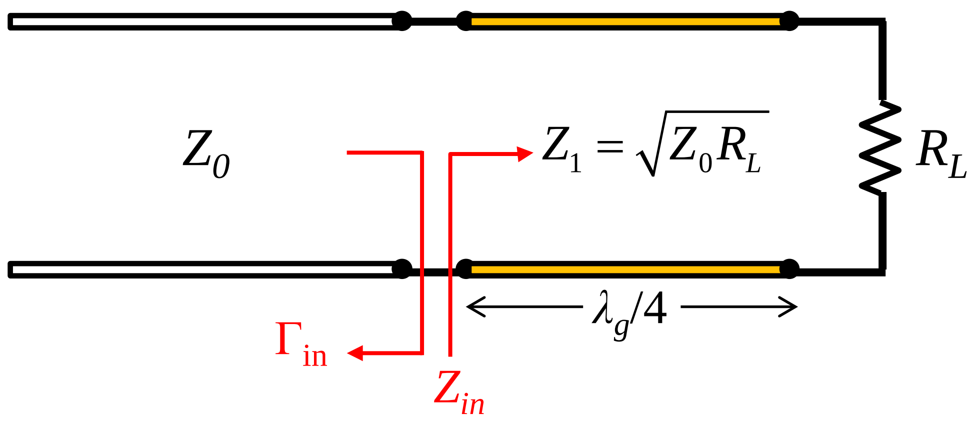
Before matching | After matching



- RL = 5.37 dB
- 17% power is lost

- $d = \lambda_g/4$
- RL = ∞; No power reflects back
- Cost: the antenna size ↑

Characteristics of $\lambda_g/4$ Transformers

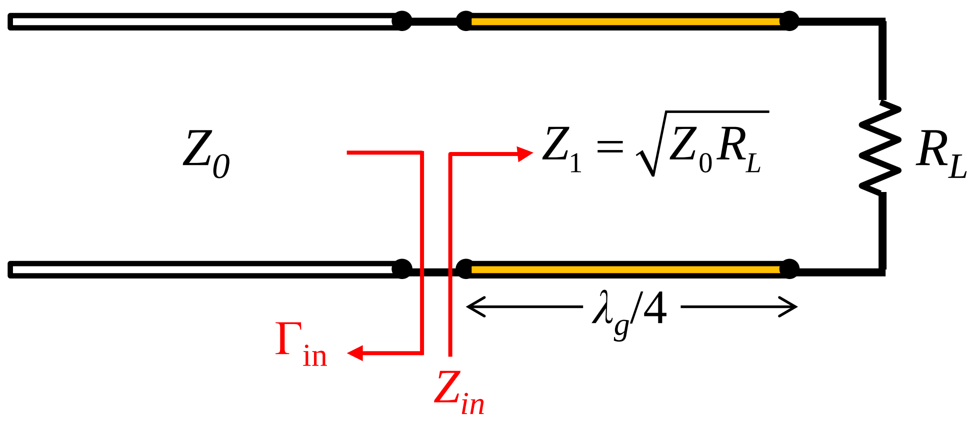


- The $\lambda_g/4$ transformers makes $\Gamma_{\text{in}} = 0$
- But $Z_1 \neq R_L$; so, reflected voltage waves exist between Z_1 and R_L
- However, all the power sent from the source successfully transmits to R_L

Drawbacks of $\lambda_g/4$ transformers:

- It applies only when the line length is $\lambda_g/4$ long
- Perfect matching is achieved at one frequency; mismatch occurs at other frequencies
- This method is limited to real-value load impedances

Frequency Response of a $\lambda_g/4$ Transformer (1/4)



- R_L = 100 Ω and Z_0 = 50 Ω are given

1. Find the characteristic impedance of the matching section (Z_1)
2. Plot $|\Gamma_{in}|$ vs. normalized frequency f/f_0, where f_0 is the frequency at which the line is $\lambda/4$ long

EX 3.1 Frequency Response of a $\lambda_g/4$ Transformer (2/4)

- The characteristic impedance of the matching section Z_1:

$$Z_1 = \sqrt{Z_0 R_L} = \sqrt{50 \times 100} = 70.71\,\Omega$$

- The magnitude of reflection coefficient Γ_{in}:

$$\left|\Gamma_{in}\right| = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right|$$

where Z_{in} is a function of frequency:

$$Z_{in} = Z_1 \left(\frac{Z_L + jZ_1 \tan \beta l}{Z_1 + jZ_L \tan \beta l} \right)$$

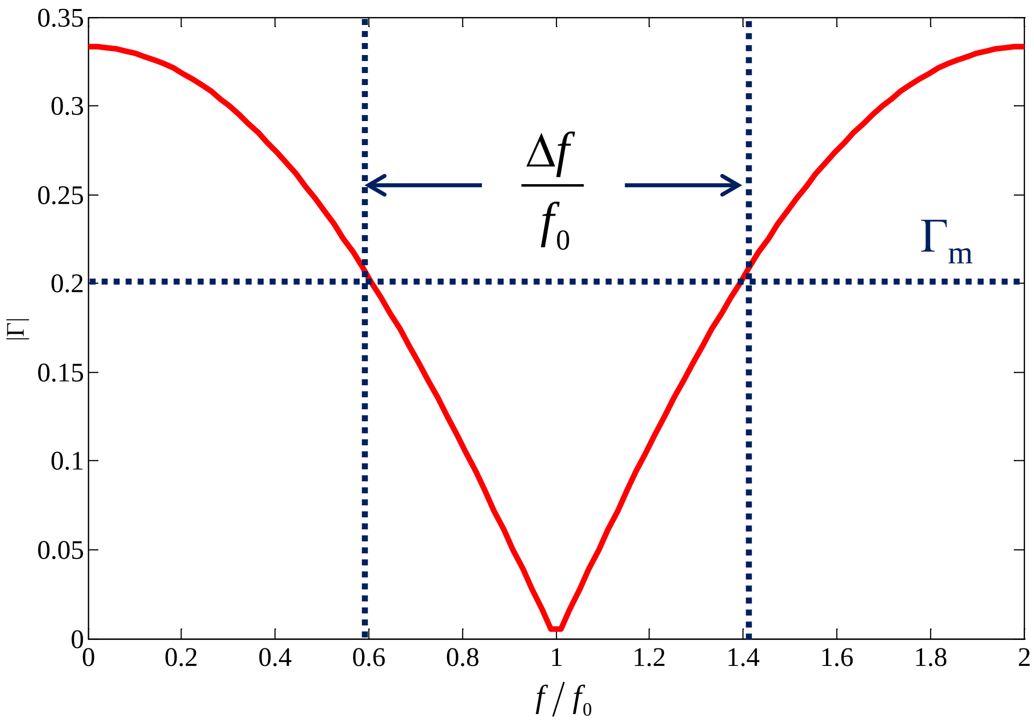
The frequency dependence comes from the βl term:

$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4 f_0}\right) = \frac{\pi f}{2 f_0}$$

- λ: variable
- λ_0: the associated wavelength of design frequency

Frequency Response of a $\lambda_g/4$ Transformer (3/4)

- Therefore, the explicit formula is: $$\left|\Gamma_{in}\left(\frac{f}{f_0}\right)\right| = \left| Z_1\left(\frac{Z_L + jZ_1 \tan\left(\frac{\pi f}{2 f_0}\right)}{Z_1 + jZ_L \tan\left(\frac{\pi f}{2 f_0}\right)}\right) - Z_0 \middle/ Z_1\left(\frac{Z_L + jZ_1 \tan\left(\frac{\pi f}{2 f_0}\right)}{Z_1 + jZ_L \tan\left(\frac{\pi f}{2 f_0}\right)}\right) + Z_0 \right|$$



- If we set a maximum value, Γ_m, that can be tolerated, we can define the bandwidth of the matching circuit:

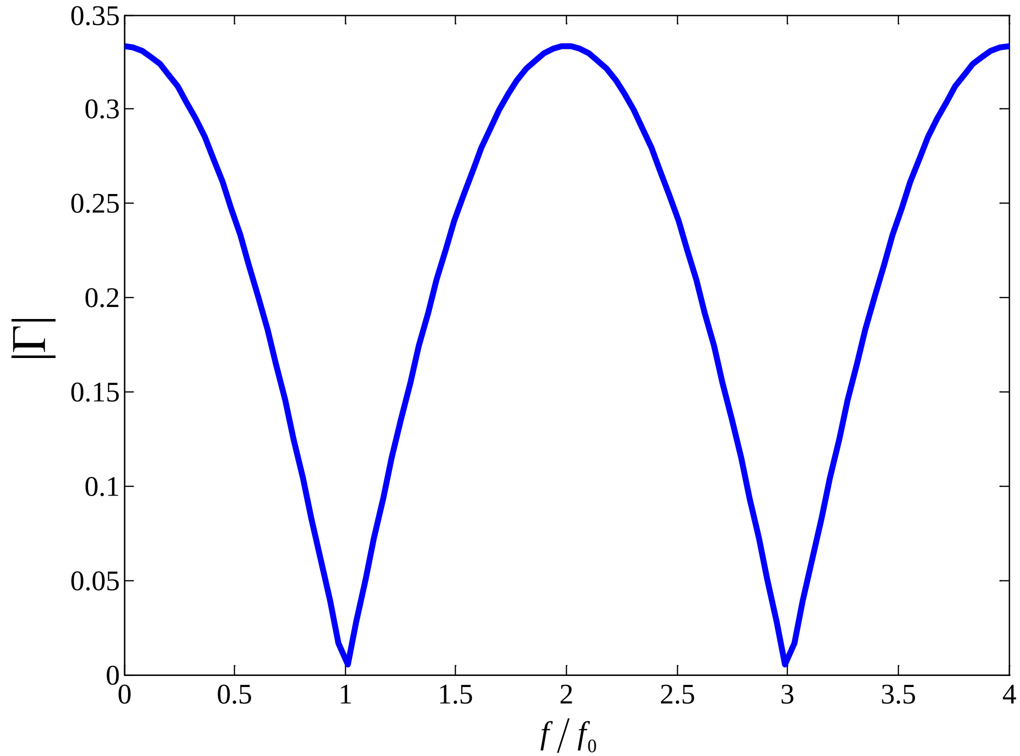
$$\frac{\Delta f}{f_0} = 0.8 \quad \text{Fractional bandwidth}$$

$$= 80\ \% \quad \text{Percentage bandwidth}$$

(However, in practical situations, Z_L is also the function of frequency!)

- For higher frequencies the line looks electrically longer
- For lower frequencies the line looks electrically shorter

Frequency Response of a $\lambda_g/4$ Transformer (4/4)



Frequency f	TL length l
$f = 0$	0
$f = 1f_0$	$l = \lambda_0/4$
$f = 2f_0$	$l = \lambda_0/2$
$f = 3f_0$	$l = 3\lambda_0/4$
$f = 4f_0$	$l = \lambda_0$

- Only at the design frequency f_0, the electrical length of the matching section is $\lambda_0/4$
- At other frequencies, the electrical lengths are different from $\lambda_0/4$
- However, when $f = 3f_0$, we have another perfect matching transformer again (why?)

Operational Bandwidth of $\lambda_g/4$ Transformers

- The fractional bandwidth of a quarter-wavelength transformer can be derived as:

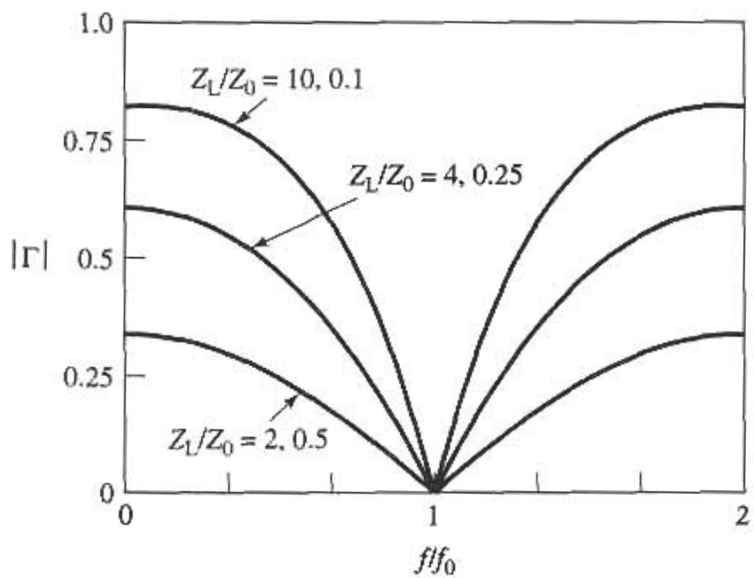
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right)$$

Γ_m Magnitude of the tolerable reflection coefficient

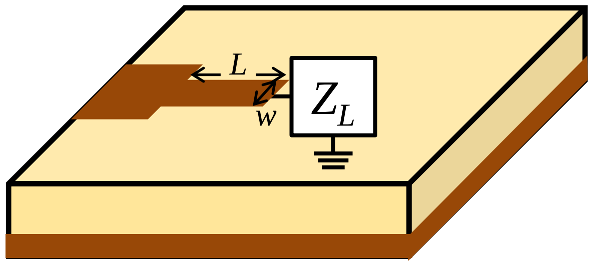
Z_L Load impedance

Z_0 Characteristic impedance of the main line

- What is the physical meaning from this formula?
  - If Γ_m becomes larger, the bandwidth increases
  - When Z_L becomes closer to Z_0, the bandwidth increases



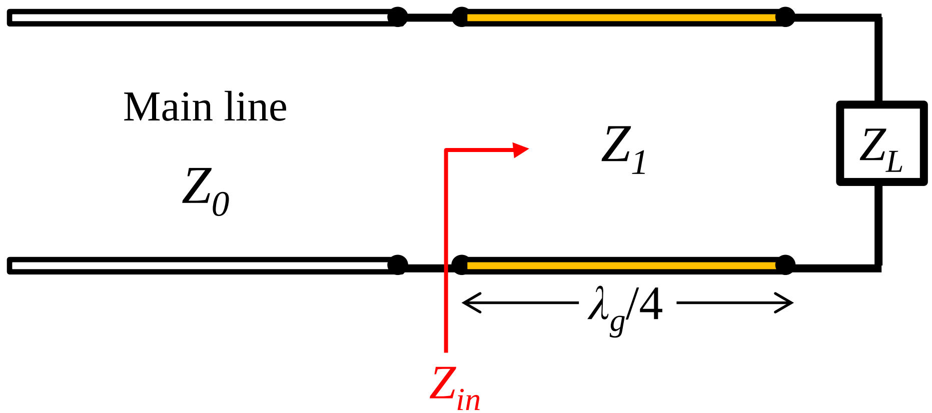
EX 3.2 A Practical Design Situation on $\lambda_g/4$ Transformers (1/4)



- Design a quarter-wavelength transformer to match $Z_L = 10\ \Omega$ to $Z_0 = 50\ \Omega$
- Design frequency: 1 GHz
- Requirement of tolerable bandwidth: VSWR < 1.5
- Materials at hand: FR4 substrate with $h = 1$ mm ($\varepsilon_r = 4.4$)

1. Find the length L
2. Find the width w
3. Find the characteristic impedance Z_1
4. Find the operational bandwidth of this transformer

A Practical Design Situation on $\lambda_g/4$ Transformers (2/4)



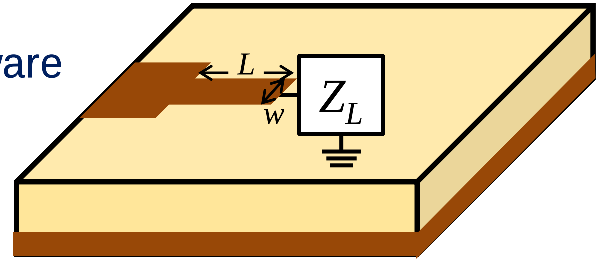
Step 1:

- Calculate the characteristic impedance of the $\lambda_g/4$ transformers :

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{50 \times 10} = 22.36\ \Omega$$

- Find the associated line width w by the TXLine software

$$w = 6.03\ \text{mm}$$



A Practical Design Situation on $\lambda_g/4$ Transformers (3/4)

Step 2:

- Find the line length by computing $\lambda_g/4$
- But, to compute λ_g, we have to know ε_{eff} first
- Again, ε_{eff} is computed by the TXLine software

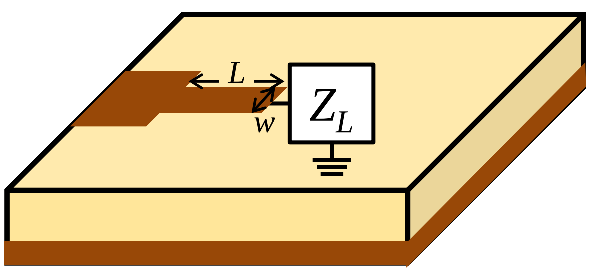
$$\varepsilon_{eff} = 3.71$$

- And the guided wavelength λ_g:

$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} = 155.7\ \text{mm}$$

- So, the line length L:

$$L = \frac{\lambda_g}{4} = 38.94\ \text{mm}$$



A Practical Design Situation on $\lambda_g/4$ Transformers (4/4)

Step 3:

- Transform VSWR = 1.5 to the associated Γ_m

$$\Gamma_m = \frac{VSWR - 1}{VSWR + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2$$

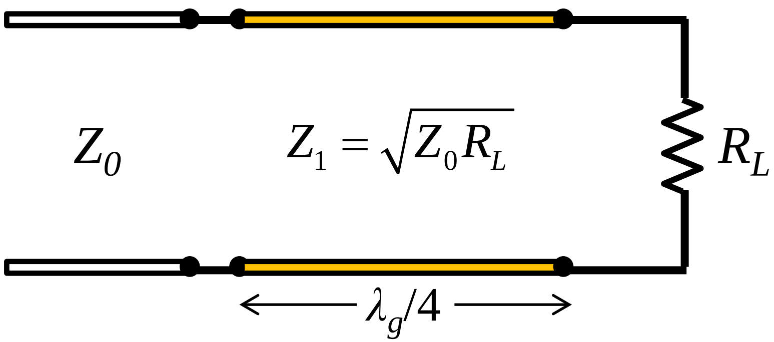
- Finally, the operational bandwidth can be calculated by the formula:

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right) = 2 - \frac{4}{\pi}\cos^{-1}\left(\frac{0.2}{\sqrt{1-0.2^2}}\frac{2\sqrt{10\times 50}}{|10-50|}\right) = 0.29$$

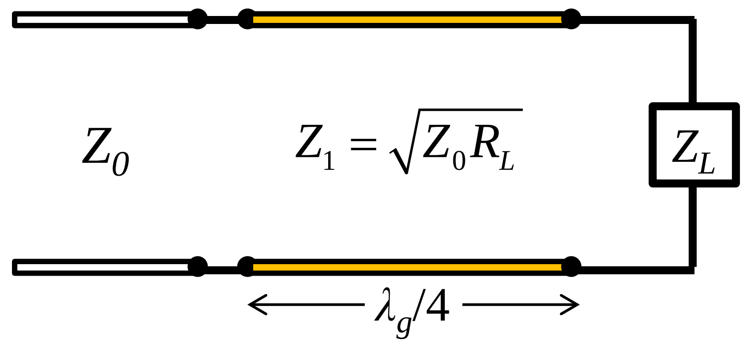
- The percentage bandwidth is 29%

If the Loading Impedance Is Not Purely Real

Previous scenario:

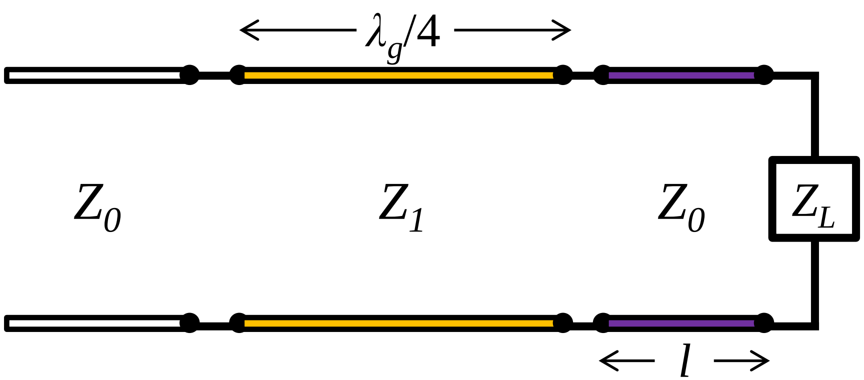


New situation:

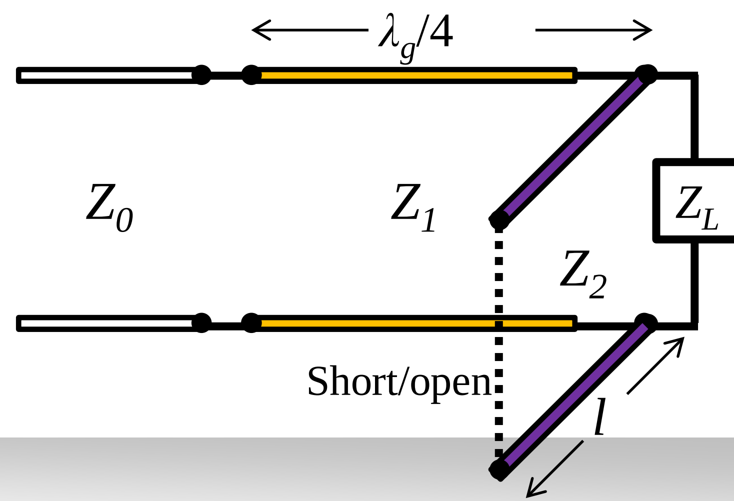


If we still want to match a complex Z_L to Z_0 by a $\lambda/4$ transformer:

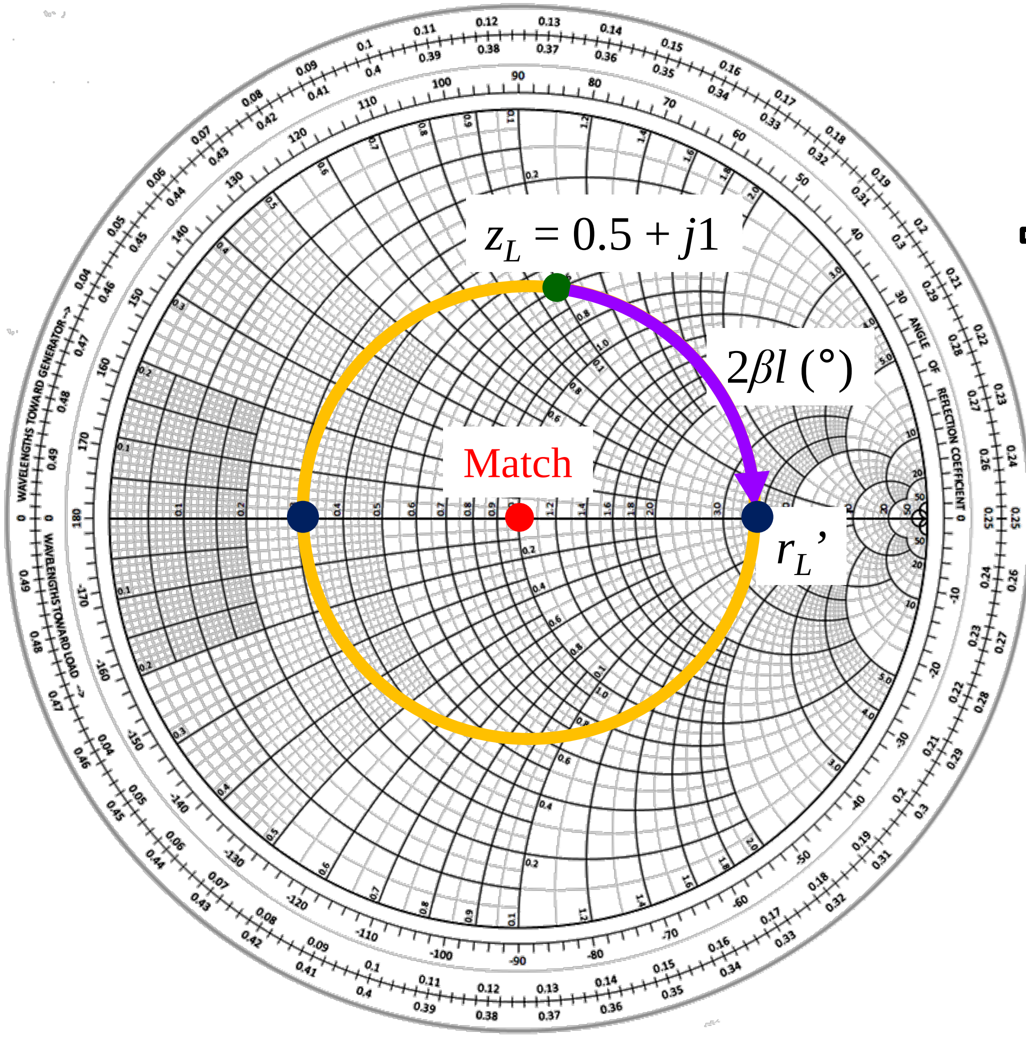
1. Inserting a series section

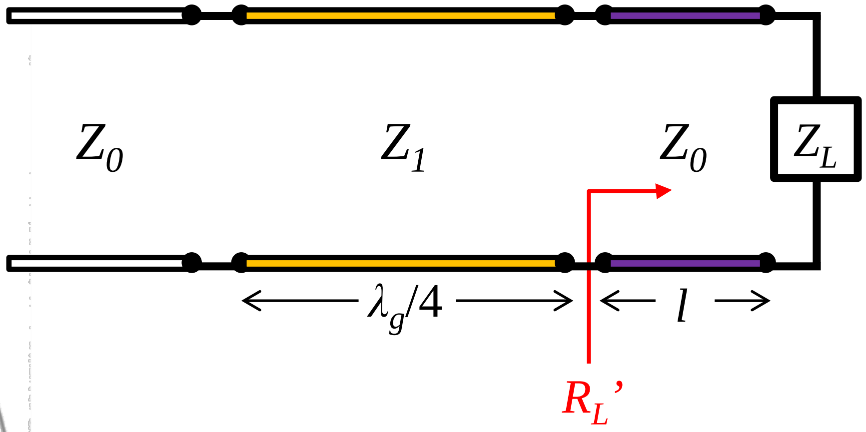


2. Inserting a shunt stub



Method 1: Inserting a Series Section



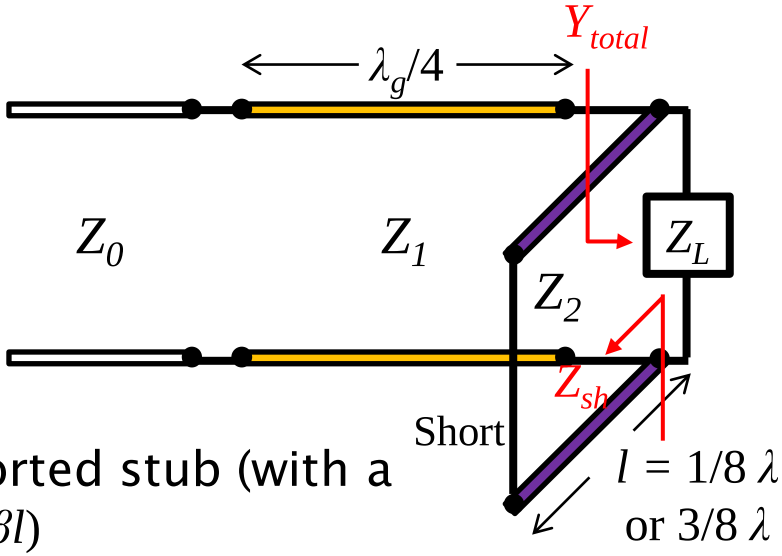


Producing a real-value R_L':

- Scenario: Z_L and Z_0 are given; we'd like to find Z_1 and l
- Considering $Z_L = 25 + j50\ \Omega$ and $Z_0 = 50\ \Omega$ (so, $z_L = 0.5 + j1$)
- Inserting a series section: rotating clockwise along the constant $|\Gamma|$ circle
- The two blue points give $x = 0$; choosing the minimum l as our design
- Finally, $Z_1 = \sqrt{Z_0 R_L'}$

Method 2: Inserting a Shunt Stub (1/4)

Using a short-circuit stub to cancel out the admittance



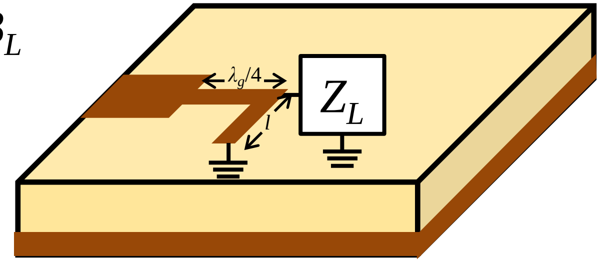
- Scenario: Z_L and Z_0 are given; we'd like to find Z_1 and Z_2
- Considering $Y_L = G_L + jB_L$ be the load admittance
- Recall that the input impedance of a shorted stub (with a characteristic impedance Z_2): $Z_{sh} = jZ_2\tan(\beta l)$
- So, the input admittance of a shorted stub: $Y_{sh} = -j\cot(\beta l)/Z_2$

Total admittance: $Y_{total} = Y_L + Y_{sh} = G_L + j(B_L - \cot(\beta l)/Z_2)$

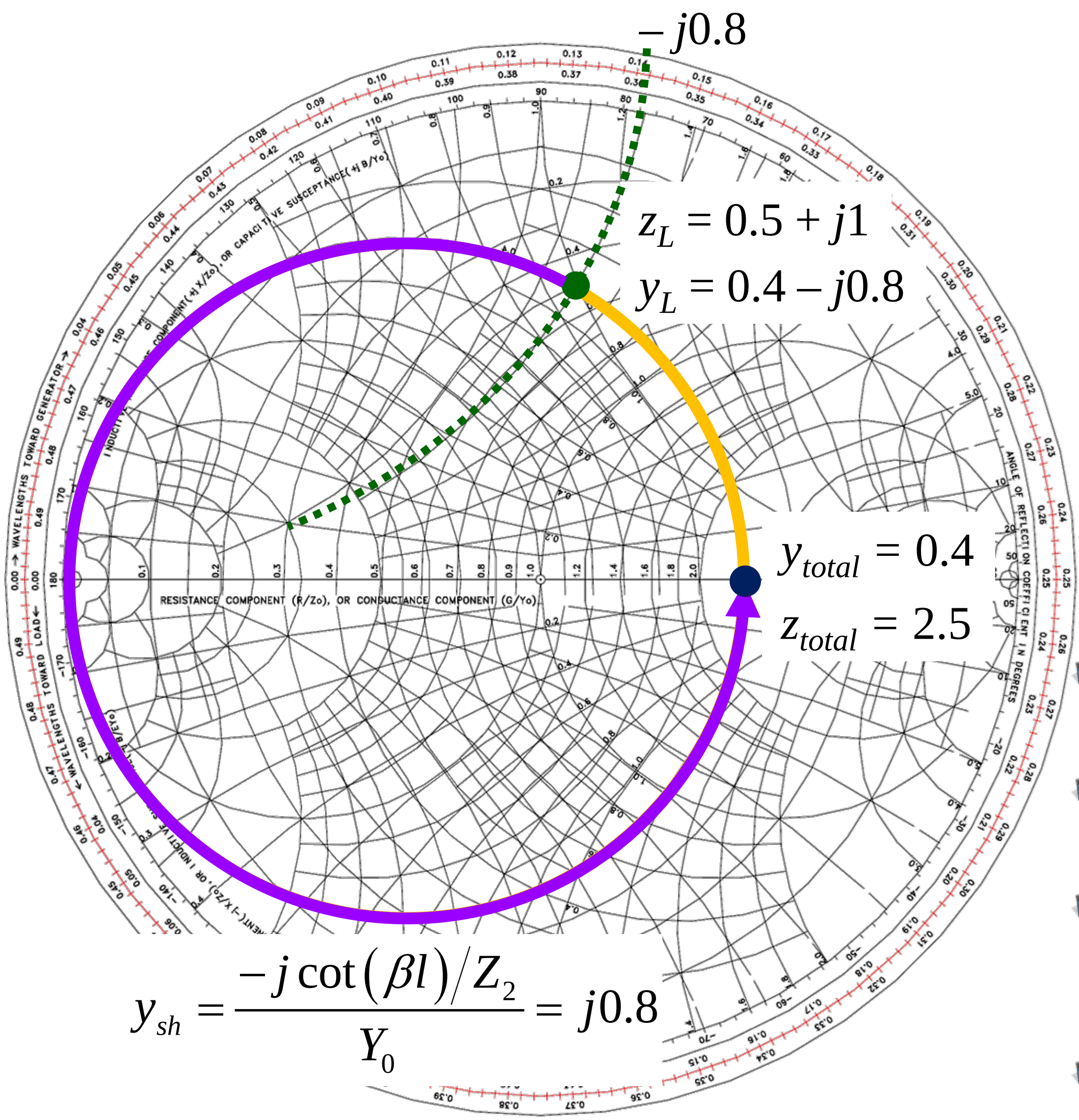
To make it purely real, we must have $Z_2 = \cot(\beta l)/B_L$

1. If $l = 1/8\,\lambda$: $Z_2 = 1/B_L$
2. If $l = 3/8\,\lambda$: $Z_2 = -1/B_L$

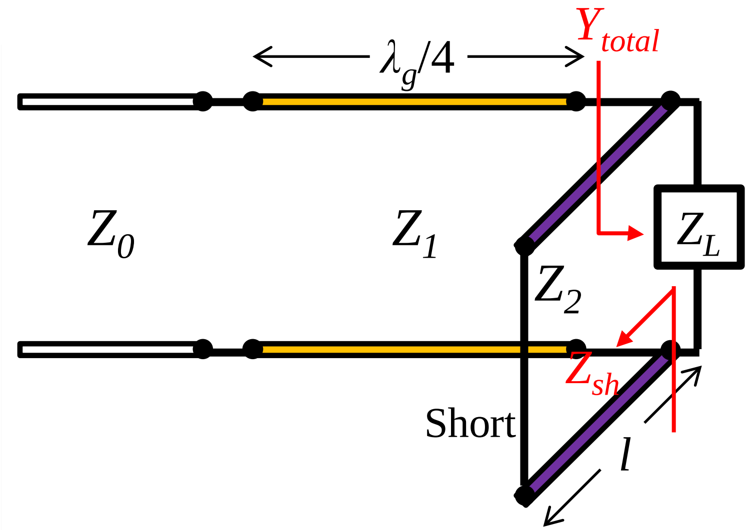
$$Z_1 = \sqrt{Z_0 \frac{1}{Y_{total}}} = \sqrt{\frac{Z_0}{G_L}}$$



Method 2: Inserting a Shunt Stub (2/4)



$$y_{sh} = \frac{-j\cot(\beta l)/Z_2}{Y_0} = j0.8$$



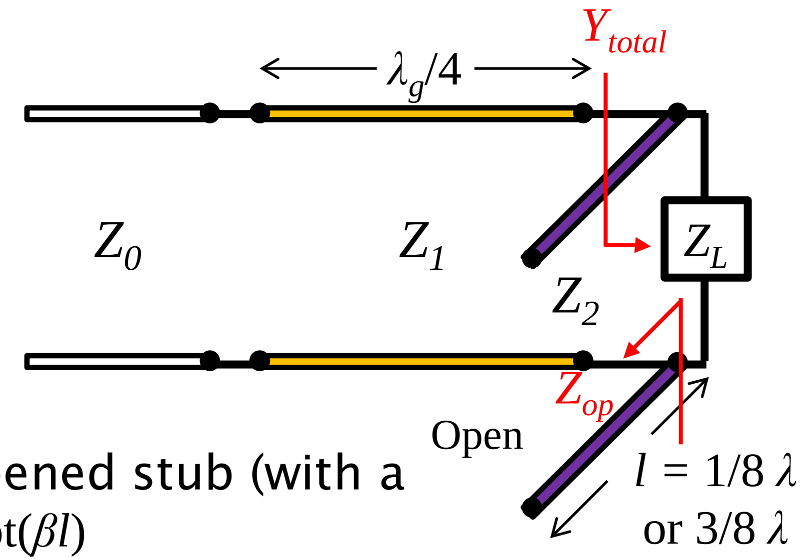
Representation on Z-Y chart:

- Considering $Z_L = 25 + j50\ \Omega$ and $Z_0 = 50\ \Omega$ (so, $z_L = 0.5 + j1$; inductive)
- Connection in parallel: reading the Y chart; so, $y_L = 0.4 - j0.8$
- Connecting a short-circuit stub in parallel: rotating counterclockwise along the constant g circle
- $z_{total} = 2.5$ gives $Z_{total} = 125\ \Omega$

$$Z_1 = \sqrt{Z_0 Z_{total}} = 79.06\ \Omega$$

Method 2: Inserting a Shunt Stub (3/4)

Using the opened stub to cancel out the admittance



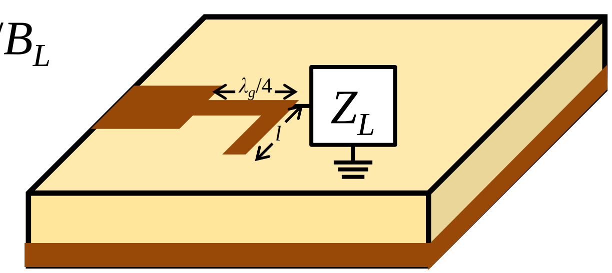
- Scenario: Z_L and Z_0 are given; we'd like to find Z_1 and Z_2
- Considering $Y_L = G_L + jB_L$ be the load admittance
- Recall that the input impedance of a opened stub (with a characteristic impedance Z_2): $Z_{op} = -jZ_2\cot(\beta l)$
- So, the input admittance of a opened stub: $Y_{op} = j\tan(\beta l)/Z_2$

Total admittance: $Y_{total} = Y_L + Y_{op} = G_L + j(B_L + \tan(\beta l)/Z_2)$

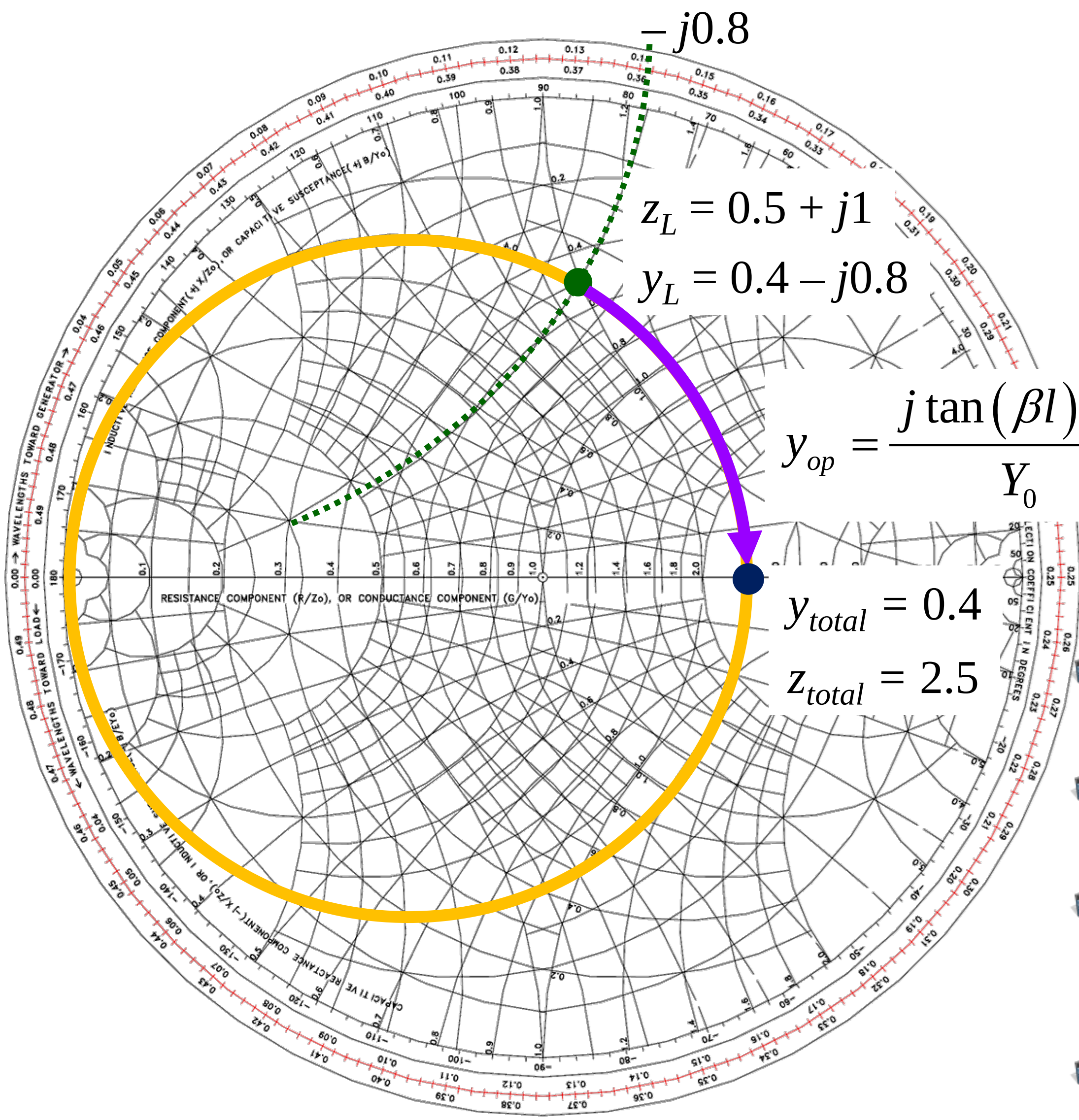
To make it purely real, we must have $Z_2 = -\tan(\beta l)/B_L$

1. If $l = 1/8\ \lambda$: $Z_2 = -1/B_L$
2. If $l = 3/8\ \lambda$: $Z_2 = 1/B_L$

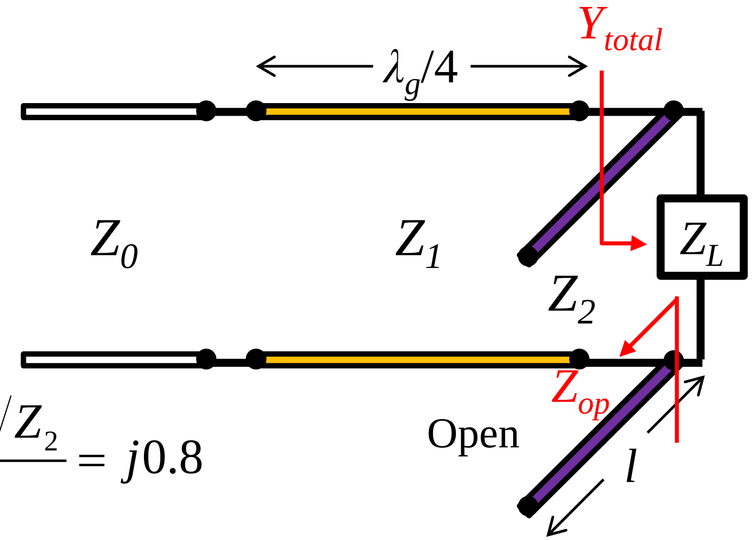
$$Z_1 = \sqrt{Z_0 \frac{1}{Y_{total}}} = \sqrt{\frac{Z_0}{G_L}}$$



Method 2: Inserting a Shunt Stub (4/4)



$$y_{op} = \frac{j\tan(\beta l)/Z_2}{Y_0} = j0.8$$



Representation on Z-Y chart:

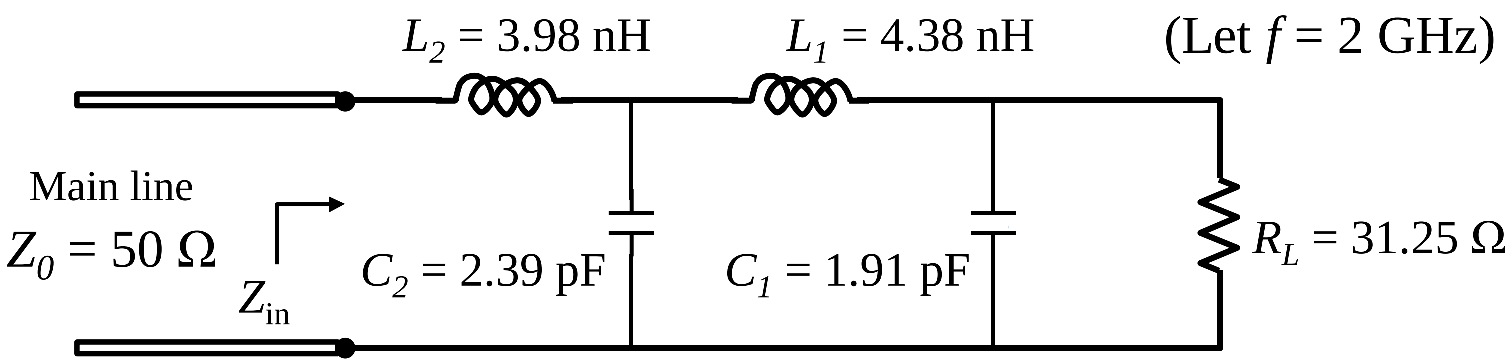
- Considering $Z_L = 25 + j50\ \Omega$ and $Z_0 = 50\ \Omega$ (so, $z_L = 0.5 + j1$; inductive)
- Connection in parallel: reading the Y chart; so, $y_L = 0.4 - j0.8$
- Connecting a opened stub in parallel: rotating clockwise along the constant g circle
- $z_{total} = 2.5$ gives $Z_{total} = 125\ \Omega$.

$$Z_1 = \sqrt{Z_0 Z_{total}} = 79.06\ \Omega$$

Contents

3.3 Matching with Lumped Elements

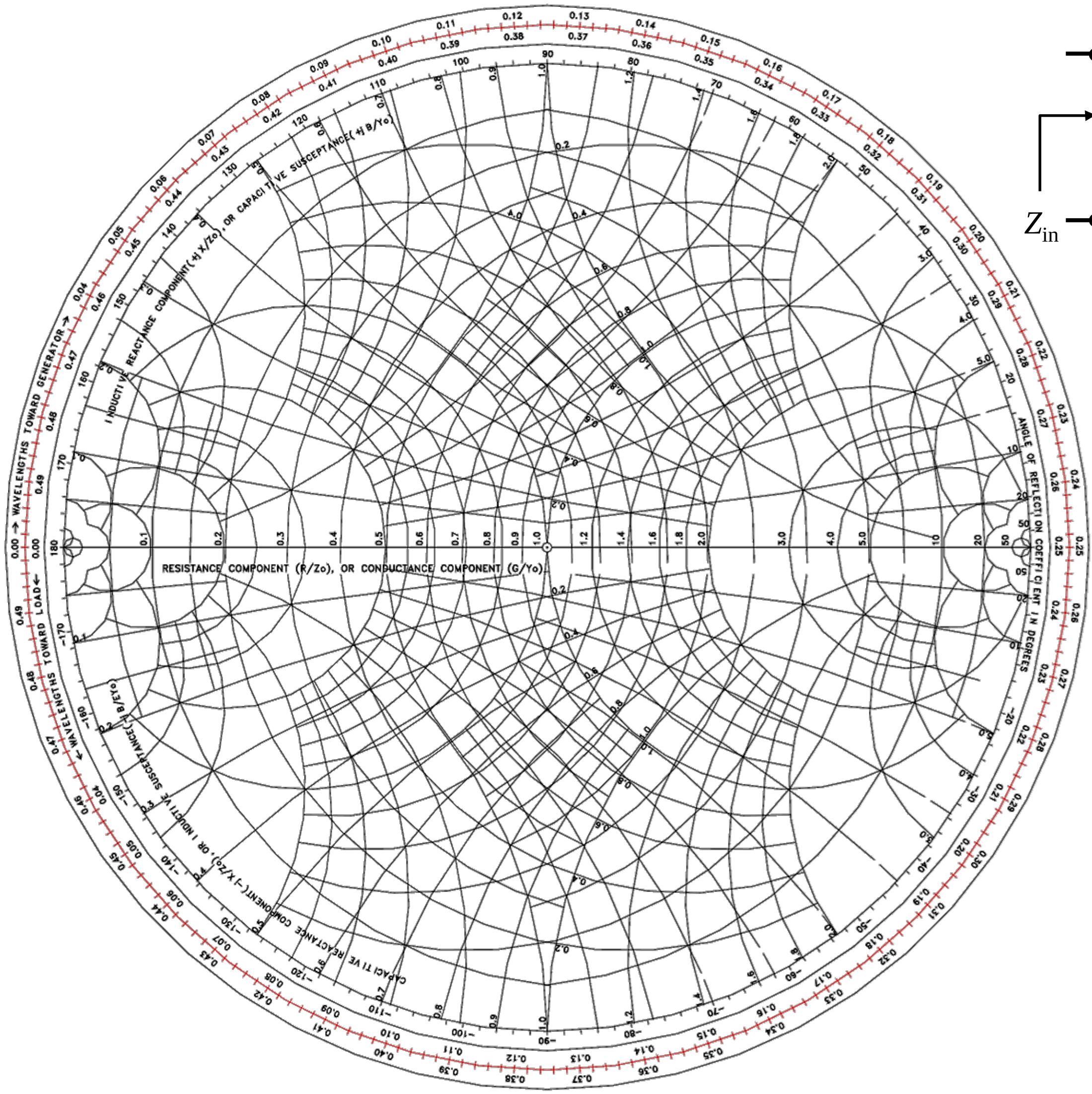
A Review of Smith Chart Operation (1/6)

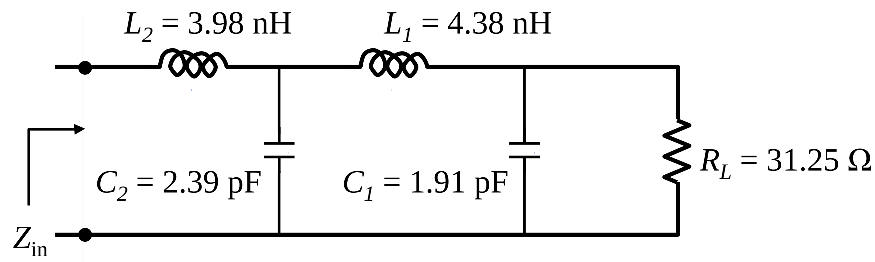


How do we calculate Z_{in}?

