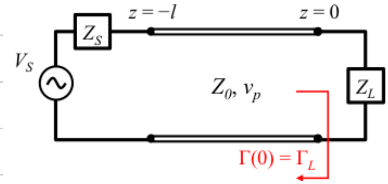
§ Impedance Matching Network.

3.1 Objective of Impedance Matching



Reflection Coefficient

$$\Gamma(z)=\frac{V^- e^{j\beta z}}{V^+ e^{-j\beta z}}=\Gamma_L e^{2j\beta z}.$$

$$\Gamma_L=\frac{Z_L-Z_0}{Z_L+Z_0}$$

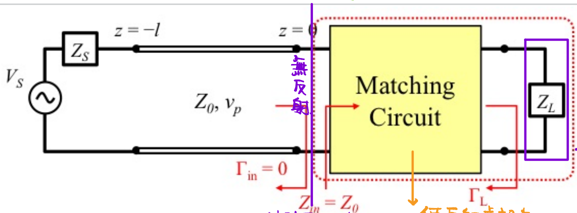
Voltage Wave

$$V(z)=V^+ e^{-j\beta z}+V^- e^{j\beta z}.$$

$$I(z)=\frac{1}{Z_0}\left(V^+ e^{-j\beta z}-V^- e^{j\beta z}\right)$$

when $Z_0 \neq Z_L \Rightarrow$ create reflected voltage wave $V^- e^{j\beta z}$.

wastes transmitting power $P_{av}(z)=\frac{|V^+|^2}{2Z_0}\left(1-|\Gamma(z)|^2\right)$



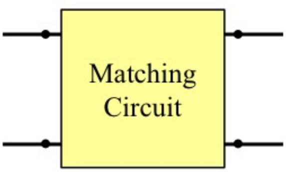
無反射.

仍然有反射.

消除反射端0.

使反射虛部為0.

- with well design. making $Z_{in}=Z_0$ $|\Gamma_{in}|=0$. no reflected voltage wave to source.
- Γ_L exists. matching circuits like buffer. storing transmitting power from Vs.



matching circuit must be lossless components.

- Lump Inductors. Capacitors
- Transmittion-line stubs

Factors selecting matching circuit.

- complexity. 複雜度
- Implementation. 实作性
- Size 尺寸.
- Bandwidth. 頻寬
- Adjustability. 可調性.

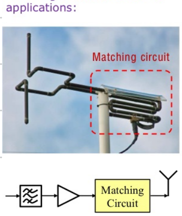
Technique of Impedance Matching.

Z_L is purely real: quarter wavelength transformer.

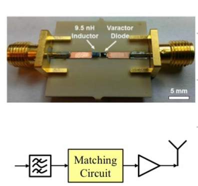
Z_L is complex. lump component.

single / double stubs.

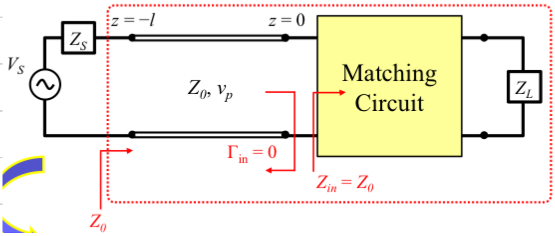
Matching circuit for antenna applications:



Matching circuit for power amplifiers:



Power Delivered to Load.



transmittion line connects to match load.
⇒ impedance on any point Z is Z_0.

not all power generated by source transmit to load.

to achieve maximum power. $Z_0 = Z_S^*$

Pratical Scenario.

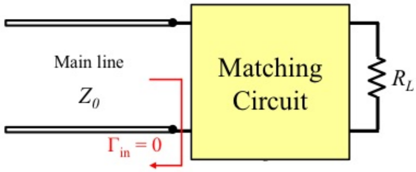
V_s. Z_s (assuming purely real). Z_L are given.