- Z_{in} is calculated stage by stage; from the right side to the left
- Series connection:
  - Using Z chart
  - The locus of impedance moves along a constant r circle
- Shunt connection:
  - Using Y chart
  - The locus of admittance moves along a constant g circle

A Review of Smith Chart Operation (2/6)





Step 1:

- Calculate the normalized impedances and admittances of all components:

$$r = 0.625$$

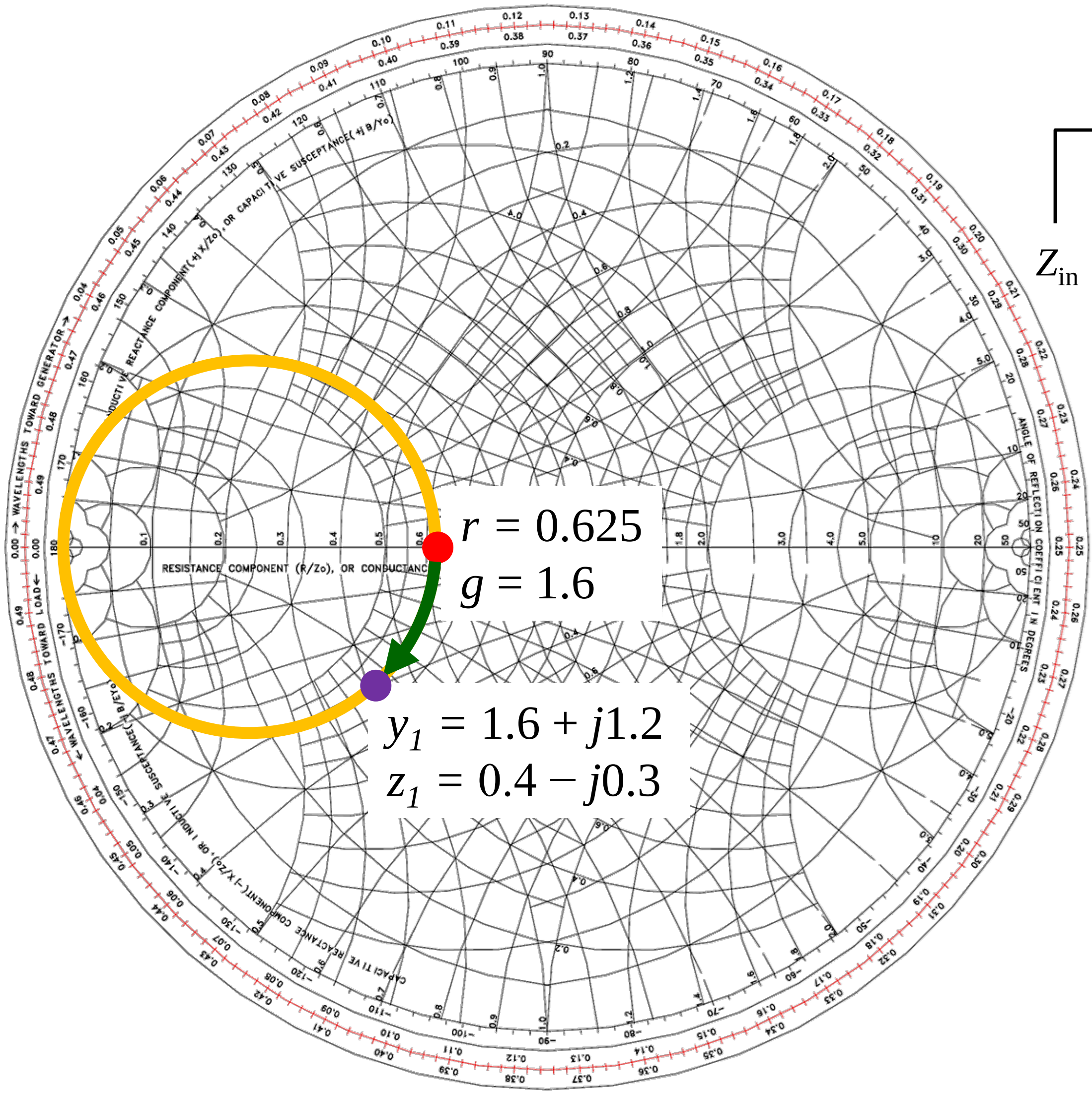
$$jx_{L1} = j\frac{\omega L}{Z_0} = j\frac{2\pi \times 2\times10^9 \times 4.38\times10^{-9}}{50} = j1.1$$

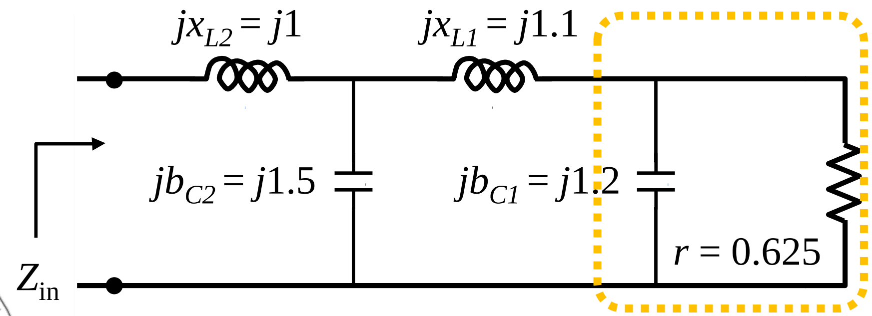
$$jx_{L2} = j\frac{2\pi \times 2\times10^9 \times 3.98\times10^{-9}}{50} = j1$$

$$jb_{C1} = j\frac{\omega C}{Y_0} = j\frac{2\pi \times 2\times10^9 \times 1.91\times10^{-12}}{0.02} = j1.2$$

$$jb_{C2} = j\frac{2\pi \times 2\times10^9 \times 2.39\times10^{-12}}{0.02} = j1.5$$

A Review of Smith Chart Operation (3/6)





Step 2:

- Begin from the right side of the circuit
- Mark $r = 0.625$ on the Z chart
- Since we are going to connect r to jb_{C1} in parallel, read its admittance on Y chart: $g = 1.6$

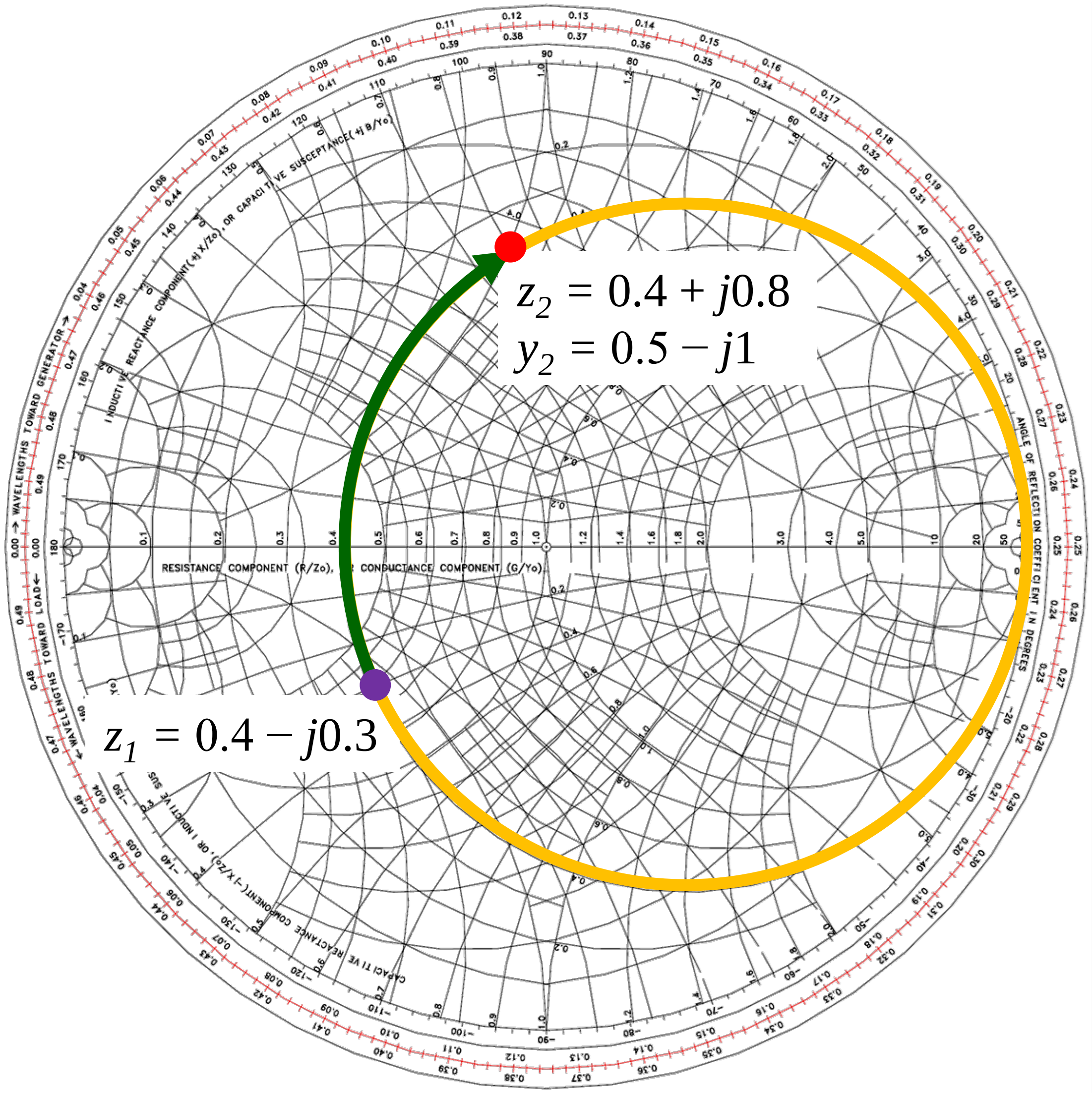
Step 3:

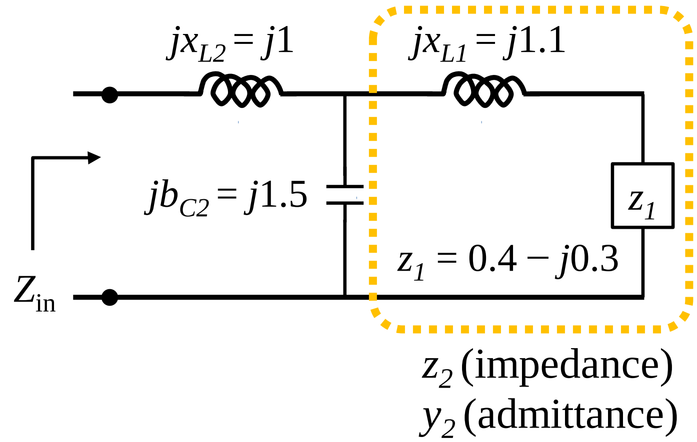
- The overall admittance $y_1 = g \parallel jb_{C1}$

$$y_1 = 1.6 + j1.2$$

$$z_1 = 0.4 - j0.3$$

A Review of Smith Chart Operation (4/6)





Step 4:

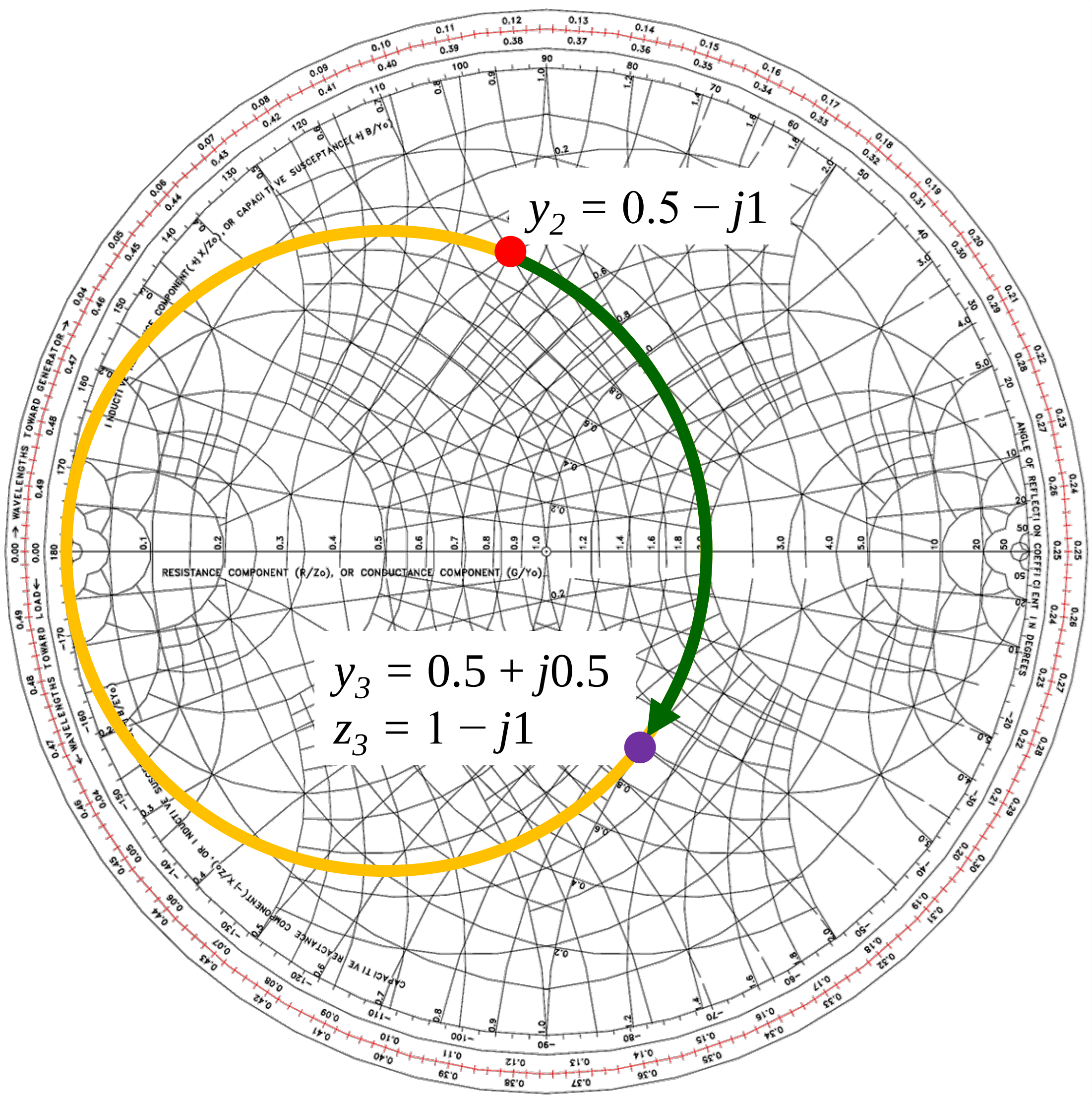
- The overall impedance $z_2 = z_1 \parallel jx_{L1}$
- This is done by the tracing the constant r circle on Z chart

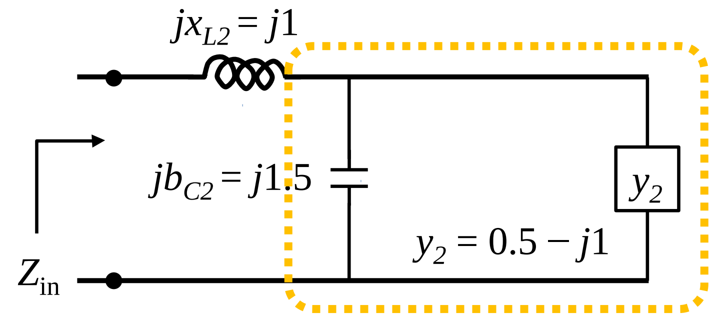
$$z_2 = (0.4 - j0.3) + j1.1 = 0.4 + j0.8$$

- Since we are going to connect z_2 to jb_{C2} in parallel, read its admittance on Y chart:

$$y_2 = 0.5 - j1$$

A Review of Smith Chart Operation (5/6)





z_3(impedance)
y_3(admittance)

Step 5:

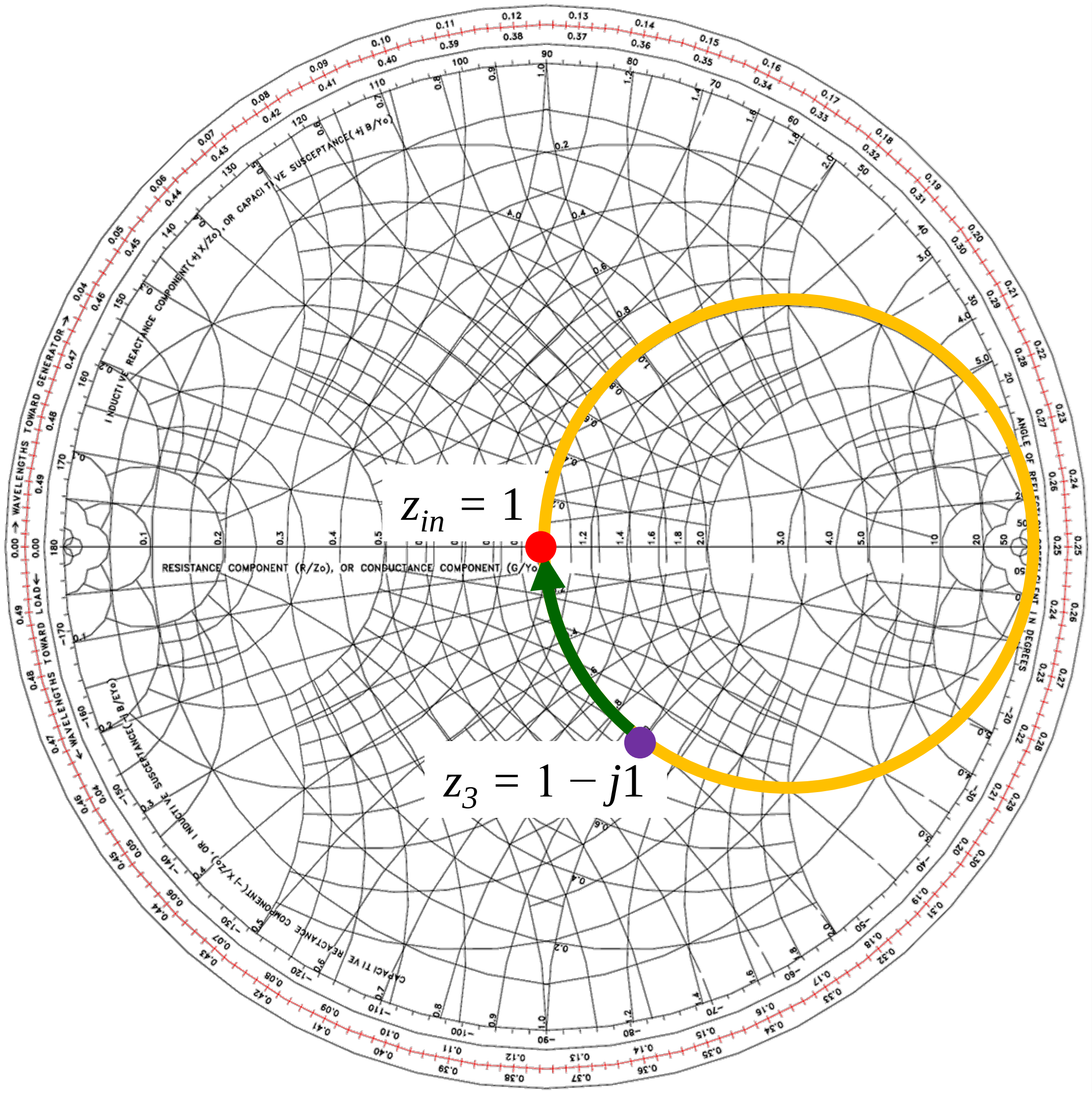
- The overall admittance $y_3 = y_2 \parallel jb_{C2}$
- This is done by the tracing the constant g circle on y chart

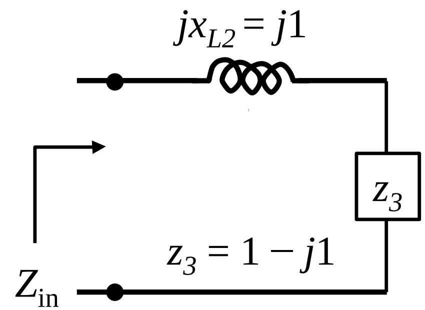
$$y_3 = (0.5 - j1) + j1.5 = 0.5 + j0.5$$

- Since we are going to connect y_3 to jx_{L2} in series, read its impedance on Z chart:

$$z_3 = 1 - j1$$

A Review of Smith Chart Operation (6/6)





Step 5:

- The normalized input impedance $z_{in} = z_3 \parallel jx_{L2}$

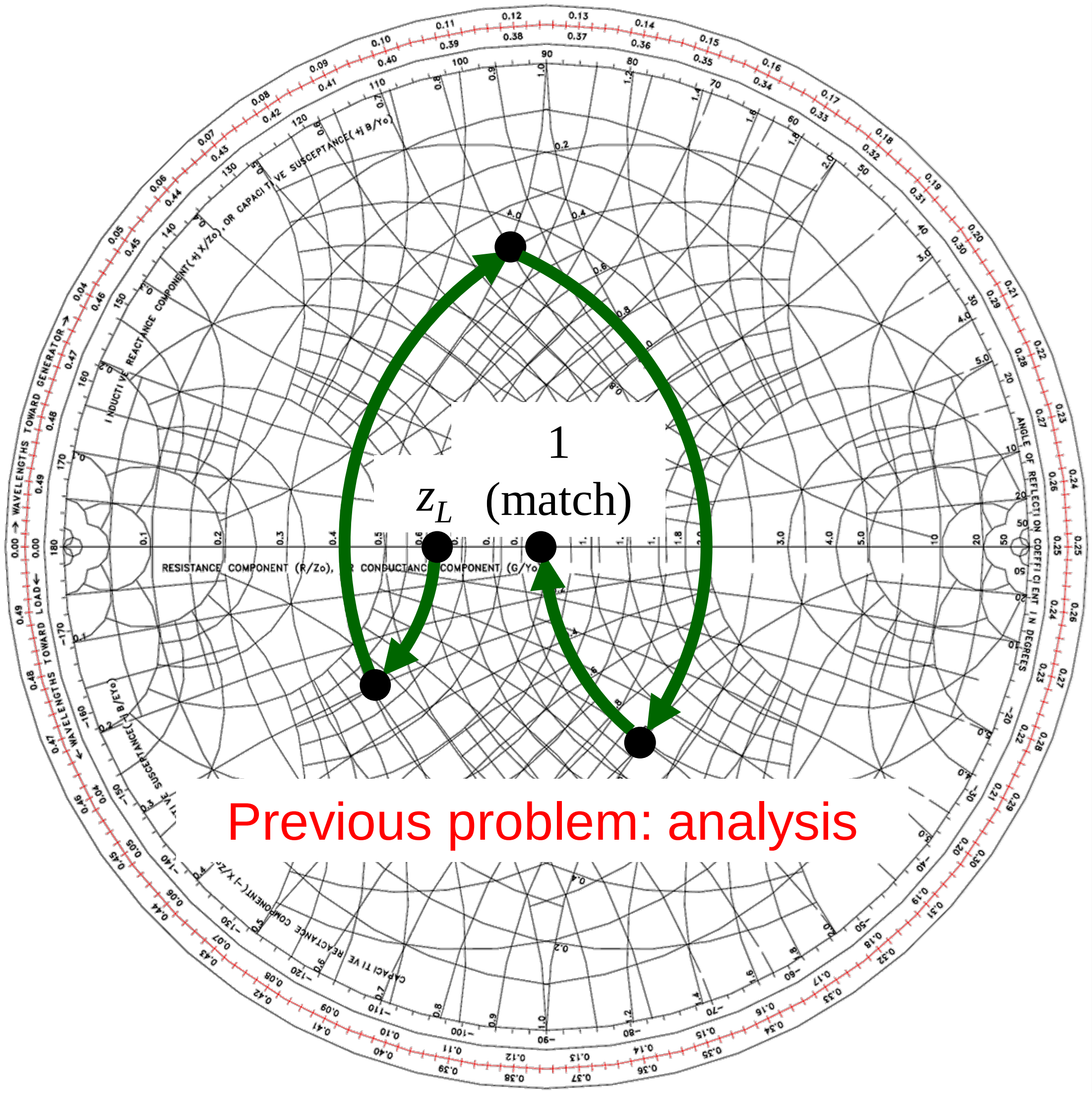
$$z_{in} = (1 - j1) + j1 = 1$$

- Finally, we reach the match point!

How about the inverse problem?

1. The circuitry is given; find the input impedance
2. The required input impedance is specified; find the associated circuitry

Analysis vs. Synthesis



Analysis

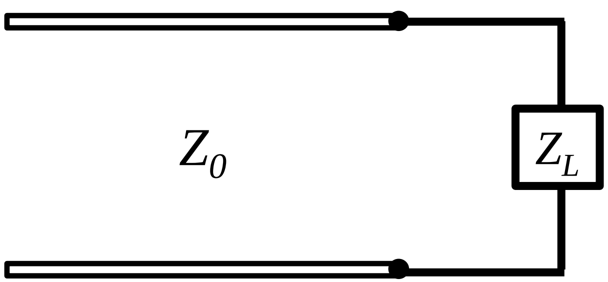
- Giving you an input, and asking you about the output
- Giving you a circuitry, and asking you to find the Smith chart locus

Synthesis

- Telling you the output requirement, and asking you how to do it
- Telling you the terminals on Smith chart, and asking you to design a locus
- Aka “Design”

Matching with Lumped Elements

Operational scenario:

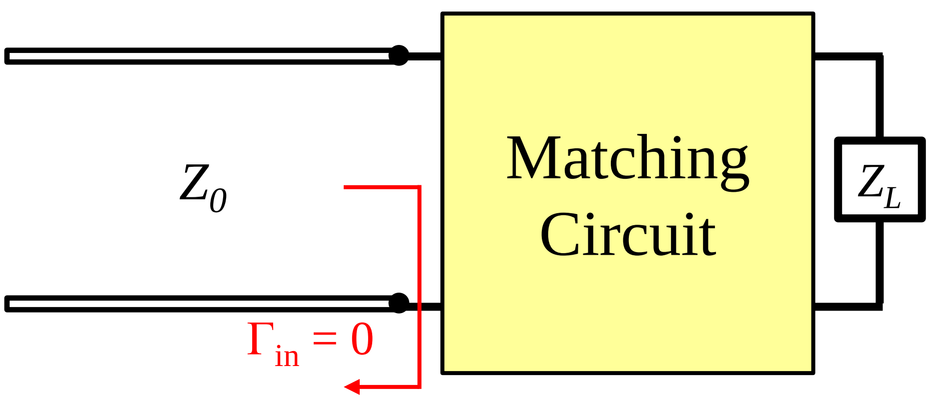


- Z_0 and Z_L are given (Z_0 is real and Z_L is arbitrary)
- If $Z_0 \neq Z_L$, reflected voltage waves occur:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- We'd like to match the load to the Z_0 line

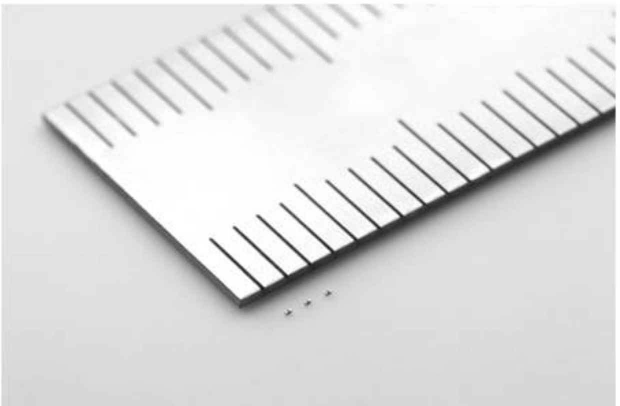
To make $\Gamma_{in} = 0$:



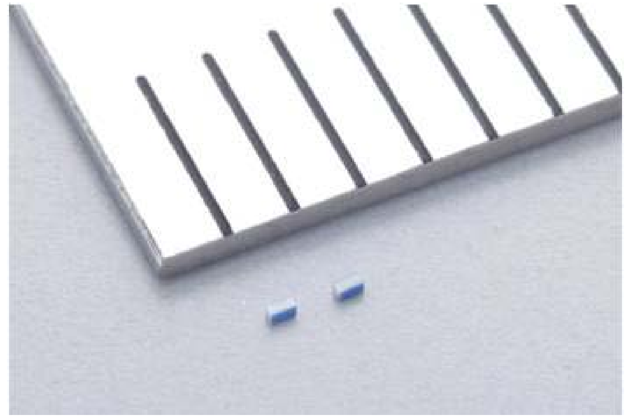
- Solution: by designing a combination of reactive elements
- The reactive elements (jX and jB) may be either inductors or capacitors

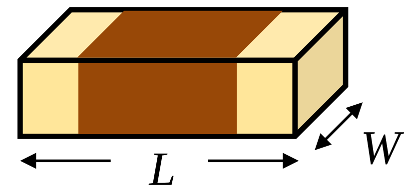
Lumped Elements for Microwave IC

Chip capacitor



Chip inductor

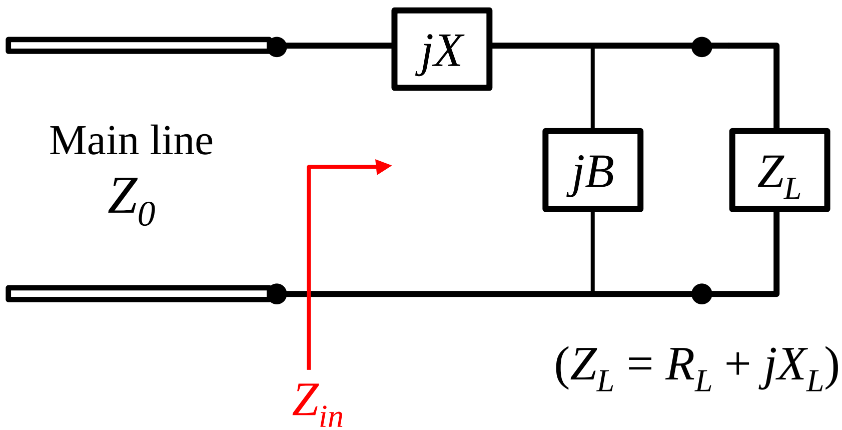


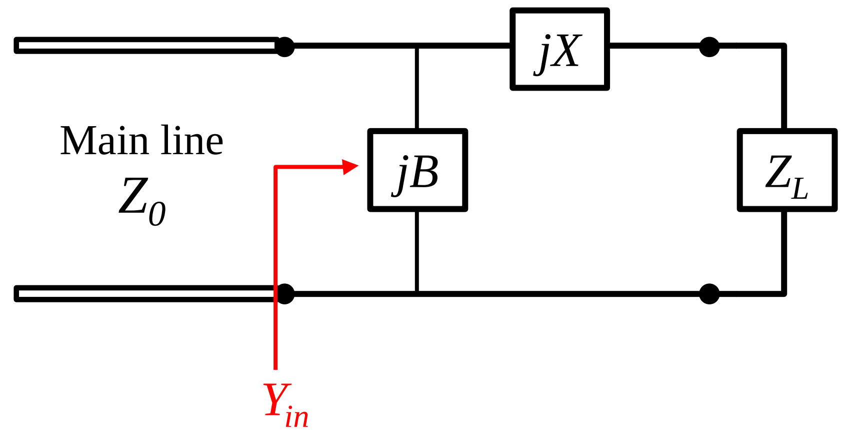


- Specification: 0603, 0402, 0201, 0805, etc.
- These components are very small relative to the operating frequency ($L < \lambda/10$)
- But the characteristics are far from ideal because of the undesirable effects:
  - Parasitic inductor and capacitor
  - Loss
  - Fringing fields
  - Grounding

Two Simplest Frames of Matching Problems

L-section: Using 2 reactive elements





- Goal: $Z_{in} = Z_0$ (so that $\Gamma_{\text{in}} = 0$)
- Used for $R_L > Z_0$
- Analytic solutions (2 solutions):

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}\left(R_L^2 + X_L^2 - Z_0 R_L\right)}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$

- Goal: $Y_{in} = Y_0$ (so that $\Gamma_{\text{in}} = 0$)
- Used for $R_L < Z_0$
- Analytic solutions (2 solutions):

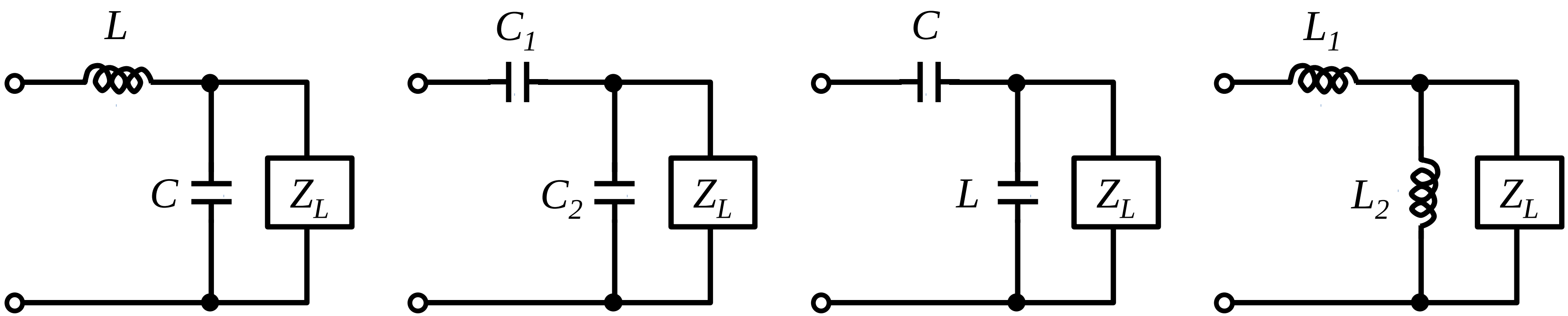
$$X = \pm\sqrt{R_L\left(Z_0 - R_L\right)} - X_L$$

$$B = \frac{X + X_L}{Z_0 R_L}$$

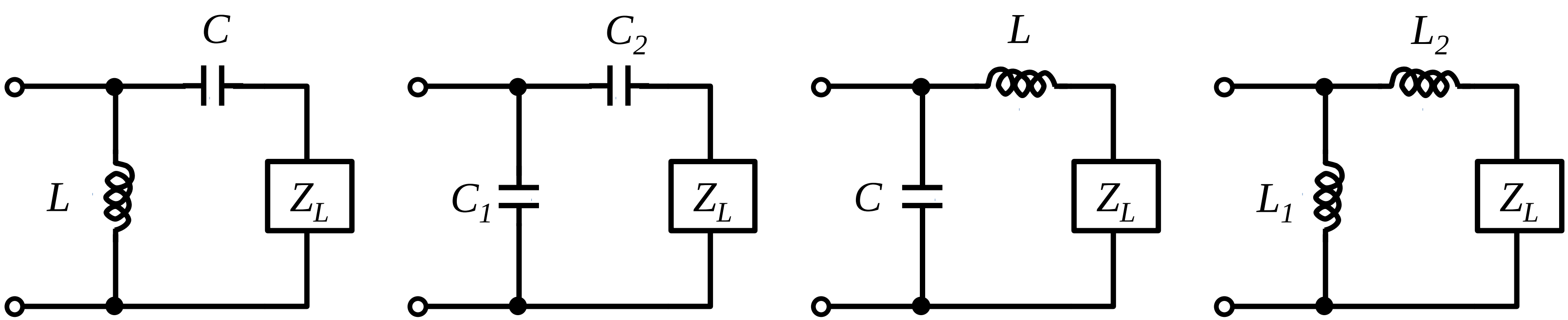
Characterization of L-Section Networks

8 types of L-section network:

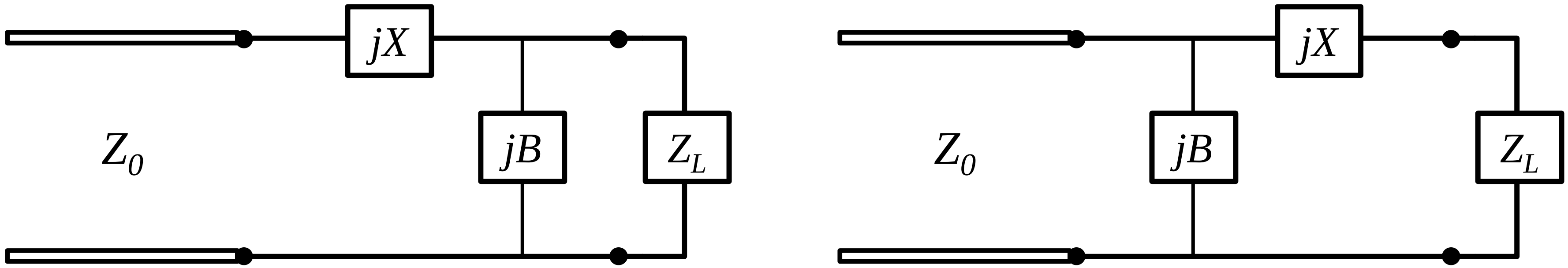
1. $R_L > Z_0$:



2. $R_L < Z_0$:

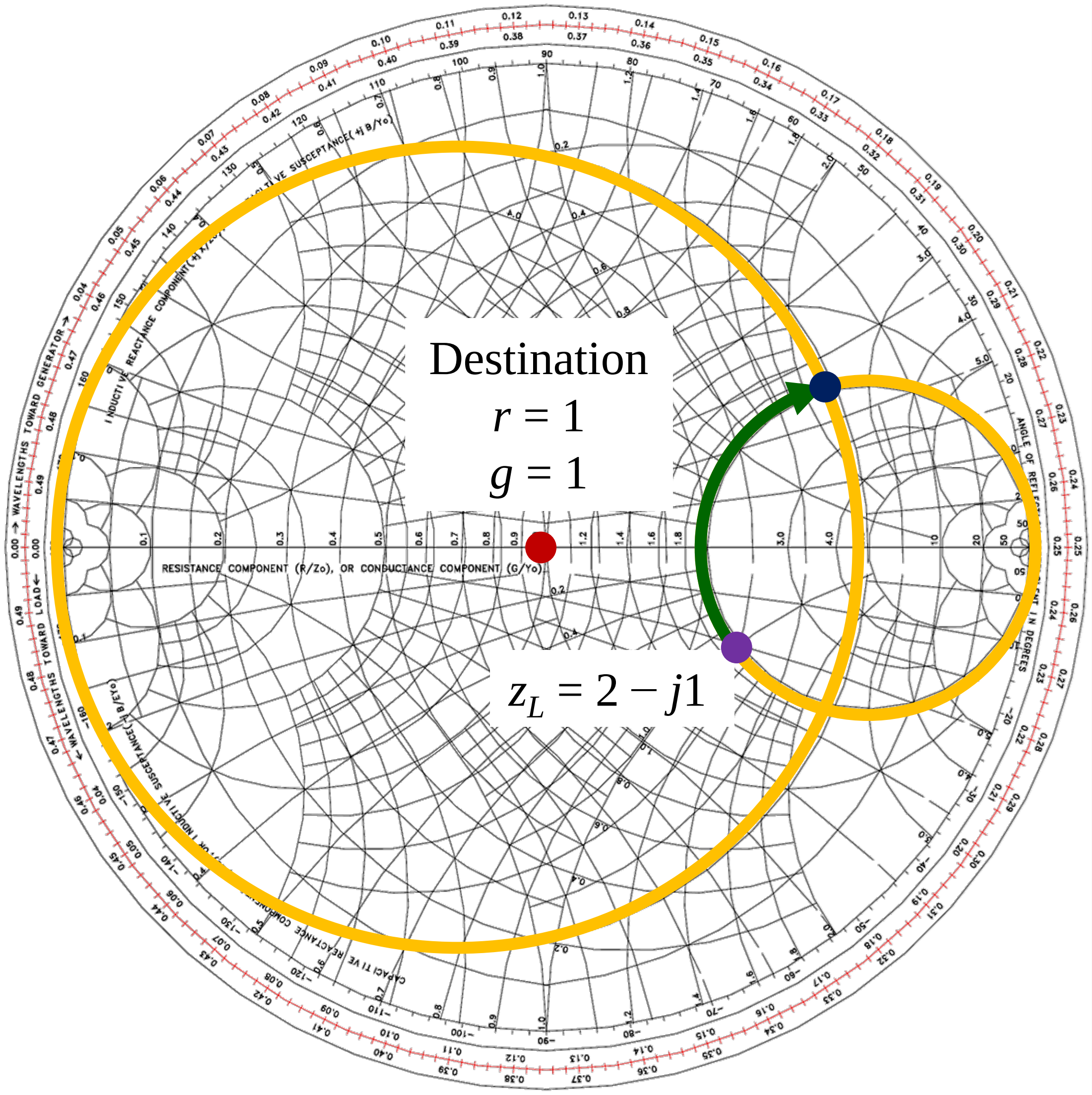


EX 3.3 Smith Chart Solutions (1/9)



1. Design an L-section matching circuit to match $Z_L = 200 - j100\ \Omega$ to a 100 Ω line (Operational frequency: 500 MHz)
2. Compare the operational bandwidth between the 2 solutions

Smith Chart Solutions (2/9)



Step 1:

- The normalized load impedance:

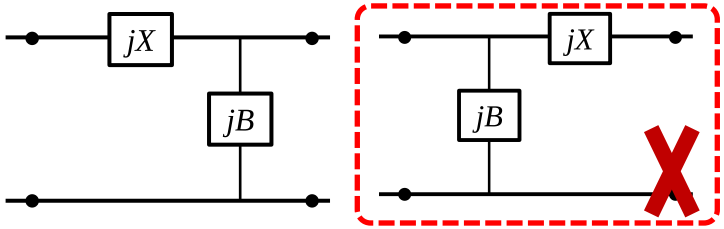
  $$z_L = 2 - j1$$

- The associated normalized admittance:

  $$y_L = 0.4 + j0.2$$

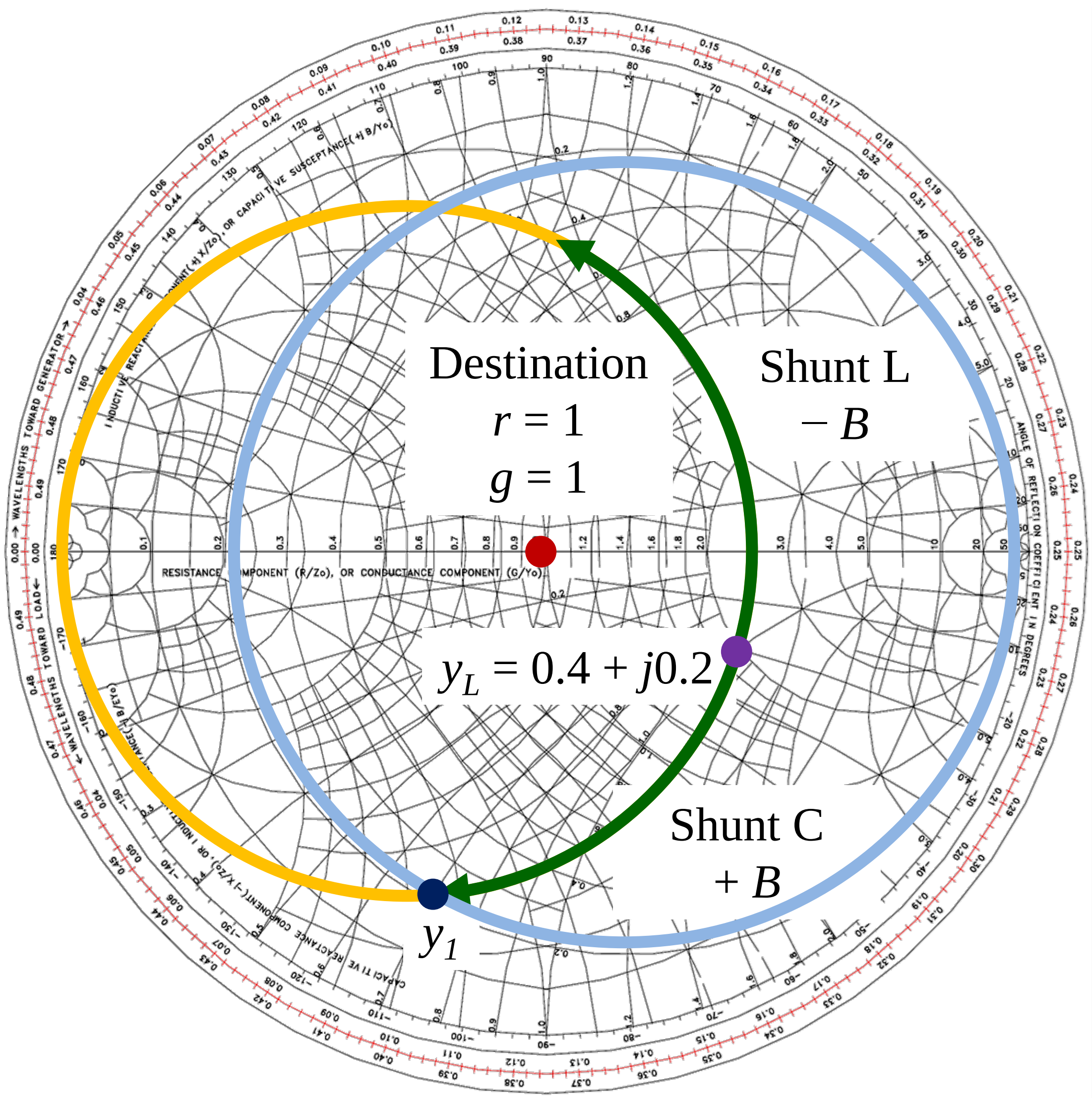
Step 2:

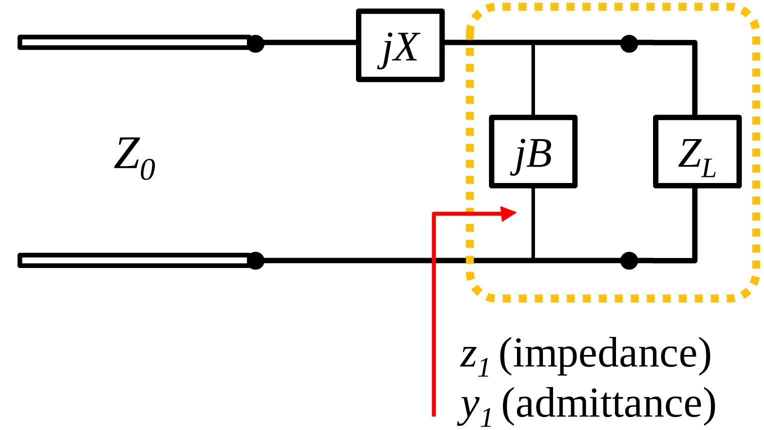
- Which method should we apply?