⇒ choose proper Z_0. matching circuit that maximum power deliver to load.

choosing $Z_0 = Z_S$. design matching circuit that $Z_{in} = Z_S$.

power efficiency = 50% (maximum power to load)

3.2 Quarter Wavelength Transformers



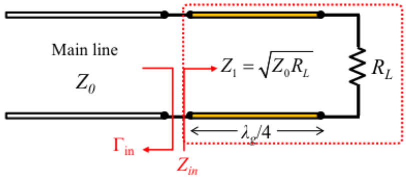
Z_0, R_L, both real and given.

if $Z_0 \neq R_L$, (no matching network), then,

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

⇒ by using lossless peice of transmission line of characteristic impedance Z_1, length $\frac{\lambda}{4}$ where $Z_1 = \sqrt{Z_0 R_L}$

Theorem of quarter wavelength matching.

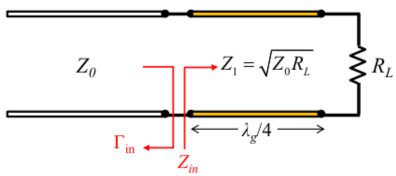


to having perfect matching. ⇒ $Z_{in} = Z_0$

$$Z_{in} = Z_1\left(\frac{R_L + jZ_1 \tan\beta l}{Z_1 + jR_L \tan\beta l}\right) \Rightarrow l = \frac{\lambda}{4} \quad \therefore Z_{in} = \frac{Z_1^2}{R_L}$$

if $Z_1 = \sqrt{Z_0 R_L}$ $Z_{in} = Z_0$, $\Gamma_{in} = 0$.

Characteristics of $\lambda/4$ transformers



$\lambda/4$ transformers makes $\Gamma_{in} = 0$.

$\because Z_1 \neq R_L$ $\therefore$ reflected wave still exists.

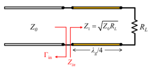
but all power sucessfully transmitts to R_L.

☆ Drawbacks.

only when line length $= \lambda/4$.

perfect match only in one frequency.

limited to real-value load impedance



- $R_L = 100\ \Omega$ and $Z_0 = 50\ \Omega$ are given
  1. Find the characteristic impedance of the matching section (Z_1)
  2. Plot $|\Gamma_{in}|$ vs. normalized frequency f/f_0, where f_0 is the frequency at which the line is $\lambda/4$ long

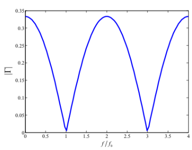
characteristic impedance: $Z_1 = \sqrt{Z_0 R_L}$.

$Z_1 = 70.71\Omega$.

$$|\Gamma_{in}| = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right|$$

設計頻率的相對波長.

$$Z_{in} = Z_1 \frac{R_L + Z_1 j \tan\beta l}{Z_1 + R_L j \tan\beta l} \qquad \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda_0}{4} = \left(\frac{2\pi f}{v_p} \cdot \frac{v_p}{4 f_0}\right) = \frac{\pi f}{2 f_0}.$$



Frequency f	TL length l
$f = 0$	0
$f = 1f_0$	$l = \lambda_0/4$
$f = 2f_0$	$l = \lambda_0/2$
$f = 3f_0$	$l = 3\lambda_0/4$
$f = 4f_0$	$l = \lambda_0$

- Only at the design frequency f_0, the electrical length of the matching section is $\lambda_0/4$
- At other frequencies, the electrical lengths are different from $\lambda_0/4$
- However, when $f = 3f_0$, we have another perfect matching transformer again (why?)

在 $f = f_0$ 與 $f = 3f_0$ 時.

$\because \tan\beta l = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow Z_1 = \sqrt{Z_0 R_L}$ (match)

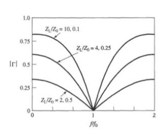
$\therefore$ 此時會有最小的Γ.