- RHS method: moving along the constant r circle
- No points on their constant g circle give us a locus to get to the destination
- So we choose the LHS method

EX 3.3 Smith Chart Solutions (3/9)





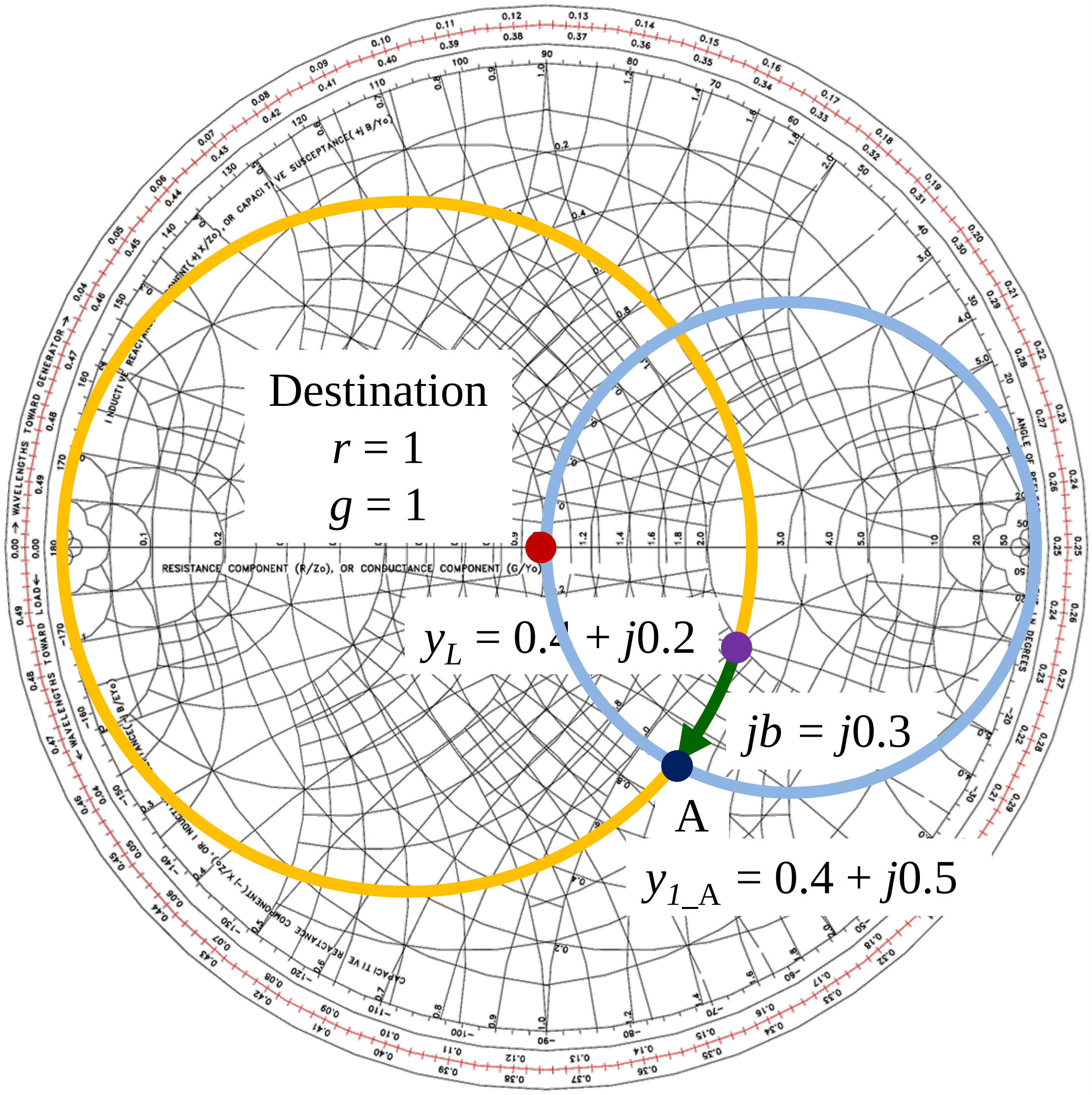
Step 3:

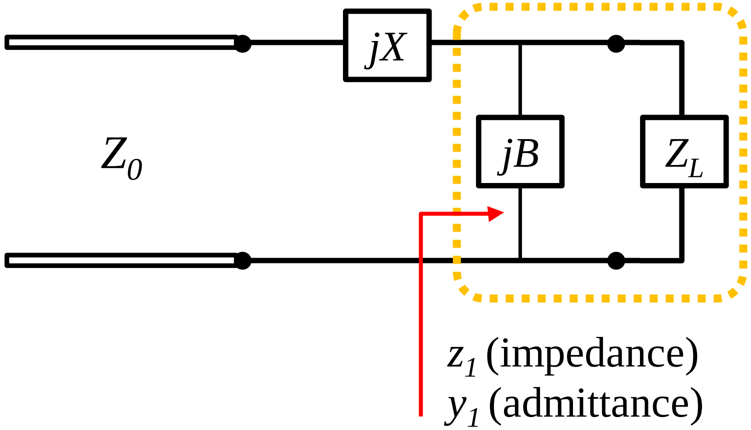
- Connecting jB in parallel: moving along its constant g circle
- How can we get to the destination?

Destination: $r = 1$ (or $g = 1$)

- y_1 (or its associated impedance z_1) has to be located on the constant r = 1 circle
- Clearly, we have two possible solutions

Smith Chart Solutions (4/9)





Solution 1:

- If y_L connects to a C in parallel, y_1 = A:

  $y_{1\_A} = 0.4 + j0.5$

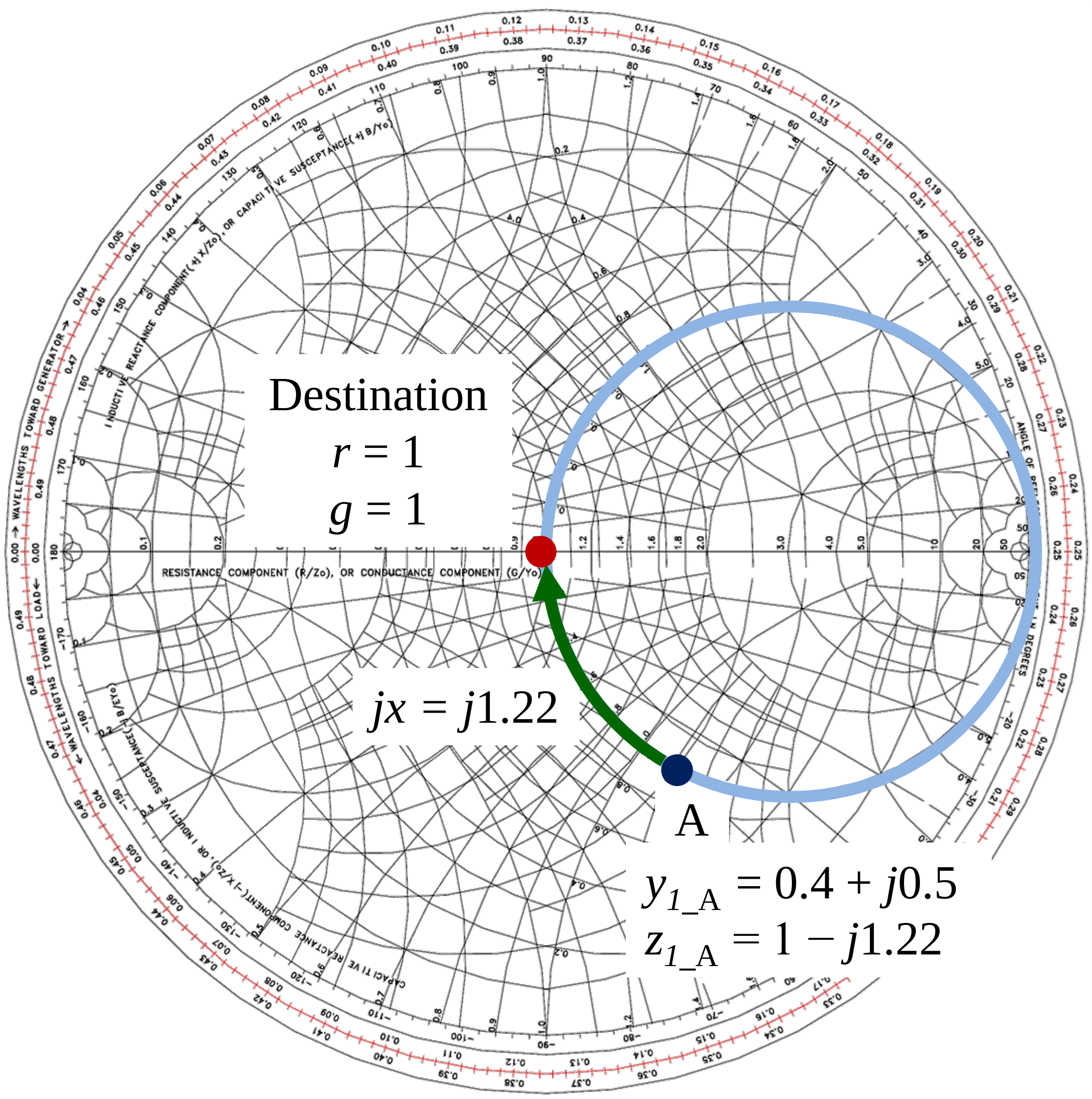
- How to find the corresponding capacitance?

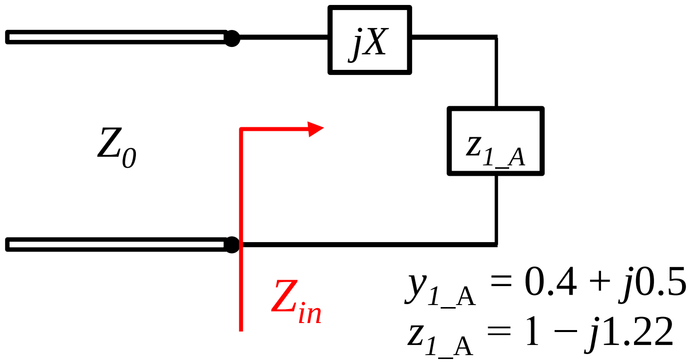
$$\frac{j\omega C}{Y_0} = (0.4 + j0.5) - (0.4 + j0.2) = j0.3$$

$$C = \frac{0.3 \times Y_0}{\omega} = \frac{0.3 \times 0.01}{2\pi \times 500 \times 10^6}$$

$$= 9.54 \times 10^{-13}\ \text{F} = 0.95\ \text{pF}$$

Smith Chart Solutions (5/9)





Step 4: (Continuing Solution 1)

- Associated impedance of A:

$$z_{1\_A} = 1 - j1.22$$

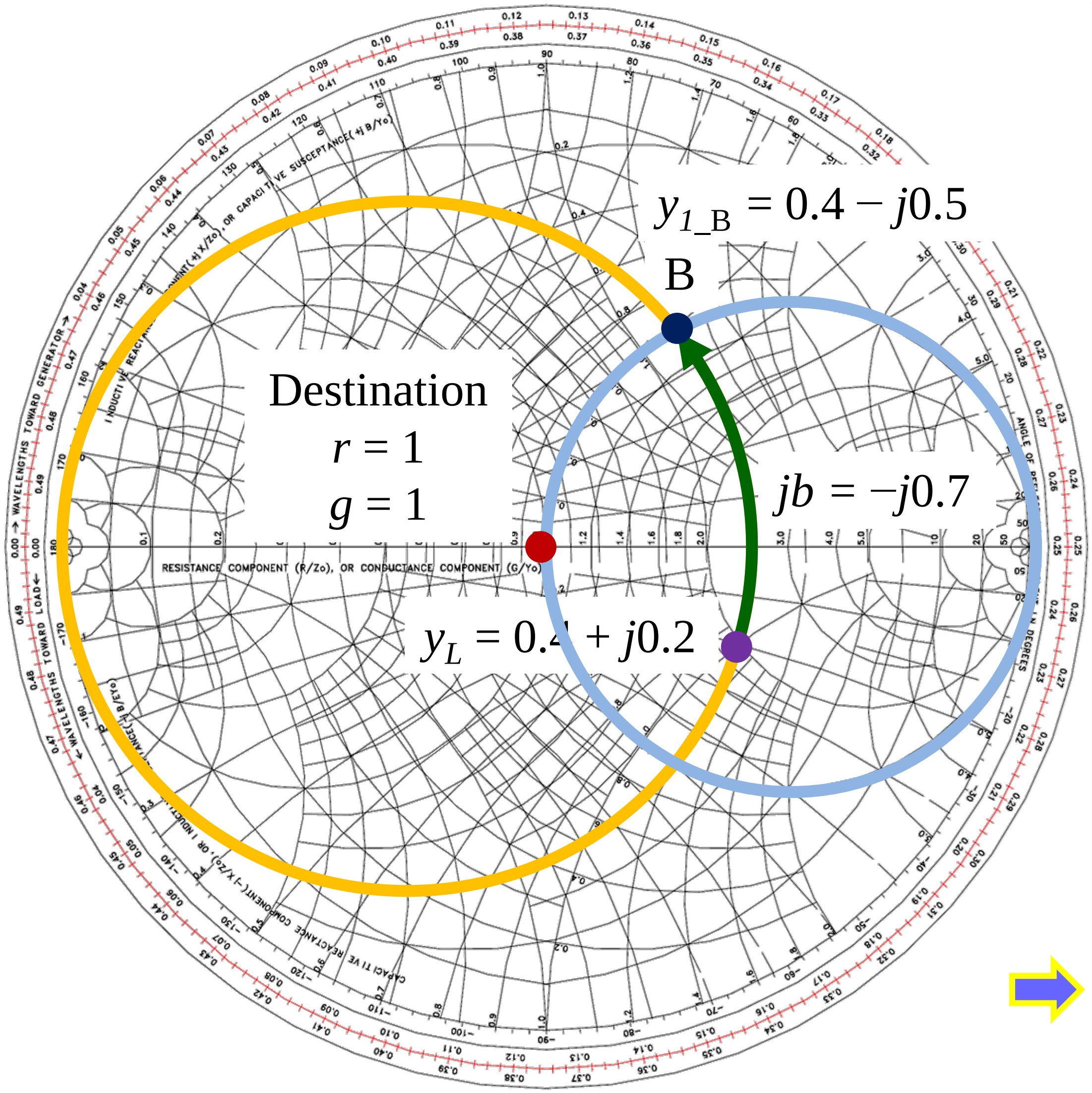
- If we connect an L in series, it brings us to the destination: r =1
- How to find the corresponding inductance?

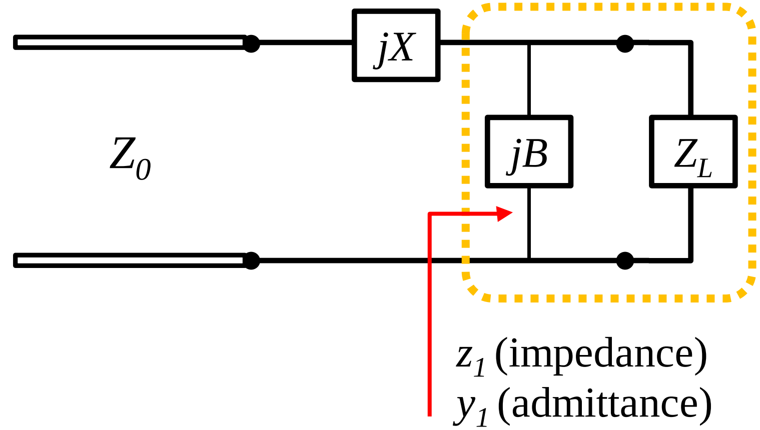
$$\frac{j\omega L}{Z_0} = (1) - (1 - j1.22) = j1.22$$

$$L = \frac{1.22 \times Z_0}{\omega} = \frac{1.22 \times 50}{2\pi \times 500 \times 10^6}$$

$$= 3.88 \times 10^{-8}\ \mathrm{H} = 38.8\ \mathrm{nH}$$

Smith Chart Solutions (6/9)





Solution 2:

- If y_L connects to a L in parallel, y_1 = B:
  $y_{1\_B} = 0.4 - j0.5$
- How to find the corresponding inductance?

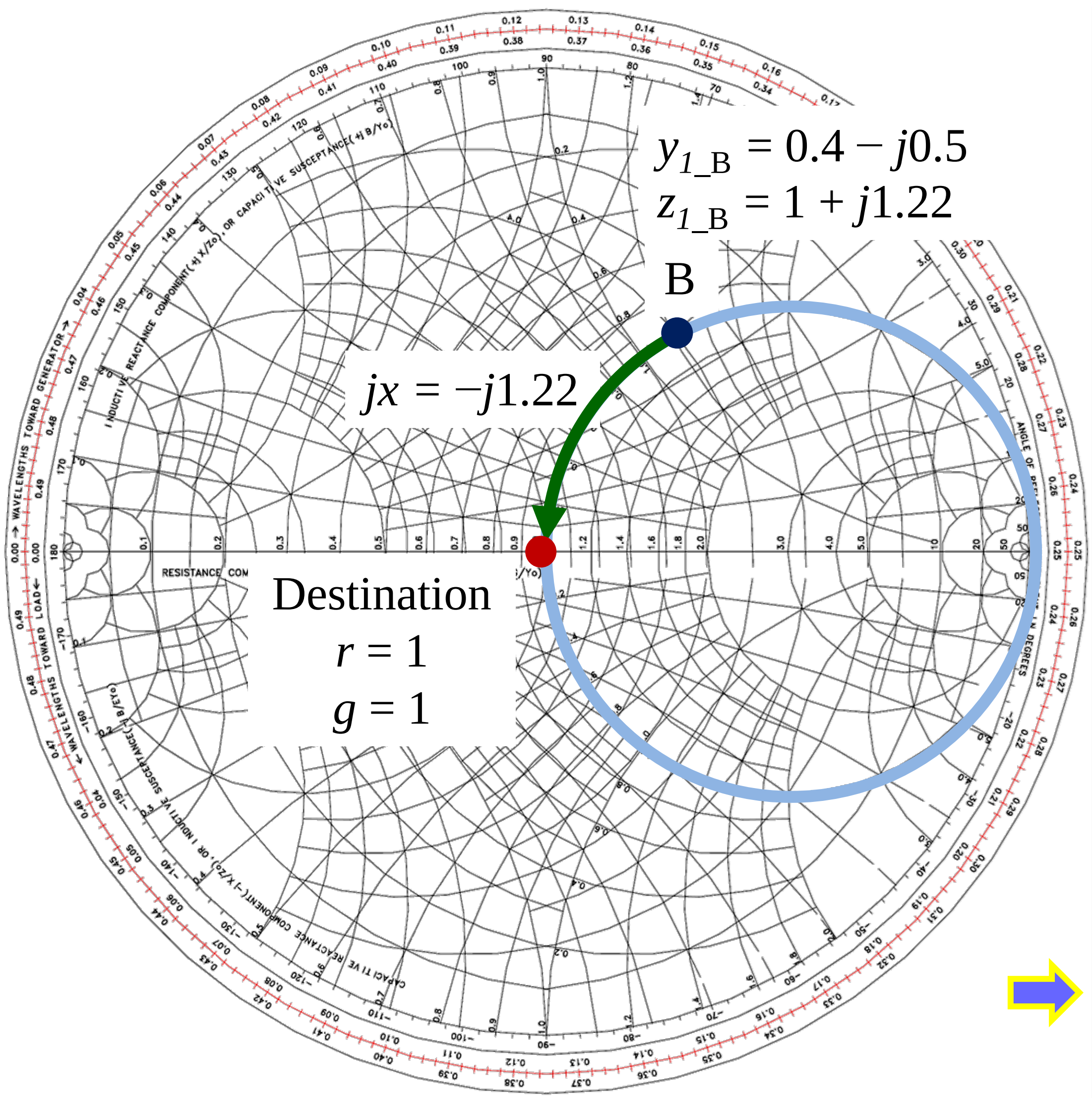
$$-j\frac{1}{\omega L Y_0} = (0.4 - j0.5) - (0.4 + j0.2) = -j0.7$$

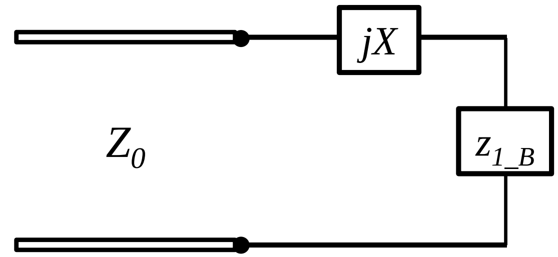
$$L = \frac{1}{0.7 \times \omega \times Y_0} = \frac{1}{0.7 \times 2\pi \times 500 \times 10^6 \times 0.01}$$

$$= 4.55 \times 10^{-8}\ \text{H} = 45.5\ \text{nH}$$

EX 3.3

Smith Chart Solutions (7/9)





$y_{1\_B} = 0.4 - j0.5$
$z_{1\_B} = 1 + j1.22$

Step 4: (Continuing Solution 2)

- Associated impedance of B:

  $z_{1\_B} = 1 + j1.22$

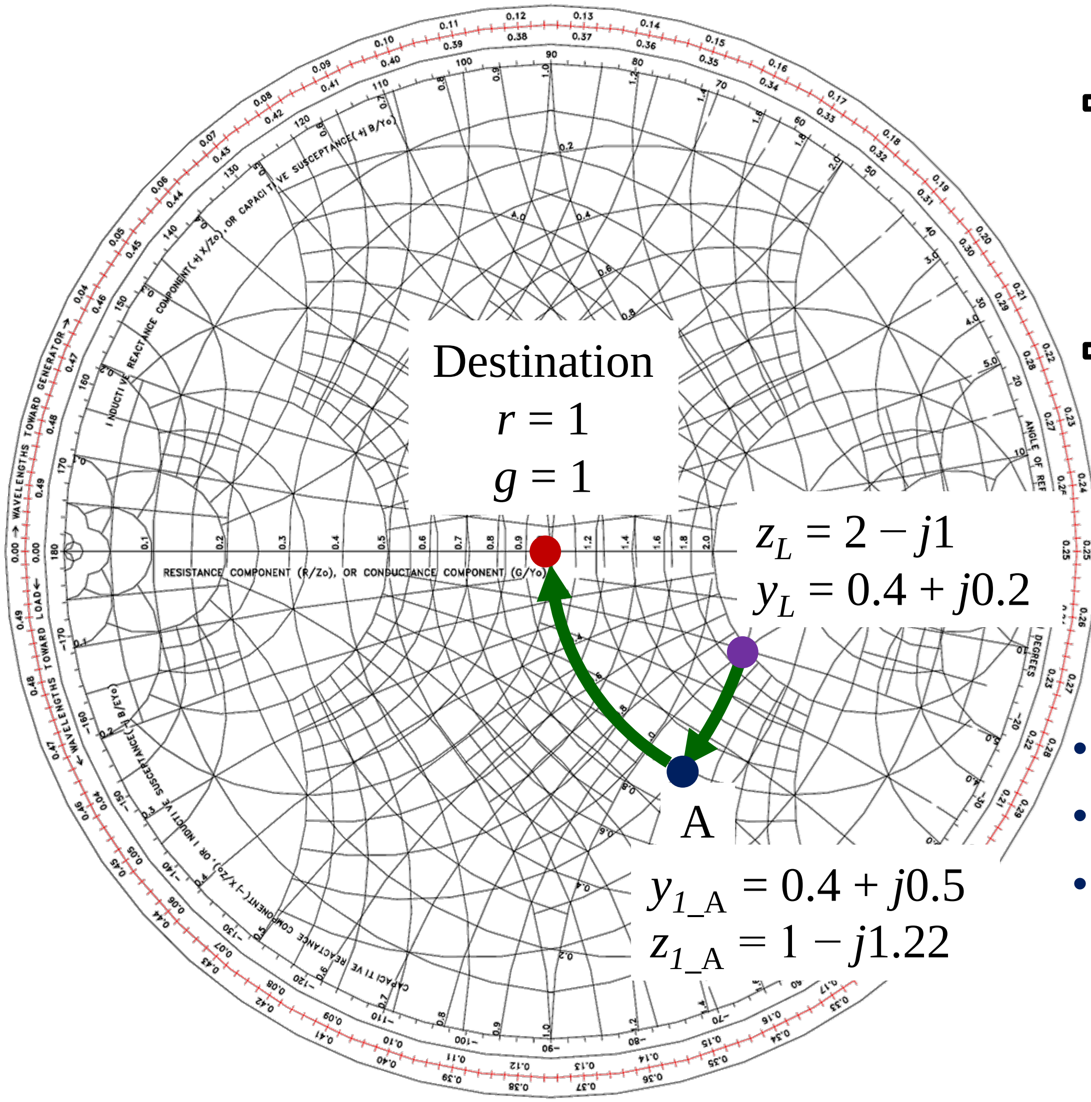
- If we connect a C in series, it brings us to the destination: r =1
- How to find the corresponding inductance?

$$-j\frac{1}{\omega C Z_0} = (1) - (1 + j1.22) = -j1.22$$

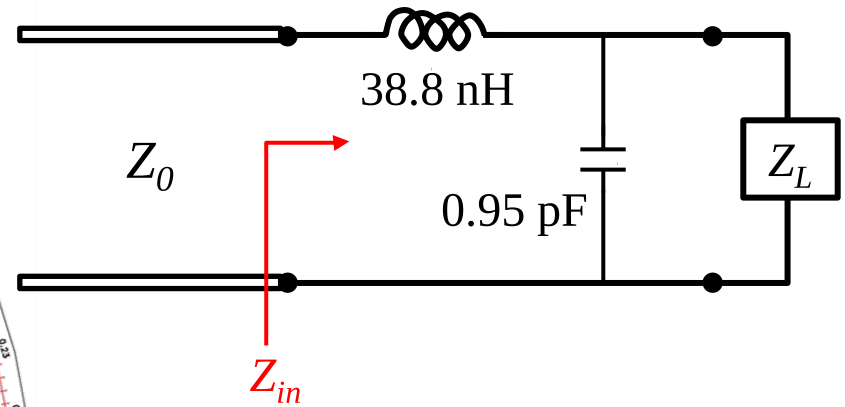
$$C = \frac{1}{\omega \times 1.22 \times Z_0} = \frac{1}{2\pi \times 500 \times 10^6 \times 1.22 \times 100}$$

$$= 2.61 \times 10^{-12}\ \mathrm{F} = 2.61\ \mathrm{pF}$$

Smith Chart Solutions (8/9)



Summary of Solution 1:

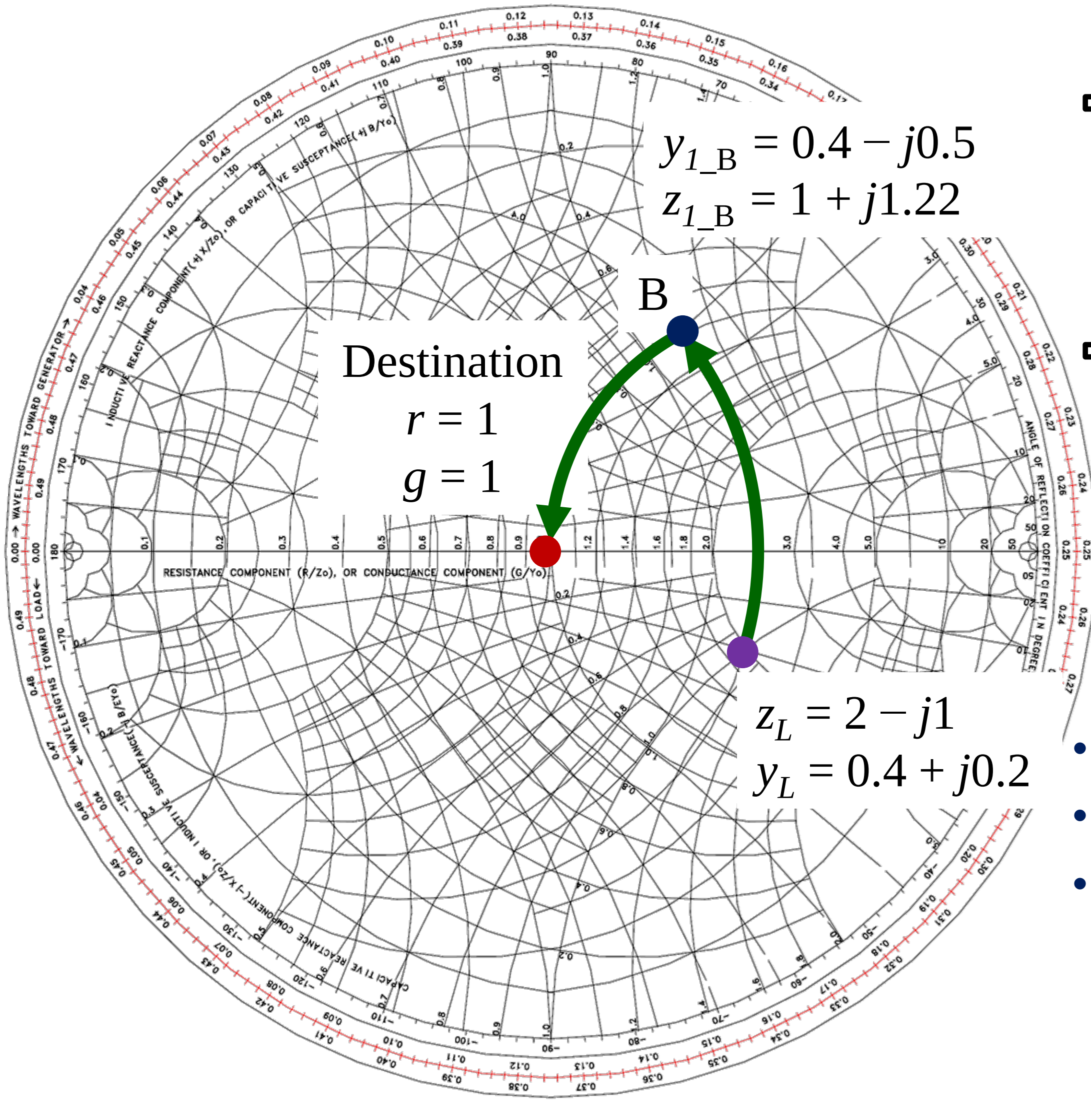


$$Z_{in} = \left[Z_L \parallel \left(-j \frac{1}{2\pi \times f \times 0.95 \times 10^{-12}} \right) \right] + j2\pi \times f \times 38.8 \times 10^{-9}$$

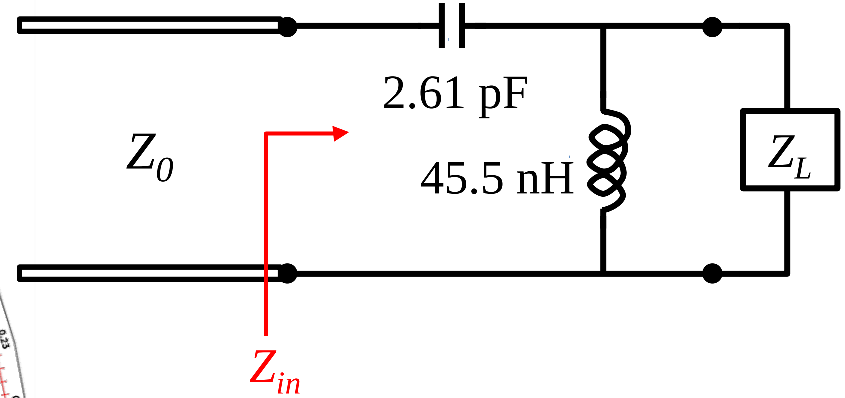
- Z_{in} is the function of frequency
- At 500 MHz: Z_{in} = 100 Ω
- Other than 500 MHz, the magnitude of reflection coefficient can be calculated by:

$$|\Gamma| = \left| \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right|$$

Smith Chart Solutions (9/9)



Summary of Solution 2:

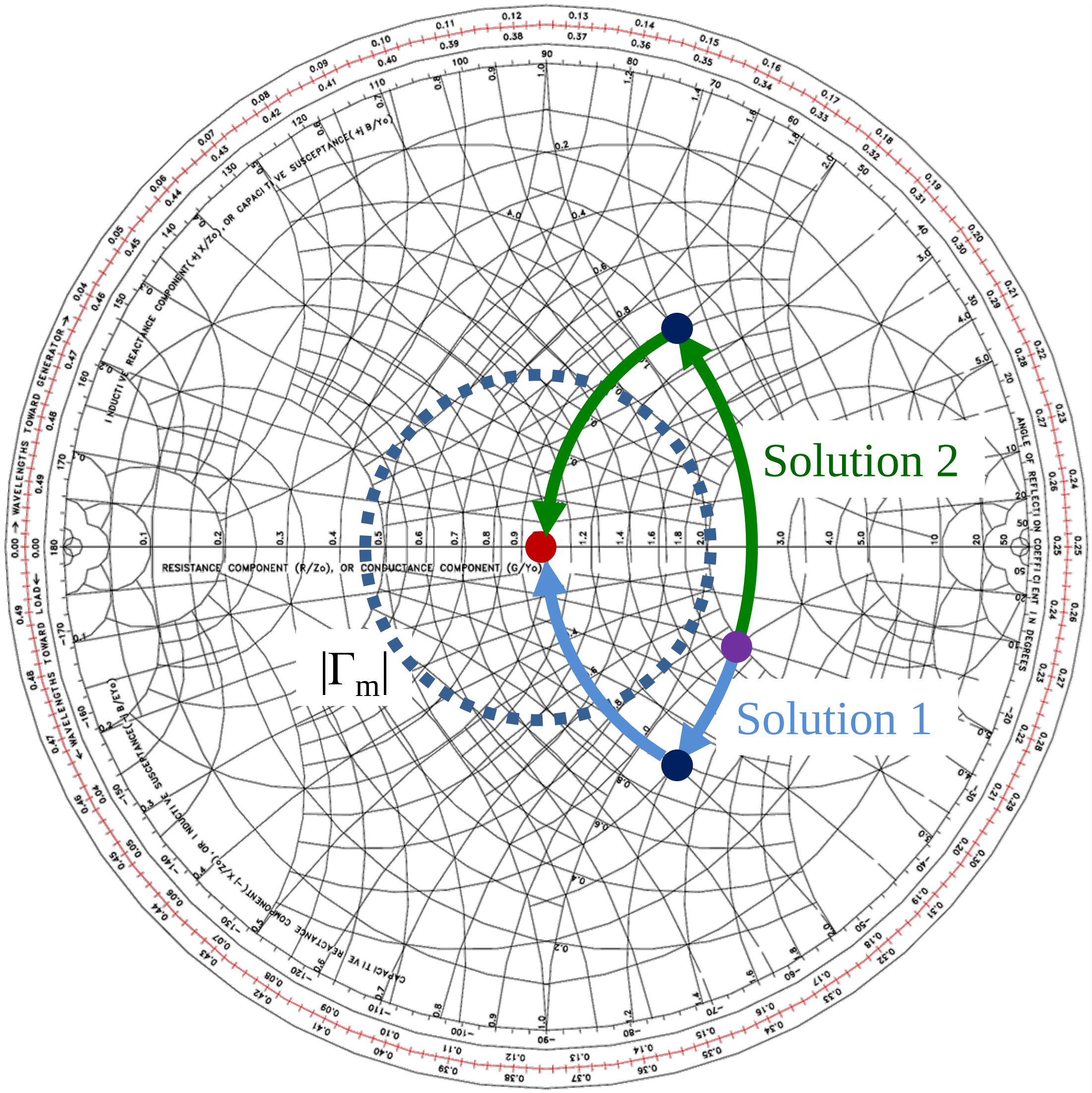


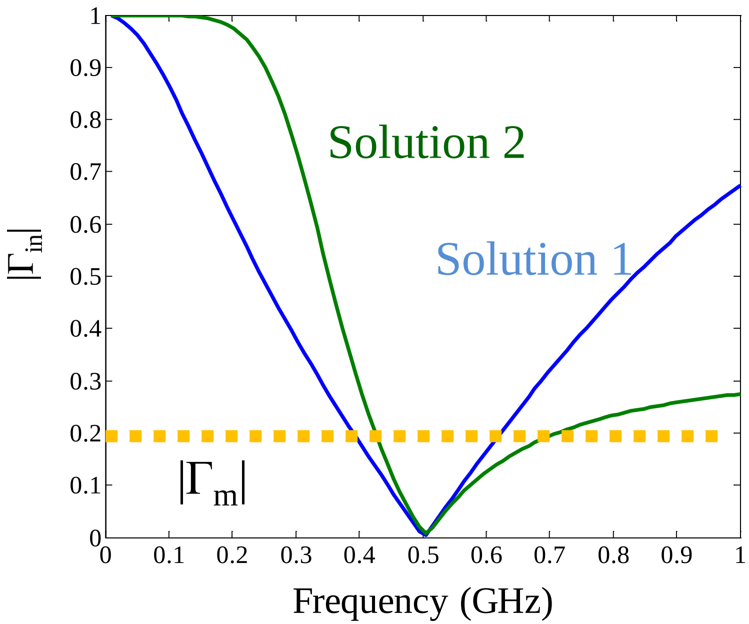
$$Z_{in} = \left[Z_L \parallel \left(j2\pi \times f \times 45.5 \times 10^{-9} \right) \right] - j\frac{1}{2\pi \times f \times 2.61 \times 10^{-12}}$$

- Z_{in} is the function of frequency
- At 500 MHz: Z_{in} = 100 Ω
- Other than 500 MHz, the magnitude of reflection coefficient can be calculated by:

$$|\Gamma| = \left| \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right|$$

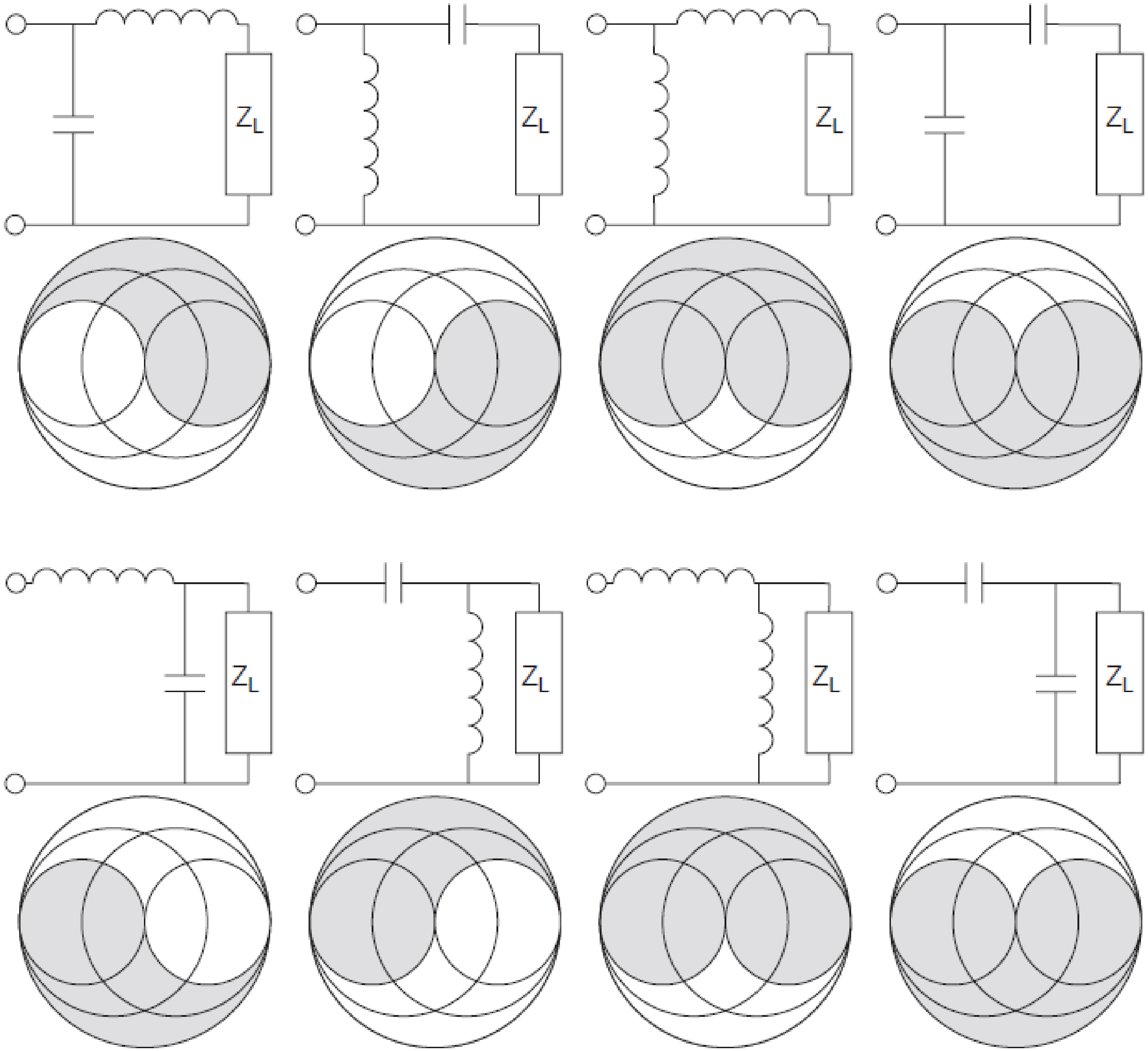
Which One Has Better Bandwidth?