- The fractional bandwidth of a quarter-wavelength transformer can be derived as:

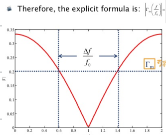
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right)$$

Γ_m Magnitude of the tolerable reflection coefficient
Z_L Load impedance
Z_0 Characteristic impedance of the main line

- What is the physical meaning from this formula?
  - If Γ_m becomes larger, the bandwidth increases
  - When Z_L becomes closer to Z_0, the bandwidth increases



- Therefore, the explicit formula is: $\left|\Gamma_{in}\left(\frac{f}{f_0}\right)\right| = \left| Z_1\left(\frac{Z_L + jZ_1\tan\left(\frac{\pi f}{2 f_0}\right)}{Z_1 + jZ_L\tan\left(\frac{\pi f}{2 f_0}\right)}\right) - Z_0 \middle/ Z_1\left(\frac{Z_L + jZ_1\tan\left(\frac{\pi f}{2 f_0}\right)}{Z_1 + jZ_L\tan\left(\frac{\pi f}{2 f_0}\right)}\right) + Z_0 \right|$



Γ_m 可接受反射最小值.

- If we set a maximum value, Γ_m, that can be tolerated, we can define the bandwidth of the matching circuit:

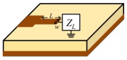
$\frac{\Delta f}{f_0} = 0.8$ Fractional bandwidth

$= 80\ \%$ Percentage bandwidth

(However, in practical situations, Z_L is also the function of frequency!)

- For higher frequencies the line looks electrically longer
- For lower frequencies the line looks electrically shorter

故定傳輸線長度 (看f).



- Design a quarter-wavelength transformer to match $Z_L = 10\ \Omega$ to $Z_0 = 50\ \Omega$
- Design frequency: 1 GHz
- Requirement of tolerable bandwidth: VSWR < 1.5
- Materials at hand: FR4 substrate with $h = 1$ mm ($\varepsilon_r = 4.4$)

1. Find the length L
2. Find the width w
3. Find the characteristic impedance Z_l
4. Find the operational bandwidth of this transformer

$Z_1 = \sqrt{Z_0 Z_L} = 22.36\,\Omega.$

freq $= 1\text{G} \Rightarrow \lambda = 0.3\text{m}.$

$W = 6.03\text{mm}.$

$\varepsilon_{eff} = 3.71.$

$\lambda_g = \dfrac{\lambda_0}{\sqrt{\varepsilon_{eff}}} = 155.7\text{mm}.$

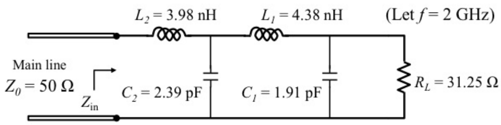
$L = \dfrac{\lambda_g}{4} = 38.94\text{mm}.$

at VSWR $= 1.5.$

$\Gamma_m = \dfrac{1.5-1}{1.5+1} = 0.2.$

$\dfrac{\Delta f}{f_0} = 29\%.$

3.3 Matching with Lump Elements



	L	C	
Impedance	$j\omega L$	$-j\frac{1}{\omega C}$	Series 串接
Nor-Impedance	$\frac{j\omega L}{Z_0}$	$-j\frac{1}{\omega C Z_0}$	
Admittance	$-j\frac{1}{\omega L}$	$j\omega C$	Shunt 跨接
Nor-Admittance	$-j\frac{1}{\omega L Y_0}$	$\frac{j\omega C}{Y_0}$	

For series connection.

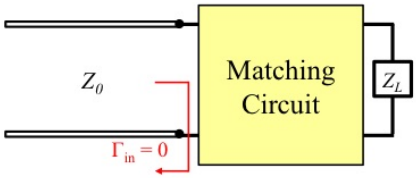
Using Z chart

Impedance moves constant r circle

For shunt connection.

Using Y chart

Admittance moves constant g circle



if $Z_0 \neq Z_L$.

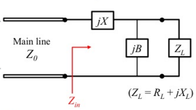
$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ (mismatch)

by applying matching circuit

designing combination reactive elements < L型>

reactive components be inductors or capacitors

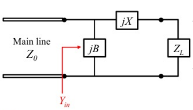
L-section: Using 2 reactive elements



- Goal: $Z_{in} = Z_0$ (so that $\Gamma_{in} = 0$)
- Used for $R_L > Z_0$
- Analytic solutions (2 solutions):

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}\left(R_L^2 + X_L^2 - Z_0 R_L\right)}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$



- Goal: $Y_{in} = Y_0$ (so that $\Gamma_{in} = 0$)
- Used for $R_L < Z_0$
- Analytic solutions (2 solutions):

$$X = \pm\sqrt{R_L (Z_0 - R_L)} - X_L$$

$$B = \frac{X + X_L}{Z_0 R_L}$$

1. $R_L < Z_0$.

∵在匹配時倒數第2步要回到 r.g=1 circles

∴若 $R_L < Z_0$，用串联使 R_L' 变大回到 r.g=1 circles.

⇒先串再並.

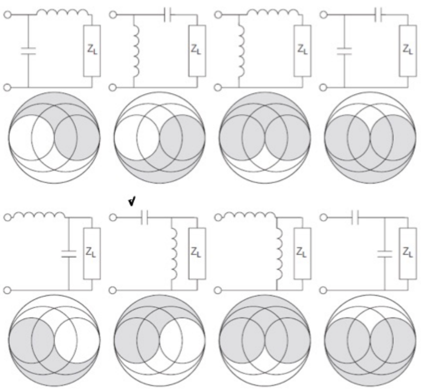
2. $R_L > Z_0$

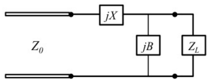
同理要使 R_L' 回到 r.g=1 circles

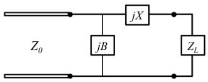
用並使 R_L' 变小回到 r.g=1 circles.

⇒先並再串.

Dynamic Range (white area)





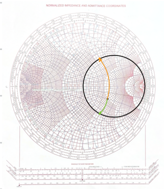


1. Design an L-section matching circuit to match $Z_L = 200 - j100\ \Omega$ to a 100 Ω line (Operational frequency: 500 MHz)
2. Compare the operational bandwidth between the 2 solutions

$Z_L = 200 - j100.$
$Z_0 = 100\Omega$
$f = 0.5G$

① $Z_n = 2 - j1$ (先並再串)

② 最後一步再串.(看Z chart)
→先從y chart著手.



① y-chart 順時旋轉.

$j0.5 - j0.2 = j0.3$ (但∵並Y chart ∴為C)

$\frac{j\omega C}{Y_0} = \frac{j2\pi \times 0.5G \times C_1}{0.01} = j0.3$ $C = 0.6 \times 0.01 \frac{1}{\pi} \times 10^{-9} = 0.95pF$

再串.看Z chart.

$0 - (-j1.22) = j1.22$ (∵串Z chart ∴為L)

$\frac{j\omega L}{Z_0} = j1.22$

$L = 122 \times \frac{1}{2\pi \times 0.5G} = 38.3nH.$

② y-chart 逆時針旋轉.

$-j0.5 - j0.2 = -j0.7$

(但∵並看Y chart ∴為L.)