- Z_L is modeled as a 200 Ω resistor and a 3.18 pF capacitor in series
- No big differences among the 2 solutions
- For a fixed Z_L and Z_0, once the L-section framework is chosen, the bandwidth is fixed uniquely

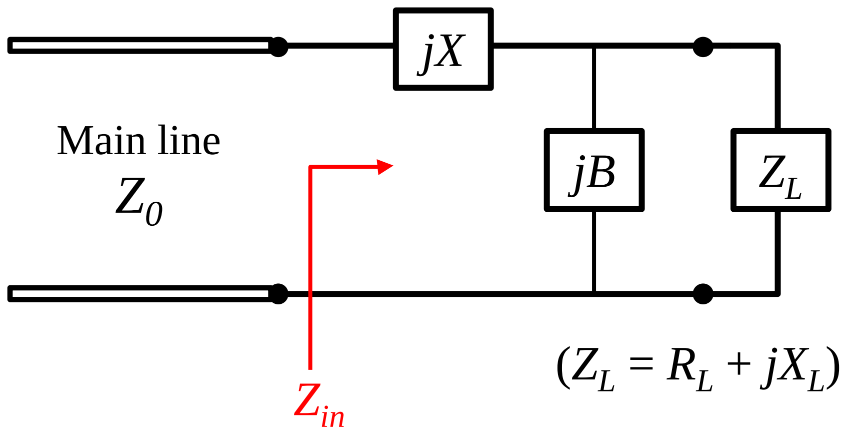
Dynamic Ranges of L-Shape Network



- **Dynamic range**: the range of the load impedance that can be matched to Z_0 by this circuit
- Each of the **white area** is the dynamic range of the associated L-shape network

Derivation of Analytic Solutions (1/5)

Case 1: The matching circuit used for $R_L > Z_0$



- The expression of Z_{in}:

$$Z_{in} = Z_L \parallel \frac{1}{jB} + jX = \frac{Z_L \dfrac{1}{jB}}{Z_L + \dfrac{1}{jB}} + jX = \frac{\left(R_L + jX_L\right)\left(-j\dfrac{1}{B}\right)}{R_L + jX_L - j\dfrac{1}{B}} + jX$$

$$= \frac{X + X_L - BXX_L + j\left(BR_L X - R_L\right)}{BR_L + j\left(BX_L - 1\right)}$$

Derivation of Analytic Solutions (2/5)

- Z_{in} must be equal to Z_0; so, the real part of Z_{in} must be Z_0 and its imaginary part must be 0

$$\begin{cases} \text{Real part:} & B\left(XR_L - X_L Z_0\right) = R_L - Z_0 \\ \text{Imaginary part:} & X\left(1 - BX_L\right) = BZ_0 R_L - X_L \end{cases}$$

- After a tedious computation, B and X are solved:

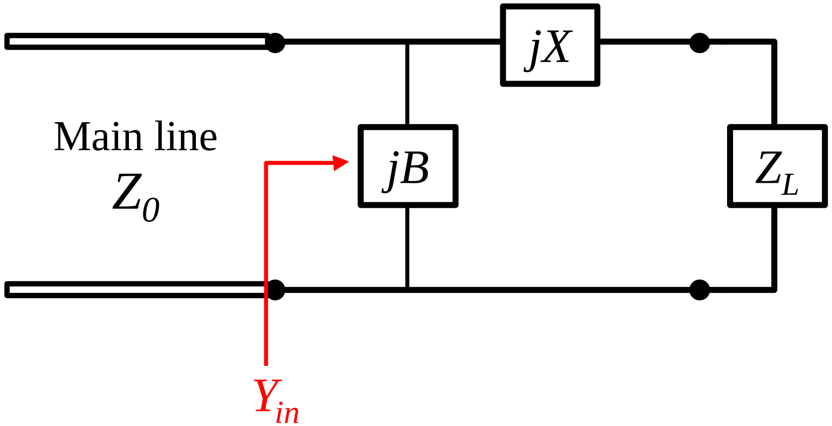
$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}\left(R_L^2 + X_L^2 - Z_0 R_L\right)}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L}$$

- $\because R_L > Z_0$, $\therefore R_L^2 + Z_L^2 - Z_0 R_L > 0$
- B and X must have two real solutions
- Smith chart solutions reach the same conclusion

Derivation of Analytic Solutions (3/5)

Case 2: The matching circuit used for $R_L < Z_0$



- The expression of Y_{in}:

$$Y_{in} = jB + \frac{1}{R_L + j\left(X + X_L\right)} = \frac{jB\left[R_L + j\left(X + X_L\right)\right] + 1}{R_L + j\left(X + X_L\right)}$$

$$= \frac{-B\left(X + X_L\right) + jBR_L}{R_L + j\left(X + X_L\right)}$$

Derivation of Analytic Solutions (4/5)

- Y_{in} must be equal to Y_0; so, the real part of Y_{in} must be Y_0 and its imaginary part must be 0

$$\begin{cases} \text{Real part:} & BZ_0\left(X + X_L\right) = Z_0 - R_L \\ \text{Imaginary part:} & X + X_L = BZ_0R_L \end{cases}$$

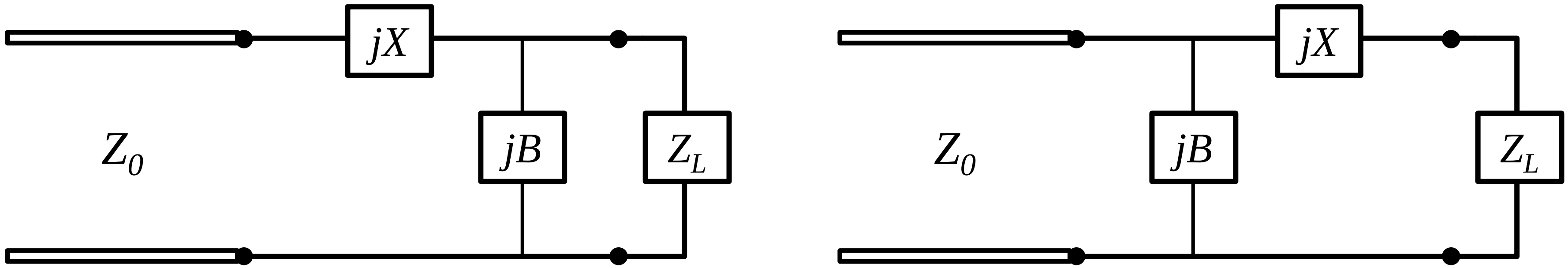
- Solving B and X gives:

$$X = \pm\sqrt{R_L\left(Z_0 - R_L\right)} - X_L$$

$$B = \frac{X + X_L}{Z_0R_L}$$

- $\because R_L < Z_0, \ \therefore Z_0 - R_L > 0$
- B and X must have two real solutions
- Smith chart solutions reach the same conclusion

Derivation of Analytic Solutions (5/5)



How to select a series element jX?

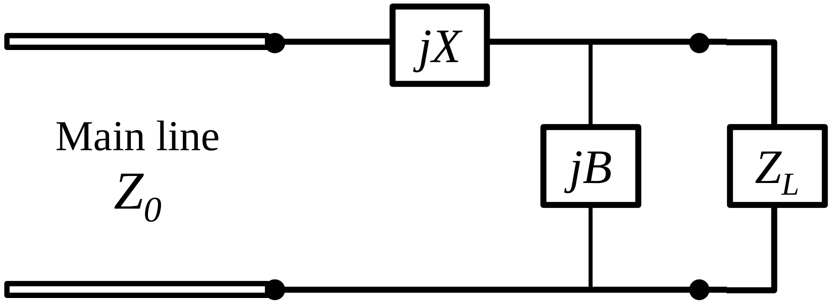
1. $X > 0$: Use L as a solution so that $Z = jX = j\omega L$
2. $X < 0$: Use C as a solution so that $Z = jX = -j/\omega C$

How to select a shunt element jB?

1. $B > 0$: Use C as a solution so that $Y = jB = j\omega C$
2. $B < 0$: Use L as a solution so that $Y = jB = -j/\omega L$

EX 3.4 Analytic Solutions (1/2)

1. Design an L-section matching circuit to match $Z_L = 200 - j100\ \Omega$ to a 100 Ω line (Operational frequency: 500 MHz)



$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}\left(R_L^2 + X_L^2 - Z_0 R_L\right)}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$

Analytic Solutions (2/2)

- Since $R_L > Z_0$, choose the first case for the L-section matching circuit:

$$B=\frac{-100\pm\sqrt{\frac{200}{100}\left(200^2+(-100)^2-100\times200\right)}}{200^2+(-100)^2}=\begin{cases}2.89\times10^{-3}\ \mathrm{S}\\ -6.9\times10^{-3}\ \mathrm{S}\end{cases}$$

$$X=\frac{1}{B}+\frac{(-100)\times100}{200}-\frac{100}{B\times200}=\begin{cases}122\ \Omega\\ -122\ \Omega\end{cases}$$

- Solution 1: $C=\frac{B}{\omega}=\frac{2.89\times10^{-3}}{2\pi\times500\times10^{6}}=9.2\times10^{-13}\ \mathrm{F}=0.92\ \mathrm{pF}$

$$L=\frac{X}{\omega}=\frac{122}{2\pi\times500\times10^{6}}=3.88\times10^{-8}\ \mathrm{H}=38.8\ \mathrm{nH}$$

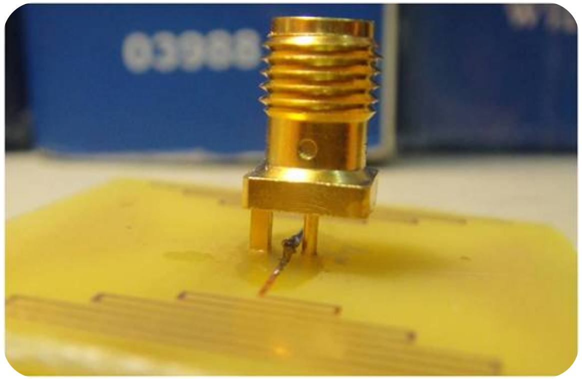
- Solution 2: $L=\frac{-1}{\omega B}=\frac{-1}{2\pi\times500\times10^{6}\times\left(-6.9\times10^{-3}\right)}=4.61\times10^{-8}\ \mathrm{H}=46.1\ \mathrm{nH}$

$$C=\frac{-1}{\omega X}=\frac{-1}{2\pi\times500\times10^{6}\times(-122)}=2.61\times10^{-12}\ \mathrm{F}=2.61\ \mathrm{pF}$$

Characteristics of L-Shape Circuits

Advantage:

- Lumped elements have smaller size
- Lumped elements provide better adjustability
- They provide wider bandwidth

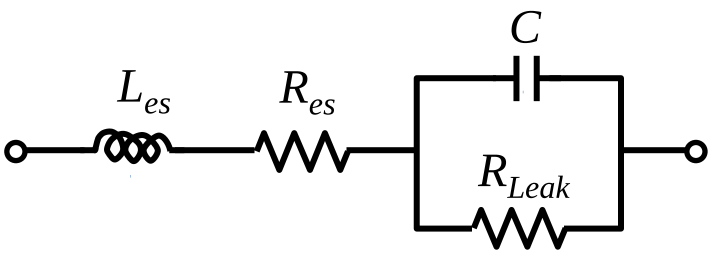


Limitation:

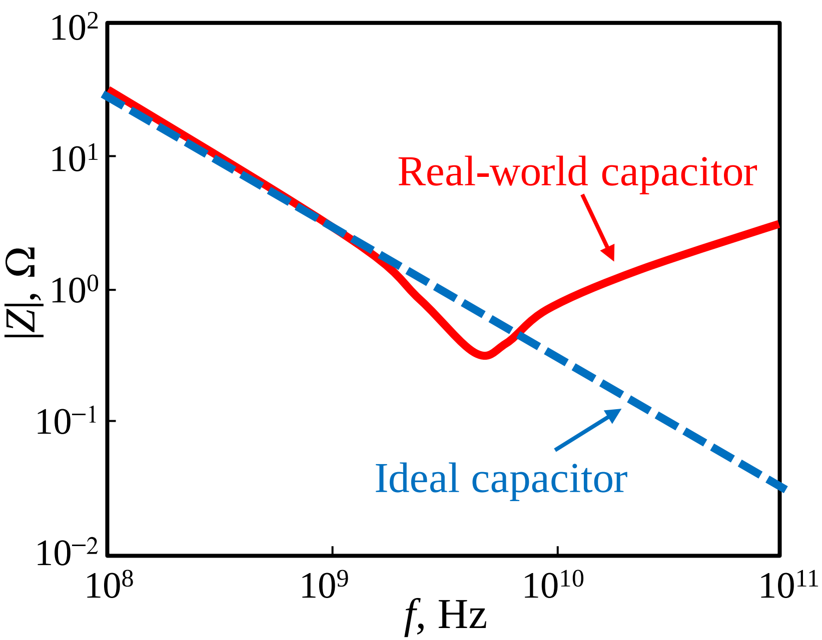
- Lumped elements may not be realizable at higher frequencies
- The values of inductance and capacitance are discrete; they can be chosen over only a limited ranges of values

Real-World Capacitors

The equivalent circuit of a real-world capacitor:



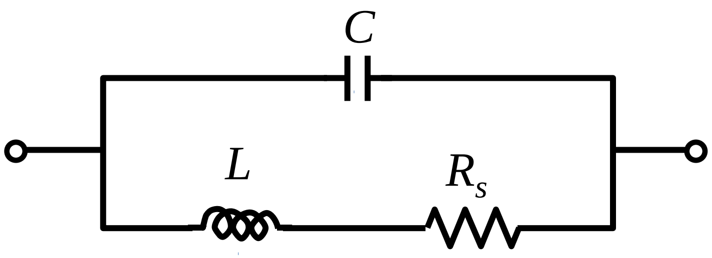
Frequency response in log scale:



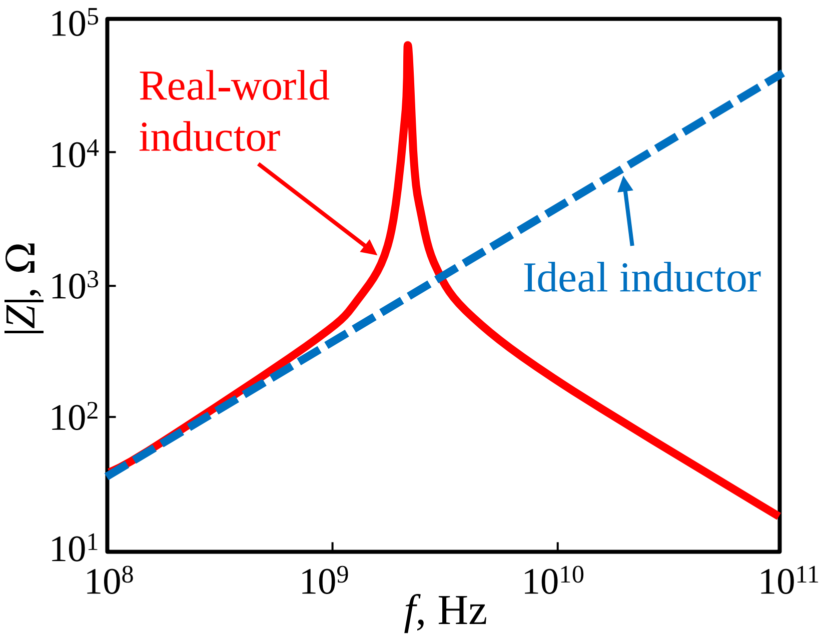
	Source and details
Equivalent series resistance R_{es}	• It comes from the connections of wires and the plate • It produces heat
Equivalent series inductance L_{es}	• It determines when the capacitor acts like an inductor • The value of L_{es} depends on the package type
Leakage resistance R_{leak}	• It results from the dielectric loss

Real-World Inductors

The equivalent circuit of a real-world inductor:



Frequency response in log scale:

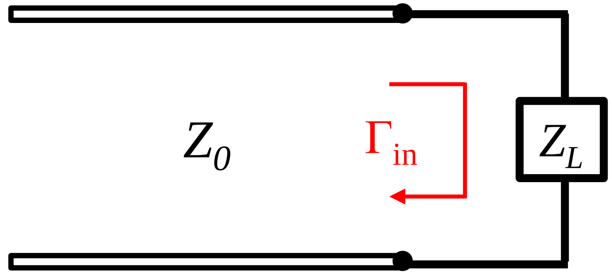


	Source and details
Equivalent series resistance R_s	• It comes from the loss of conductor
Equivalent shunt capacitance C	• It results from the effect of winding • The value is determined by the method of construction of the component • It determines the behavior of the component in high frequencies

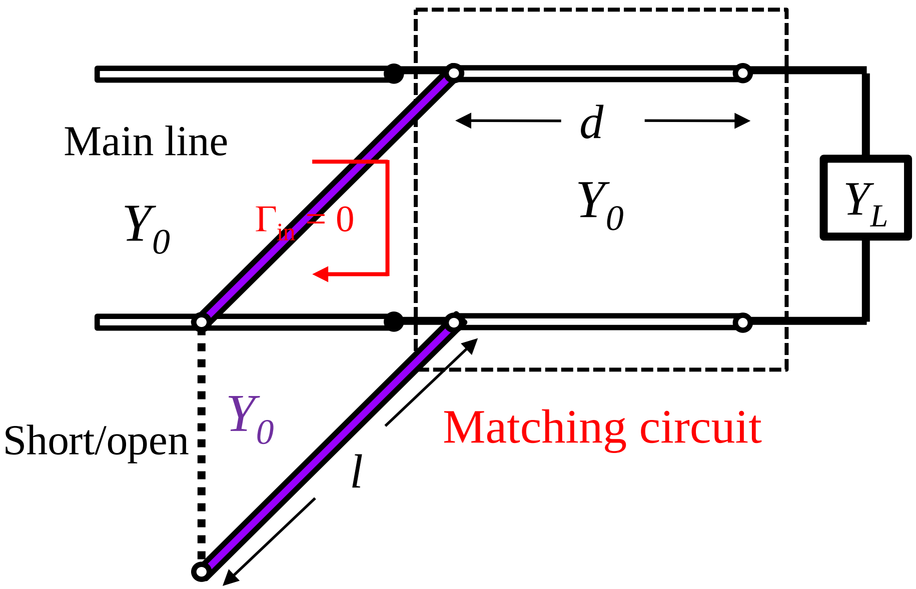
Contents

3.4 Matching with TL Stubs

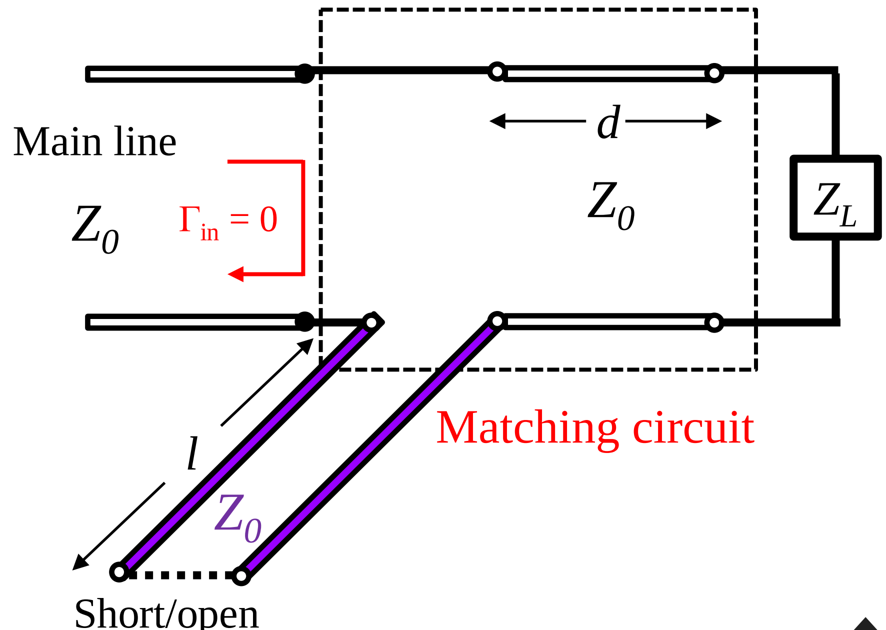
Matching with Single Stub



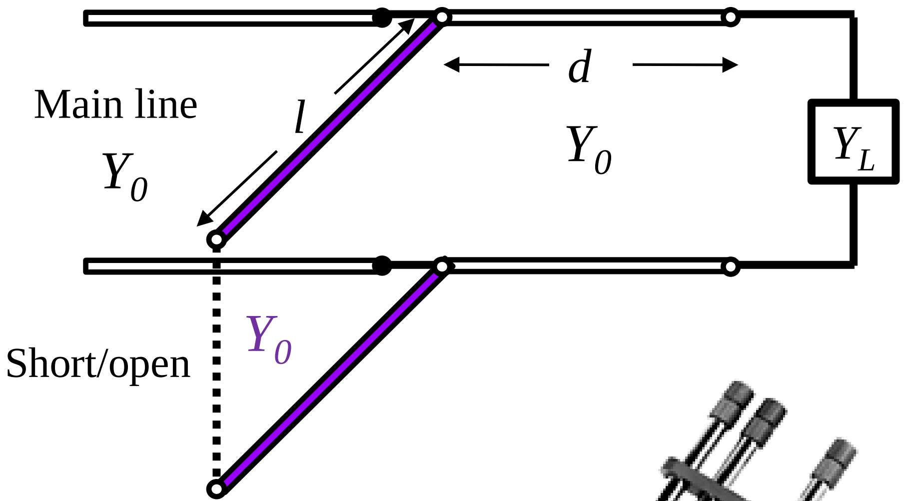
Shunt stub matching



Series stub matching



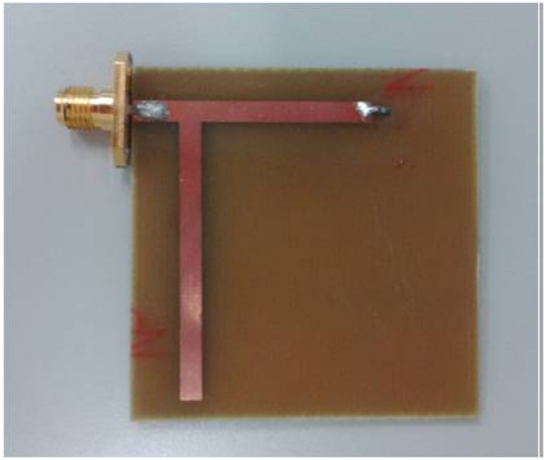
Shunt Stubs



$$Z_{short} = jZ_0 \tan \beta l$$

$$Z_{open} = -jZ_0 \cot \beta l$$

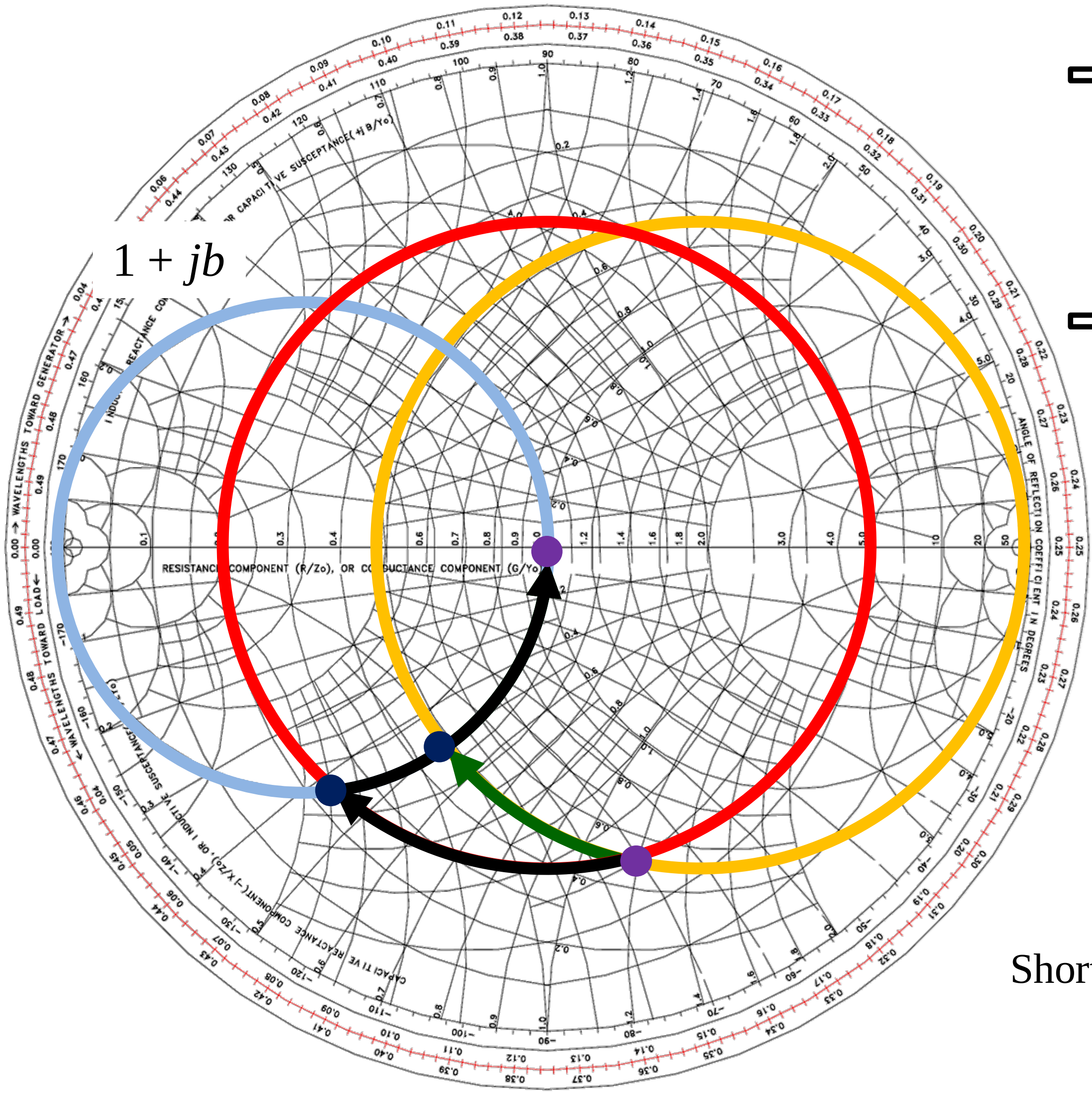
- Parameter: the distance d and the stub length l
- Shunt stubs are more popular than series stubs because of less electrical connection problems

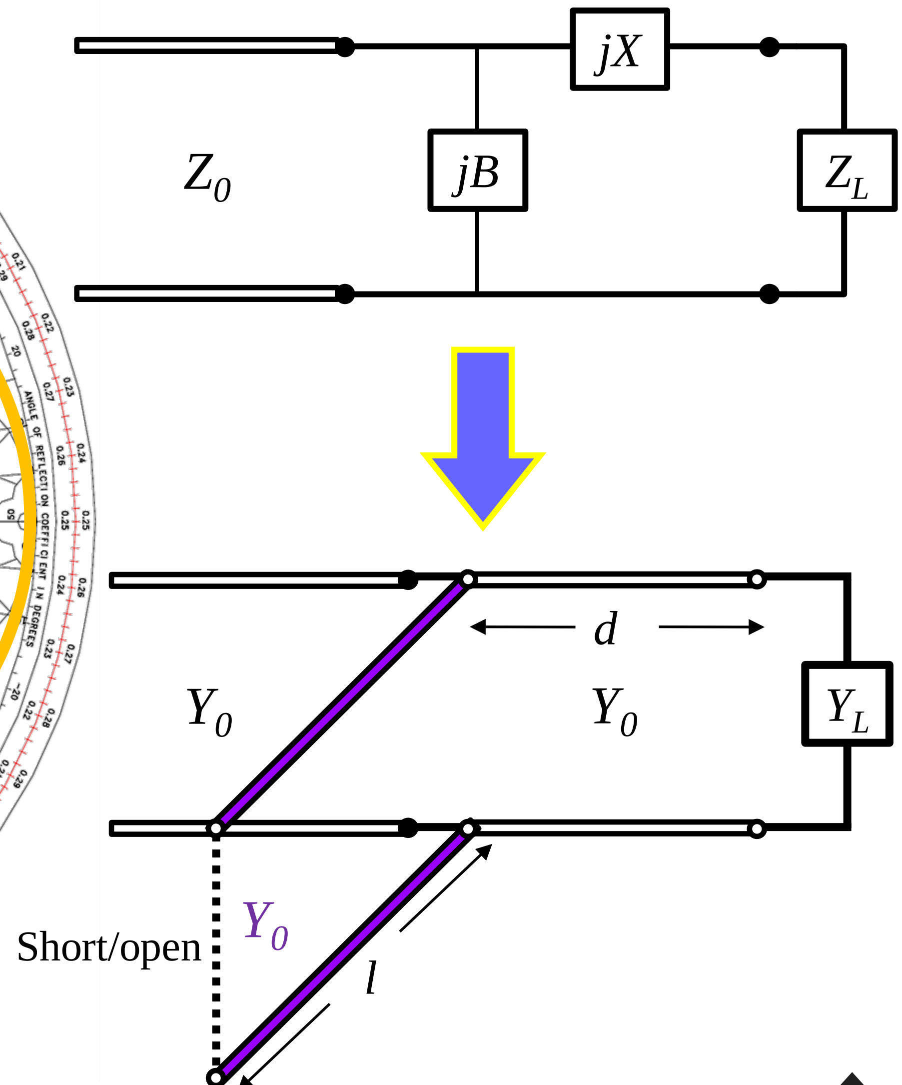


- For stubs using coaxial cable:
  - short-circuited stubs are more popular than the open-circuited stubs because of less radiation
- For stubs using microstrip lines:
  - Open-circuited stubs are easier to fabricate than the short-circuited stubs

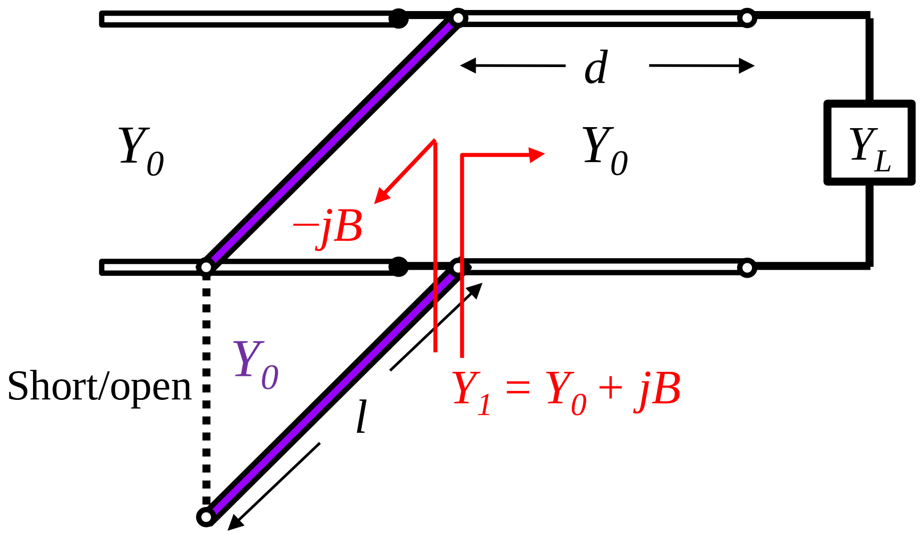
HIS Global Spec. [Online] Available: http://www.globalspec.com/

From LC to Single-Stub Matching Circuit





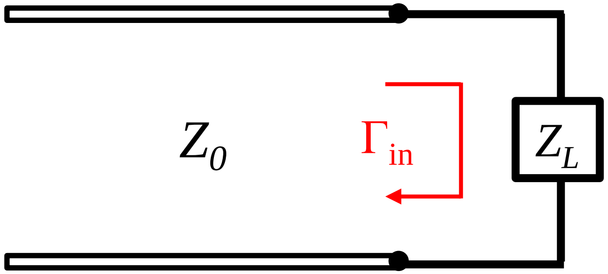
Design Idea of Shunt Stubs



1. To select d so that $Y_1 = Y_0 + jB$
2. To select l so that the susceptance of the shunt stub is $-jB$

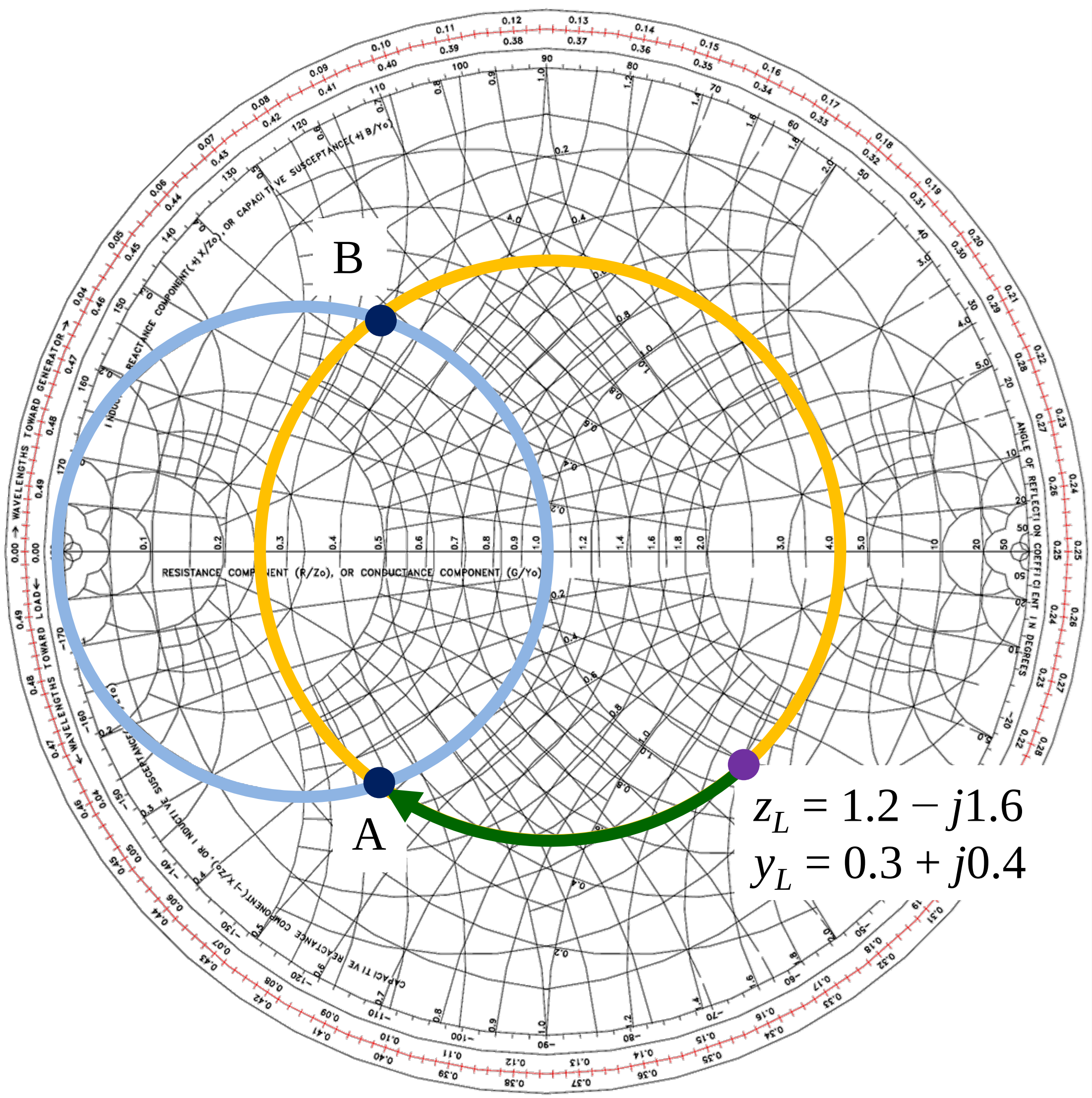
The matching condition is achieved!

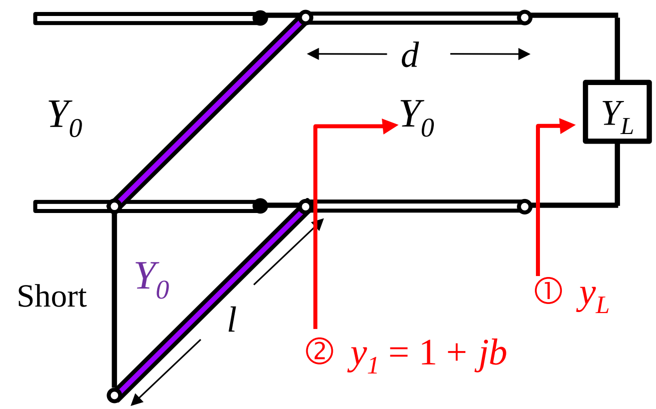
Smith Chart Solutions of Shunt-Stub Matching (1/8)



1. For a load impedance $Z_L = 60 - j80\ \Omega$, design 2 single-stub (short-circuit) shunt matching circuits to match this load to a $Z_0 = 50\ \Omega$ line (Operational frequency: 2 GHz)
2. If Z_L is composed of a resistor and a capacitor in series, plot the reflection coefficient magnitude from 1 GHz to 3 GHz for each solution

Smith Chart Solutions of Shunt-Stub Matching (2/8)





Step 1:

- The normalized impedance:

$$z_L = 1.2 - j1.6$$

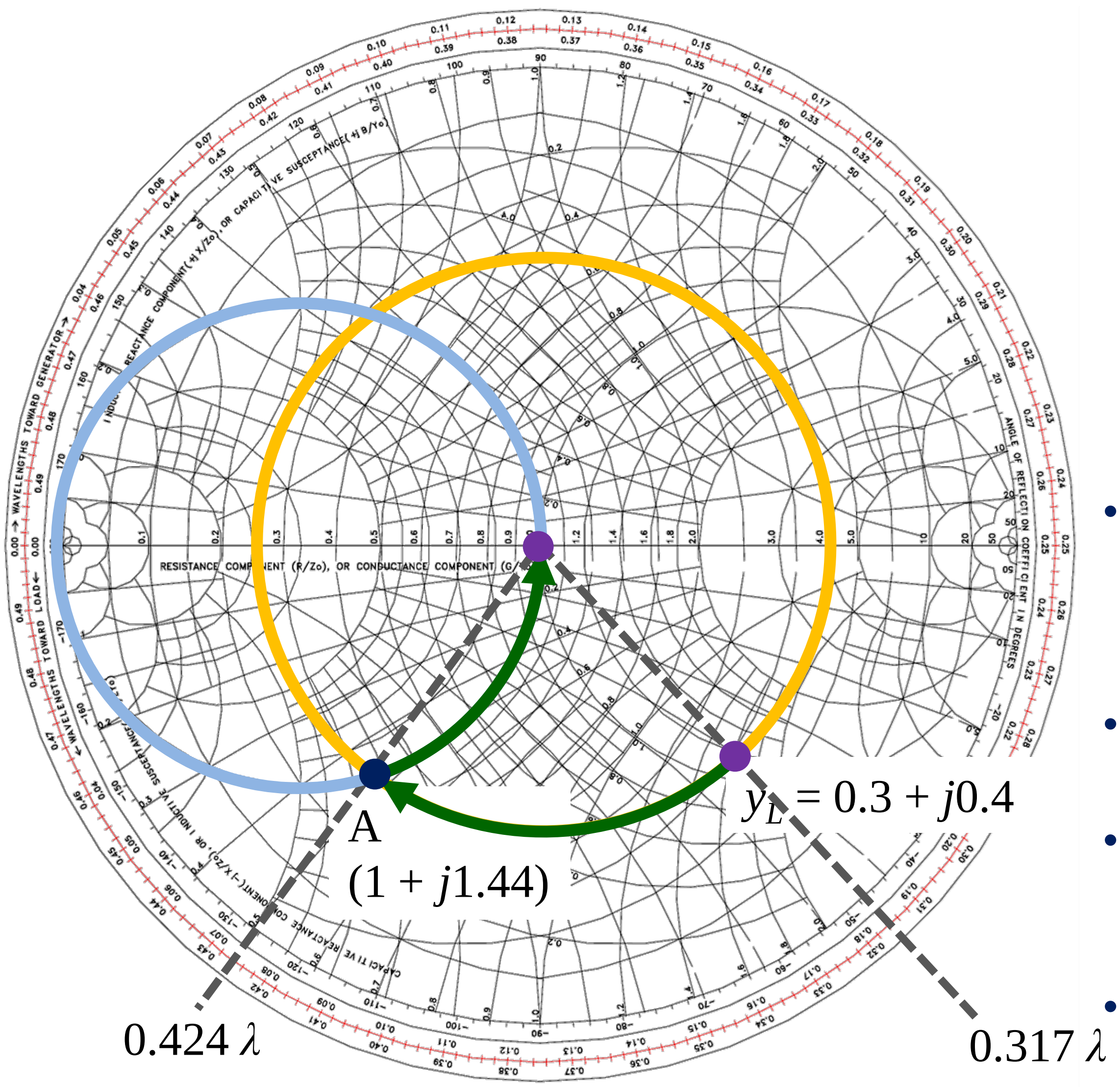
- Read its normalized admittance:

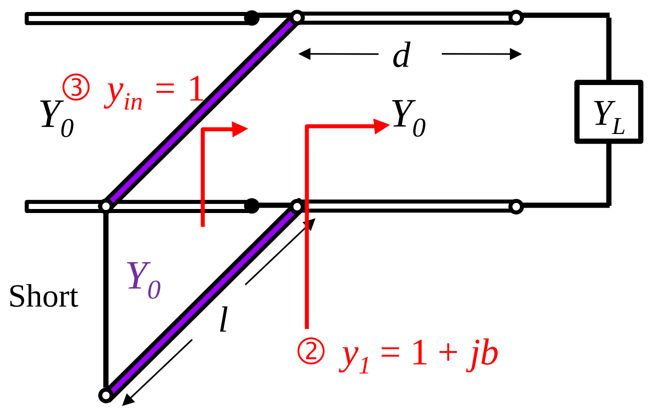
$$y_L = 0.3 + j0.4$$

Step 2:

- Connecting y_L to a transmission line: rotating y_L along the constant $|\Gamma|$ circle
- Where do we stop?

EX 3.5 Smith Chart Solutions of Shunt-Stub Matching (3/8)





Solution 1 (for point A):

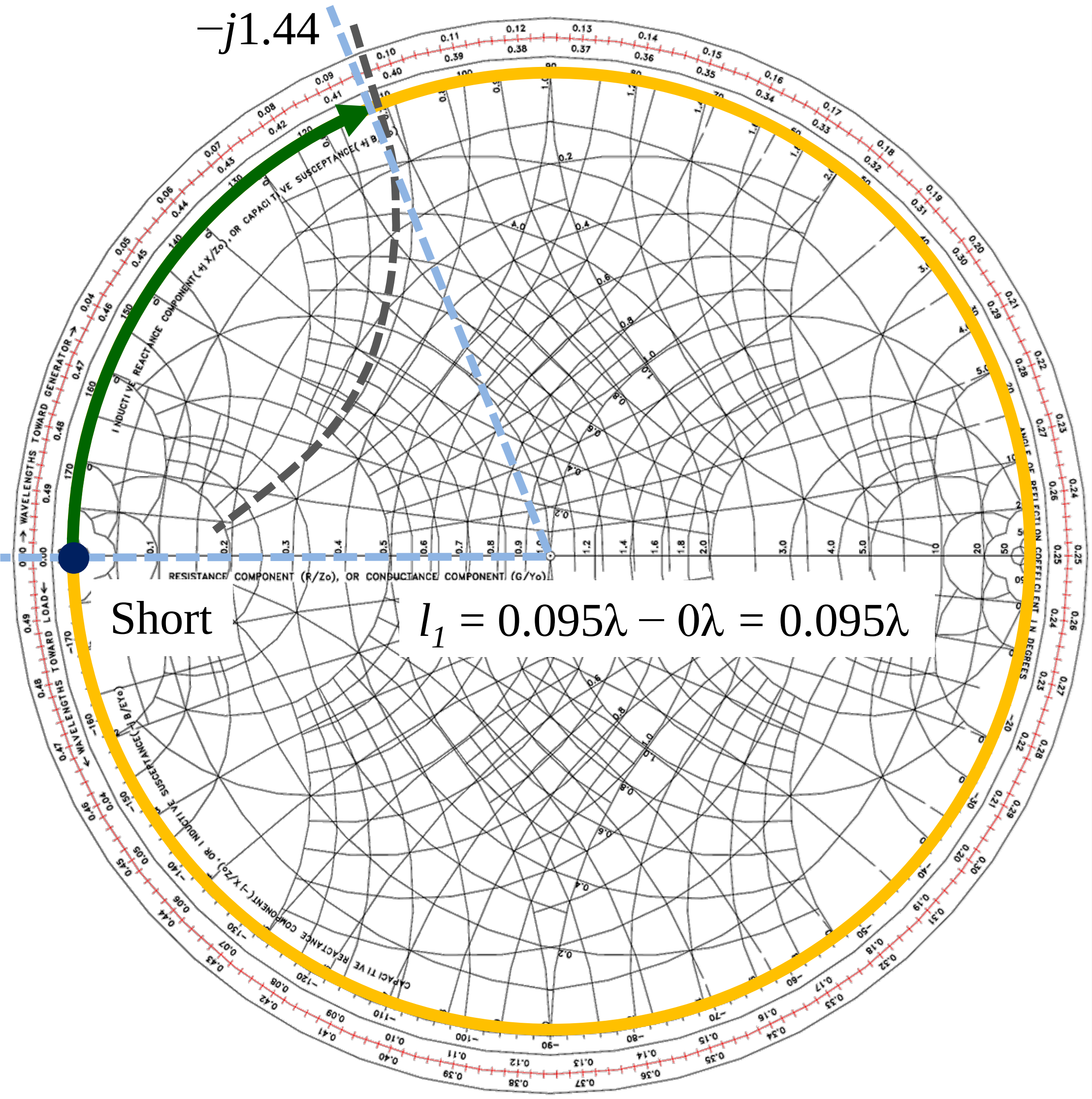
- How to rotate y_L to A $(1 + j1.44)$?

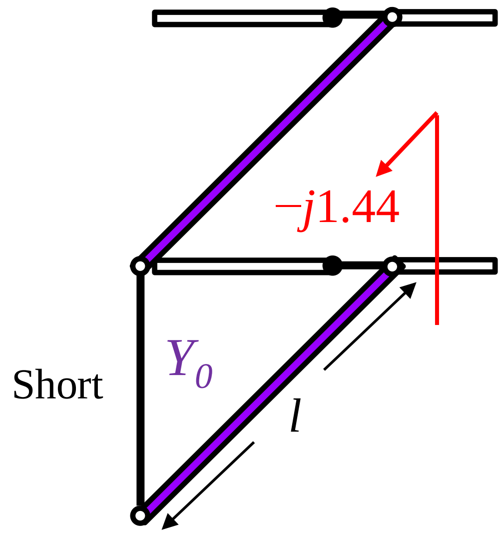
$$d_1 = 0.424\lambda - 0.317\lambda = 0.107\lambda$$

Step 3:

- Using a shunt stub to eliminate the susceptance $j1.44$
- Which way to take the turn? Counterclockwise or clockwise? Counterclockwise! (But why?)
- The shorted stub should provide a susceptance of $-j1.44$

Smith Chart Solutions of Shunt-Stub Matching (4/8)

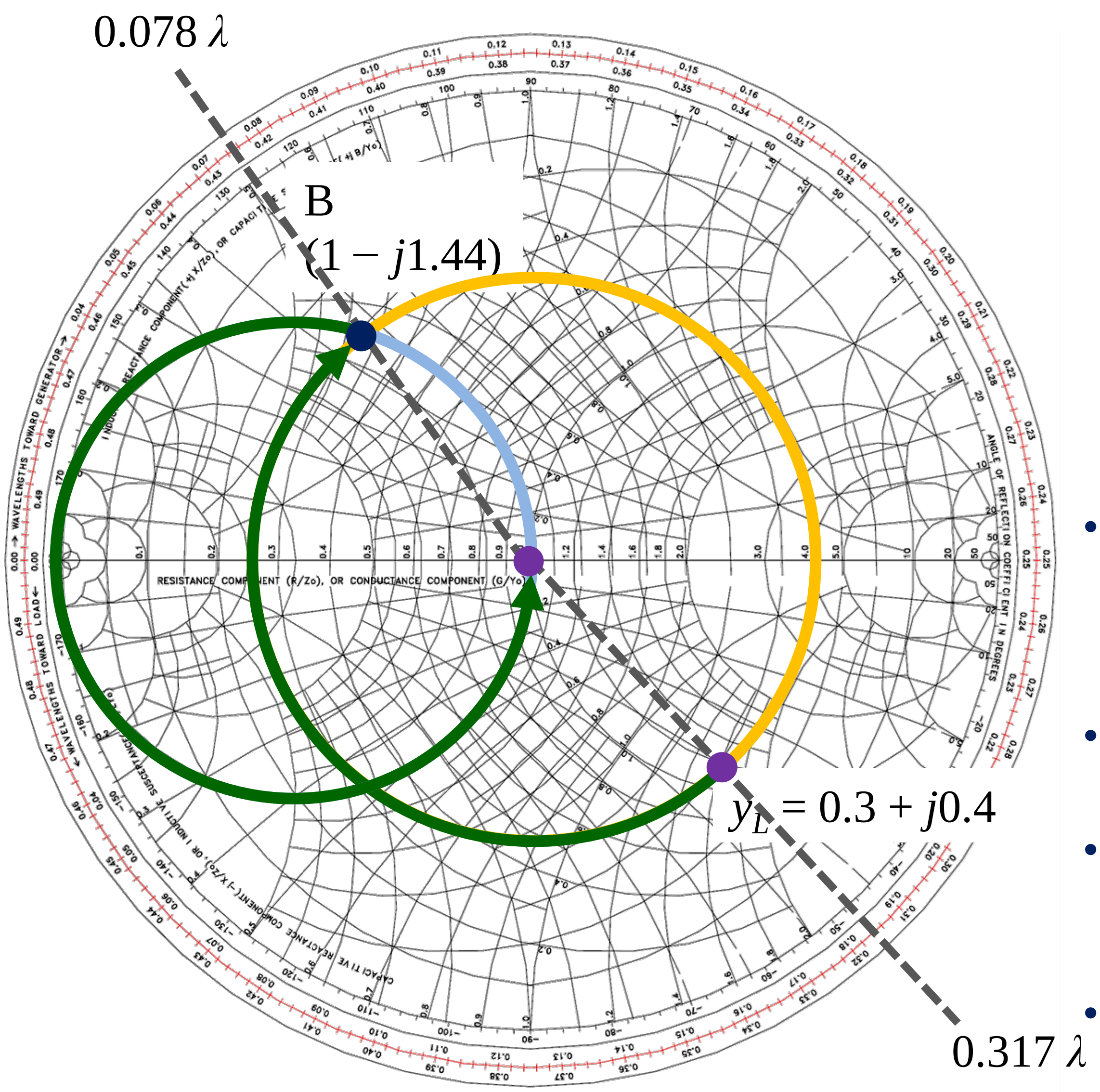


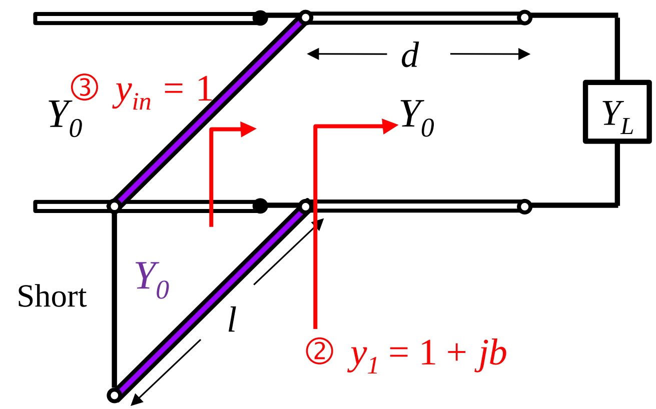


- What's the associated length l to make a $-j1.44$ susceptance?
- The input admittance of y_L: moving the point along its constant $|\Gamma|$ circle
- Read the distance that we have moved:

$$l_1 = 0.095\lambda - 0 = 0.095\lambda$$

Smith Chart Solutions of Shunt-Stub Matching (5/8)





Solution 2 (for point B):

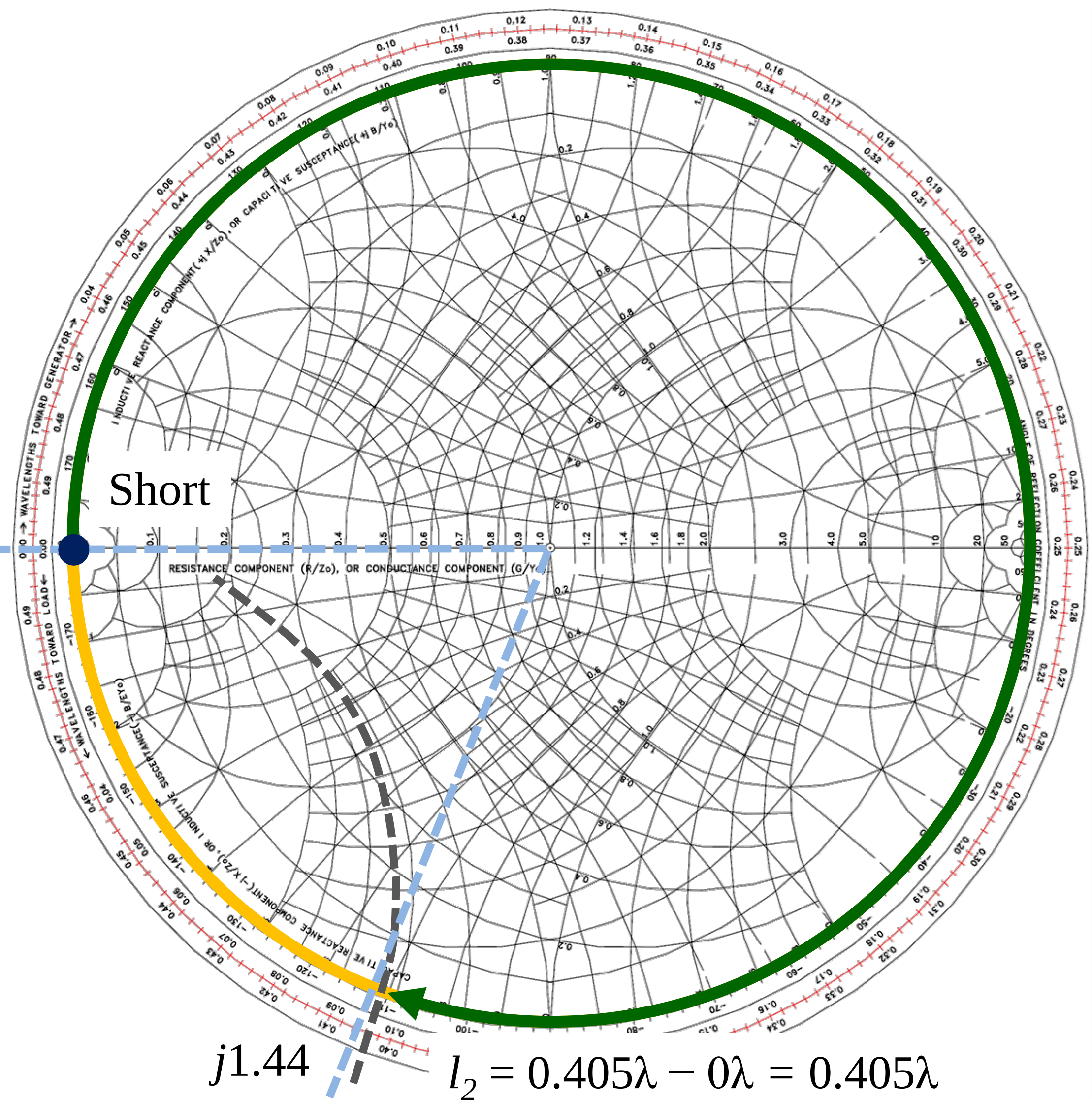
- How to rotate y_L to B $(1 - j1.44)$?

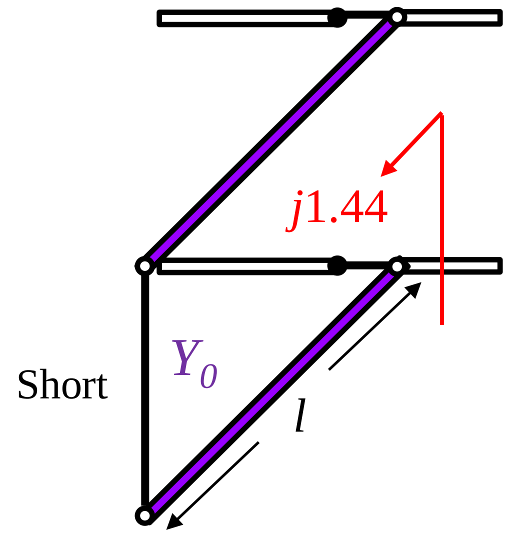
$$d_2 = 0.578\lambda - 0.317\lambda = 0.261\lambda$$

Step 3:

- Using a shunt stub to eliminate the susceptance $-j1.44$
- Which way to take the turn? Counterclockwise or clockwise? Counterclockwise! (But why?)
- The shorted stub should provide a susceptance of $j1.44$

Smith Chart Solutions of Shunt-Stub Matching (6/8)



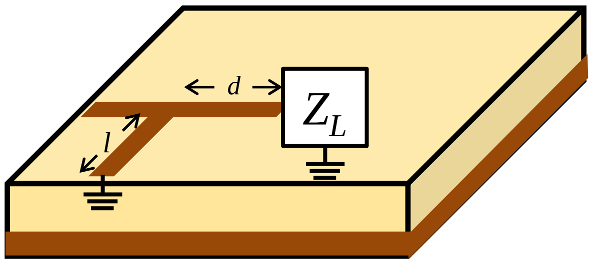


- What's the associated length l to make a $j1.44$ susceptance?
- The input admittance of y_L: moving the point along its constant $|\Gamma|$ circle
- Read the distance that we have moved:

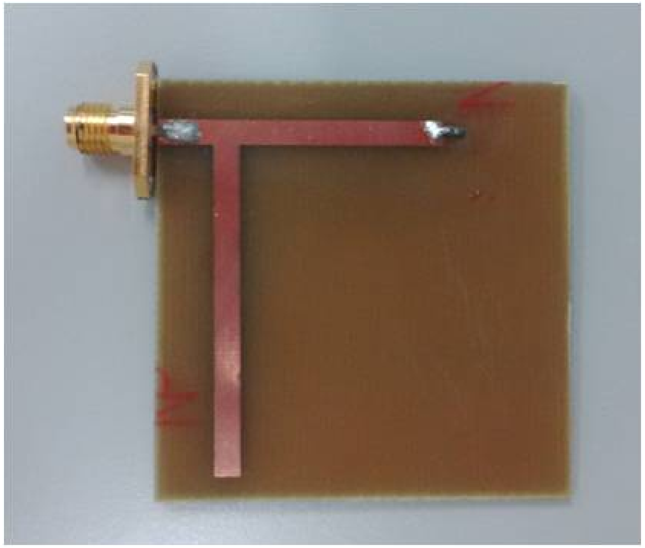
$$l_2 = 0.405\lambda - 0 = 0.405\lambda$$

Smith Chart Solutions of Shunt-Stub Matching (7/8)

Physical geometry:



(λ @ 2 GHz: 150 mm)



- Solution 1:

$$d_1 = 0.107\lambda = 16.05 \text{ mm}$$
$$l_1 = 0.095\lambda = 14.25 \text{ mm}$$

- Solution 2:

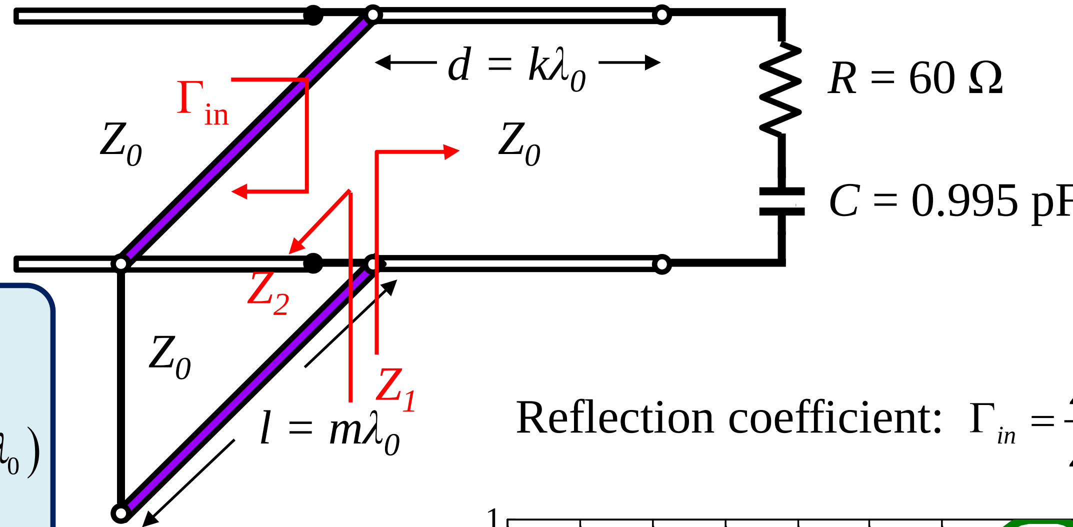
$$d_2 = 0.261\lambda = 39.15 \text{ mm}$$
$$l_2 = 0.405\lambda = 60.75 \text{ mm}$$

- In order to analyze the frequency response of the matching circuit, we model Z_L as series-RC with

$$R = 60\ \Omega$$
$$C = \frac{1}{80 \times 2\pi \times 2 \times 10^9} = 0.995 \text{ F}$$
$$(\text{because } X = -j80 = -j\frac{1}{\omega C})$$

Smith Chart Solutions of Shunt-Stub Matching (8/8)

(λ_0: associated wavelength of 2 GHz)



$$Z_2 = jZ_0 \tan \beta l$$

where $\tan \beta l = \tan\left(\frac{2\pi}{\lambda}\right)(m\lambda_0)$

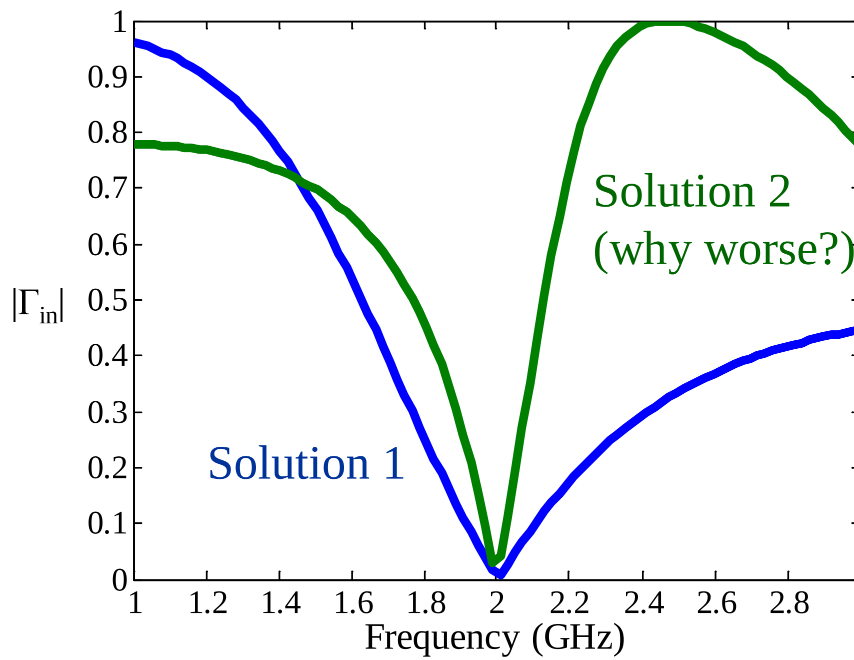
$$= \tan \frac{2\pi f \times m}{f_0}$$

$$Z_1 = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d} \right)$$

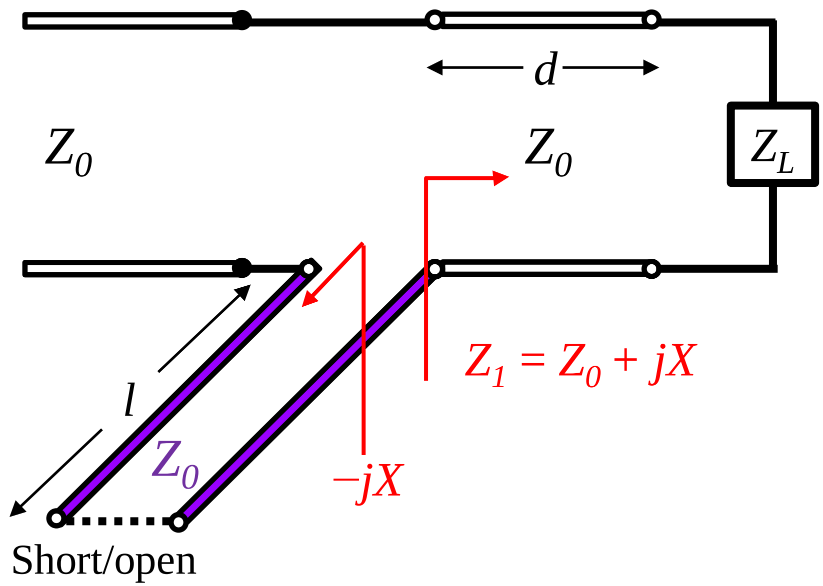
where $Z_L = 60 - j\dfrac{1}{2\pi \times f \times 0.995 \times 10^{-12}}$

$$\tan \beta d = \tan\left(\frac{2\pi}{\lambda}\right)(k\lambda_0) = \tan \frac{2\pi f \times k}{f_0}$$

Reflection coefficient: $\Gamma_{in} = \dfrac{Z_1 \parallel Z_2 - Z_0}{Z_1 \parallel Z_2 + Z_0}$



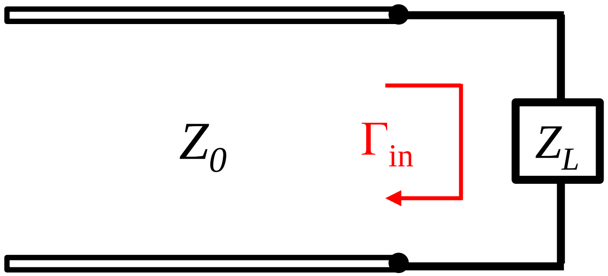
Design Idea of Series Stubs



1. To select d so that $Z_1 = Z_0 + jX$
2. To select l so that the reactance of the series stub is $-jX$

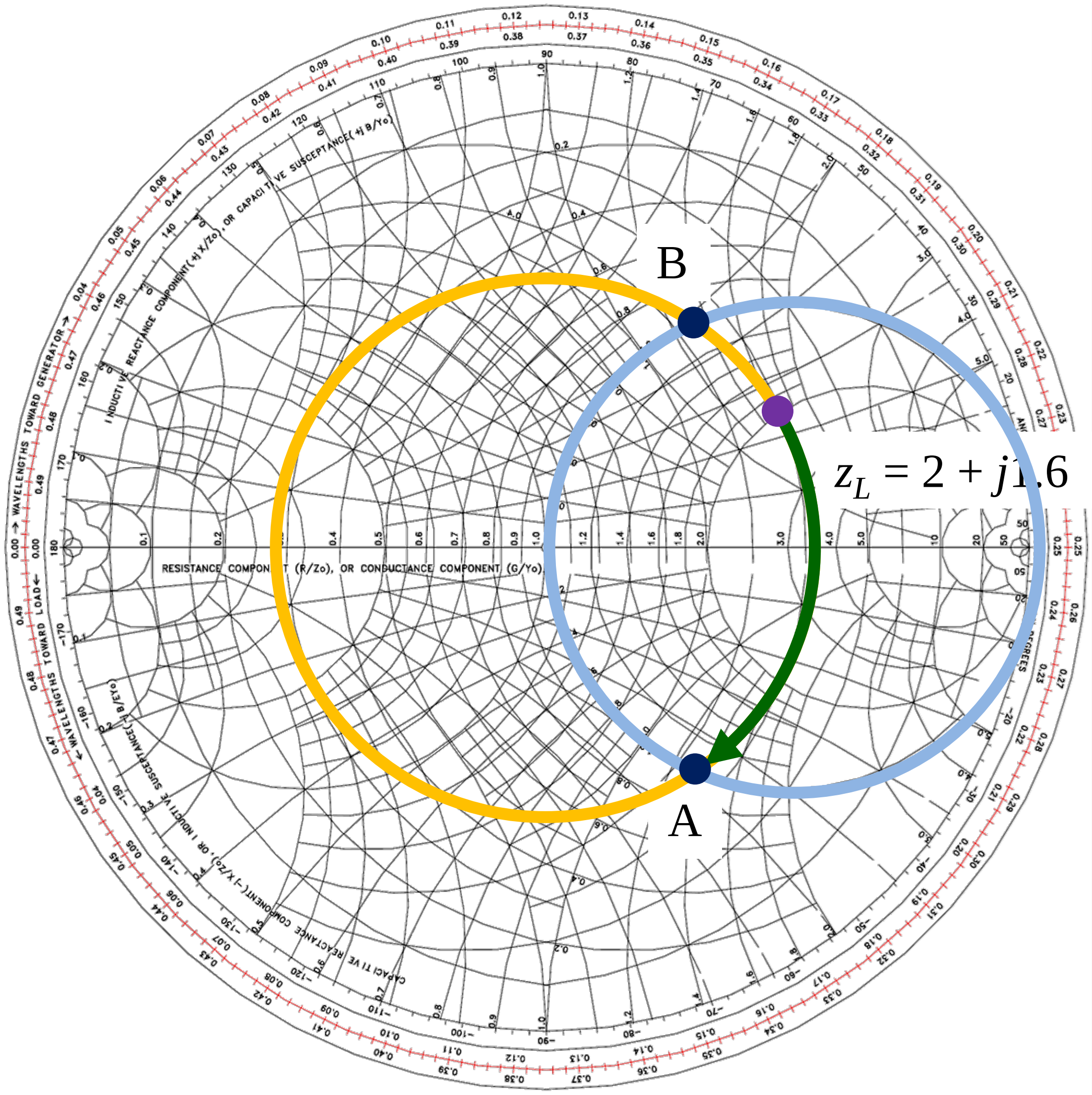
➡ The matching condition is achieved!

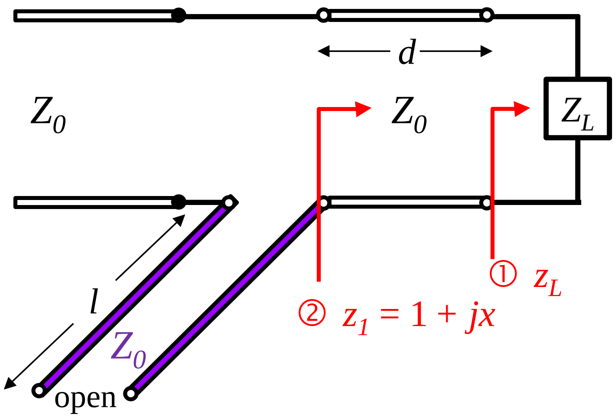
Smith Chart Solutions of Series-Stub Matching (1/7)



1. For a load impedance $Z_L = 100 + j80\ \Omega$, design 2 single-stub (open-circuit) series matching circuits to match this load to a $Z_0 = 50\ \Omega$ line (Operational frequency: 2 GHz)
2. If Z_L is composed of a resistor and a inductor in series, plot the reflection coefficient magnitude from 1 GHz to 3 GHz for each solution

Smith Chart Solutions of Series-Stub Matching (2/7)





Step 1:

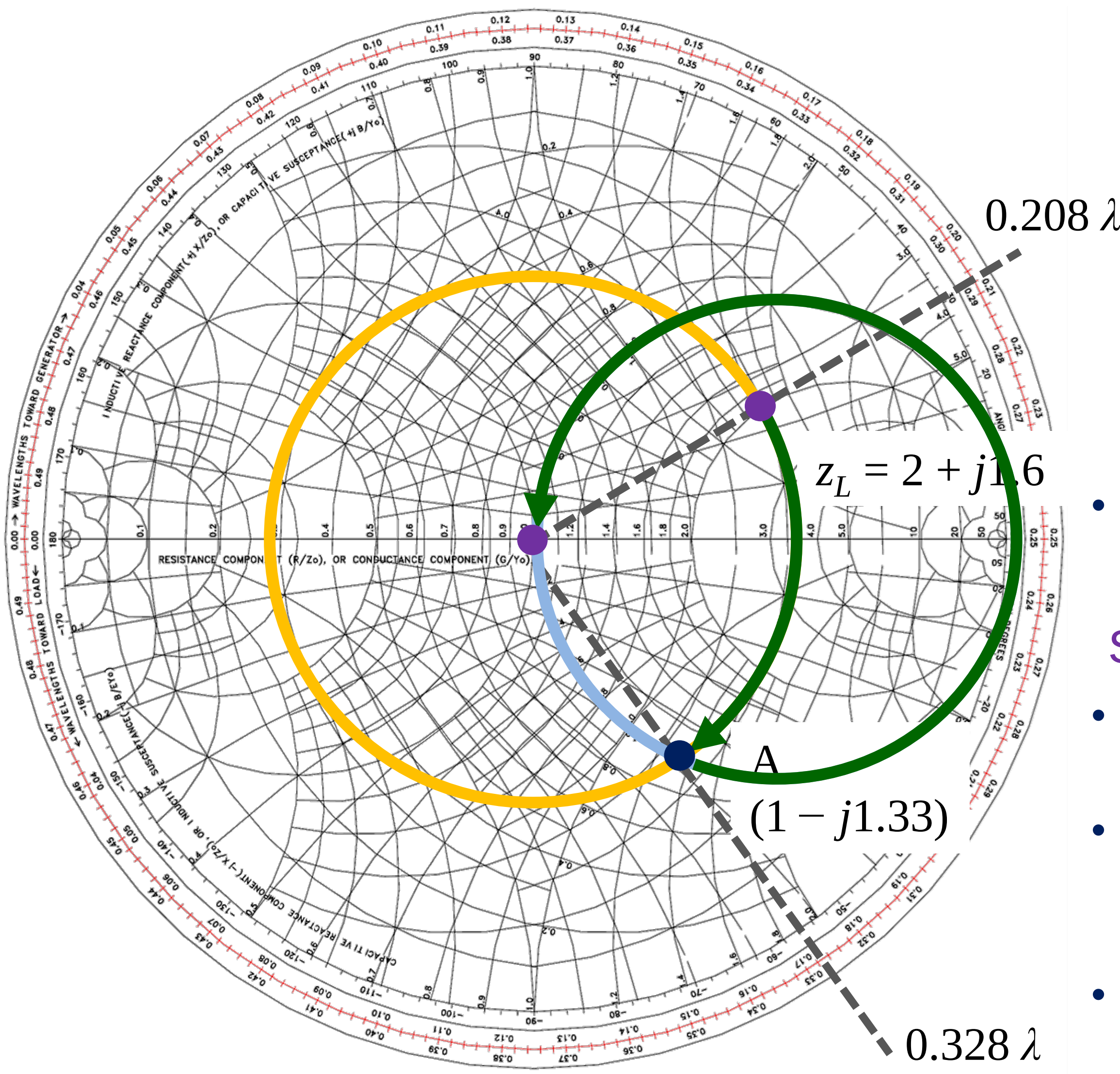
- The normalized impedance:

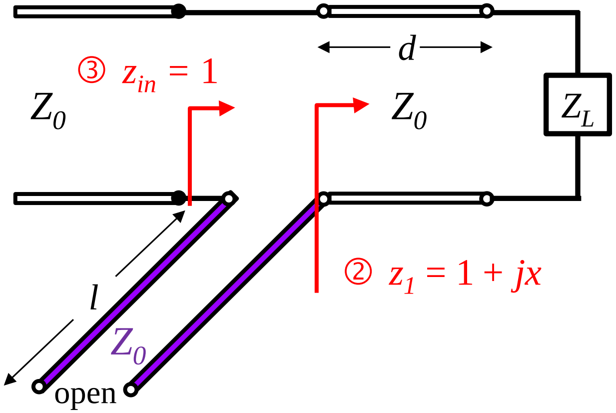
$$z_L = 2 + j1.6$$

Step 2:

- Connecting z_L to a transmission line: rotating z_L along the constant $|\Gamma|$ circle
- Where do we stop?

Smith Chart Solutions of Series-Stub Matching (3/7)





Solution 1 (for point A):

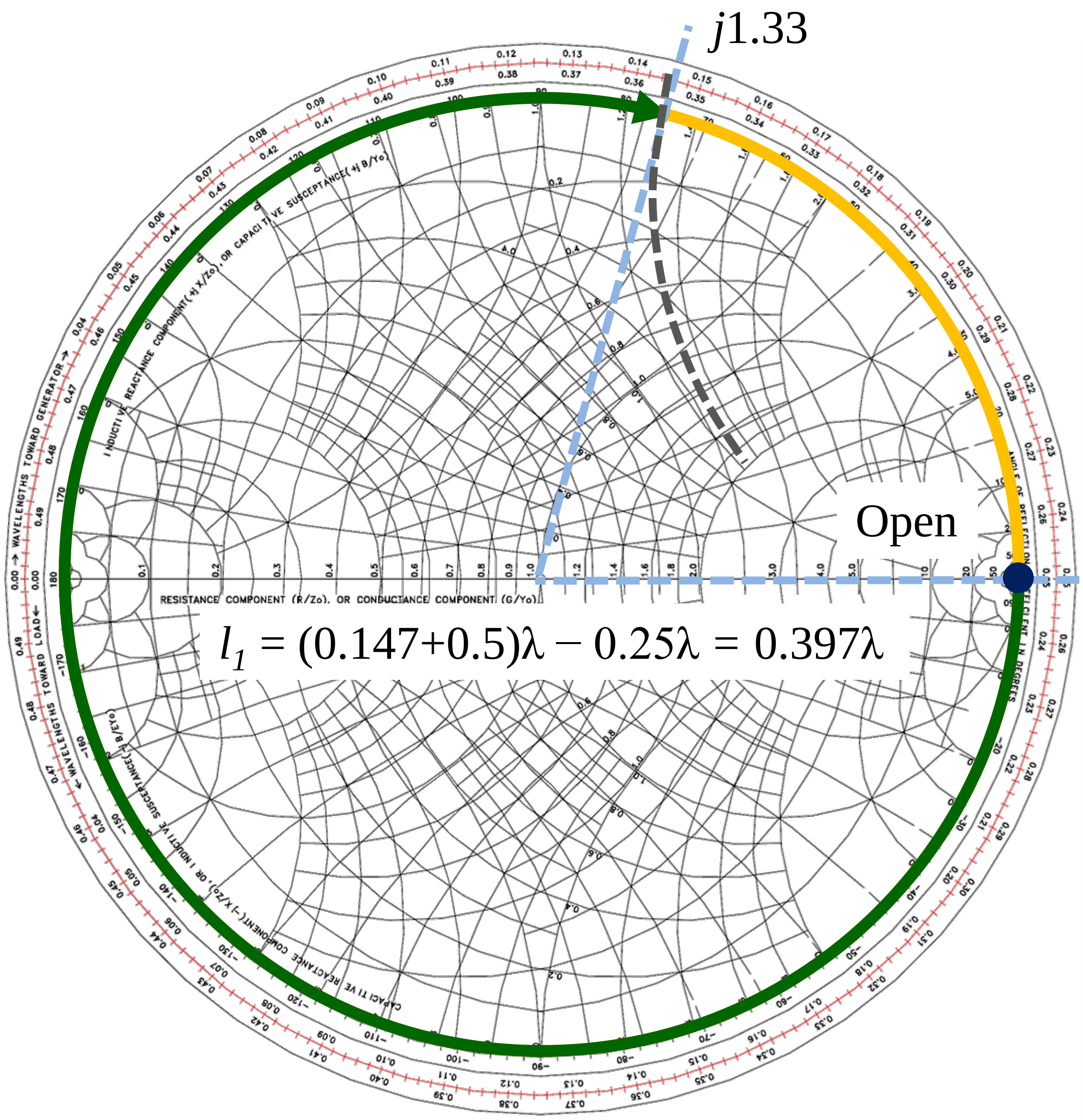
- How to rotate z_L to A $(1-j1.33)$?

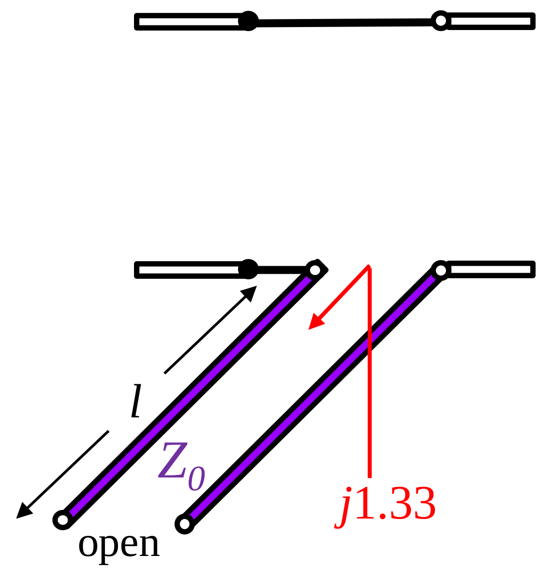
$$d_1 = 0.328\lambda - 0.208\lambda = 0.120\lambda$$

Step 3:

- Using a series stub to eliminate the reactance $-j1.33$
- Which way to take the turn? Counterclockwise or clockwise? Counterclockwise! (But why?)
- The opened stub should provide a reactance of $j1.33$

Smith Chart Solutions of Series-Stub Matching (4/7)



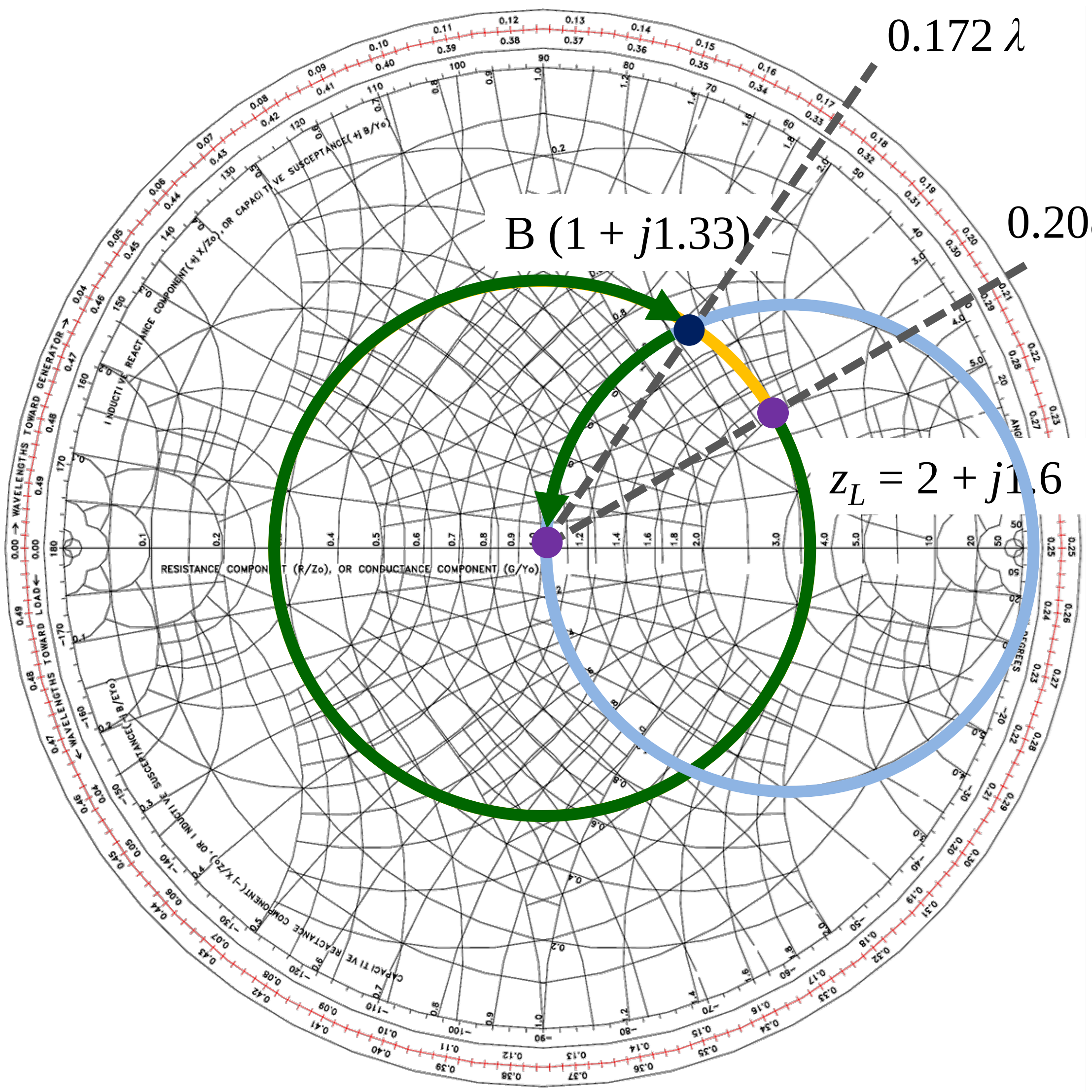


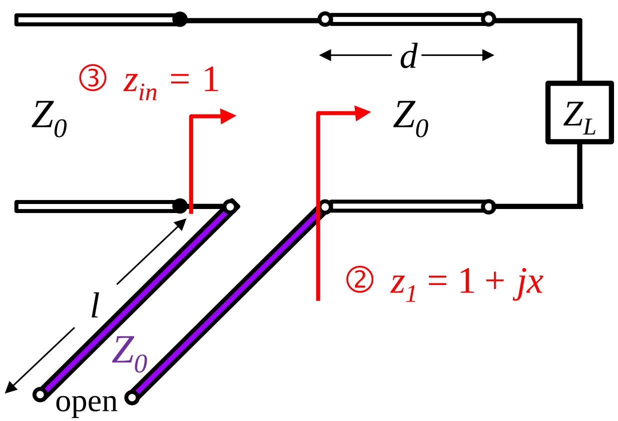
- What's the associated length l to make a $j1.33$ reactance?
- The input impedance of z_L: moving the point along its constant $|\Gamma|$ circle
- Read the distance that we have moved:

$$l_1 = 0.647\lambda - 0.25\lambda = 0.397\lambda$$

EX 3.6

Smith Chart Solutions of Series-Stub Matching (5/7)





Solution 2 (for point B):

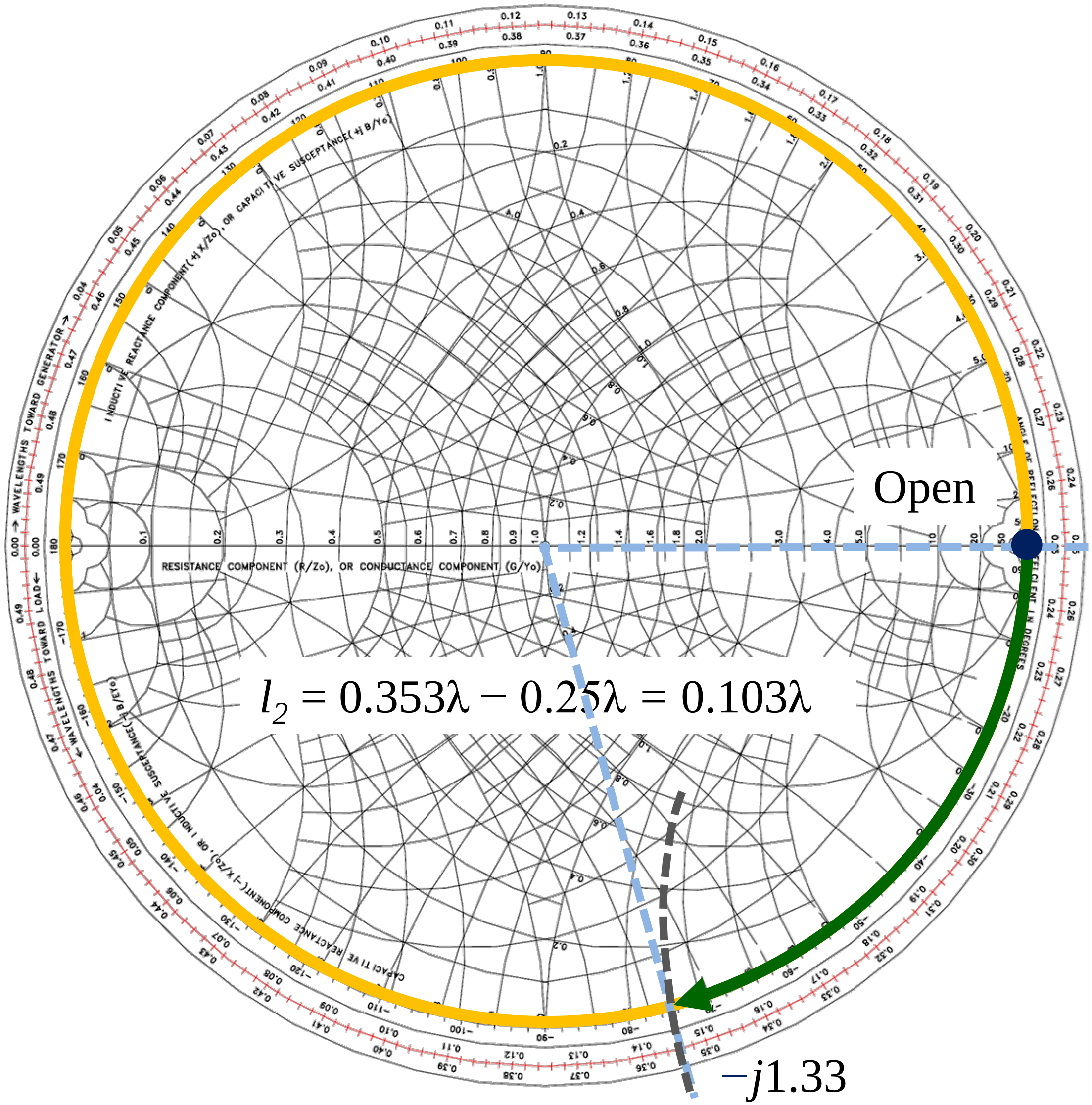
- How to rotate z_L to B $(1 + j1.33)$?