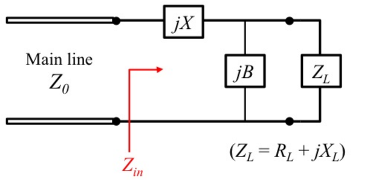
$-j\frac{1}{\omega L Y_0} = -j0.7$

$L = \frac{100}{2\pi \times 0.5G \times 0.7} = 45.5\ nH.$

再串看Z chart
(但∵串看Z chart ∴為C)
$0 - j1.22 = -j1.22$

$-j\frac{1}{\omega C Z_0} = -j1.22.$

$C = \frac{1}{\omega Z_0 1.22} = 2.6pF$



$$Z_{in} = \frac{(R_L + jX_L)\frac{1}{jB}}{(R_L + jX_L) + \frac{1}{jB}} + jX$$

$$= \frac{R_L + jX_L}{1 - BX_L + jBR_L} + jX.$$

$$= \frac{(R_L + jX_L)(1 - BX_L - jBR_L)}{(1 - BX_L)^2 + B^2R_L^2} + jX.$$

$$= \frac{R_L - BR_LX_L - jBR_L^2 + jX_L - jBX_L^2 + BR_LX_L}{(1 - BX_L)^2 + B^2R_L^2} + jX.$$

$$= \frac{R_L}{(1 - BX_L)^2 + B^2R_L^2} + j\left(X + \frac{X_L - B(X_L^2 + R_L^2)}{(1 - BX_L)^2 + B^2R_L^2}\right)$$

$$\frac{R_L}{(1 - BX_L)^2 + B^2R_L^2} = Z_0.$$

$$R_L = Z_0 - 2BX_L + B^2(X_L^2 + R_L^2)Z_0.$$

$$(X_L^2 + R_L^2)\ B^2 - 2B\ X_L + 1 - \frac{R_L}{Z_0} = 0$$

$$B = \frac{2X_L \pm \sqrt{4X_L^2 - 4(X_L^2 + R_L^2)(1 - \frac{R_L}{Z_0})}}{2(X_L^2 + R_L^2)}$$

$$= \frac{X_L \pm \sqrt{X_L^2 - X_L^2 + \frac{R_L}{Z_0}X_L^2 - R_L^2 + \frac{R_L}{Z_0}R_L^2}}{X_L^2 + R_L^2}$$

$$= \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}(X_L^2 + R_L^2 - Z_0R_L)}}{R_L^2 + X_L^2}$$

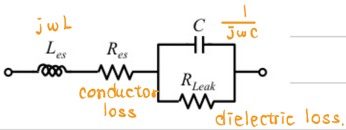
Characteristic of L-shape circuit

- Lump elements have smaller size
- Lump elements better adjustability
- provide wider bandwidth.

⇒ may not be realizable at high frequency.

⇒ inductance, capacitance discrete. chosen over limited ranges values

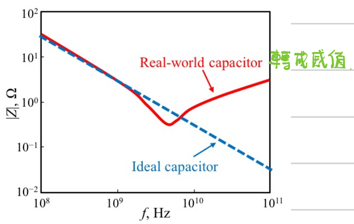
Real-World Capacitors



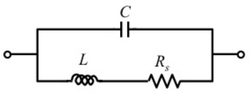
Res：平行金屬板與導体的連接.

Les：當頻率↑其等效电路容值↓,造成感值為主導.

Rleak：介电材質損耗.



Real-World Inductors.

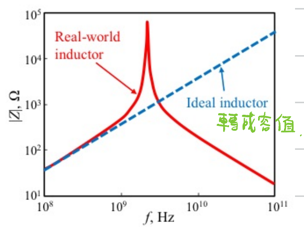


Res：導体損耗.

C：电感繞線間線距感應电容.

當頻率↑其等效电路並联感值↓,並联容值↑.

Frequency response in log scale:

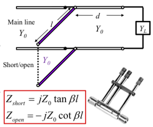


3.4 Matching with TL stubs

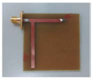
由 $Z_{in} = Z_0 \dfrac{Z_L - jZ_0 \tan\beta z}{Z_0 - jZ_L \tan\beta z}$

$\Rightarrow Z_L = 0$ 時，$Z_{in} = -jZ_0 \tan\beta z$．(電感)

$Z_L = \infty$ 時 $Z_{in} = jZ_0 \cot\beta z$．(電容)．

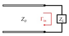


- Parameter: the distance d and the stub length l
- Shunt stubs are more popular than series stubs because of less electrical connection problems



- For stubs using coaxial cable:
  - short-circuited stubs are more popular than the open-circuited stubs because of less radiation
- For stubs using microstrip lines:
  - Open-circuited stubs are easier to fabricate than the short-circuited stubs

ex.