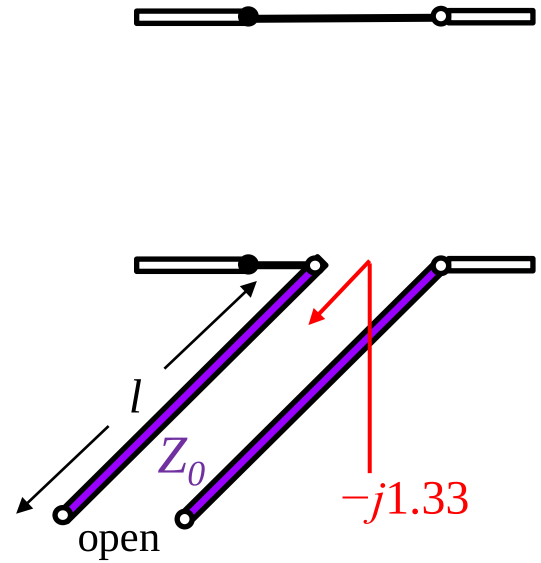
$$d_2 = (0.5\lambda + 0.172\lambda) - 0.208\lambda = 0.463\lambda$$

Step 3:

- Using a series stub to eliminate the reactance $j1.33$
- Which way to take the turn? Counterclockwise or clockwise? Counterclockwise! (But why?)
- The opened stub should provide a reactance of $-j1.33$

EX 3.6 Smith Chart Solutions of Series-Stub Matching (6/7)



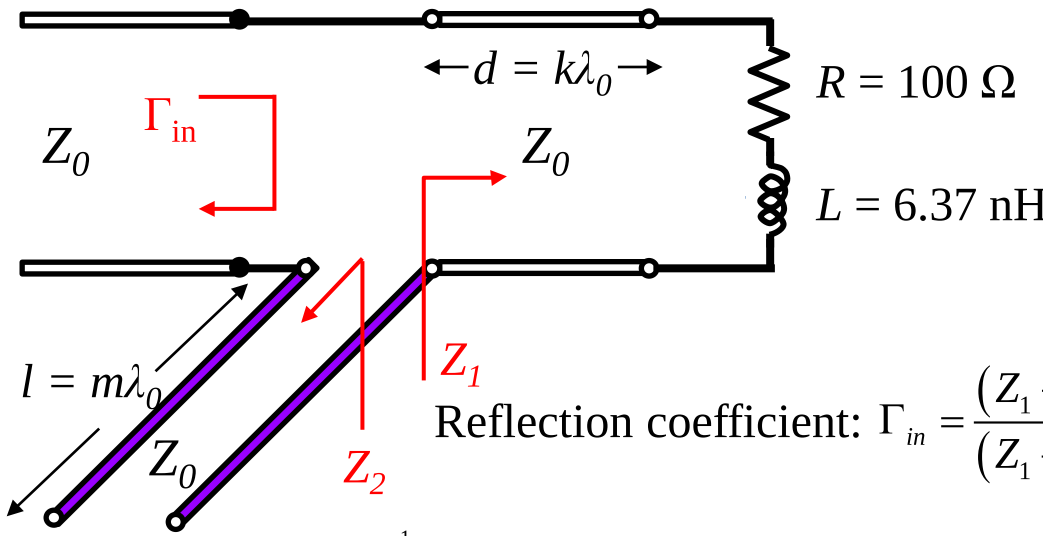


- What's the associated length l to make a $-j1.33$ reactance?
- The input impedance of z_L: moving the point along its constant $|\Gamma|$ circle
- Read the distance that we have moved:

$$l_2 = 0.353\lambda - 0.25\lambda = 0.103\lambda$$

Smith Chart Solutions of Series-Stub Matching (7/7)

(λ_0: associated wavelength of 2 GHz)



Reflection coefficient: $\Gamma_{in} = \dfrac{(Z_1 + Z_2) - Z_0}{(Z_1 + Z_2) + Z_0}$

$$Z_2 = -jZ_0 \cot \beta l$$

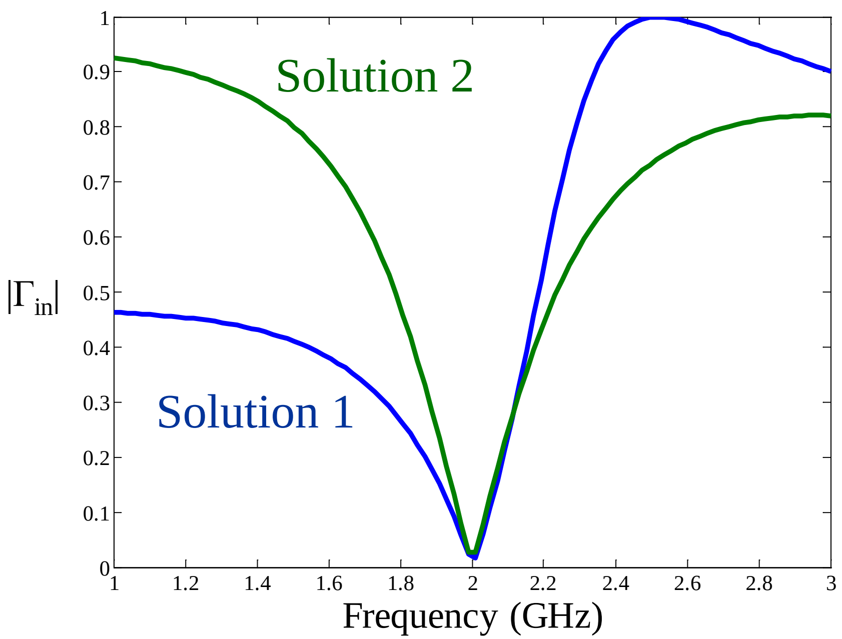
where $\cot \beta l = \cot\left(\dfrac{2\pi}{\lambda}\right)(m\lambda_0)$

$$= \cot \frac{2\pi f \times m}{f_0}$$

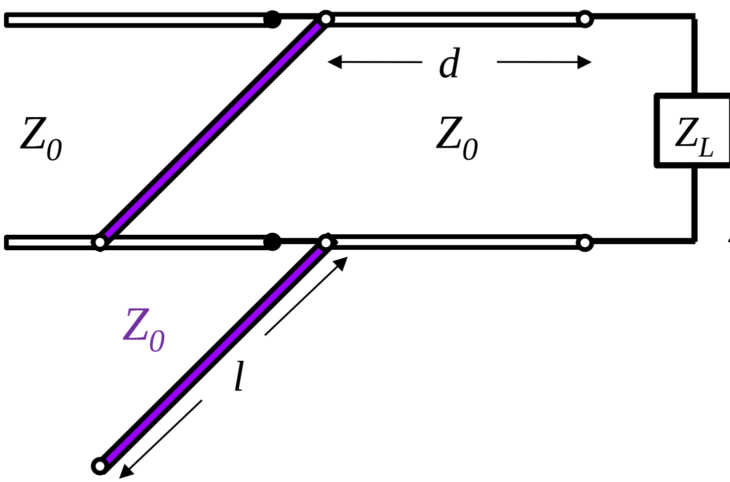
$$Z_1 = Z_0\left(\frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d}\right)$$

where $Z_L = 100 + j2\pi \times f \times 6.37 \times 10^{-9}$

$$\tan \beta d = \tan\left(\frac{2\pi}{\lambda}\right)(k\lambda_0) = \tan \frac{2\pi f \times k}{f_0}$$



Analytic Solutions of Shunt Stub (Opened-Circuit)



$Z_L = R_L + jX_L$

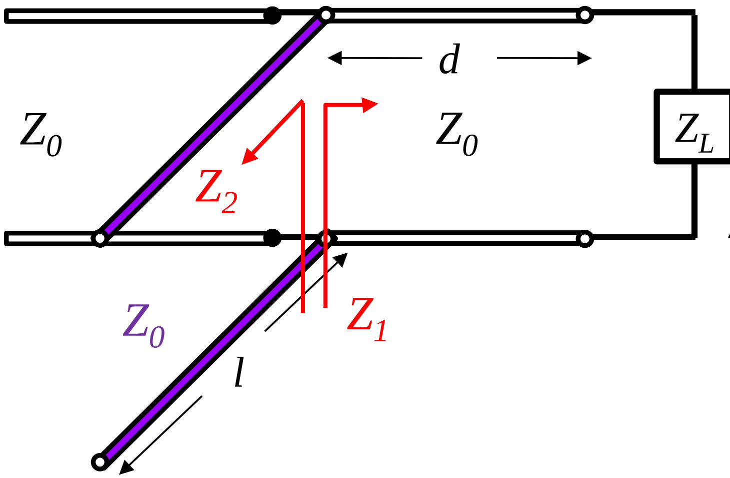
$Y_0 = 1/Z_0$

The analytic form of d an l:

$$\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t, \text{ for } t \geq 0 \\ \dfrac{1}{2\pi}\left(\pi + \tan^{-1} t\right), \text{ for } t < 0 \end{cases} \quad \text{where} \quad t = \begin{cases} \dfrac{X_L \pm \sqrt{R_L\left[\left(Z_0 - R_L\right)^2 + X_L^2\right]/Z_0}}{R_L - Z_0}, \text{ for } R_L \neq Z_0 \\ \dfrac{X_L}{-2Z_0}, \text{ for } R_L = Z_0 \end{cases}$$

$$\frac{l}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1}\left(\dfrac{B_s}{Y_0}\right), \text{ for } B_s \geq 0 \\ \dfrac{1}{2\pi}\left(\pi + \tan^{-1}\left(\dfrac{B_s}{Y_0}\right)\right), \text{ for } B_s < 0 \end{cases} \quad \text{where} \quad B_s = -\frac{R_L^2 t - \left(Z_0 - X_L t\right)\left(X_L + Z_0 t\right)}{Z_0\left[R_L^2 + \left(X_L + Z_0 t\right)^2\right]}$$

Derivation of Analytic Solutions (1/2)



$Z_L = R_L + jX_L$

$Y_0 = 1/Z_0$

1. For Z_1 part:

$$Z_1 = Z_0\left(\frac{(R_L + jX_L) + jZ_0\tan\beta d}{Z_0 + j(R_L + jX_L)\tan\beta d}\right) \xrightarrow{\text{Associated admittance}} Y_1 = \frac{1}{Z_1} = G + jB$$

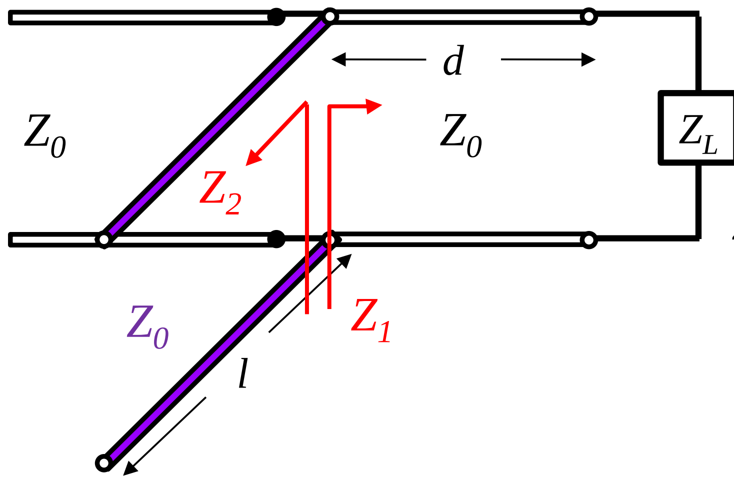
$$\boxed{G = \frac{R_L\left(1+\tan^2\beta d\right)}{R_L^2 + \left(X_L + Z_0\tan\beta d\right)^2}}$$

$$B = \frac{R_L^2\tan\beta d - \left(Z_0 - X_L\tan\beta d\right)\left(X_L + Z_0\tan\beta d\right)}{Z_0\left[R_L^2 + \left(X_L + Z_0\tan\beta d\right)^2\right]}$$

Matching equation: To solve d so that $G = Y_0$

$$\Rightarrow \tan\beta d = \begin{cases} \dfrac{X_L \pm \sqrt{R_L\left[\left(Z_0 - R_L\right)^2 + X_L^2\right]/Z_0}}{R_L - Z_0}, & \text{for } R_L \neq Z_0 \\ \dfrac{X_L}{-2Z_0}, & \text{for } R_L = Z_0 \end{cases}$$

Derivation of Analytic Solutions (2/2)



$Z_L = R_L + jX_L$

$Y_0 = 1/Z_0$

2. For Z_2 part:

$$Z_2 = jZ_0 \tan\beta l \xrightarrow{\text{Associated admittance}} Y_2 = \frac{1}{Z_2} = -jY_0 \cot\beta l \equiv jB_s$$

Matching equation: To solve l so that $Y_2 = jB_s = -jB$

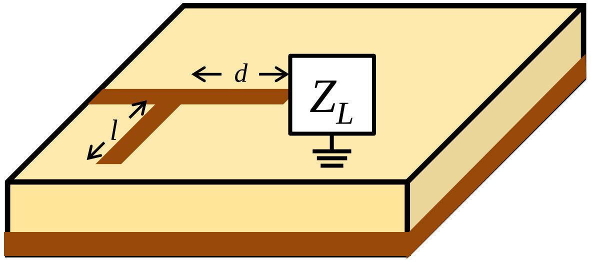
$$\Rightarrow \cot\beta l = -\frac{R_L^2 \tan\beta d - (Z_0 - X_L \tan\beta d)(X_L + Z_0 \tan\beta d)}{Z_0\left[R_L^2 + (X_L + Z_0 \tan\beta d)^2\right]} \Rightarrow \frac{l}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1}\left(\dfrac{B_s}{Y_0}\right), & \text{for } B_s \geq 0 \\ \dfrac{1}{2\pi}\left(\pi + \tan^{-1}\left(\dfrac{B_s}{Y_0}\right)\right), & \text{for } B_s < 0 \end{cases}$$

The other methods of single-stub matching also have analytic forms (Textbook Ch5.2)

Characteristics of Matching with Single Stubs

Advantage:

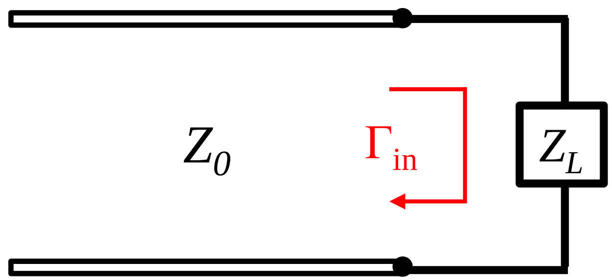
- They are the most widely used matching circuit
- Shunt tuning stubs are easy to fabricate in microstrip form
- They can be realized at any frequency
- They are capable of matching any load impedance to a transmission line



Limitation:

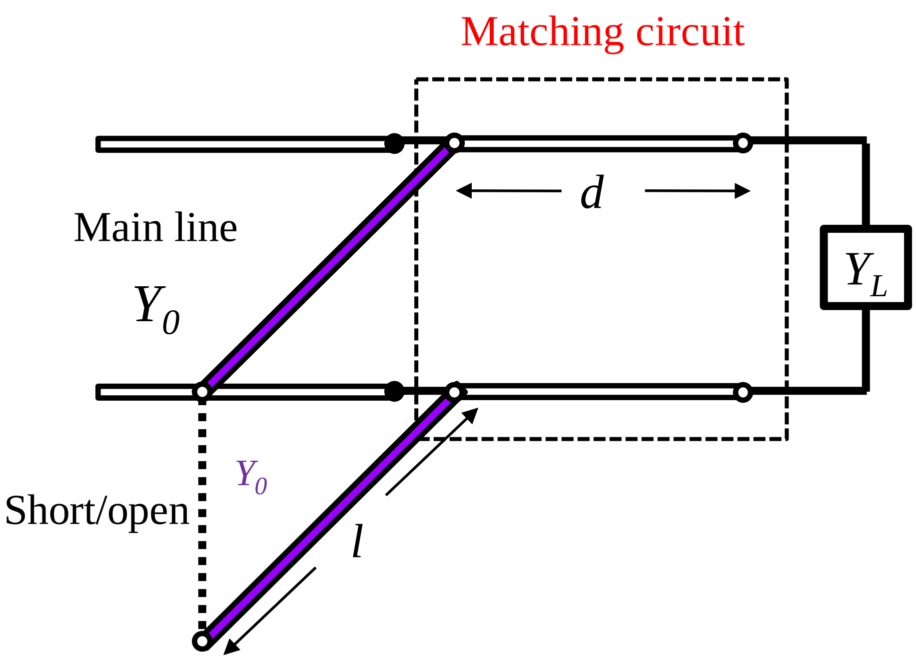
- They result in a larger size
- The matching bandwidth is quite narrow
- They have poor adjustability. They require a variable length of line between the load and the stub

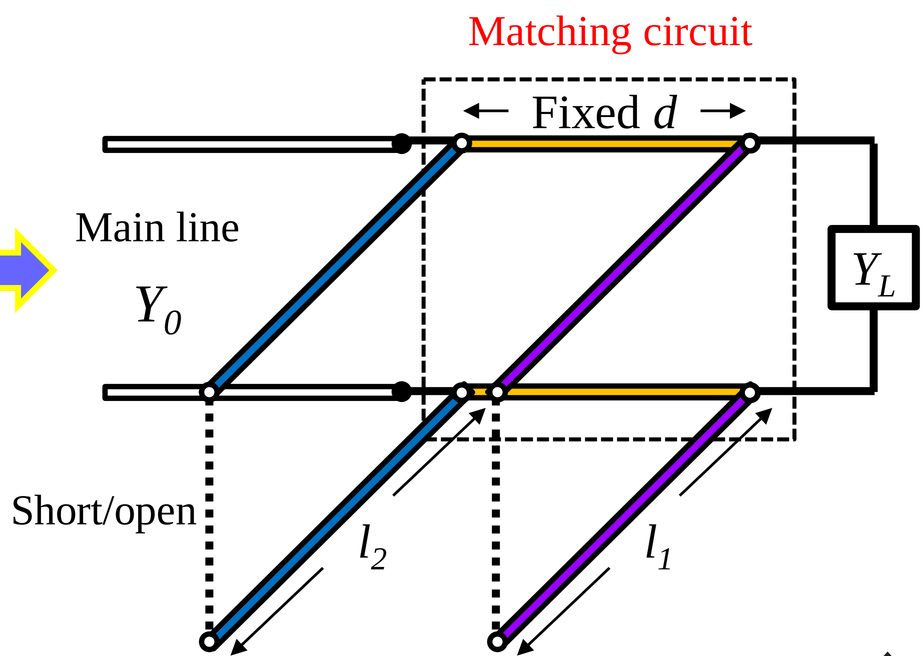
Matching with Double Stub



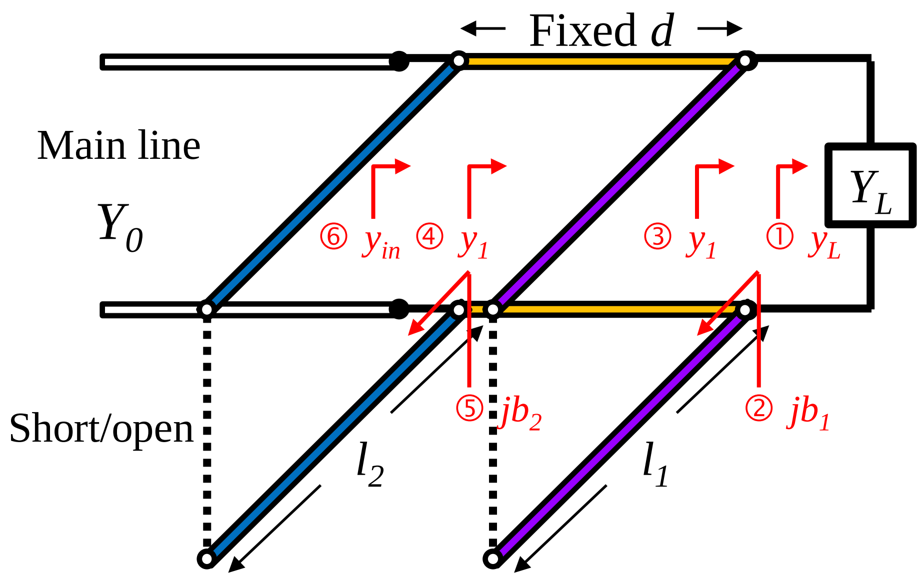
Single stub matching

Double stub matching



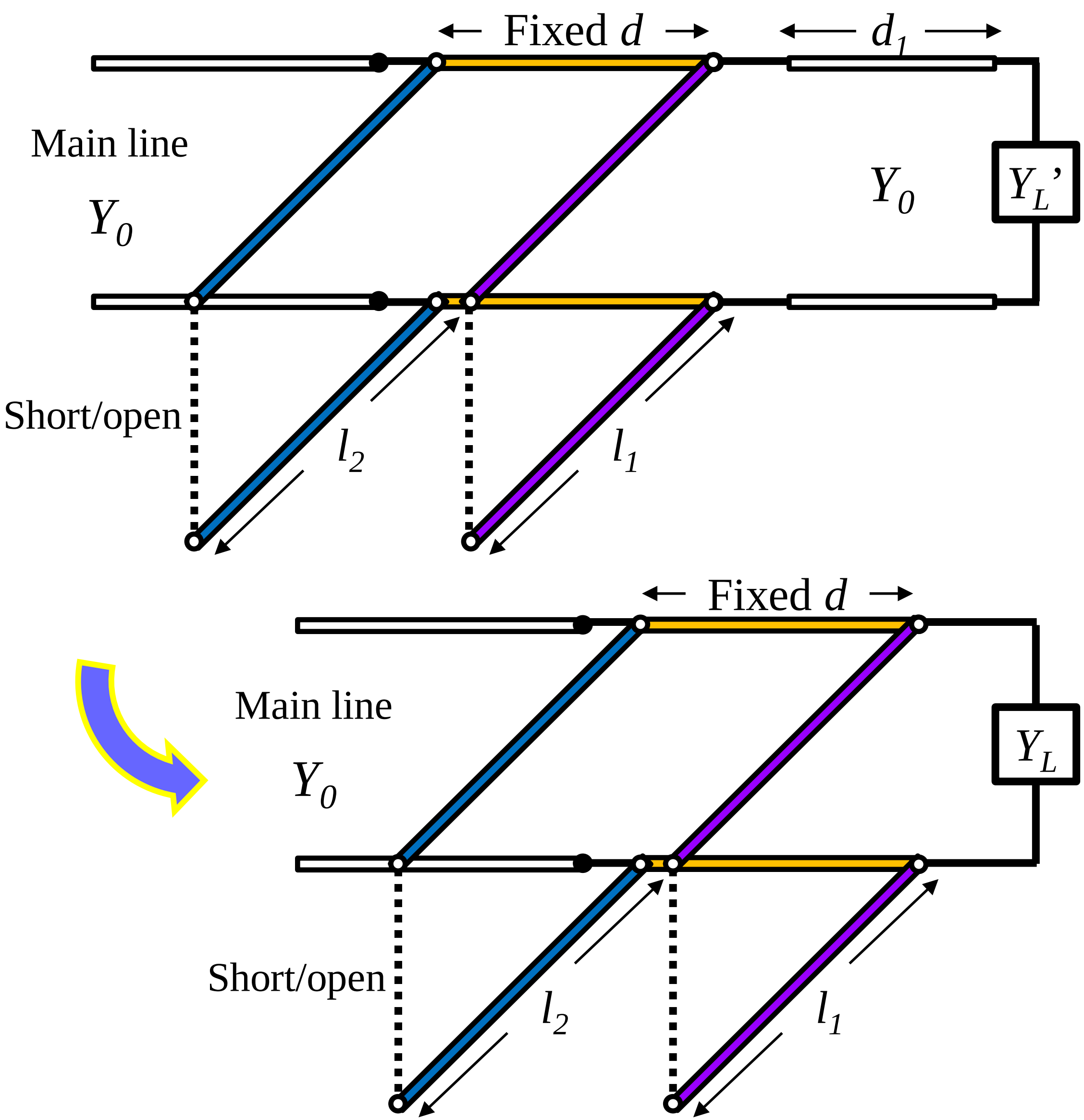


Design Idea of Double-Stub Matching (1/2)



- Parameters: the length of the first stub l_1
  the length of the second stub l_2
- By controlling l_1 and l_2, the double-stub tuner can match *any* load impedance Z_L to the characteristic impedance Z_0 of the main line
- Note that every section and stub has the same characteristic impedance Z_0

Design Idea of Double-Stub Matching (2/2)

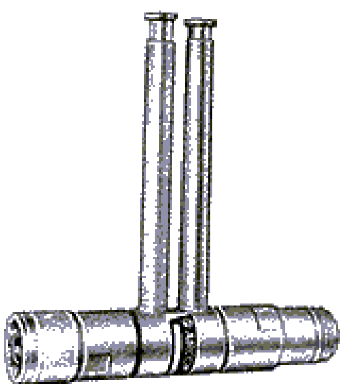


- The load may be an arbitrary distance from the first stub
- But, without loss of generality, we transform Y_L' back to the position of the first stub
- Usually, d is chosen to be $\lambda_0/8$ or $3\lambda_0/8$ in this lecture
- λ_0: the associated wavelength of operational frequency

Why Fix the Spacing d between the Two Stubs?

Fixing d is good for better adjustability:

- We only need to alter the length of the two stubs
- The matching circuit becomes adjustable
- Stub tuners are basic laboratory tools used for matching load impedances to provide for maximum power transfer between a source and a load
- The more stubs a tuner has, the larger impedance matching range it can achieve

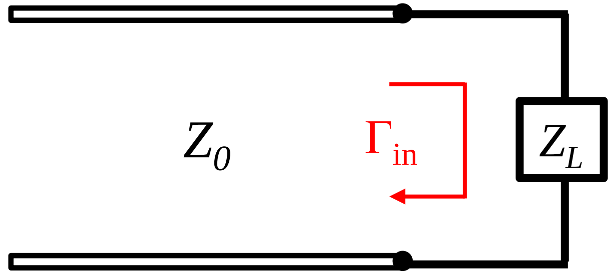


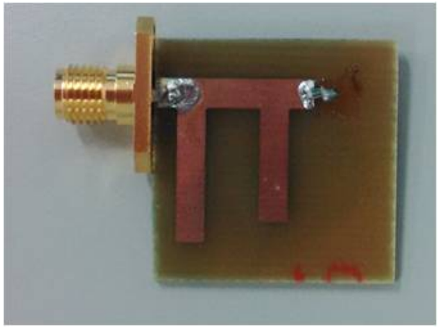
Double stub tuners for waveguide

- Frequency range: 1–10 GHz
- This double-stub tuner uses two spaced adjustable-length shorted-line sections
- It's a widely-used component for matching other units such as coaxial detector mounts
- It tunes out mismatches as great as 20:1

Hofstra Group, Weinschel DS-109LL Double Stub Tuner 0.2-2.0 GHz, [Online] Available: http://www.hofstragroup.com/
Aircom Mocrowave, Waveguide Component: Double Stub Tuners, [Online] Available: http://www.waveguide-components.com/waveguide-components-tuner11.htm

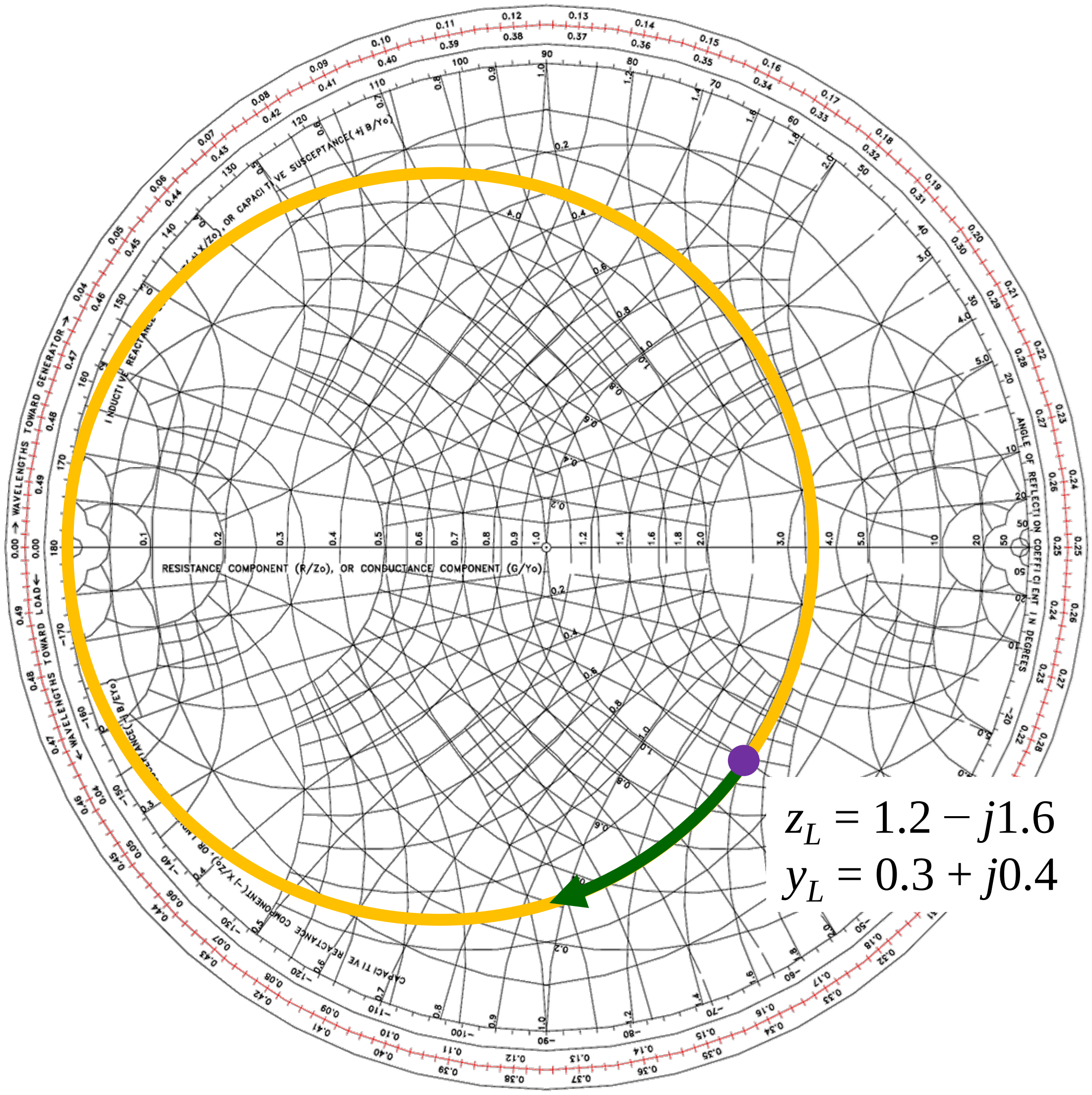
Smith Chart Solutions of Double-Stub Matching (1/11)

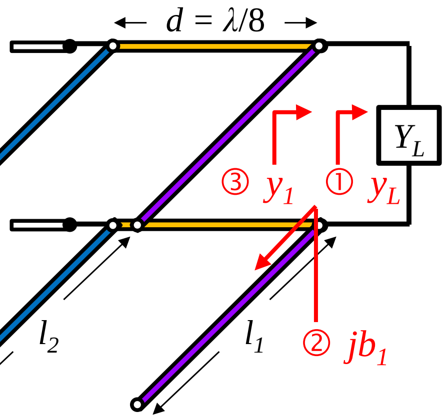




1. Design a double-stub shunt tuner to match a load impedance $Z_L = 60 - j80\ \Omega$ to a $Z_0 = 50\ \Omega$ line
   - The stubs are open-circuited stubs
   - The stubs are separated $\lambda_0/8$ apart
   - Operational frequency: 2 GHz
2. If Z_L is composed of a resistor and a capacitor in series, plot the reflection coefficient magnitude from 1 GHz to 3 GHz for each solution

Smith Chart Solutions of Double-Stub Matching (2/11)





Step 1:

- The normalized impedance:

$$z_L = 1.2 - j1.6$$

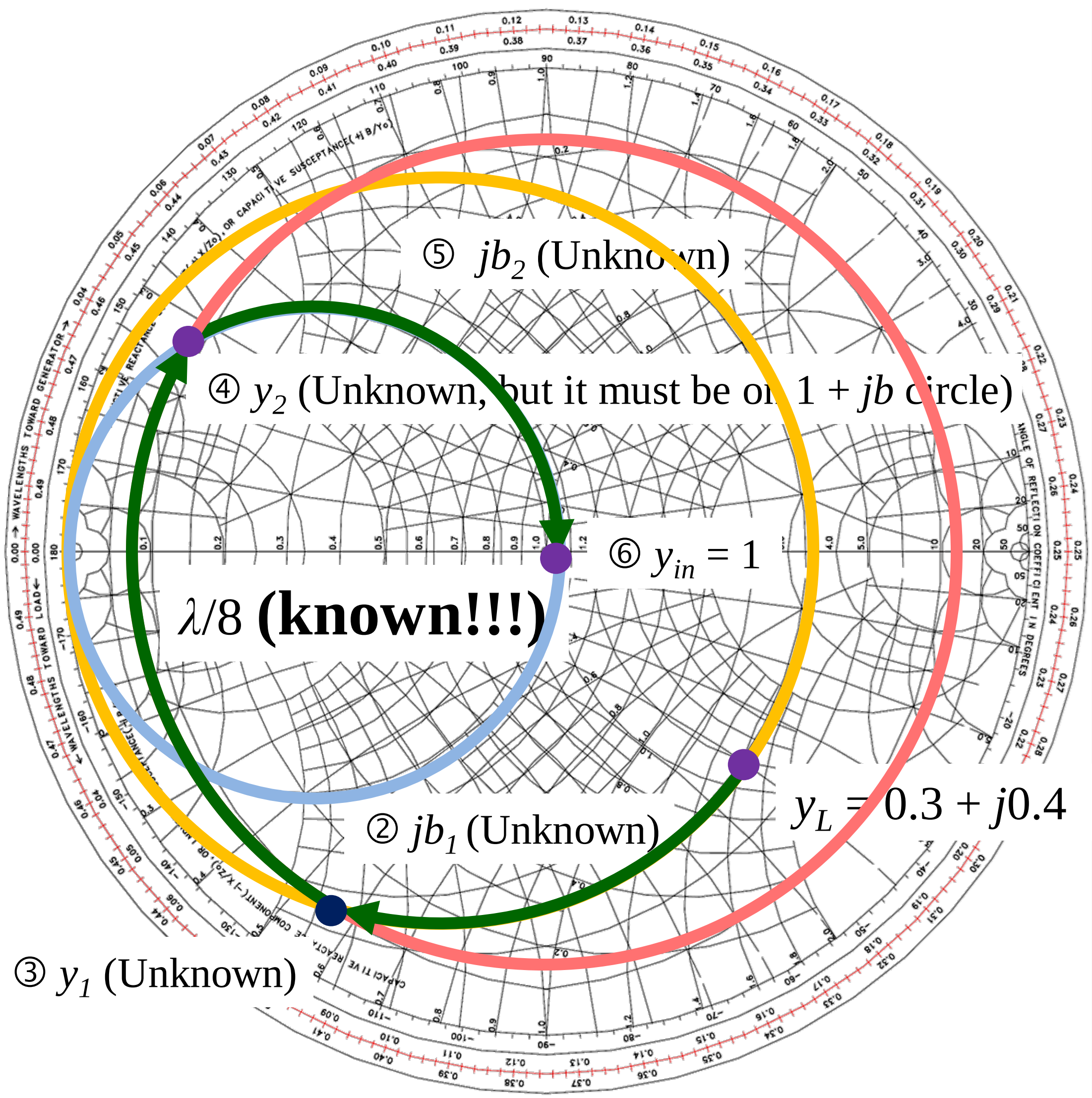
- Read its normalized admittance:

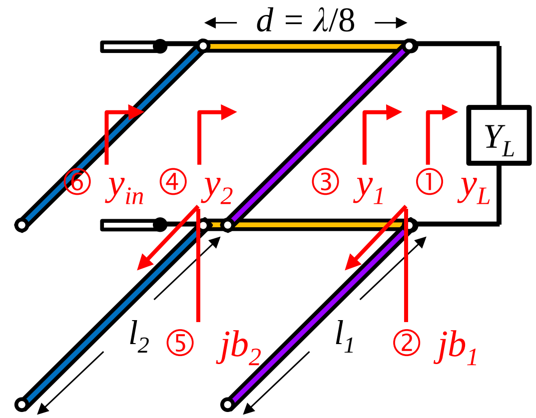
$$y_L = 0.3 + 0.4i$$

Step 2:

- Connecting y_L to a shunt open-circuited line (jb_1): rotating y_L along the constant g circle ($y_1 = y_L \| jb_1$)
- y_L moves clockwise; but, where should it stop?

Smith Chart Solutions of Double-Stub Matching (3/11)

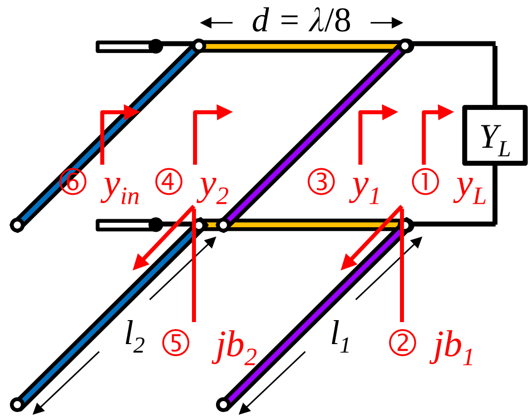


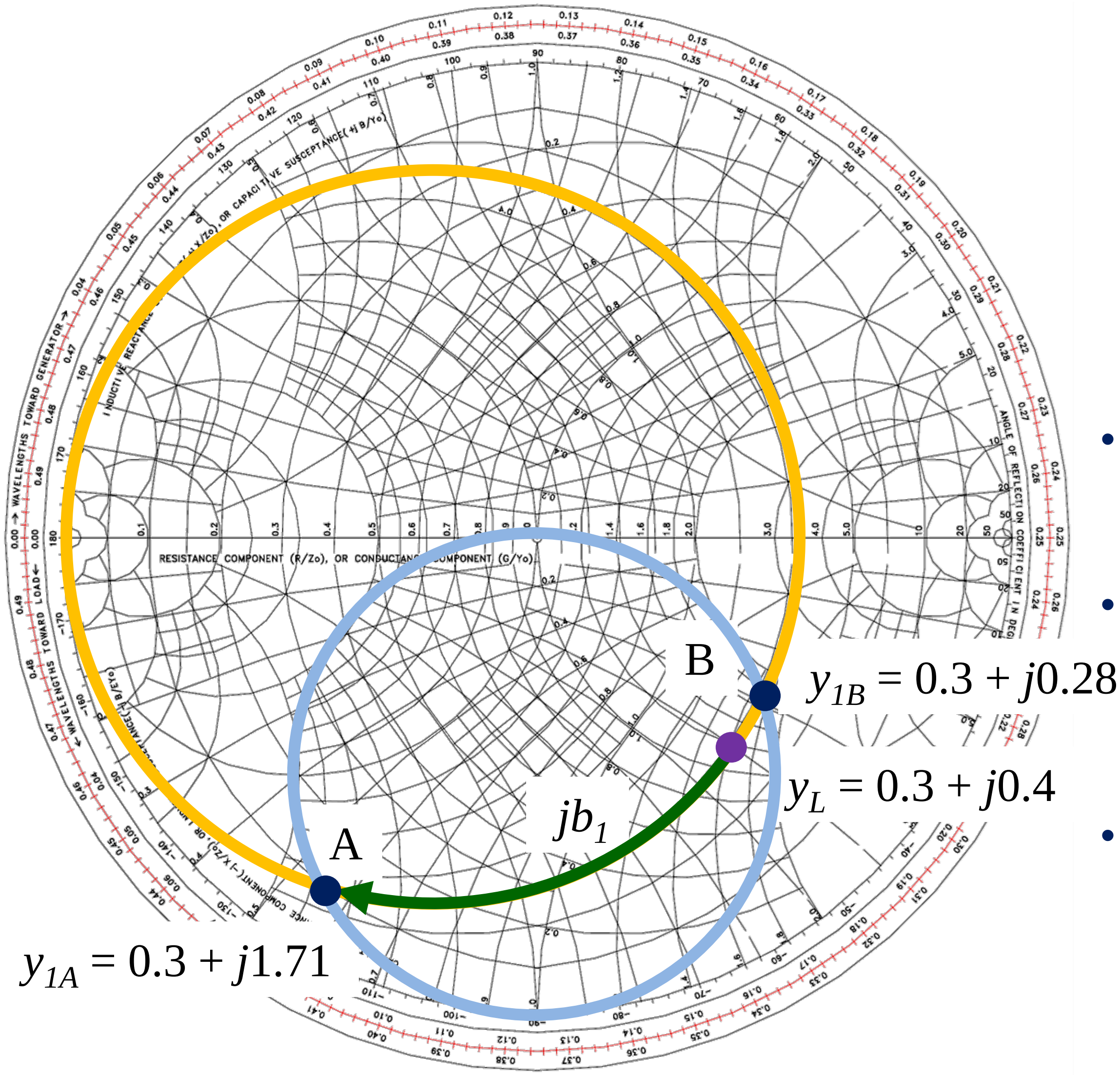


- Observing that y_1 is going to connect with a $\lambda/8$ long transmission line
- So y_1 will definitely be moved along a constant $|\Gamma|$ circle, and it becomes y_2
- When it becomes y_2, y_2 will be connected to jb_2, resulting in y_{in}
- Since y_{in} must equal 1, so y_2 must be located on the $1 + jb$ circle

So, how to get y_L to y_1 and make everything straightforwardly happen?

EX 3.7 Smith Chart Solutions of Double-Stub Matching (4/11)





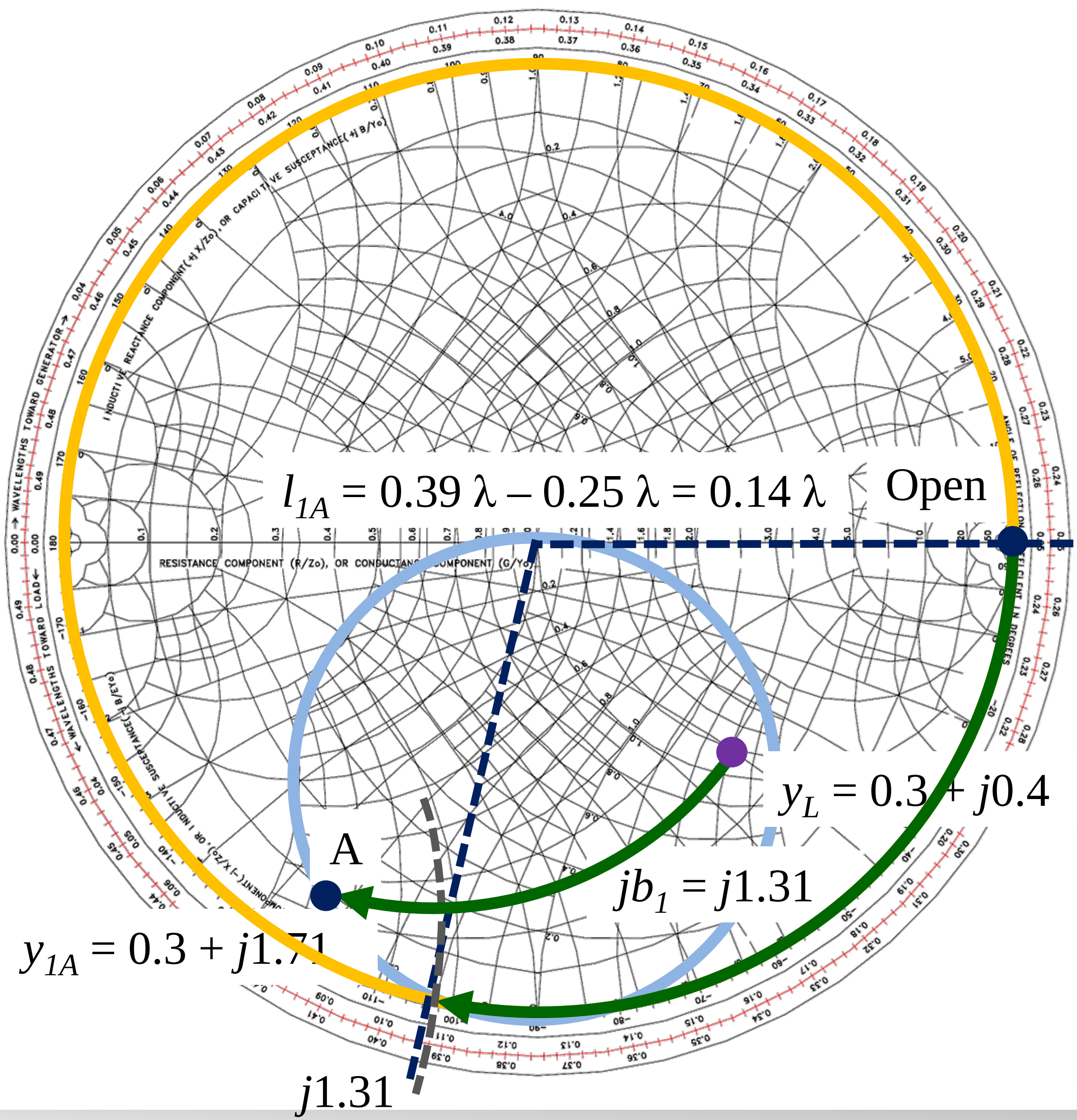
- If y_1 is on the blue circle, then it will be shift to the $1 + jb$ circle by the $\lambda/8$ long transmission line
- Two solutions:

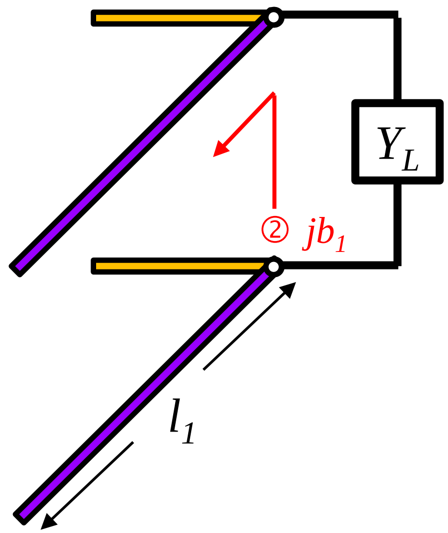
  A: $y_{1A} = 0.3 + j1.71$
  B: $y_{1B} = 0.3 + j0.28$

- So, the shunt (opened-circuit) stub should provide a susceptance:

  A: $jb_{1A} = j1.31$
  B: $jb_{1B} = -j0.12$

Smith Chart Solutions of Double-Stub Matching (5/11)



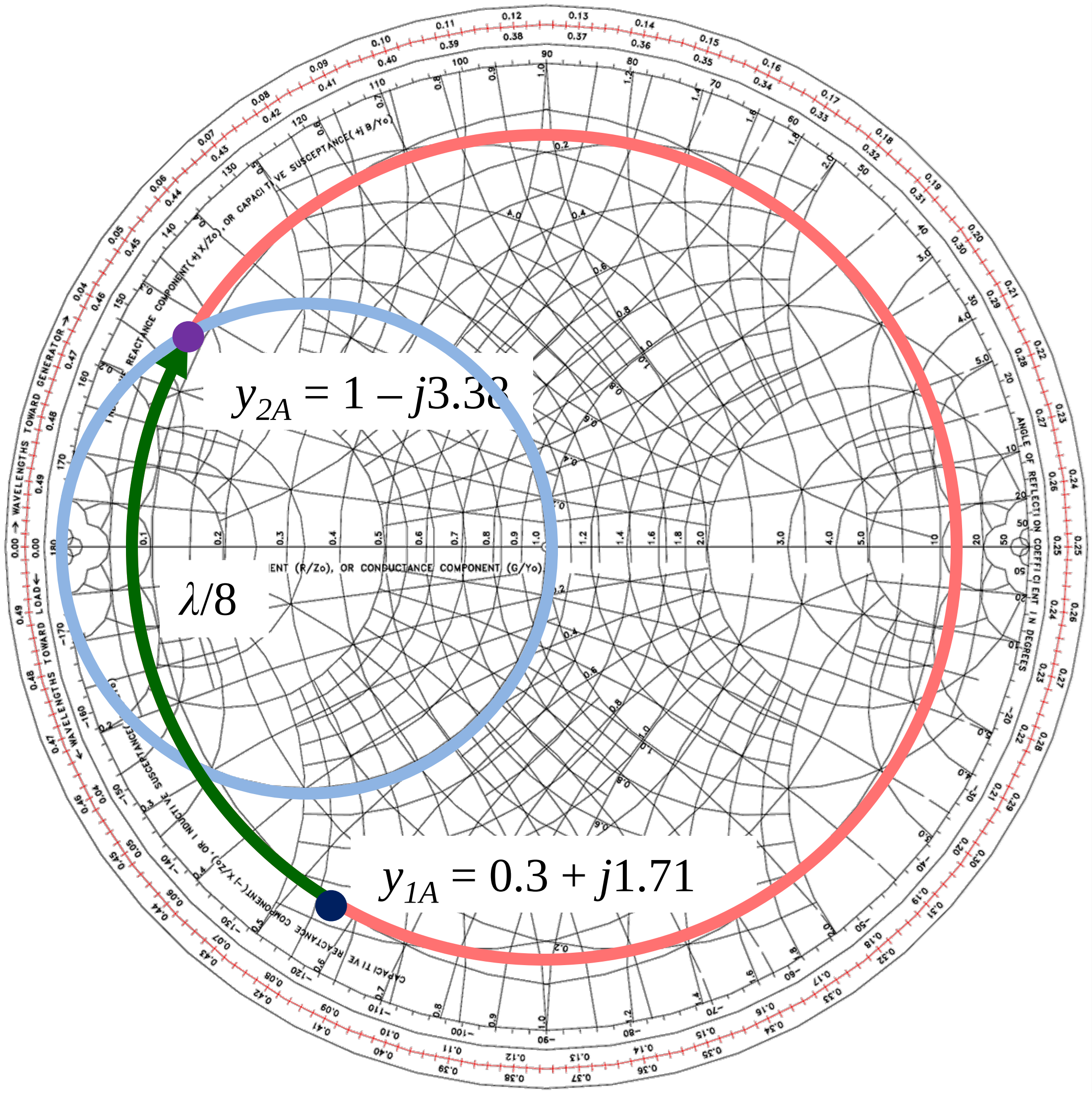


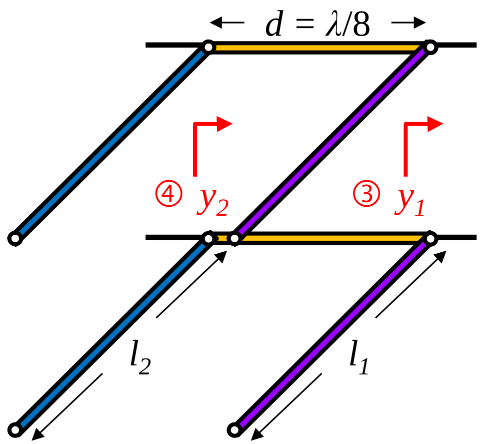
Solution 1 (for point A):

- What's the associated length l to make a $j1.31$ susceptance?
- The input admittance of y_L: moving the point along its constant $|\Gamma|$ circle
- Read the distance that we have moved:

A: $l_{1A} = 0.396\,\lambda - 0.25\,\lambda = 0.146\,\lambda$

Smith Chart Solutions of Double-Stub Matching (6/11)





Continuing with Solution 1 (for point A):

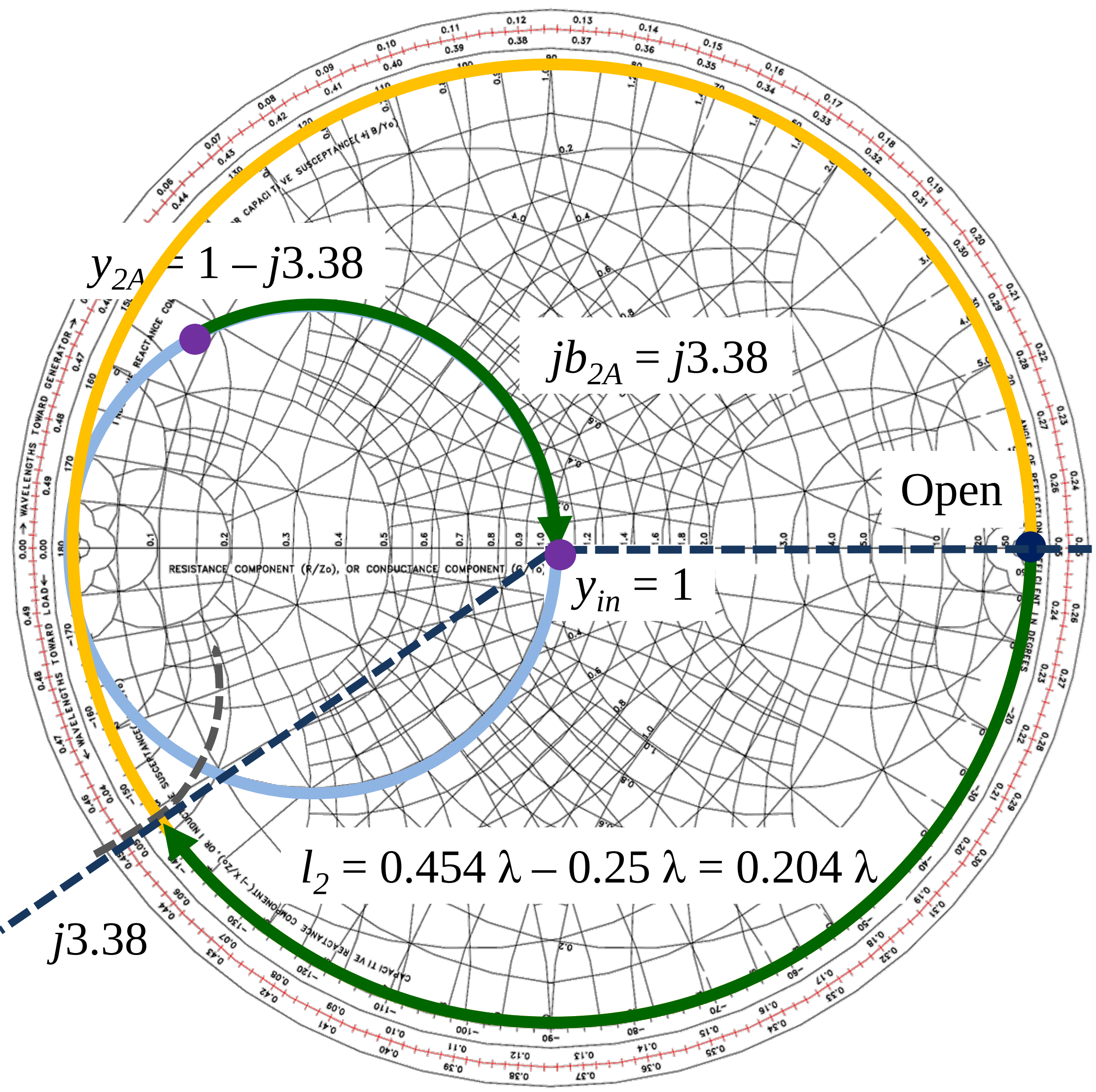
Step 3:

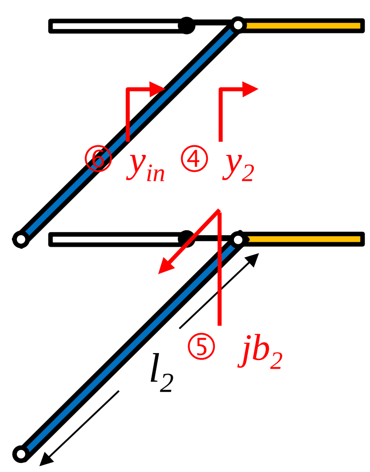
- y_2: Moving y_1 along the constant $|\Gamma|$ circle for $\lambda/8$ long
- Read y_2:

$$\text{A:} \quad y_{2A} = 1 - j3.38$$

- y_2 is on the $1 + jb$ circle
- We need a shunt stub to eliminate the susceptance $-j3.38$

Smith Chart Solutions of Double-Stub Matching (7/11)



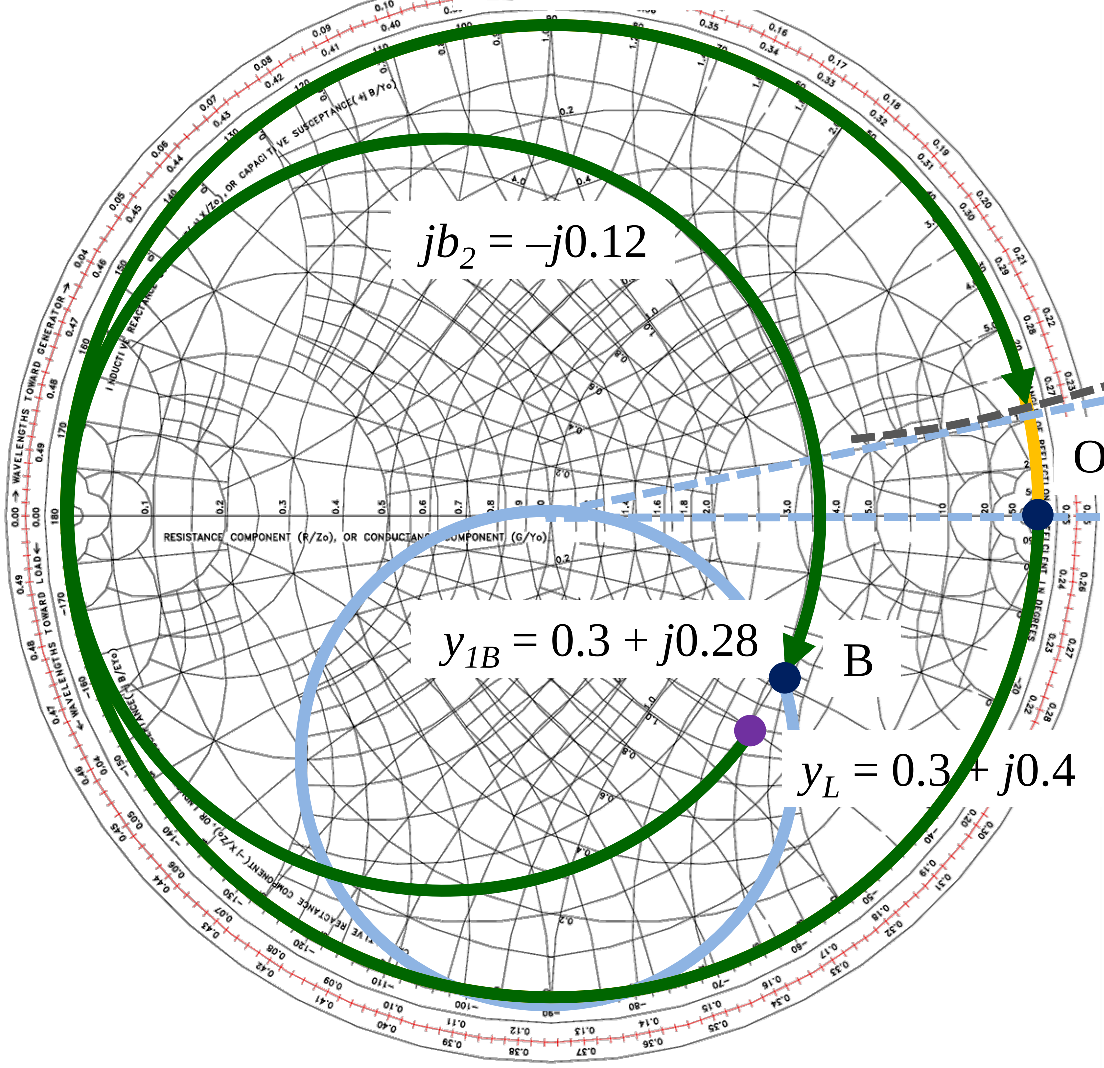


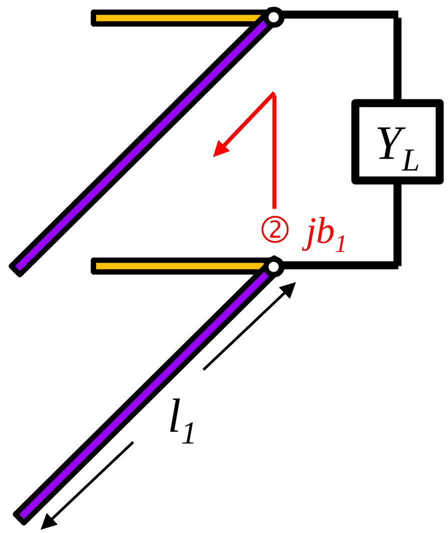
Step 4:

- $y_{in} = y_2 + jb_2$
- So, y_2 is moved along the constant g circle clockwise
- The opened stub should provide a susceptance of $j3.38$
- The corresponding length:

A: $l_{2A} = 0.454\,\lambda - 0.25\,\lambda = 0.204\,\lambda$

Smith Chart Solutions of Double-Stub Matching (8/11)



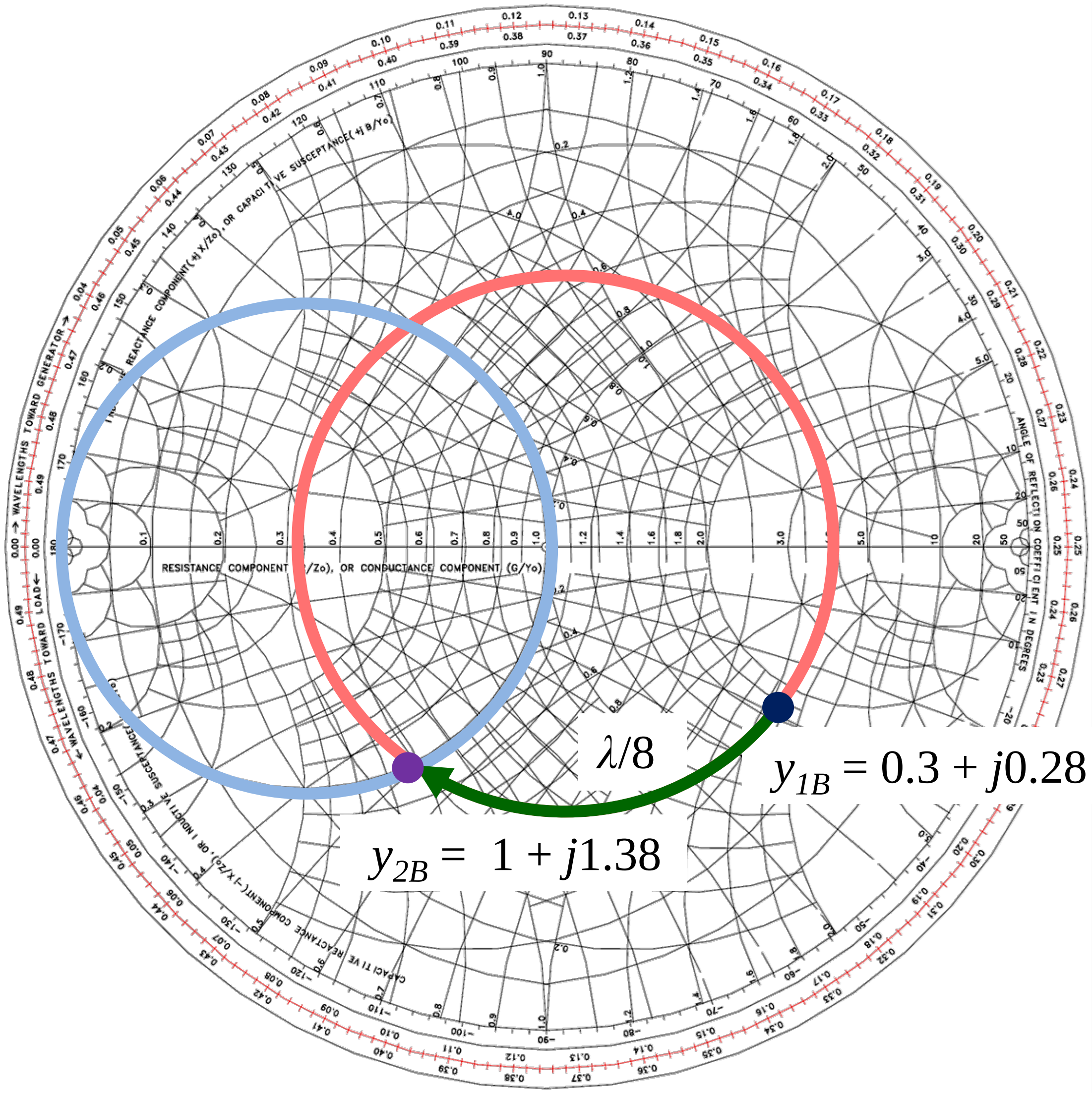


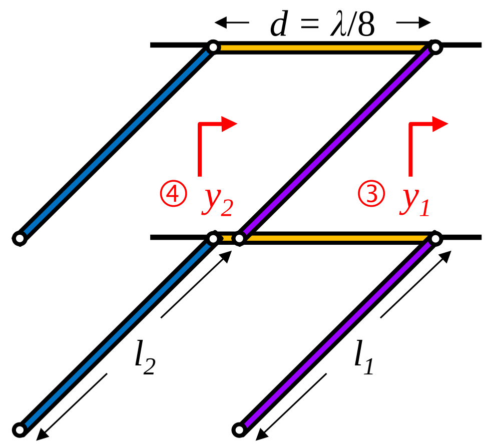
Solution 2 (for point B):

- What's the associated length l to make a $-j0.12$ susceptance?
- The input admittance of y_L: moving the point along its constant $|\Gamma|$ circle
- Read the distance that we have moved:

B: $l_{1B} = (0.5 + 0.23)\lambda - 0.25\,\lambda = 0.48\,\lambda$

Smith Chart Solutions of Double-Stub Matching (9/11)





Continuing with Solution 2 (for point B):

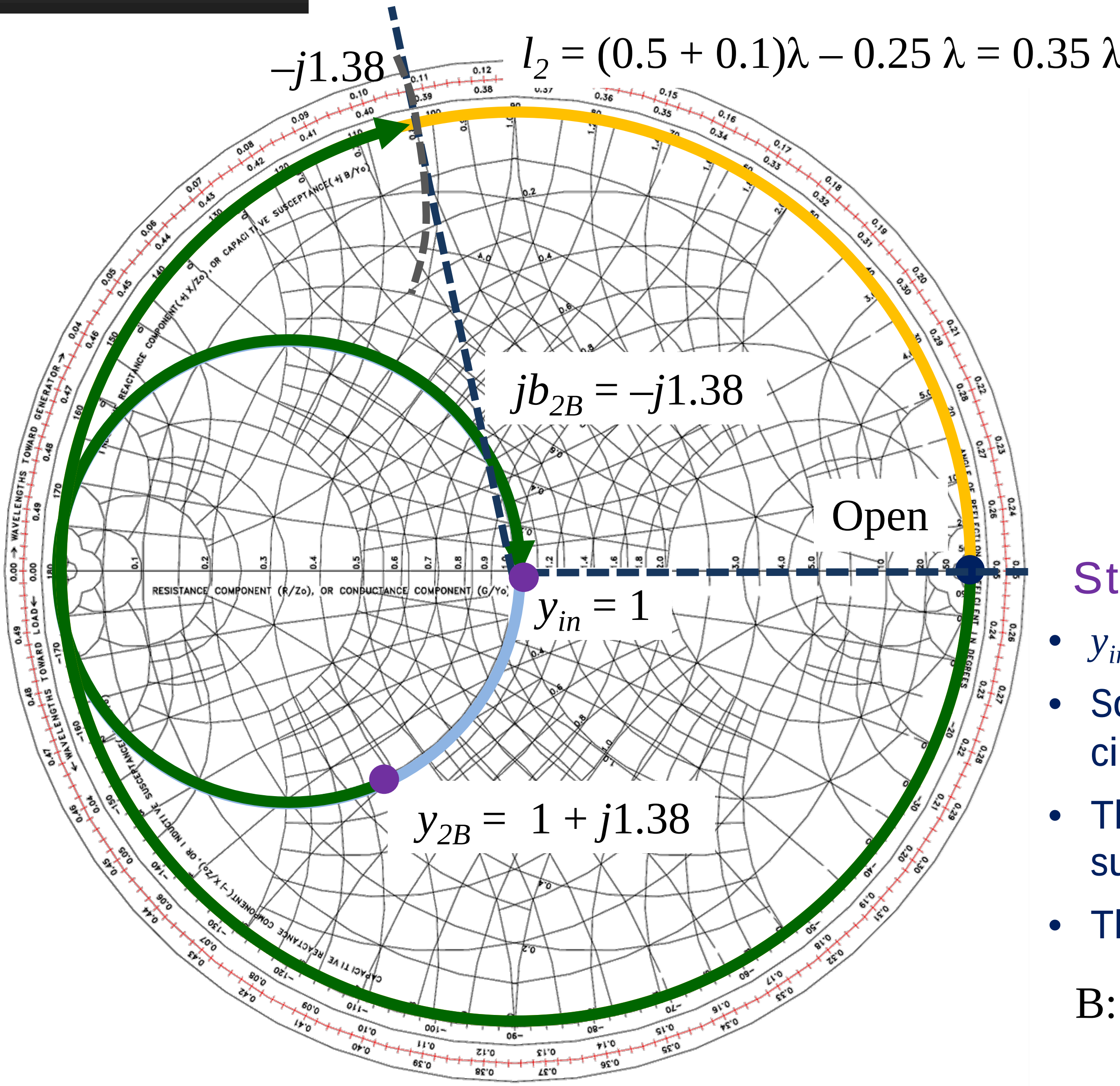
Step 3:

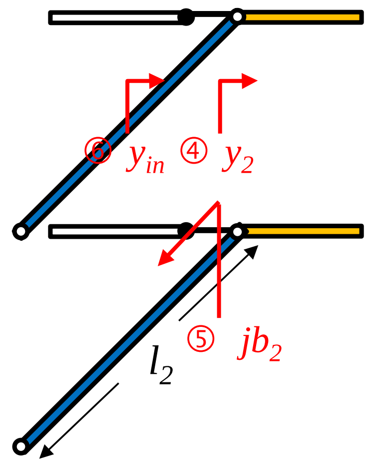
- y_2: Moving y_1 along the constant $|\Gamma|$ circle for $\lambda/8$ long
- Read y_2:

$$\text{B:}\ \ y_{2B} = 1 + j1.38$$

- y_2 is on the $1 + jb$ circle
- We need a shunt stub to eliminate the susceptance $j1.38$

EX 3.7 Smith Chart Solutions of Double-Stub Matching (10/11)



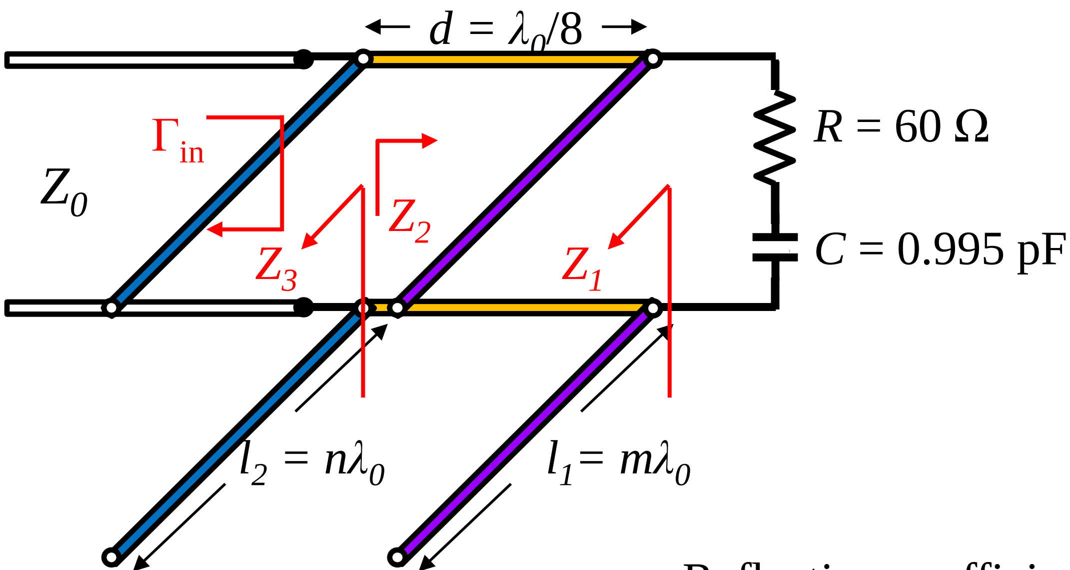


Step 4:

- $y_{in} = y_2 + jb_2$
- So, y_2 is moved along the constant g circle clockwise
- The opened stub should provide a susceptance of $-j1.38$
- The corresponding length:

B: $l_{2B} = (0.5 + 0.1)\lambda - 0.25\,\lambda = 0.35\,\lambda$

EX 3.7 Smith Chart Solutions of Double-Stub Matching (11/11)



$$Z_2 = Z_0\left(\frac{(Z_L \parallel Z_1) + jZ_0 \tan\beta d}{Z_0 + j(Z_L \parallel Z_1)\tan\beta d}\right)$$

$$\text{where } Z_L = 60 - j\frac{1}{2\pi \times f \times 0.995 \times 10^{-12}}$$

$$\tan\beta d = \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{8}\right) = \tan\frac{\pi f}{4 f_0}$$

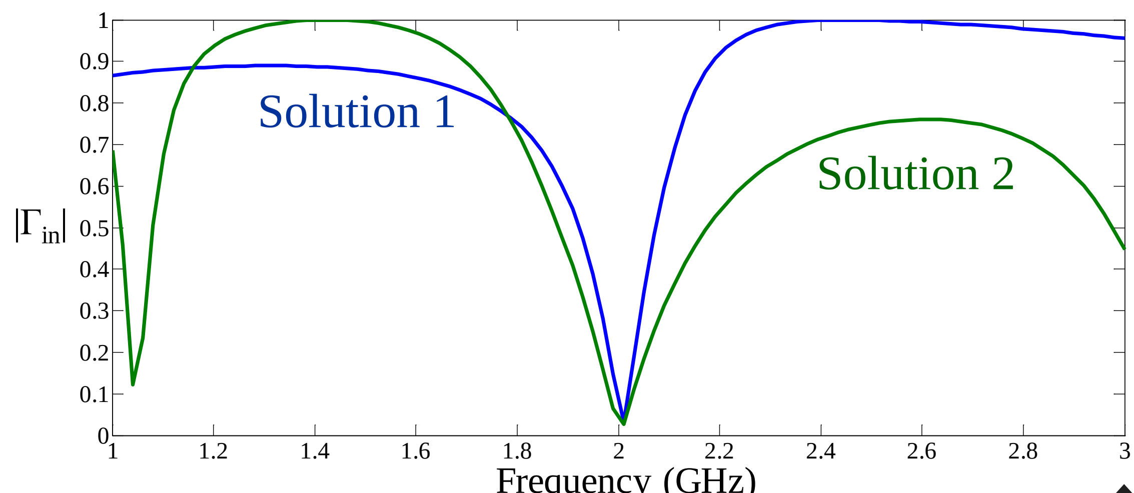
Reflection coefficient: $\Gamma_{in} = \dfrac{Z_3 \parallel Z_2 - Z_0}{Z_3 \parallel Z_2 + Z_0}$

$$Z_1 = -jZ_0 \cot\beta l_1$$

$$\text{where } \cot\beta l_1 = \cot\left(\frac{2\pi}{\lambda}\right)(m\lambda_0) = \cot\frac{2\pi f \times m}{f_0}$$

$$\text{Similarly, } Z_3 = -jZ_0 \cot\beta l_2$$

$$\cot\beta l_2 = \cot\frac{2\pi f \times n}{f_0}$$



Contents

3.5 Improving the Matching Bandwidth

Q Factor

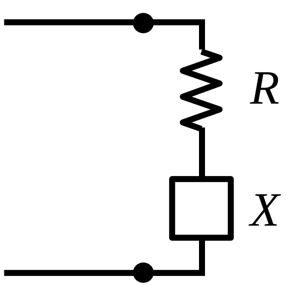
Original definition of quality factor (Q factor):

$$Q = 2\pi \times \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$$

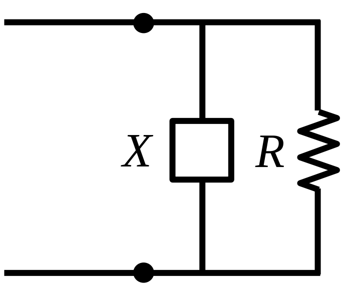
- In microwave engineering the Q factor is a dimensionless parameter that describes how underdamped a resonator or load impedance is
- It characterizes the bandwidth of the load impedance

	Physical meaning	Damping situation
Higher Q	A lower rate of energy loss relative to the stored energy of the resonator	Q > ½: Underdamped
Lower Q	The energy tends to dissipate rather than store in the circuit	Q < ½: Overdamped

Q factor in electrical system:

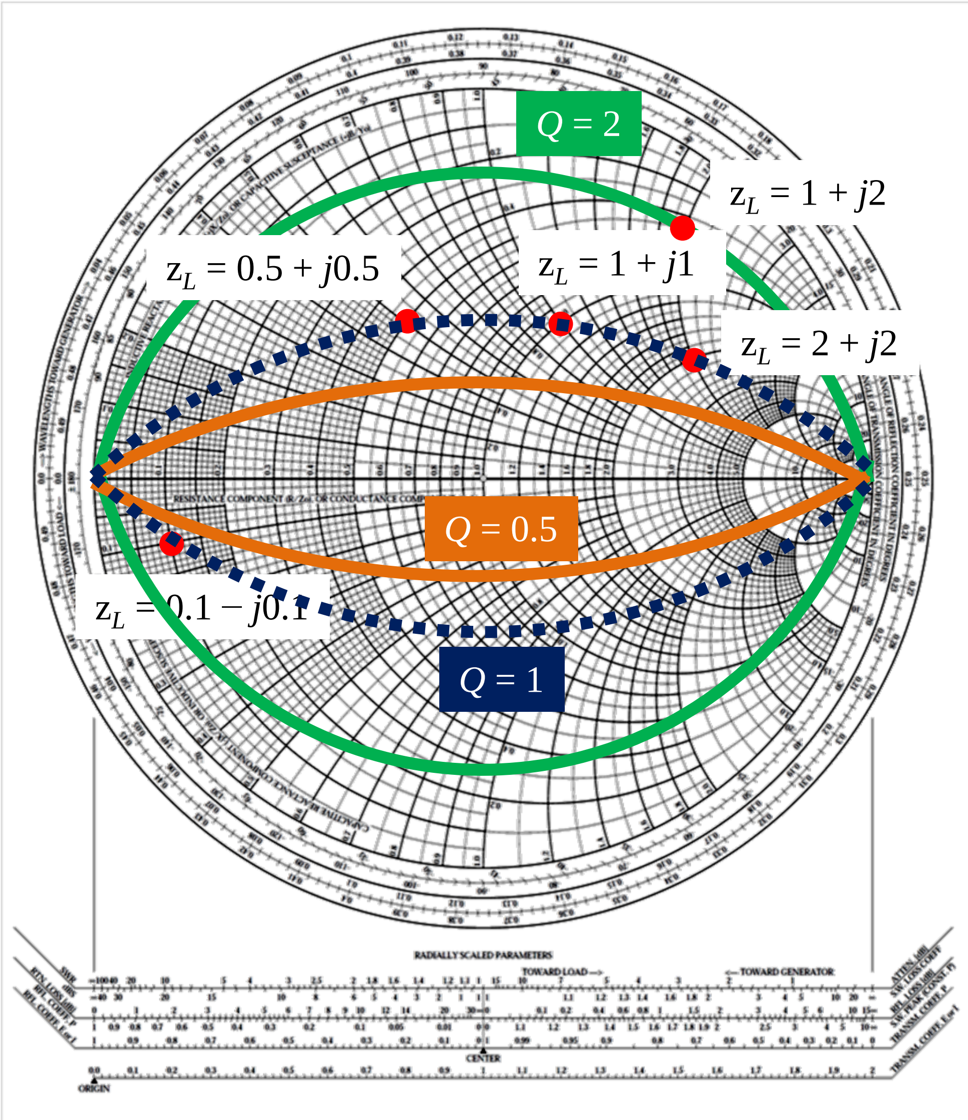


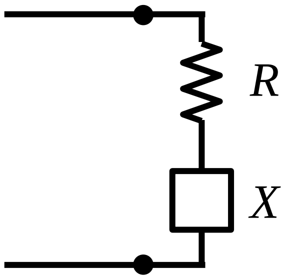
$$Q = \frac{|X|}{R} = \frac{\omega_0 L}{R} = \frac{1}{R\omega_0 C}$$



$$Q = \frac{|B|}{G} = \frac{R}{|X|} = \frac{R}{\omega_0 L} = R\omega_0 C$$

Constant Q Path on Smith Chart





$$Q = \frac{X}{R} = \frac{\omega_0 L}{R} = \frac{1}{R\omega_0 C}$$

- Since the Q factor is related to the impedance, we can locate the Q factor on Smith chart
  1. $z_L = 1 + j1$
  2. $z_L = 0.5 + j0.5$
  3. $z_L = 2 + j2$
  4. $z_L = 0.1 - j0.1$

  Constant $Q = 1$ path on Smith chart

- Following similar operations:
  1. Constant $Q = 2$ path
  2. Constant $Q = 0.5$ path

The Relation between Q and Bandwidth

After some manipulation, the Q factor has following relation to BW:

$$Q = \frac{f_{res}}{f_2 - f_1}$$

(valid for Q ≫ 1)

- f_2 and f_1 are the frequencies that have a 3-dB return loss
- $f_2 - f_1$ is the half-power bandwidth
- Therefore, bandwidth is inversely proportional to the Q factor

Another viewpoint of Q-BW relation:

$$Q(\omega_0) \approx \frac{\omega_0}{2R(\omega_0)} X'(\omega_0)$$

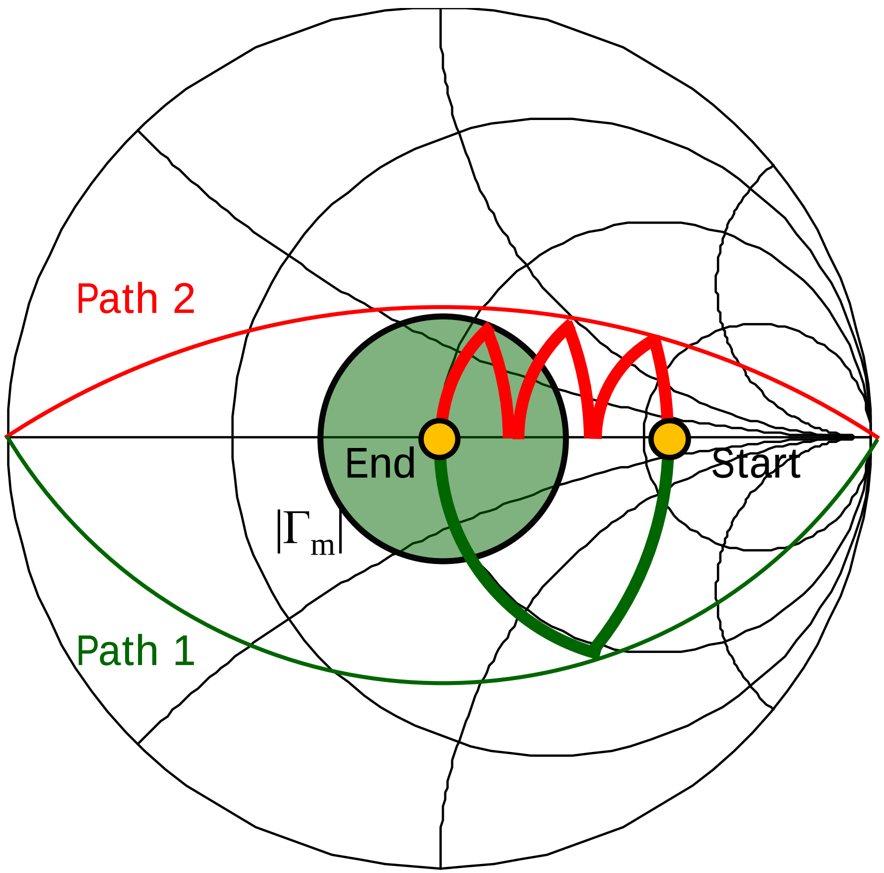
(valid for resonance region)

- The smaller the Q factor, the slower the variation of the reactance is
- So, the variation of reflection coefficient becomes more gently

If we'd like to achieve broadband impedance matching, the associated Q factor must be as small as possible

A. D. Yaghjian and S. R. Best, "Impedance, bandwidth, and *Q* of antennas," *IEEE Trans. Antennas Propag.*, vol. 53, no. 4, pp. 1298-1324, Apr. 2005.

How to Expand the Impedance Bandwidth?



Two matching paths on Smith chart:

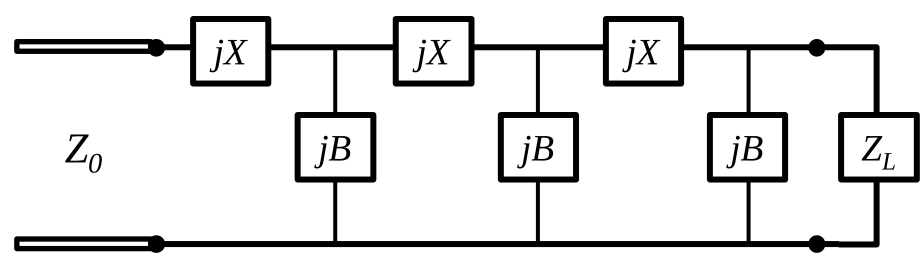
- Path 2 has the lower value of Q factor
- Therefore, do impedance matching via path 2 has a greater bandwidth

If we use single L-section network:

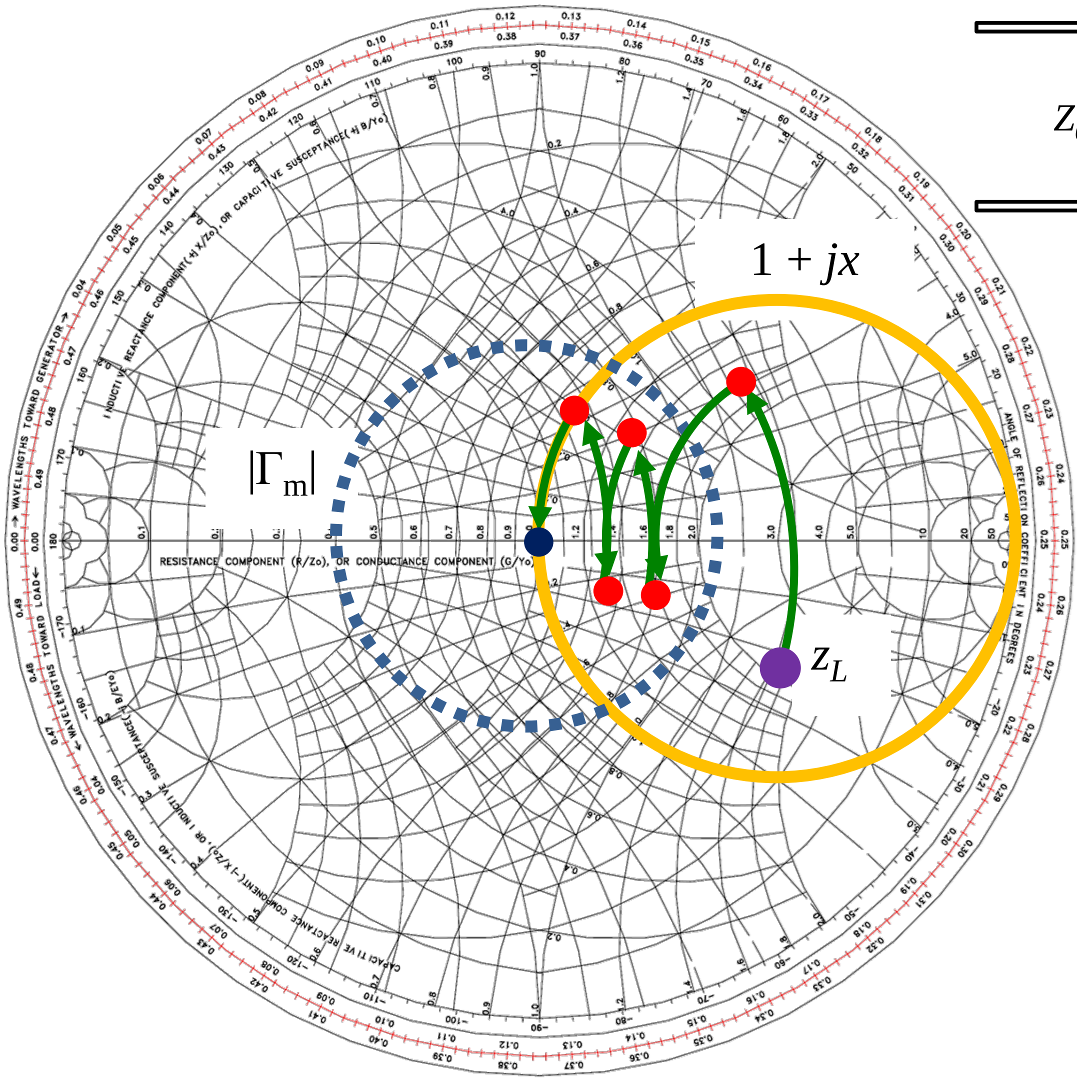
- The path of Q factor has no degree of freedom to broaden the impedance bandwidth

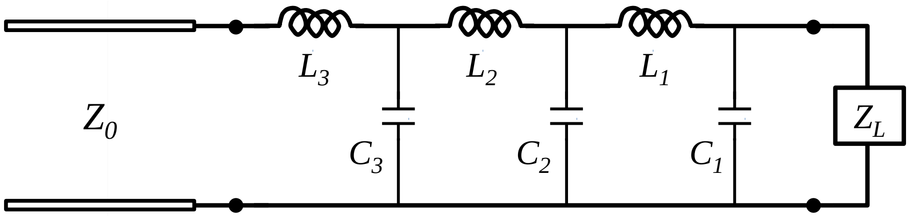
How to select Path 1 as the locus?

- It utilizes a 3 L-C section matching, resulting in a smaller Q path, which is related to a wider bandwidth



Example Multi-L-Section Matching Circuit

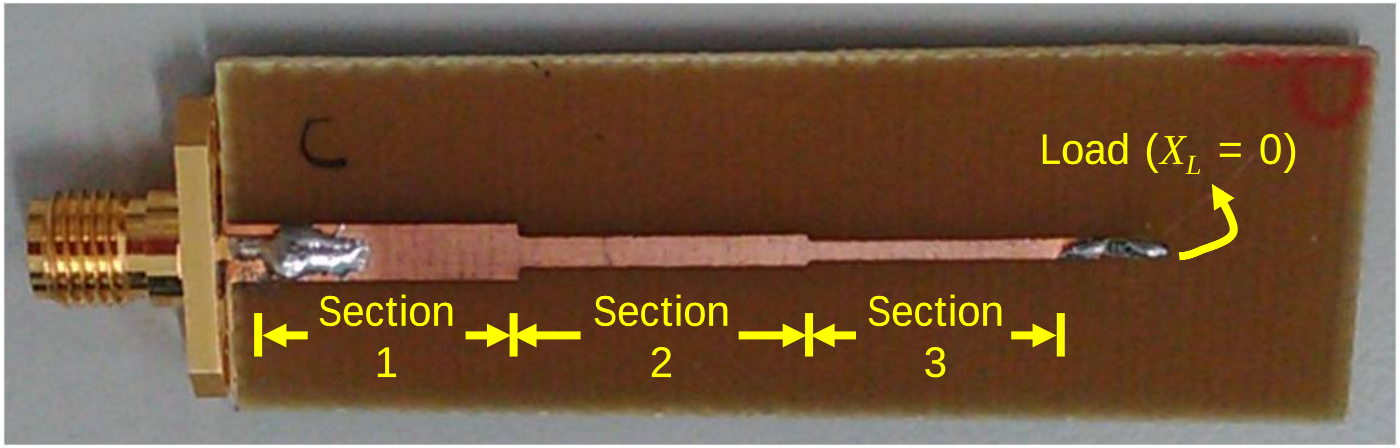




- Designing such a locus requires 3 capacitors and 3 inductors
- The values of the reactive elements can be found using the Smith chart approach
- Another viewpoint to have a greater bandwidth:
  - To make the path as short as possible
  - To make the locus locate as near the center as possible

Example

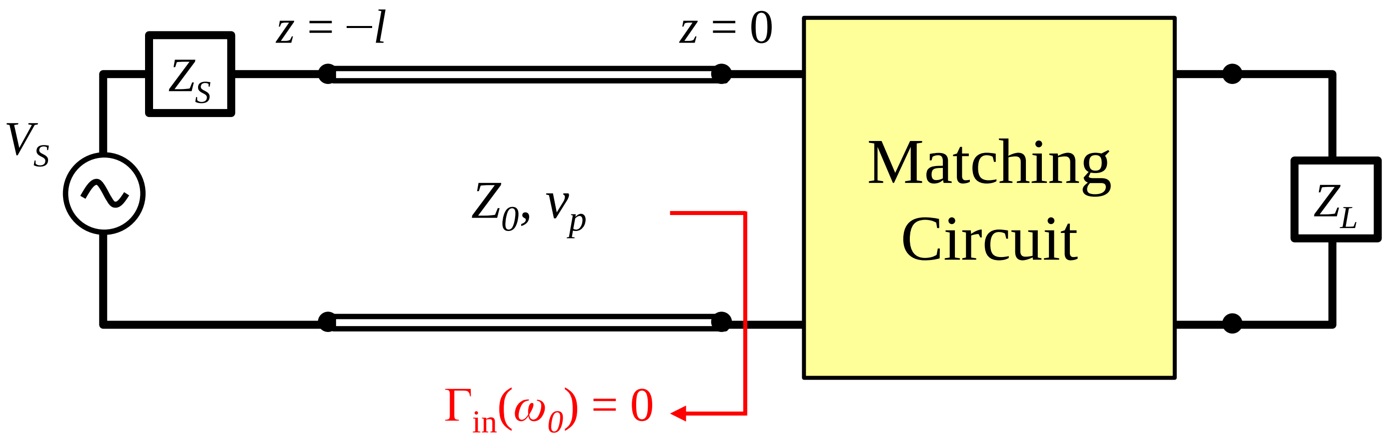
Multi-Section Matching Transformers



Three-stage quarter-wavelength transformer:

- Putting more sections together can expand the bandwidth
- The length of each section is $\lambda_g/4$; the associated characteristic impedance has specific design approaches
- Two design approaches are widely used:
  - **Binomial multi-section matching transformer:** The frequency response is as flat as possible near the design frequency
  - **Chebyshev multi-section matching transformer:** It results in a larger bandwidth than the binomial design approach; however, it optimizes the bandwidth at the expense of pass-band ripples

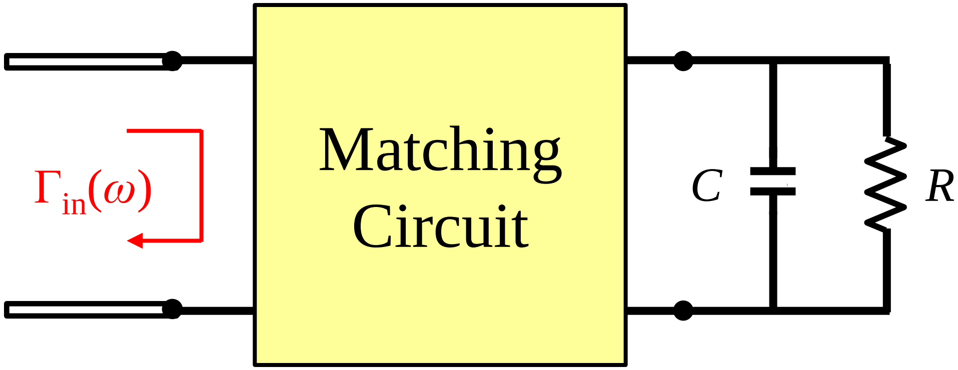
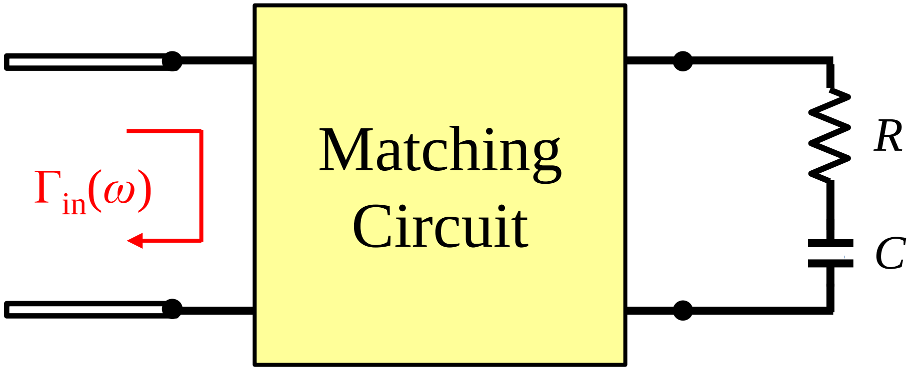
Matching Circuits Have Their Limits



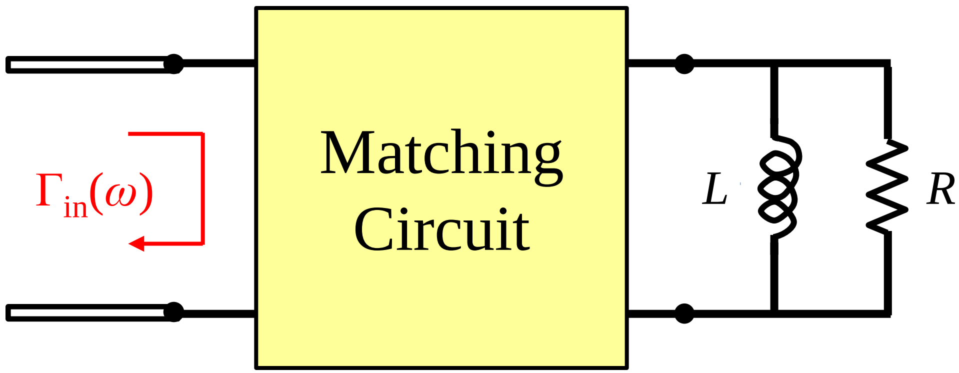
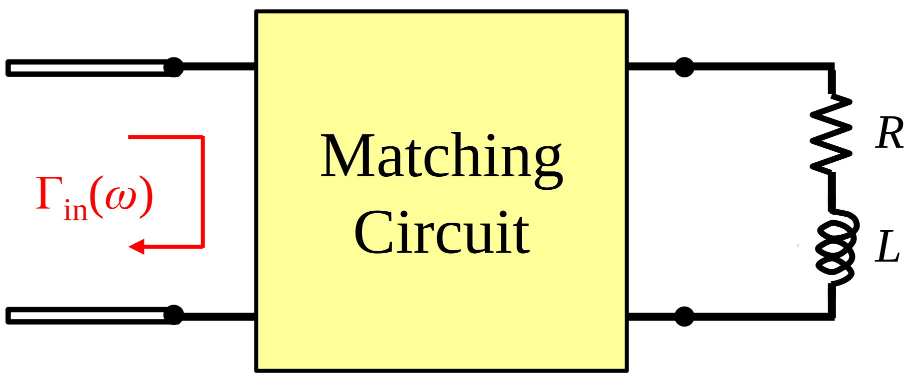
Do impedance matching circuits have performance limits?