1. For a load impedance $Z_L = 60 - j80\ \Omega$, design 2 single-stub (short-circuit) shunt matching circuits to match this load to a $Z_0 = 50\ \Omega$ line (Operational frequency: 2 GHz)
2. If Z_L is composed of a resistor and a capacitor in series, plot the reflection coefficient magnitude from 1 GHz to 3 GHz for each solution

Problem statement:

1. (5 pts) A load impedance $Z_L = 30 - j40\ \Omega$ is to be matched to a 50-Ω transmission line using a single series stub tuner. Find two solutions (d_1, l_1, d_2, and l_2) using open-circuited stubs by the Smith chart.

1. $Z_0 = 50$．$Z_L = 60 - j80$．$f = 2\,GHz$．

$Z_n = 1.2 - j1.6$．

1. 並联短路

看 A點：$0.427\lambda - 0.317\lambda = 0.11\lambda$．

此時 Y-chart $= 1 + j1.44$．

$\Rightarrow$ 用 $-j1.44$ 消除 $\Rightarrow 0.095\lambda$

看 B點：$(0.073 + 0.5 - 0.317)\lambda = 0.256\lambda$．

此時 Y-chart $= 1 - j1.44$．

$\Rightarrow$ 用 $+j1.44$ 消除 $\Rightarrow 0.403\lambda$．

2. 並联开路．

看 A點：$0.427\lambda - 0.317\lambda = 0.11\lambda$．

此時 Y-chart $1 + j1.44$

$\Rightarrow$ 用 $-j1.44$ 消除 $\Rightarrow (0.095 + 0.5 - 0.25)\lambda$

$= 0.595 - 0.25 = 0.34\lambda$．

看 B點：$(0.073 + 0.5 - 0.317)\lambda = 0.256\lambda$．

此時 Y-chart $1 - j1.44$．

$\Rightarrow$ 用 $-j1.44$ 消除．$(0.403 - 0.25)\lambda = 0.153\lambda$．

3. 串联短路．

看 A點：$(0.324 - 0.315)\lambda = 0.09\lambda$．

此時 Z-chart $1 - j1.5$．

用 $+j1.5$ 消除．$0.156 - 0 = 0.156\lambda$．

看 B點 $(0.175 + 0.5 - 0.315)\lambda = 0.675 - 0.315$

$= 0.36\lambda$

此時 Z-chart $1 + j1.5$

用 $-j1.5$ 消除．$0.405 - 0 = 0.405\lambda$．

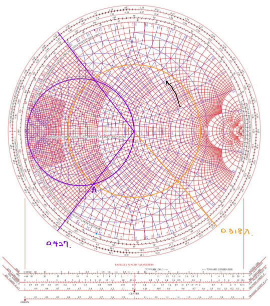
4. 串联开路．

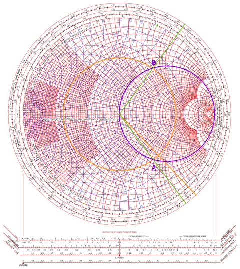
看 A點 $= 0.09\lambda$．

用 $+j1.5$ 消除 $0.156\lambda + 0.5\lambda - 0.25\lambda = 0.406\lambda$．

看 B點 $= 0.36\lambda$．

用 $-j1.5$ 消除．$0.405\lambda - 0.25\lambda = 0.155\lambda$．





Characteristic of Matching Single Stubs．

- 對於任意頻率皆可使用．
- 簡易製作．
- 可用於高頻．

$\Rightarrow$ limitation．

- 大尺寸．
- 靈活度低．

Matching with double