1. Can we achieve a perfect match ($\Gamma_{\text{in}} = 0$) over a specified bandwidth?
2. If not, what is the best we can do?
3. What's the tradeoff between Γ_{m} (the maximum allowable Γ) and the bandwidth?
4. How complex must the matching circuit be for a given specification?

The Bode-Fano Criterion (1/2)

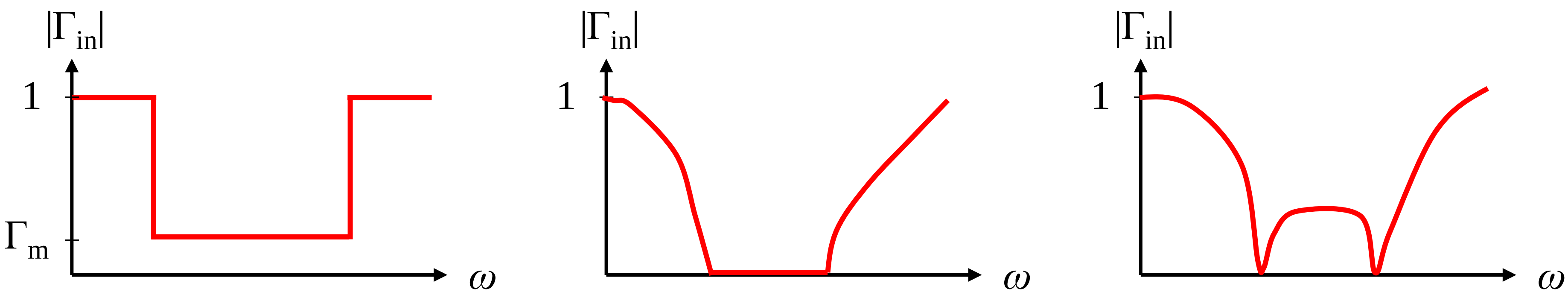
Circuit	Bode-Fano Limit
	$\int_0^{\infty} \ln \frac{1}{\lvert\Gamma(\omega)\rvert} d\omega < \frac{\pi}{RC}$
	$\int_0^{\infty} \frac{1}{\omega^2} \ln \frac{1}{\lvert\Gamma(\omega)\rvert} d\omega < \pi RC$

The Bode-Fano Criterion (2/2)

Circuit	Bode-Fano Limit
	$\int_0^\infty \frac{1}{\omega^2} \ln \frac{1}{\lvert\Gamma(\omega)\rvert} d\omega < \frac{\pi L}{R}$
	$\int_0^\infty \ln \frac{1}{\lvert\Gamma(\omega)\rvert} d\omega < \frac{\pi R}{L}$

Tradeoff Between BW and Γ_m

These formulas imply:



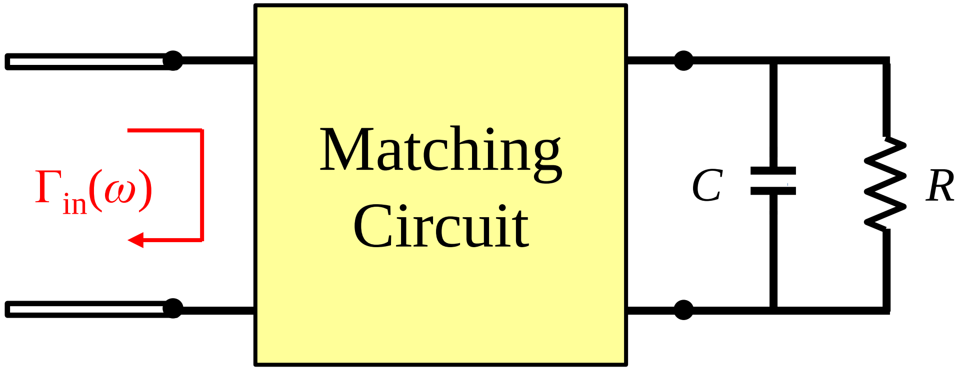
Optimum matching circuit | Not realizable | Realizable

- For a given load, a broader bandwidth $(\Delta\omega)$ can be achieved only at the expense of a higher Γ_m
- Γ_m cannot be 0 over the entire operational band unless $\Delta\omega = 0$
- A perfect match can be achieved only at a finite number of frequencies
- Assuming $Z_L = R + jX$. As R and/or X increases, the quality of matching $(\Delta\omega)$ must decreases

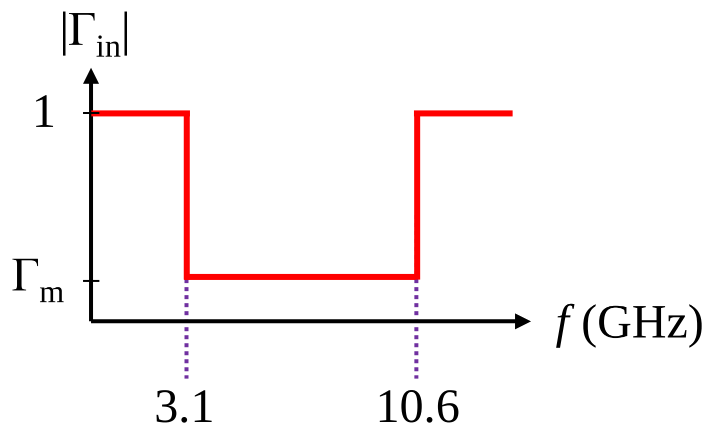
EX 3.8 The Bode-Fano Criterion (1/2)

- An ultra wideband (UWB) transmitter, operating from 3.1 to 10.6 GHz, drives a parallel RC load with $R = 75\ \Omega$ and $C = 0.6$ pF

➡ What is the return loss that can be obtained with an optimum matching circuit?



The Bode-Fano Criterion (2/2)



- For a parallel RC circuit: $\int_0^{\infty} \ln \frac{1}{|\Gamma(\omega)|} d\omega < \frac{\pi}{RC}$

- The optimum scenario: $\left(\ln \frac{1}{\Gamma_m}\right) \Delta\omega < \frac{\pi}{RC}$

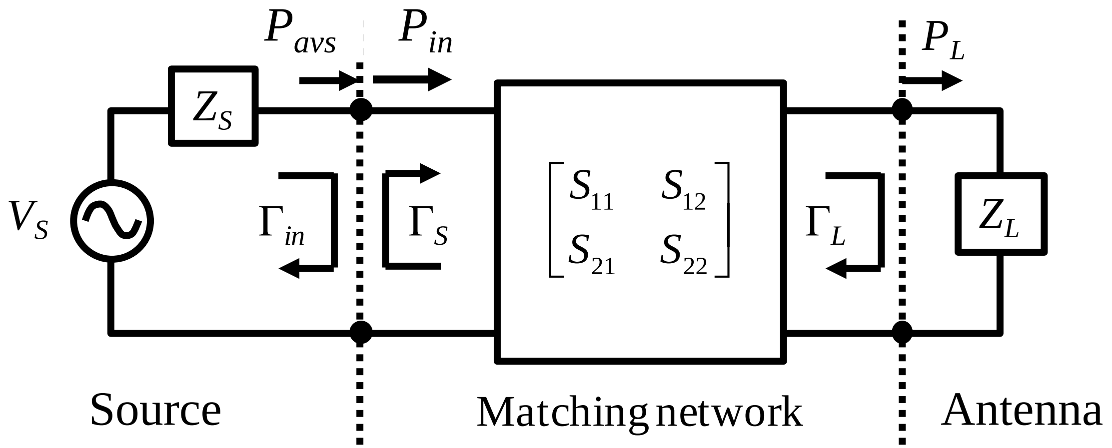
$$\left(\ln \frac{1}{\Gamma_m}\right) < \frac{\pi}{\Delta\omega RC} = \frac{\pi}{2\pi \times (10.6 - 3.1) \times 10^9 \times 75 \times 0.6 \times 10^{-12}} = 1.481$$

$$\Gamma_m > 0.228, \ \text{Return loss} < -20 \log \Gamma_m = 12.86 \ dB$$

Contents

3.6 Additional Loss

Insertion Loss Characterization

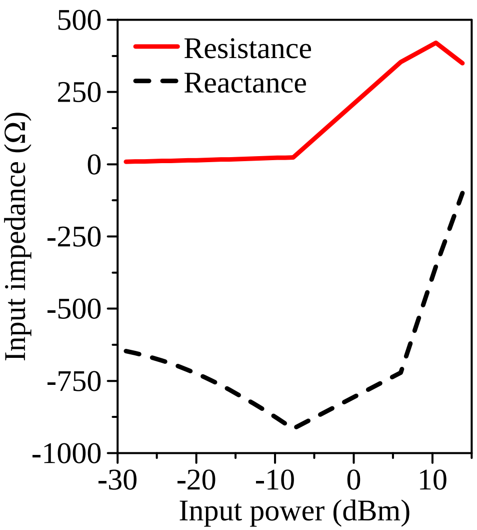


The additional loss of the matching circuit is computed by the operational power gain:

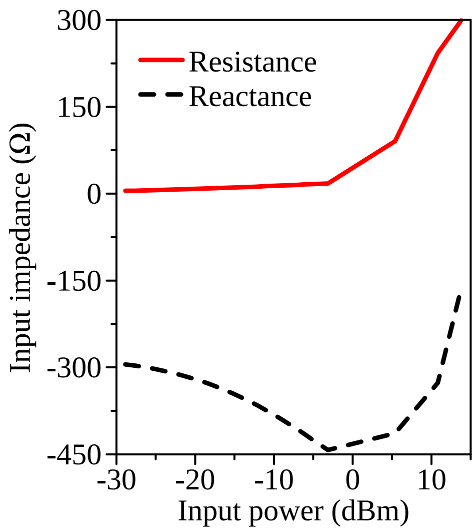
$$G_p = \frac{P_L}{P_{in}} = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

Input Impedance of the Rectifier

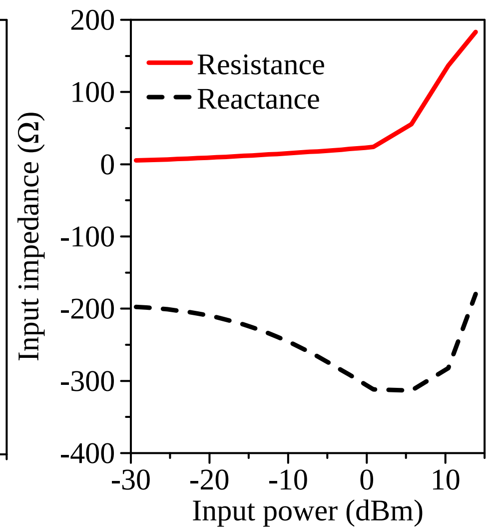
900 MHz



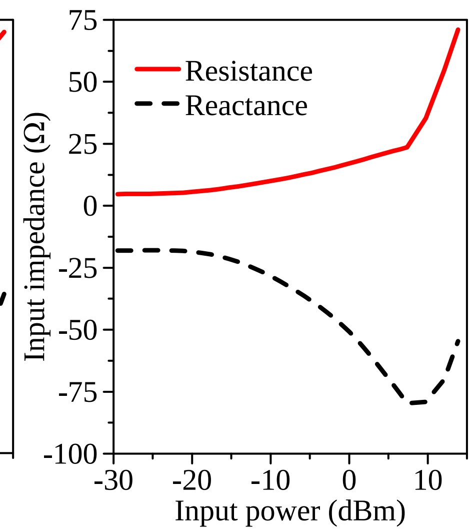
1.8 GHz

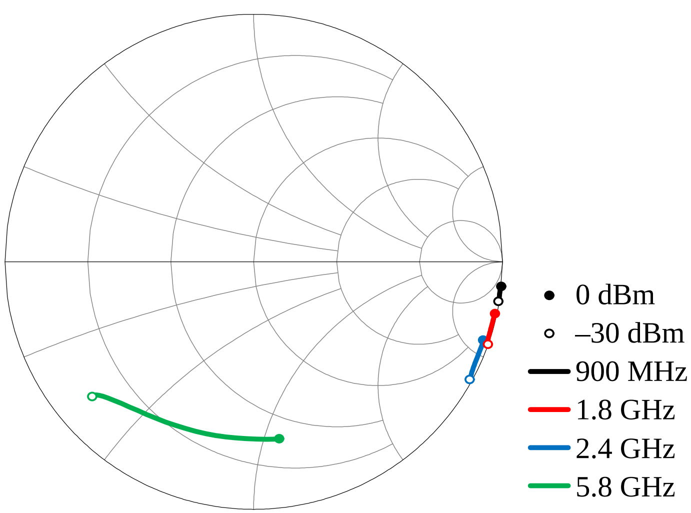


2.4 GHz



5.8 GHz

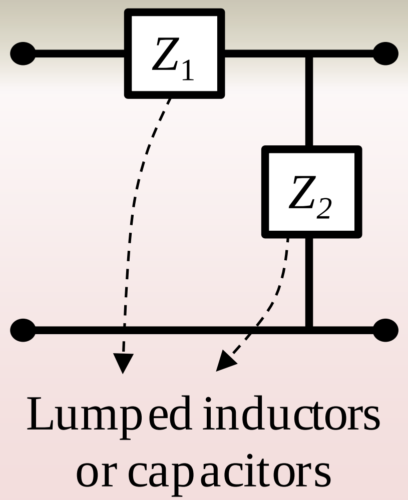




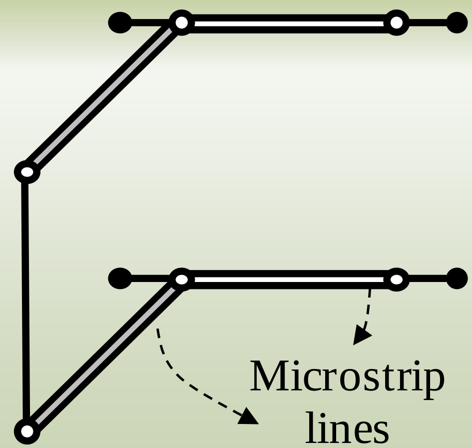
- These impedances are poorly matched to Z_S and highly capacitive
- The input resistances are particularly small as compared to the magnitude of input reactances
- The reactance becomes more capacitive as the frequency decreases
- The input reactance of the 900-MHz rectifier is more significant and capacitive than that of the other rectifiers

Topology of Matching Circuits

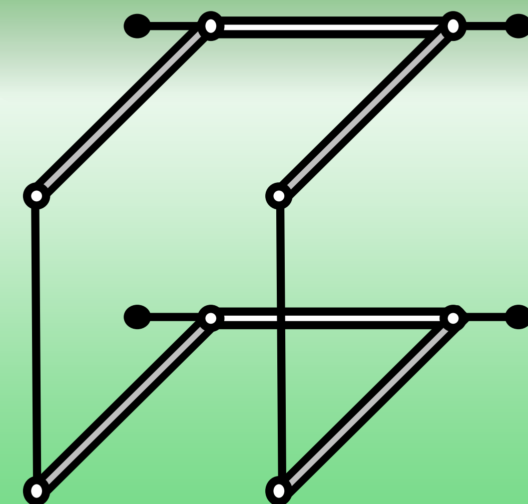
LC networks



Single-stub tuners

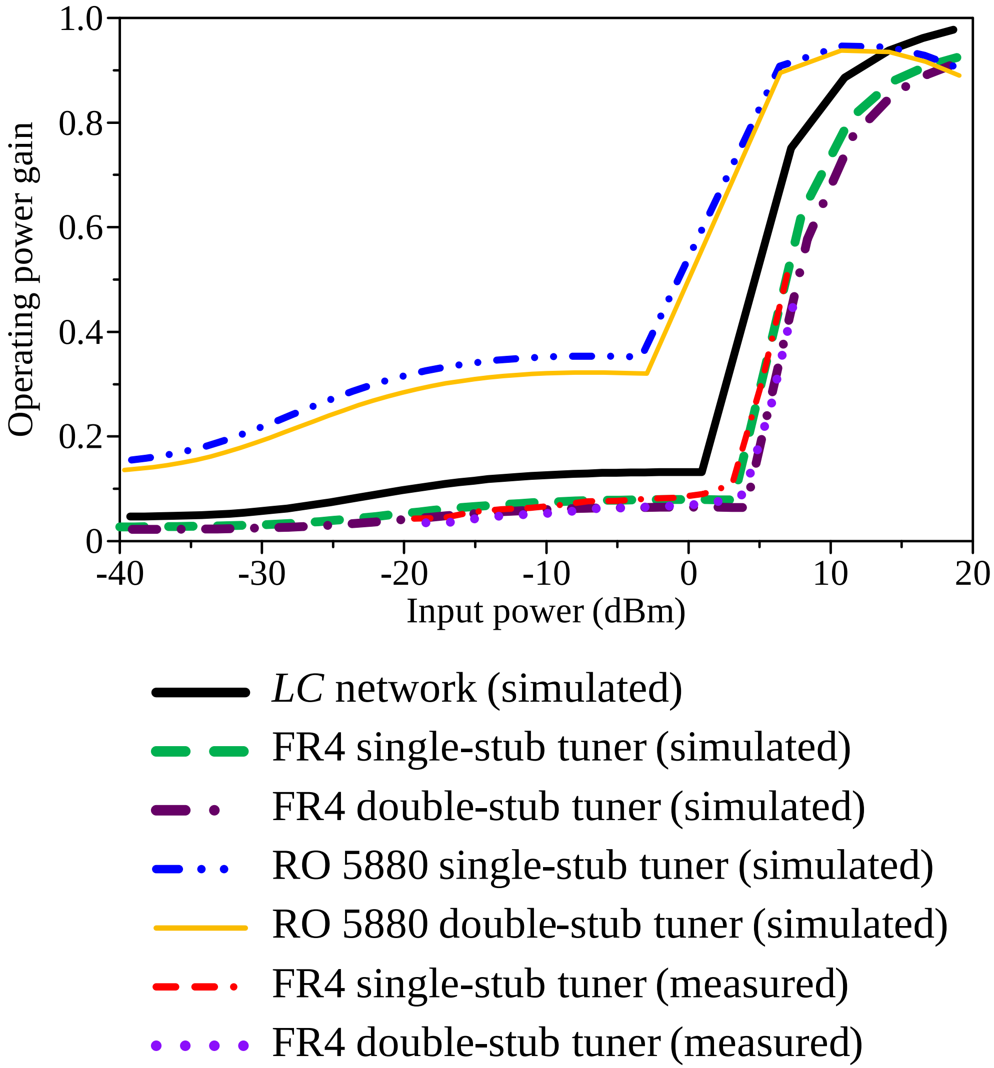


Double-stub tuners



- The design goal is to minimize $|\Gamma_{in}|$ at P_{in} = –10 dBm
- The *LC* networks are modeled by using Murata capacitors (GRM 15 series) and inductors (LQP 15M series)
- The transmission-line-based techniques are constructed by using microstrip lines fabricated on either FR4 or Rogers Corp. Duroid 5880 substrates

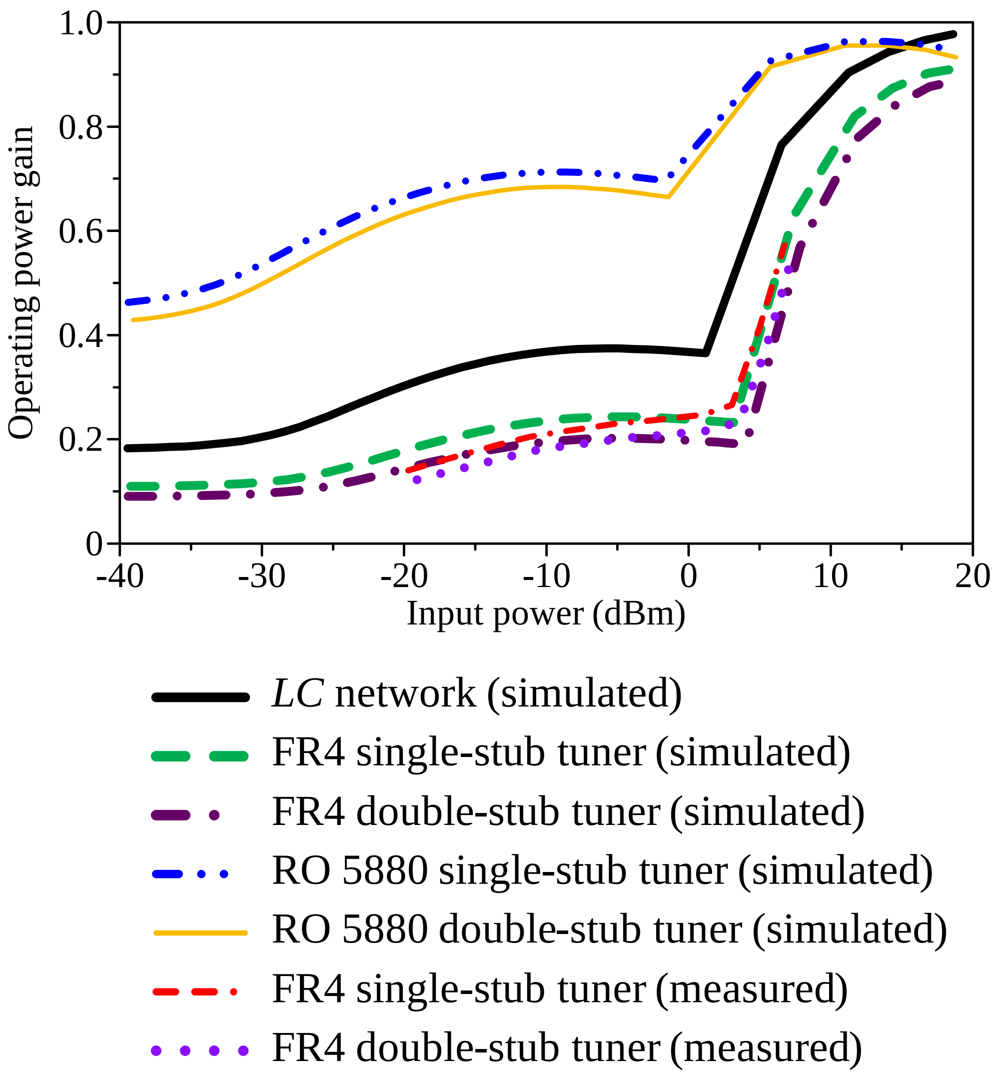
Results of 900-MHz Networks



- Less than 10% of power is transferred to the rectifier if *LC* networks or transmission-line-based impedance tuners fabricated on FR4 are used
- Only when the input power is larger than 10 dBm can energy successfully transmit to the rectifier
- Even though a less lossy substrate is used, in the interested power region more than 65% of power is dissipated

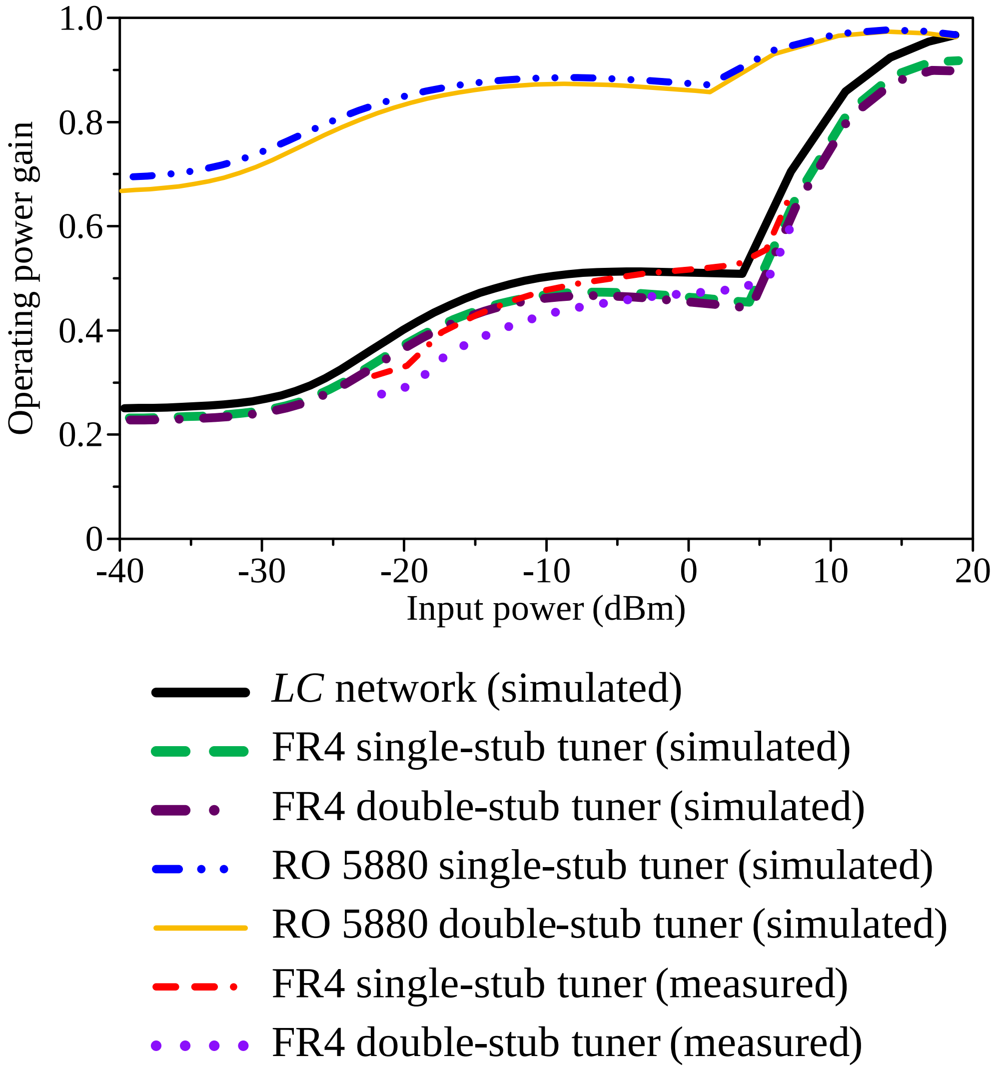
Y.-S. Chen and C.-W. Chiu, "Insertion loss characterization of impedance matching networks for low-power rectennas," *IEEE Trans. Compon. Packag. Manuf. Technol.*, vol. 8, no, 9, pp. 1632–1641, Sep. 2018.

Results of 1.8-GHz Networks



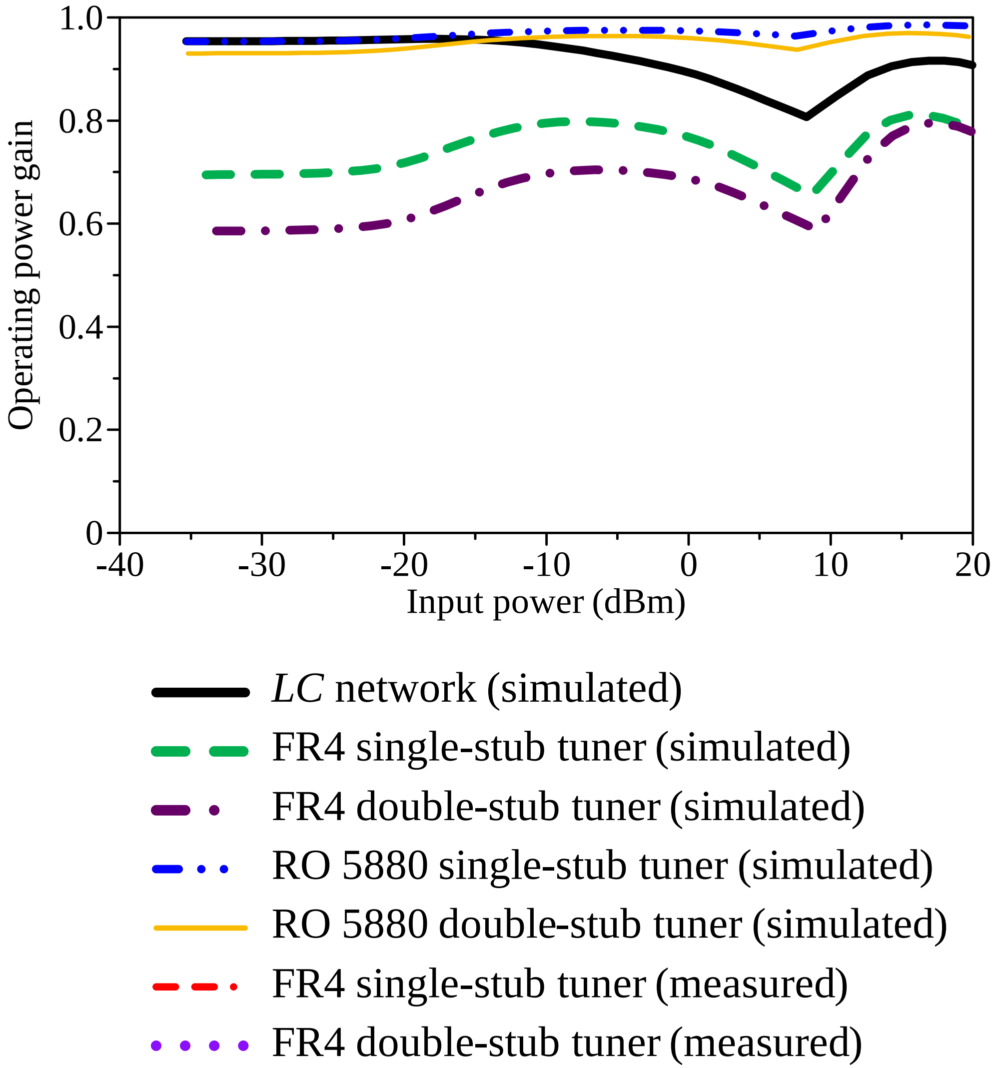
- Similar results can be observed as operating the rectennas at 1.8 GHz
- If *LC* networks or transmission-line-based impedance tuners fabricated on FR4 are used, over 50% of power is consumed by the lossy elements of the matching networks
- Such additional loss cannot be reduced unless Duroid 5880 is applied

Results of 2.4-GHz Networks



- Similar results can be observed as operating the rectennas at 2.4 GHz
- The additional loss is closely related to the operational frequency

Results of 5.8-GHz Networks



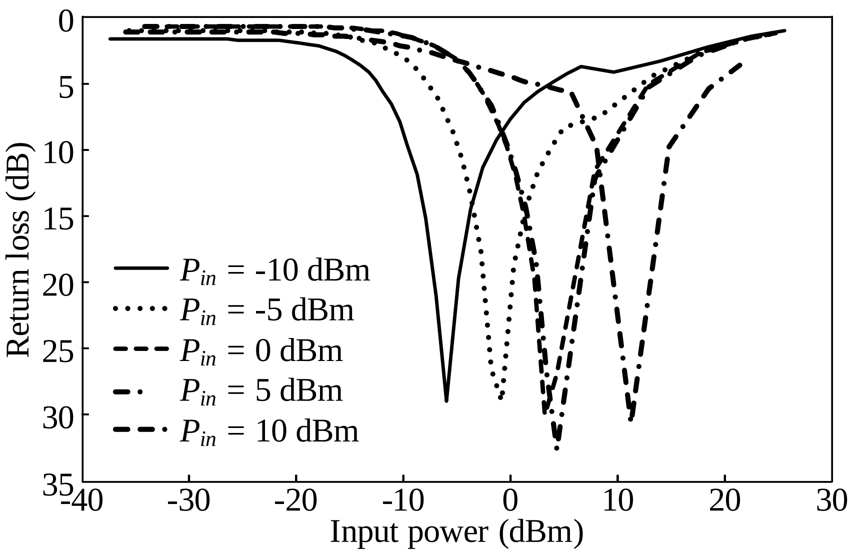
- Only the 5.8-GHz matching circuits provide slight loss
- But the rectifying efficiency has the poorest performance at this frequency

$$G_p = \frac{P_L}{P_{in}} = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

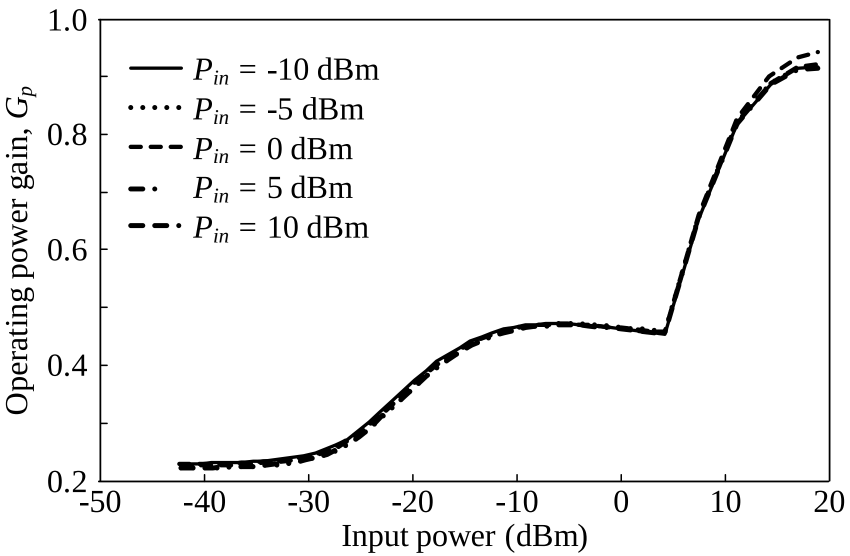
- G_p is not the function of power
- But G_p is the function of the input impedance of the rectifier

Associated Return Loss

Return loss

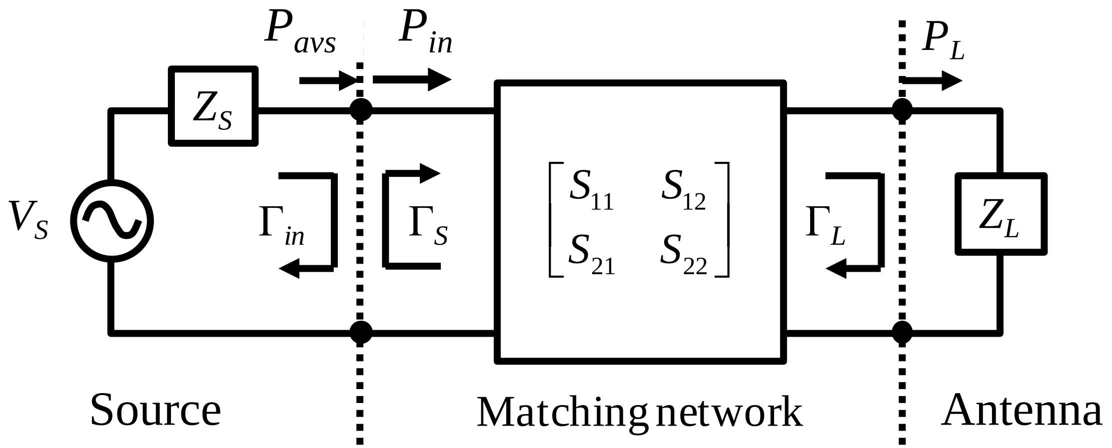


Operating power gain



All the results are valid if the matching circuit is designed at other levels of input power

Transducer Power Gain

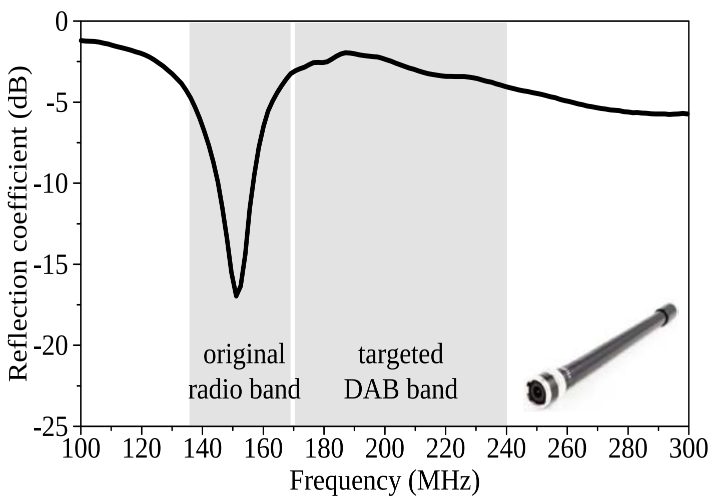


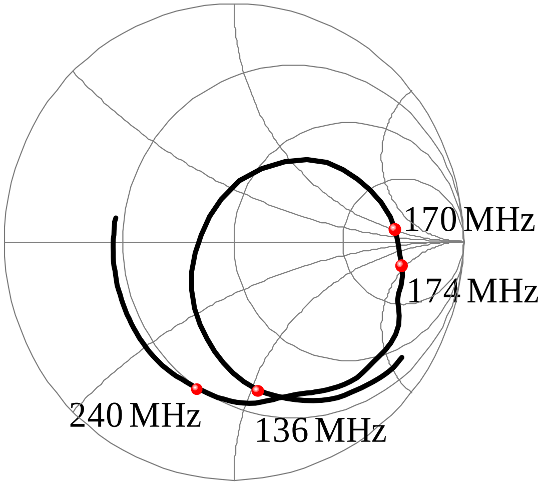
The overall matching effect is determined by the transducer power gain:

$$G_T = \frac{P_L}{P_{avs}} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|}$$

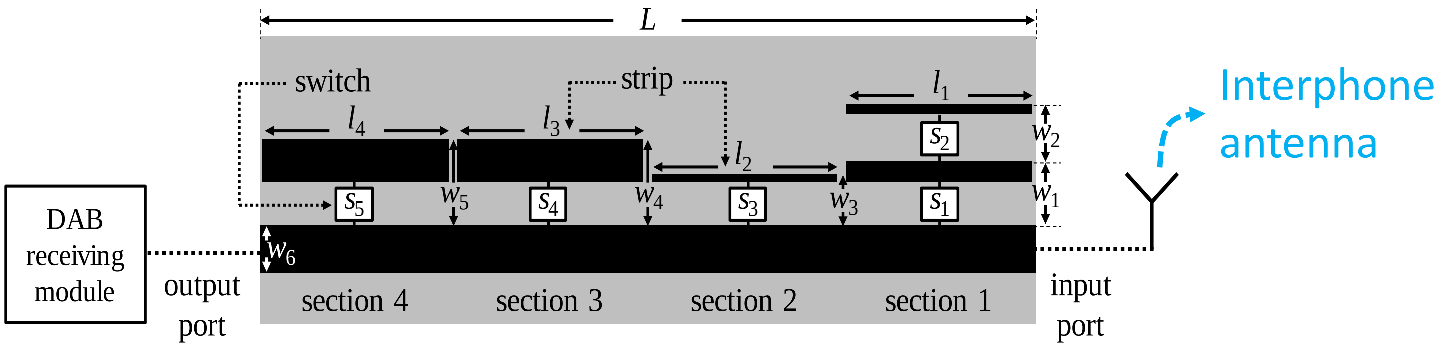
Example: Reconfigurable Matching Network

- An interphone antenna that originally operates at 140–170 MHz:

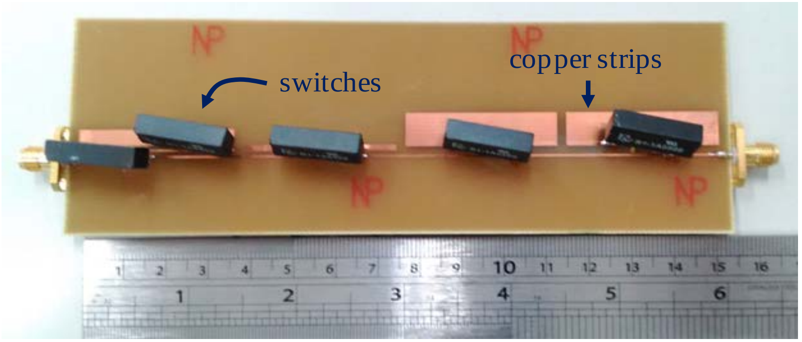


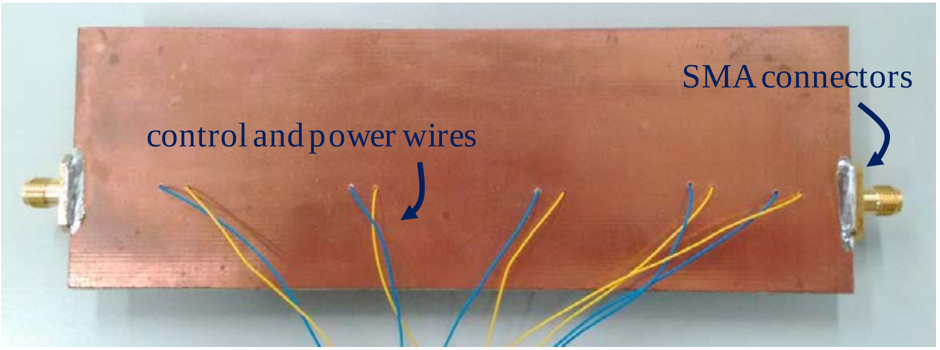


- A reconfigurable matching network that attempts to recover the matching at 170–240 MHz:

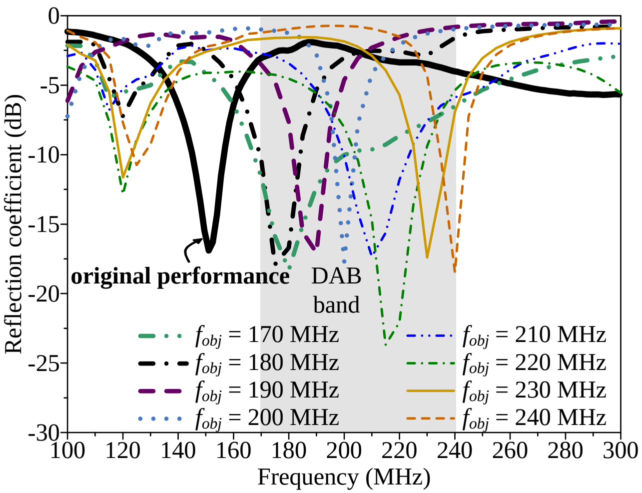


Performance Characterization





- Input reflection coefficient:



- Transducer power gain:

f (MHz)	Return loss (dB)	G_T (dB)	Bandwidth (MHz)
170	11.61	−1.83	30.1
180	16.71	−1.52	17.3
190	17.08	−1.51	12.5
200	17.74	−1.48	9.2
210	17.36	−0.97	22.7
220	22.09	−0.79	26.3
230	17.44	−1.66	12.6
240	18.49	−0.84	10.8

- Far-field efficiency: