Índice

1. algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	*void* ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	*void* llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	*it* al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	*bool* esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	*it* encuentra i $\in$[f,l) tq. i=elem, pred(i), i$\in$[f2,l2)
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2,l2) $\in$ [f,l)
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	*it* min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	*bool* con [f1,l1]¡[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f,l), hace un heap de [f,l)
is_heap	f,l	*bool* es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum$/oper de [f,l)
inner_product	f1, l1, f2, i	T = i + [f1, l1) . [f2, ...)
partial_sum	f, l, r, [op]	r+i = $\sum$/oper de [f,f+i] $\forall i \in$[f,l)
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

2. Estructuras

2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL $\geq$ ceil(logn); Usar [ ] para llenar arreglo y luego build().

```
struct RMQ{
  #define LVL 10
  tipo vec[LVL][1<<(LVL+1)];
  tipo &operator[](int p){return vec[0][p];}
  tipo get(int i, int j) {//intervalo [i,j)
    int p = 31-__builtin_clz(j-i);
    return min(vec[p][i],vec[p][j-(1<<p)]);
  }
  void build(int n) {//O(nlogn)
    int mp = 31-__builtin_clz(n);
    forn(p, mp) forn(x, n-(1<<p))
      vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
  }};
```

2.2. Segment Tree

2.2.1. Segment Tree Recursivo

```
//inclusive segment tree [L,R]
int T[4 * N];
void init(int node = 1,int l = 0,int r = n - 1){
  if(l == r)T[node] = v[l];
  else{
    int mid = (l + r) >> 1;
    init(2 * node,l,mid);
    init(2 * node + 1,mid + 1,r);
    T[node] = op(T[2 * node],T[2 * node + 1]);
  }
}
void update(int pos,int val,int node = 1,int l = 0,int r = n - 1){
  if(r < pos || l > pos)return;
  if(l == r)T[node] = val;
  else{
    int mid = (l + r) >> 1;
    update(pos,val,2 * node,l,mid);
    update(pos,val,2 * node + 1,mid + 1,r);
    T[node] = op(T[2 * node],T[2 * node + 1]);
```

```
  }
}
int query(int x,int y,int node = 1,int l = 0,int r = n - 1){
  if(r < x || l > y)return NEUTRO;
  if(x <= l && r <= y)return T[node];
  else{
    int mid = (l + r) >> 1;
    return op(query(x,y,2 * node,l,mid),query(x,y,2 * node + 1,mid + 1,r
        ));
  }
}
```

2.2.2. ST Iterativo - (Consulta en rango, modificacion a posicion)

```
//Segment tree iterative [l,r)
int T[2 * N];
void init(){
  for(int i = n; i < 2 * n;i++)T[i] = val[i];
  for(int i = n - 1; i >= 1; i--)T[i] = op(T[i << 1],T[i << 1 | 1]);
}
int op(int a,int b){
  //an asociative function
  return a + b;
}
void update(int pos,int u){
  pos += n;
  for(pos >>= 1; pos >= 1; pos >>= 1)T[pos] = op(T[pos << 1],T[pos << 1
      | 1]);
}

int query(int l,int r){
  l += n, r += n;
  int ans = NEUTRO;
  while(l  < r){
    if(l & 1)ans = op(ans,T[l++]);
    if(r & 1)ans = op(ans,T[--r]);
    l >>= 1,r >>= 1;
  }
  return ans;
}
```

2.2.3. ST Iterativo - (Consulta a posicion, modificacion en rango)

```
/*Segment Tree modificar un rango, acceder a una posicion
  solo sirve cuando la operacion que realizamos es conmutativa
  por ejemplo la suma, pero no funciona con la asignacion
*/
//adiciona value al rango [l, r)
void modify(int l, int r, int value) {// rango [l, r)
  for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
    if (l&1) t[l++] += value;
    if (r&1) t[--r] += value;
  }
}
//acceder a la posicion
int query(int p) {
  int res = 0;
  for (p += n; p > 0; p >>= 1) res += t[p];
  return res;
}
//Si necesitamos actualizar todo lo podemos hacer en O(n)
//Y luego acceder a las hojas en O(1)
void push() {
  for (int i = 1; i < n; ++i) {
    t[i<<1] += t[i];
    t[i<<1|1] += t[i];
    t[i] = 0;
  }
}
```

2.2.4. Segment Tree con Punteros

```
/*La creacion y las queries son [0, n)
[cerrado, abierto)
Por alguna razon el destructor hace que se borre todo
inmediatamente despues de que termina el constructor*/
const int maxn = 1000000;
tipo v[maxn];

const tipo NEUTRO = PONGA_AQUI_EL_NEUTRO;
tipo oper(tipo a, tipo b) {
  return min(a, b);
}
struct segment_tree {
  segment_tree *L, *R;
  int l, r;
  tipo value;
```

```
  segment_tree(): L(nullptr), R(nullptr), value(NEUTRO) {}
  segment_tree(int _l, int _r) : l(_l), r(_r) {
    if (l + 1 == r) {
      value = v[l];
    } else {
      int mid = (l + r) >> 1;
      L = new segment_tree(l, mid);
      R = new segment_tree(mid, r);
      value = oper(L->value, R->value);
    }
  }
  //~segment_tree() {delete L; delete R;}//NO PONER!!! ERROR!!!
  void update(int pos, int val) {
    if (r <= pos || l > pos) return;
    if (l + 1 == r) {
      v[pos] = value = val;
    } else {
      L->update(pos, val);
      R->update(pos, val);
      value = oper(L->value, R->value);
    }
  }
  tipo query(int a, int b) {
    if (a <= l && r <= b) return value;
    if (l >= b || r <= a) return NEUTRO;
    return oper(L->query(a, b), R->query(a, b));
  }
} tree;
```

2.2.5. Segment Tree 2D

```
typedef long long ll;
struct segmetree{
  int n;
  vector<ll>T;
  segmetree(){n = 0;}
  segmetree(int _){
    n = _;
    T.resize(2 * n + 1);
  }
  void rupdate(int pos,int value){
    pos += n;
    T[pos] = value;
    for(pos >>= 1; pos >= 1; pos >>= 1)
      T[pos] = T[pos << 1] + T[pos << 1 | 1];
  }
  void update(int pos,int value){
    pos += n;
    T[pos] += value;
    for(pos >>= 1; pos >= 1; pos >>= 1)
      T[pos] = T[pos << 1] + T[pos << 1 | 1];
  }
  int query(int l,int r){
    l += n;r+= n;
    int ans = 0;
    while(l < r){
      if(l & 1)ans += T[l++];
      if(r & 1)ans += T[--r];
      l >>= 1, r >>= 1;
    }
    return ans;
  }
};
struct st{
  int n;
  vector<segmetree>T;
  st(){}
  st(int _){
    n = _;
    for(int i = 0;i < 2 * n;i++){
      T.push_back(segmetree(n));
    }
  }
  void update(int x,int y,int val){
    x += n;
    T[x].update(y,val);
    segmetree ok;
    for(x >>= 1; x >= 1; x >>= 1){
      T[x].rupdate(y,T[x << 1].query(y,y + 1));
      T[x].update(y,T[x << 1 | 1].query(y,y + 1));
    }
  }
  ll query(int l,int b,int r,int t){
    l += n;
    r += n;
    r++,t++;
```

```
    ll ans = 0LL;
    while(l < r){
      if(l & 1)ans += T[l++].query(b,t);
      if(r & 1)ans += T[--r].query(b,t);
      l >>= 1, r >>= 1;
    }
    return ans;
  }
};
```

2.2.6. Segment Tree Lazy - Suma

```
//Todo es [l, r)
void build(int id = 1,int l = 0,int r = n){
  if(r - l < 2){  //  l + 1 == r
    s[id] = a[l];
    return ;
  }
  int mid = (l+r)/2;
  build(id * 2, l, mid);
  build(id * 2 + 1, mid, r);
  s[id] = s[id * 2] + s[id * 2 + 1];
}
//aqui poner los arrays lazy and id
void upd(int id,int l,int r,int x){// increase all members in this
    interval by x
  lazy[id] += x;
  s[id] += (r - l) * x;
}
//A function to pass the update information to its children :
void shift(int id,int l,int r){//pass update information to the children
  int mid = (l+r)/2;
  upd(id * 2, l, mid, lazy[id]);
  upd(id * 2 + 1, mid, r, lazy[id]);
  lazy[id] = 0;// passing is done
}
//A function to perform increase queries :
void increase(int x,int y,int v,int id = 1,int l = 0,int r = n){
  if(x >= r or l >= y)  return ;
  if(x <= l && r <= y){
    upd(id, l, r, v);
    return ;
  }
  shift(id, l, r);
  int mid = (l+r)/2;
  increase(x, y, v, id * 2, l, mid);
  increase(x, y, v, id*2+1, mid, r);
  s[id] = s[id * 2] + s[id * 2 + 1];
}
//(We should call increase(l r x))
int sum(int x,int y,int id = 1,int l = 0,int r = n){
  if(x >= r or l >= y)  return 0;
  if(x <= l && r <= y)  return s[id];
  shift(id, l, r);
  int mid = (l+r)/2;
  return sum(x, y, id * 2, l, mid) +
         sum(x, y, id * 2 + 1, mid, r);
}
```

2.2.7. Segment Tree Lazy - Pintar

```
void shift(int id){
  if(lazy[id])
    lazy[2 * is] = lazy[2 * id + 1] = lazy[id];
  lazy[id] = 0;
}
//color > 1, por que se usa el 0 para decir que no hay lazy
void upd(int x,int y,int color, int id = 0,int l = 0,int r = n){//
    painting the interval [x,y) whith color "color"
  if(x >= r or l >= y)  return ;
  if(x <= l && r <= y){
    lazy[id] = color;
    return ;
  }
  int mid = (l+r)/2;
  shift(id);
  upd(x, y, color, 2 * id, l, mid);
  upd(x, y, color, 2*id+1, mid, r);
}

set <int> se;
void cnt(int id = 1,int l = 0,int r = n){
  if(lazy[id]){
    se.insert(lazy[id]);
    return ; // there is no need to see the children, because all the
        interval is from the same color
```

```
  }
  if(r - l < 2) return ;
  int mid = (l+r)/2;
  cnt(2 * id, l, mid);
  cnt(2*id+1, mid, r);
}
```

2.2.8. Segment Tree Persistente

```
int segcnt = 0;
struct segment {
    int l, r, lid, rid, sum;
} segs[2000000];
int build(int l, int r) {
    if (l > r) return -1;
    int id = segcnt++;
    segs[id].l = l;
    segs[id].r = r;
    if (l == r) segs[id].lid = -1, segs[id].rid = -1;
    else {
        int m = (l + r) / 2;
        segs[id].lid = build(l , m);
        segs[id].rid = build(m + 1, r); }
    segs[id].sum = 0;
    return id; }
int update(int idx, int v, int id) {
    if (id == -1) return -1;
    if (idx < segs[id].l || idx > segs[id].r) return id;
    int nid = segcnt++;
    segs[nid].l = segs[id].l;
    segs[nid].r = segs[id].r;
    segs[nid].lid = update(idx, v, segs[id].lid);
    segs[nid].rid = update(idx, v, segs[id].rid);
    segs[nid].sum = segs[id].sum + v;
    return nid; }
int query(int id, int l, int r) {
    if (r < segs[id].l || segs[id].r < l) return 0;
    if (l <= segs[id].l && segs[id].r <= r) return segs[id].sum;
    return query(segs[id].lid, l, r) + query(segs[id].rid, l, r); }
```

2.3. Fenwick Tree

```
vector<tipo> tree;
int maxn;
void init(int n) {
  tree = vector<tipo>(n, 0);
  maxn = n;
}
void add(int i, tipo k) { //i valid [1, n)
  for(; i < maxn; i += i&-i ) tree[i] += k;
}

tipo get(int i){//returns sum [1, i]
  tipo s = 0;
  for(; i > 0; i-=i&-i) s+=tree[i];
  return s;
}
```

2.3.1. Fenwick Tree 2D

```

ll T[1025][1025];
int n;

ll query(int x, int y)
{
  ll res = 0;
  for(int i = x; i >= 0; i = (i & (i+1)) - 1)
        for(int j = y; j >= 0; j = (j & (j+1)) - 1)
            res += T[i][j];
    return res;
}

void update(int x, int y, int val)
{
  for(int i = x; i < n; i = (i | (i+1)))
        for(int j = y; j < n; j = (j | (j+1)))
            T[i][j] += val;
}
```

2.4. Union Find con rank

```
/*=============== <Union find rangos> ==================
Complexity: O(N)
index 0 to n - 1 warning
Complexity O(N)
*/
#define MAX INSERTE_VALOR_AQUI
```

```
int padre[MAX];
int rango[MAX];
void MakeSet(int n){
    for (int i = 0 ; i < n ; ++i) {
        padre[i] = i; rango[i] = 0; }
}
int Find(int x) {
    if(x == padre[x])
        return x;
    return padre[x] = Find(padre[x]);
}
void UnionbyRank(int x , int y){
    int xRoot = Find(x);
    int yRoot = Find(y);
    //el padre de ambas componentes sera el de mayor altura
    if(rango[xRoot] > rango[yRoot])//X tiene mas altura que Y
        padre[yRoot] = xRoot;
    }else{//Y >= X
        padre[xRoot] = yRoot;
        if(rango[xRoot] == rango[yRoot])//si poseen la misma altura
            rango[yRoot]++;//incremento el rango de la nueva raiz
    }
}
```

2.5. BigInteger C++

```
// g++ -std=c++11 "bigint.cpp" -o run
/***
================ <Big Int c++ version> ==================
Contain a useful big int, overload all operators, including cin, cout,
comparator, build via string (prefer this metod) or long long, for now
    this not have a
to_string method
Problem for practice: UVA 494
*/
// base and base_digits must be consistent
const int base = 1000000000;
const int base_digits = 9;

struct bigint {
    vector<int> a;
    int sign;

    bigint() :
        sign(1) {
    }

    bigint(long long v) {
        *this = v;
    }

    bigint(const string &s) {
        read(s);
    }

    void operator=(const bigint &v) {
        sign = v.sign;
        a = v.a;
    }

    void operator=(long long v) {
        sign = 1;
        if (v < 0)
            sign = -1, v = -v;
        for (; v > 0; v = v / base)
            a.push_back(v % base);
    }

    bigint operator+(const bigint &v) const {
        if (sign == v.sign) {
            bigint res = v;

            for (int i = 0, carry = 0; i < (int) max(a.size(), v.a.size
                ()) || carry; ++i) {
                if (i == (int) res.a.size())
                    res.a.push_back(0);
                res.a[i] += carry + (i < (int) a.size() ? a[i] : 0);
                carry = res.a[i] >= base;
                if (carry)
                    res.a[i] -= base;
            }
            return res;
        }
        return *this - (-v);
    }

```

```
    bigint operator-(const bigint &v) const {
        if (sign == v.sign) {
            if (abs() >= v.abs()) {
                bigint res = *this;
                for (int i = 0, carry = 0; i < (int) v.a.size() || carry
                    ; ++i) {
                    res.a[i] -= carry + (i < (int) v.a.size() ? v.a[i] :
                         0);
                    carry = res.a[i] < 0;
                    if (carry)
                        res.a[i] += base;
                }
                res.trim();
                return res;
            }
            return -(v - *this);
        }
        return *this + (-v);
    }

    void operator*=(int v) {
        if (v < 0)
            sign = -sign, v = -v;
        for (int i = 0, carry = 0; i < (int) a.size() || carry; ++i) {
            if (i == (int) a.size())
                a.push_back(0);
            long long cur = a[i] * (long long) v + carry;
            carry = (int) (cur / base);
            a[i] = (int) (cur % base);
            //asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) : "A"(cur), "c
                "(base));
        }
        trim();
    }

    bigint operator*(int v) const {
        bigint res = *this;
        res *= v;
        return res;
    }

    friend pair<bigint, bigint> divmod(const bigint &a1, const bigint &
        b1) {
        int norm = base / (b1.a.back() + 1);
        bigint a = a1.abs() * norm;
        bigint b = b1.abs() * norm;
        bigint q, r;
        q.a.resize(a.a.size());

        for (int i = a.a.size() - 1; i >= 0; i--) {
            r *= base;
            r += a.a[i];
            int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];
            int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r.a[b.a.size() -
                 1];
            int d = ((long long) base * s1 + s2) / b.a.back();
            r -= b * d;
            while (r < 0)
                r += b, --d;
            q.a[i] = d;
        }

        q.sign = a1.sign * b1.sign;
        r.sign = a1.sign;
        q.trim();
        r.trim();
        return make_pair(q, r / norm);
    }

    bigint operator/(const bigint &v) const {
        return divmod(*this, v).first;
    }

    bigint operator%(const bigint &v) const {
        return divmod(*this, v).second;
    }

    void operator/=(int v) {
        if (v < 0)
            sign = -sign, v = -v;
        for (int i = (int) a.size() - 1, rem = 0; i >= 0; --i) {
            long long cur = a[i] + rem * (long long) base;
            a[i] = (int) (cur / v);
            rem = (int) (cur % v);
        }
        trim();
```

```
    }

    bigint operator/(int v) const {
        bigint res = *this;
        res /= v;
        return res;
    }

    int operator%(int v) const {
        if (v < 0)
            v = -v;
        int m = 0;
        for (int i = a.size() - 1; i >= 0; --i)
            m = (a[i] + m * (long long) base) % v;
        return m * sign;
    }

    void operator+=(const bigint &v) {
        *this = *this + v;
    }
    void operator-=(const bigint &v) {
        *this = *this - v;
    }
    void operator*=(const bigint &v) {
        *this = *this * v;
    }
    void operator/=(const bigint &v) {
        *this = *this / v;
    }

    bool operator<(const bigint &v) const {
        if (sign != v.sign)
            return sign < v.sign;
        if (a.size() != v.a.size())
            return a.size() * sign < v.a.size() * v.sign;
        for (int i = a.size() - 1; i >= 0; i--)
            if (a[i] != v.a[i])
                return a[i] * sign < v.a[i] * sign;
        return false;
    }

    bool operator>(const bigint &v) const {
        return v < *this;
    }
    bool operator<=(const bigint &v) const {
        return !(v < *this);
    }
    bool operator>=(const bigint &v) const {
        return !(*this < v);
    }
    bool operator==(const bigint &v) const {
        return !(*this < v) && !(v < *this);
    }
    bool operator!=(const bigint &v) const {
        return *this < v || v < *this;
    }

    void trim() {
        while (!a.empty() && !a.back())
            a.pop_back();
        if (a.empty())
            sign = 1;
    }

    bool isZero() const {
        return a.empty() || (a.size() == 1 && !a[0]);
    }

    bigint operator-() const {
        bigint res = *this;
        res.sign = -sign;
        return res;
    }

    bigint abs() const {
        bigint res = *this;
        res.sign *= res.sign;
        return res;
    }

    long long longValue() const {
        long long res = 0;
        for (int i = a.size() - 1; i >= 0; i--)
            res = res * base + a[i];
        return res * sign;
    }
```

```
226
227     friend bigint gcd(const bigint &a, const bigint &b) {
228         return b.isZero() ? a : gcd(b, a % b);
229     }
230     friend bigint lcm(const bigint &a, const bigint &b) {
231         return a / gcd(a, b) * b;
232     }
233
234     void read(const string &s) {
235         sign = 1;
236         a.clear();
237         int pos = 0;
238         while (pos < (int) s.size() && (s[pos] == '-' || s[pos] == '+'))
                {
239             if (s[pos] == '-')
240                 sign = -sign;
241             ++pos;
242         }
243         for (int i = s.size() - 1; i >= pos; i -= base_digits) {
244             int x = 0;
245             for (int j = max(pos, i - base_digits + 1); j <= i; j++)
246                 x = x * 10 + s[j] - '0';
247             a.push_back(x);
248         }
249         trim();
250     }
251
252     friend istream& operator>>(istream &stream, bigint &v) {
253         string s;
254         stream >> s;
255         v.read(s);
256         return stream;
257     }
258
259     friend ostream& operator<<(ostream &stream, const bigint &v) {
260         if (v.sign == -1)
261             stream << '-';
262         stream << (v.a.empty() ? 0 : v.a.back());
263         for (int i = (int) v.a.size() - 2; i >= 0; --i)
264             stream << setw(base_digits) << setfill('0') << v.a[i];
265         return stream;
266     }
267
```

```
268     static vector<int> convert_base(const vector<int> &a, int old_digits
            , int new_digits) {
269         vector<long long> p(max(old_digits, new_digits) + 1);
270         p[0] = 1;
271         for (int i = 1; i < (int) p.size(); i++)
272             p[i] = p[i - 1] * 10;
273         vector<int> res;
274         long long cur = 0;
275         int cur_digits = 0;
276         for (int i = 0; i < (int) a.size(); i++) {
277             cur += a[i] * p[cur_digits];
278             cur_digits += old_digits;
279             while (cur_digits >= new_digits) {
280                 res.push_back(int(cur % p[new_digits]));
281                 cur /= p[new_digits];
282                 cur_digits -= new_digits;
283             }
284         }
285         res.push_back((int) cur);
286         while (!res.empty() && !res.back())
287             res.pop_back();
288         return res;
289     }
290
291     typedef vector<long long> vll;
292
293     static vll karatsubaMultiply(const vll &a, const vll &b) {
294         int n = a.size();
295         vll res(n + n);
296         if (n <= 32) {
297             for (int i = 0; i < n; i++)
298                 for (int j = 0; j < n; j++)
299                     res[i + j] += a[i] * b[j];
300             return res;
301         }
302
303         int k = n >> 1;
304         vll a1(a.begin(), a.begin() + k);
305         vll a2(a.begin() + k, a.end());
306         vll b1(b.begin(), b.begin() + k);
307         vll b2(b.begin() + k, b.end());
308
309         vll a1b1 = karatsubaMultiply(a1, b1);
```

```
        vll a2b2 = karatsubaMultiply(a2, b2);

        for (int i = 0; i < k; i++)
            a2[i] += a1[i];
        for (int i = 0; i < k; i++)
            b2[i] += b1[i];

        vll r = karatsubaMultiply(a2, b2);
        for (int i = 0; i < (int) a1b1.size(); i++)
            r[i] -= a1b1[i];
        for (int i = 0; i < (int) a2b2.size(); i++)
            r[i] -= a2b2[i];

        for (int i = 0; i < (int) r.size(); i++)
            res[i + k] += r[i];
        for (int i = 0; i < (int) a1b1.size(); i++)
            res[i] += a1b1[i];
        for (int i = 0; i < (int) a2b2.size(); i++)
            res[i + n] += a2b2[i];
        return res;
    }

    bigint operator*(const bigint &v) const {
        vector<int> a6 = convert_base(this->a, base_digits, 6);
        vector<int> b6 = convert_base(v.a, base_digits, 6);
        vll a(a6.begin(), a6.end());
        vll b(b6.begin(), b6.end());
        while (a.size() < b.size())
            a.push_back(0);
        while (b.size() < a.size())
            b.push_back(0);
        while (a.size() & (a.size() - 1))
            a.push_back(0), b.push_back(0);
        vll c = karatsubaMultiply(a, b);
        bigint res;
        res.sign = sign * v.sign;
        for (int i = 0, carry = 0; i < (int) c.size(); i++) {
            long long cur = c[i] + carry;
            res.a.push_back((int) (cur % 1000000));
            carry = (int) (cur / 1000000);
        }
        res.a = convert_base(res.a, 6, base_digits);
        res.trim();
        return res;
    }
};

int main() {
    bigint a=0;
    //("99999999999999999999999999999999999999999999999");
    bigint b;
    //("19999999999999999999999999999999999999999999998");
    bigint n;
    while(cin >> n) {
        if(n==0){break;}
        a += n;
    }
    cout<<a<<endl;
}
```

2.6. UnorderedSet

```
//Compilar: g++ --std=c++11
struct Hash{
  size_t operator()(const ii &a)const{
    size_t s=hash<int>()(a.fst);
    return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
  }
  size_t operator()(const vector<int> &v)const{
    size_t s=0;
    for(auto &e : v)
      s ^= hash<int>()(e)+0x9e3779b9+(s<<6)+(s>>2);
    return s;
  }
};
unordered_set<ii, Hash> s;
unordered_map<ii, int, Hash> m;//map<key, value, hasher>
```

2.7. Ordered Set

```
/*
 A brief explanation about use of a powerful library: orderd_set
 Reference link: http://codeforces.com/blog/entry/11080
 and a hash for the type pair
 */
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

```
using namespace __gnu_pbds;
/*typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> ordered_set;
If we want to get map but not the set, as the second argument type must
    be used mapped type. Apparently,  the tree supports the same
    operations as the set (at least I haven't any problems with them
    before),  but also there are two new features - it is find_by_order
    () and order_of_key().
The first returns an iterator to the k-th largest element (counting from
     zero), the second - the number of items
in a set that are strictly smaller than our item. Example of use:*/
template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
int main(){
  ordered_set<int>  s;
  s.insert(1);
  s.insert(3);
  cout << *s.find_by_order(0) << endl; // print the 0-th smallest number
       in s(0-based)
  cout << s.order_of_key(2) << endl; // the number of elements in the s
      less than 2
  //find_by_order(i) devuelve iterador al i-esimo elemento
  //order_of_key(k): devuelve la pos del lower bound de k
  //Ej: 12, 100, 505, 1000, 10000.
  //order_of_key(10) == 0, order_of_key(100) == 1,
  //order_of_key(707) == 3, order_of_key(9999999) == 5
  return 0;
}
```

2.8. Treap Modo Set

```
struct treap {
  typedef struct _node {
    int x, y, cnt;
    _node *l, *r;
    _node(int _x) : x(_x), y((rand() << 16) ^ rand()), cnt(1), l(nullptr
        ), r(nullptr) {}
    ~_node() {delete l; delete r;}
    void recalc() {
      cnt = 1;
      if (l) cnt += l->cnt;
      if (r) cnt += r->cnt;
    }
  } *node;
  treap(): root(nullptr) {}
  ~treap() {delete root;}
  node root;
  /*Divide el arbol que tiene como raiz a "t", guarda en
    L todos los nodos con key menor o igual a "x"
    R todos los nodos con key mayor estricto a x
    t es destruido/modificado
   */
  void split(node t, int x, node &L, node &R) {
    if (t == nullptr) {L = R = nullptr; return;}
    if (t->x <= x) {
      split(t->r, x, t->r, R);
      L = t;
    } else {
      split(t->l, x, L, t->l);
      R = t;
    }
    t->recalc();
  }
  /*Une los dos nodos L y R en un solo arbol y los devuelve.
    L y R son modificados
   */
  node merge(node L, node R) {
    if (L == nullptr) return R;
    if (R == nullptr) return L;
    if (L->y > R->y) {
      L->r = merge(L->r, R);
      L->recalc();
      return L;
    } else {
      R->l = merge(L, R->l);
      R->recalc();
      return R;
    }
  }
  /*Inserta un solo nodo con key igual a "x"*/
  void insert(int x) {
    //verificar que no se inserten elementos repetidos
    //pueden ocasionar que ya no cumplan la propiedad de BST
    //pueden haber keys iguales a izquierda y derecha
    node L, R;
```

```
    split(root, x, L, R);
    root = merge(merge(L, new _node(x)), R);
  }
  /*Borra todos los nodos con key igual a "x"
    No pasa nada si no hay nodos con key igual a "x"*/
  void erase(int x) {
    node L, m, R;
    split(root, x, L, R);
    split(L, x - 1, L, m);
    root = merge(L, R);
  }
  int count(int x) {
    node L, m, R;
    split(root, x, L, R);
    split(L, x - 1, L, m);
    int ans = cnt(m);
    root = merge(merge(L, m), R);
    return ans;
  }
  /*Borra un nodo con key igual a "x"
    No sucede nada si no hay nodo con ese key*/
  void erase_one(int x) {
    node L, m, R;
    split(root, x, L, R);
    split(L, x - 1, L, m);
    if (m)
      m = merge(m->l, m->r);
    root = merge(merge(L, m), R);
  }
  /*k-esimo indexado desde 0
     devuelve INT_MAX si no hay k-esimo*/
  int kthElement(int k) {
    node cur = root;
    while (cur != nullptr) {
      int sizeLeft = cnt(cur->l);
      if (sizeLeft == k)
        return cur->x;
      cur = sizeLeft > k ? cur->l : cur->r;
      if (sizeLeft < k)
        k -= (sizeLeft + 1);
    }
    return INT_MAX;
  }
  /*Numero de elementos con keys menores a "x"*/
  int less(int x) {
    node cur = root;
    int ans = 0;
    while (cur != nullptr) {
      if (cur->x >= x) {
        cur = cur->l;
      } else {
        ans += cnt(cur->l) + 1;
        cur = cur->r;
      }
    }
    return ans;
  }
  int cnt(node t) const {
    return t ? t->cnt : 0;
  }
  int size() const {
    return cnt(root);
  }
/*//from e-maxx.ru son mas rapidos(no x mucho) pero menos entendibles
//solo borra un elemento
  void erase(node &t, int x) {
    if (t->x == x)
      t = merge(t->l, t->r);
    else
      erase(x < t->x?t->l:t->r, x);
    if (t)
      t->recalc();
  }
  void erase_one(int x) {erase(root, x);}
  void insert(node &t, node it) {
    if (t == nullptr)
      t = it;
    else if (it->y > t->y)
      split(t, it->x, it->l, it->r), t = it;
    else
      insert(it->x < t->x? t->l: t->r, it);
    t->recalc();
  }
  void insert(int x) {insert(root, new _node(x));}*/
};
```

2.9. Treap Implicito(Rope)

```
struct rope {
  typedef struct _node {
    int value, y, cnt;
    _node *l, *r;
    _node(int _value) : value(_value), y((rand() << 16) ^ rand()), cnt
        (1), l(nullptr), r(nullptr) {}
    ~_node() {delete l; delete r;}
    void recalc() {
      cnt = 1;
      if (l) cnt += l->cnt;
      if (r) cnt += r->cnt;
    }
  } *node;
  rope(): root(nullptr) {}
  ~rope() {delete root;}
  node root;
  /*Divide el arbol que tiene como raiz a "t", guarda en
   L los primeros "x" elementos del array
   R el primer elemento de R es el elemento en posicion x(indexado desde
       0) del array
   t es destruido/modificado
   L = [0, x)
   R = [x, n) */
  void split(node t, int x, node &L, node &R) {
    if (t == nullptr) {L = R = nullptr; return;}
    int curIndex = cnt(t->l) + 1;
    if (curIndex <= x) {
      split(t->r, x - curIndex, t->r, R);
      L = t;
    } else {
      split(t->l, x, L, t->l);
      R = t;
    }
    t->recalc();
  }
  /* Une los dos nodos L y R en un solo arbol y los devuelve.
     L y R son modificados */
  node merge(node L, node R) {
    if (L == nullptr) return R;
    if (R == nullptr) return L;
    if (L->y > R->y) {
      L->r = merge(L->r, R);
      L->recalc();
      return L;
    } else {
      R->l = merge(L, R->l);
      R->recalc();
      return R;
    }
  }
  /*Inserta "value" en la posicion "pos"(indexado desde 0) recorre todos
       los elementos a la derecha desde la posicion pos*/
  void insert(int pos, int value) {
    node L, R;
    split(root, pos, L, R);//en R esta pos
    root = merge(merge(L, new _node(value)), R);
  }
  /*Borra el elemento en posicion pos*/
  void erase(int pos) {
    node L, m, R;
    split(root, pos, L, R);
    split(R, 1, m, R);
    root = merge(L, R);
  }
  int cnt(node t) const {
    return t ? t->cnt : 0;
  }
  int size() const {
    return cnt(root);
  }
};
int main() {srand(time(nullptr));return 0;}
```

2.10. Treap - Toby and Stones

```
const int PAINT = 0, FLIP = 1, REVERSE = 2;
struct rope {
  typedef struct _node {
    int value, y, cnt;
    int negros;
    bool rev;
    bool lazy_flip;
    int lazy_pintar;
    _node *l, *r;
```

```
    _node(int _value) : value(_value), y((rand() << 16) ^ rand()), cnt
        (1), l(nullptr), r(nullptr) {
      negros = _value;
      rev = false;
      lazy_flip = false;
      lazy_pintar = -1;
    }
    ~_node() {delete l; delete r;}
    void recalc() {
      cnt = 1;
      negros = value;
      if (l) cnt += l->cnt, negros += l->negros;
      if (r) cnt += r->cnt, negros += r->negros;
    }
    void push_lazy(int type, int param = -1) {
      if (type == PAINT) {
        rev = false;
        lazy_flip = false;
        lazy_pintar = param;
        negros = param * cnt;
      }
      if (type == FLIP) {
        if (lazy_pintar != -1) {
          assert(rev == false && lazy_flip == false);
          lazy_pintar = (1 - lazy_pintar);
          negros = lazy_pintar * cnt;
        } else {
          lazy_flip ^= true;
          negros = cnt - negros;
        }
      }
      if (type == REVERSE) {
        if (lazy_pintar == -1) {
          rev ^= true;
        }
      }
    }
    void verify() {
      bool op1 = (lazy_pintar != -1);
      bool op2 = (rev || lazy_flip);
      //solo puede pasar uno de los 2 casos, o ninguno esta activado o
          solo uno de los dos
      assert((!op1 and !op2) or (op1 xor op2));
    }
    string to_string() {
      stringstream ss;
      ss << "value=" << value << ",␣negros=" << negros << ",␣cnt="<< cnt
           << "("<<rev<< "," << lazy_pintar<<","<< lazy_flip << ")";
      return ss.str();
    }
  } *node;
  rope(): root(nullptr) {}
  ~rope() {delete root;}
  node root;
  void push(node &t) {
    if (t == nullptr) return;
    bool op1 = ((t->lazy_pintar) != -1);
    bool op2 = ((t->rev) || (t->lazy_flip));
    //solo puede pasar uno de los 2 casos, o ninguno esta activado o
        solo uno de los dos
    assert((!op1 and !op2) or (op1 xor op2));
    if (op1) {
      t->value = t->lazy_pintar;
      if (t->l) t->l->push_lazy(PAINT, t->lazy_pintar);
      if (t->r) t->r->push_lazy(PAINT, t->lazy_pintar);
      t->lazy_pintar = -1;
    }
    if (op2) {//no importa el orden en que se aplique estos 2
      //reverse
      if (t->rev) {
        swap(t->l, t->r);
        t->rev = false;
        if (t->l) t->l->push_lazy(REVERSE);
        if (t->r) t->r->push_lazy(REVERSE);
      }
      //invertir colores
      if (t->lazy_flip) {
        t->value = 1 - (t->value);
        t->lazy_flip = false;
        if (t->l) t->l->push_lazy(FLIP);
        if (t->r) t->r->push_lazy(FLIP);
      }
    }
  }
  void split(node t, int x, node &L, node &R) {
    push(t);
```

```
    if (t == nullptr) {L = R = nullptr; return;}
    int curIndex = cnt(t->l) + 1;
    if (curIndex <= x) {
      split(t->r, x - curIndex, t->r, R);
      L = t;
    } else {
      split(t->l, x, L, t->l);
      R = t;
    }
    t->recalc();
  }
  node merge(node L, node R) {
    push(L); push(R);
    if (L == nullptr) return R;
    if (R == nullptr) return L;
    if (L->y > R->y) {
      L->r = merge(L->r, R);
      L->recalc();
      return L;
    } else {
      R->l = merge(L, R->l);
      R->recalc();
      return R;
    }
  }
  void insert(int pos, int value) {
    node L, R;
    split(root, pos, L, R);//en R esta pos
    root = merge(merge(L, new _node(value)), R);
  }
  int cnt(node t) {
    return t ? t->cnt : 0;
  }
  int size() {
    return cnt(root);
  }
  void reverse(int i, int j) {
    node L, m, R;
    split(root, i, L, R);
    split(R, j - i + 1, m, R);
    m->push_lazy(REVERSE);
    root = merge(merge(L, m), R);
  }
  void flip(int i, int j) {
    node L, m, R;
    split(root, i, L, R);
    split(R, j - i + 1, m, R);
    m->push_lazy(FLIP);
    root = merge(merge(L, m), R);
  }
  void pintar(int i, int j, int color) {
    node L, m, R;
    split(root, i, L, R);
    split(R, j - i + 1, m, R);
    m->push_lazy(PAINT, color);
    root = merge(merge(L, m), R);
  }
  void query(int i, int j) {
    node L, m, R;
    split(root, i, L, R);
    split(R, j - i + 1, m, R);
    int negros = m->negros;
    int blancos = m->cnt - negros;
    printf("%d␣%d\n", negros, blancos);
    root = merge(merge(L, m), R);
  }
  void print(node &t, string spacio, char lado = '␣') {
    if (t == nullptr) return;
    cout << spacio << lado <<"␣->␣" << (t->to_string()) << endl;
    print(t->l, spacio + "\t", 'L');
    print(t->r, spacio + "\t", 'R');
  }
  void print() {
    print(root, "");
  }
};
```

2.11. Convex Hull Trick Estatico

```
// g++  "convexhulltrick.cpp" -o run
/***
=============== <Convex hull trick normal version> ==================
Contain a sample about convex hull trick optimization this recivie N
    pairs:
a "value of length" and a cost, we need to minimize the value of
    grouping
```

```
this pairs taken the most large pair as the cost of the group

Problem for practice: aquire
*/
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;
int pointer; //Keeps track of the best line from previous query
vector<long long> M; //Holds the slopes of the lines in the envelope
vector<long long> B; //Holds the y-intercepts of the lines in the envelope
//Returns true if either line l1 or line l3 is always better than line l2
bool bad(int l1,int l2,int l3)
{
  /*
  intersection(l1,l2) has x-coordinate (b1-b2)/(m2-m1)
  intersection(l1,l3) has x-coordinate (b1-b3)/(m3-m1)
  set the former greater than the latter, and cross-multiply to eliminate division
  */
  return (B[l3]-B[l1])*(M[l1]-M[l2])<(B[l2]-B[l1])*(M[l1]-M[l3]);
}
//Adds a new line (with lowest slope) to the structure
void add(long long m,long long b)
{
  //First, let's add it to the end
  M.push_back(m);
  B.push_back(b);
  //If the penultimate is now made irrelevant between the antepenultimate
  //and the ultimate, remove it. Repeat as many times as necessary
  while (M.size()>=3&&bad(M.size()-3,M.size()-2,M.size()-1))
  {
    M.erase(M.end()-2);
    B.erase(B.end()-2);
  }
}
//Returns the minimum y-coordinate of any intersection between a given vertical
//line and the lower envelope
long long query(long long x)
{
  //If we removed what was the best line for the previous query, then the
  //newly inserted line is now the best for that query
  if (pointer>=M.size())
    pointer=M.size()-1;
  //Any better line must be to the right, since query values are
  //non-decreasing
  while (pointer<M.size()-1&&
    M[pointer+1]*x+B[pointer+1]<M[pointer]*x+B[pointer])
    pointer++;
  return M[pointer]*x+B[pointer];
}
int main()
{
  int M,N,i;
  pair<int,int> a[50000];
  pair<int,int> rect[50000];
  scanf("%d",&M);
  for (i=0; i<M; i++)
    scanf("%d %d",&a[i].first,&a[i].second);
  //Sort first by height and then by width (arbitrary labels)
  sort(a,a+M);
  for (i=0,N=0; i<M; i++)
  {
    /*
    When we add a higher rectangle, any rectangles that are also
    equally thin or thinner become irrelevant, as they are
    completely contained within the higher one; remove as many
    as necessary
    */
    while (N>0&&rect[N-1].second<=a[i].second)
      N--;
    rect[N++]=a[i]; //add the new rectangle
  }
  long long cost;
  add(rect[0].second,0);
  //initially, the best line could be any of the lines in the envelope,
  //that is, any line with index 0 or greater, so set pointer=0
  pointer=0;
  for (i=0; i<N; i++)
  {
    cost=query(rect[i].first);
```

```
    if (i<N)
      add(rect[i+1].second,cost);
  }
  printf("%lld\n",cost);
  return 0;
}
```

2.12. Convex Hull Trick Dinamico

```
// g++ -std=c++11 "convexhulltrick_dynamic.cpp" -o run
/***
=============== <Convex hull trick dynamic version version> ==================
warning with the use of this, this is a black box, try to use only in an emergency.
Problem for practice: aquire
*/
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const ll is_query = -(1LL<<62);
  struct Line {
  ll m, b;
  mutable multiset<Line>::iterator it;
  const Line *succ(multiset<Line>::iterator it) const;
  bool operator<(const Line& rhs) const {
    if (rhs.b != is_query) return m < rhs.m;
    const Line *s=succ(it);
    if(!s) return 0;
    ll x = rhs.m;
    return b - s->b < (s->m - m) * x;
  }
};
struct HullDynamic : public multiset<Line>{ // will maintain upper hull for maximum
  bool bad(iterator y) {
    iterator z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y->m == z->m && y->b <= z->b;
    }
    iterator x = prev(y);
    if (z == end()) return y->m == x->m && y->b <= x->b;
    return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
  }
  iterator next(iterator y){return ++y;}
  iterator prev(iterator y){return --y;}
    void insert_line(ll m, ll b) {
    iterator y = insert((Line) { m, b });
    y->it=y;
    if (bad(y)) { erase(y); return; }
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }
  ll eval(ll x) {
    Line l = *lower_bound((Line) { x, is_query });
    return l.m * x + l.b;
  }
}h;
const Line *Line::succ(multiset<Line>::iterator it) const{
return (++it==h.end()? NULL : &*it);}
```

2.13. Misof Tree

```
/*
http://codeforces.com/blog/entry/10493#comment-159335
Sirve para encontrar el i-esimo numero de un conjunto de numeros que vamos insertando en el arbol.
Sirve solo si nuestros numeros son del 0 al n-1 (pero podemos mapearlos antes de usarlos)
La idea es esta:
Funcionamiento:
  - En el fondo sigue siendo un Segment-Tree (hacemos que 'n' sea 2^x)
  - Cada nodo guarda cuantos numeros hay en el intervalo (entonces en tree[1] dice cuantos numeros tenemos en total)
  - Se sigue representando los hijos del nodo 'i' con '2 * i' (izq) y '2 * i + 1' (der);
Query:
  - si kth es mas grande que todos los que tenemos(tree[1]) o es negativo entonces -1
  - siempre nos mantenemos en el nodo de la izquierda y si es necario avanzamos al de la derecha
                    'i <<= 1'
    - si kth es mas grande que el nodo de la izquierda(el actual) quiere decir que podemos quitarle todos esos
    numeros a nuestra busqueda 'kth - tree[i]' y buscar el nuevo kth en
```

```
        el arbol de la derecha
      if (kth > tree [i]) kth -= tree [i++];
    - Ojo en el 'i++' ahi es donde avanzamos al nodo de la derecha
  - luego hace su formula rara que aun no entendi xD:
      'i - leaf + (kth > tree [i])';
*/
const int MaxN = 1e6;

int a [MaxN], s [MaxN];
int leaf, tree [100 + MaxN << 2];

void bld (int n) { leaf = 1 << (32 - __builtin_clz (n)); }
void add (int x) { for (int i = leaf + x; i; i >>= 1) ++tree [i]; }//
    Podemos insertar mas de una copia la vez tree [i] += xcopies;
void del (int x) { for (int i = leaf + x; i; i >>= 1) --tree [i]; }//
    Podemos eliminar mas de una copia la vez tree [i] -= xcopies;
// en "leaf + x" esta cuantas copias tenemos de "x"
//Cuidado con intentar hacer del con mas copias de las disponibles, el
    kth() no funcionaria
long kth  (int kth, int i = -1) {
    if (kth > tree [1] || kth <= 0) return i;
  for (i = 1; i < leaf; i <<= 1) if (kth > tree [i]) kth -= tree [i++];
    return i - leaf + (kth > tree [i]);
}
```

2.14. SQRT Decomposition Basic

```
const int maxn = 500010;
int n;

tipo v[maxn];//vector principal

tipo lazy[maxn];
pair<tipo, tipo> t[maxn];//para poder reordenar los elementos

int SQRT;
int N;//nro. de buckets

//Recalcula y aplica el lazy al bucket con indice idx
//guarda la informacion necesaria del bucket en otros vectores
//podria ser la suma del bucket, o el min/max del bucket
void recalc(int idx) {
  int a = idx * SQRT, b = min(n, (idx + 1) * SQRT);
  for (int i = a; i < b; i++) {
    v[i] += lazy[idx];
    t[i] = make_pair(v[i], i);
  }
  lazy[idx] = 0;
  sort(t + a, t + b);
}

//adiciona delta a todos los elementos
//en el intervalo cerrado [a, b]
void add(int a, int b, tipo delta) {
  int idx_a = a / SQRT, idx_b = b / SQRT;
  if (idx_a == idx_b) {
    for (int i = a; i <= b; i++)
      v[i] += delta;
    recalc(idx_a);
  } else {
    //head
    for (int i = a, lim = min(n, (idx_a + 1) * SQRT); i < lim; i++)
      v[i] += delta;
    recalc(idx_a);//OJO puede ser necesario
    //body
    for (int i = idx_a + 1; i < idx_b; i++)
      lazy[i] += delta;
    //tail
    for (int i = idx_b * SQRT; i <= b; i++)
      v[i] += delta;
    recalc(idx_b);//OJO puede ser necesario
  }
}

//tambien podria ser en un rango como en el add
tipo query(tipo val) {
  tipo ans = 0;
  //recorro todos los buckets
  for (int idx = 0; idx < N; idx++) {
    int a = idx * SQRT, b = min(n, (idx + 1) * SQRT);
    //... hacer algo ...
  }
  return ans;
}
int main() {
  //leer n, q y los elementos de v
```

```
60
61    SQRT = (int)sqrt(n) + 1;
62    N = (n + SQRT - 1) / SQRT;//nro. de buckets
63    //construir cada bucket
64    for (int idx = 0; idx < N; idx++)
65      recalc(idx);
66
67    //resto del programa
68    return 0;
69  }
```

2.15. Nro. Elementos menores o iguales a x en $O(log(n))$

```
1  //insersion y consulta de cuantos <= en log n
2  struct leqset {
3    int maxl; vector<int> c;
4    int pref(int n, int l) { return (n>>(maxl-l))|(1<<l); }
5    void ini(int ml) { maxl=ml; c=vector<int>(1<<(maxl+1)); }
6    //inserta c copias de e, si c es negativo saca c copias
7    void insert(int e, int q=1) { forn(l,maxl+1) c[pref(e,l)]+=q; }
8    int leq(int e) {
9      int r=0,a=1;
10     forn(i,maxl) {
11       a<<=1; int b=(e>>maxl-i-1)&1;
12       if (b) r+=c[a]; a|=b;
13     } return r + c[a]; //sin el c[a] da los estrictamente menores
14   }
15   int size() { return c[1]; }
16   int count(int e) { return c[e|(1<<maxl)]; }
17 };
```

3. Algos

3.1. LIS en O(n log n) con Reconstruccion

```
1  //Para non-increasing, cambiar comparaciones y revisar busq binaria
2  //Given an array, paint it in the least number of colors so that each
       color turns to a non-increasing subsequence.
3  //Solution:Min number of colors=Length of the longest increasing
       subsequence
4
5  // Las lineas marcadas con // Camino no son necesarias si no se desea
       reconstruir el camino.
6  #define MAXN 1000000
7  int v[MAXN]; // INPUT del algoritmo.
8  int mv[MAXN];
9  int mi[MAXN] ,p[MAXN]; // Camino
10 int l[MAXN]; // Aca apareceria la maxima subsecuencia creciente(los
       indices)
11 int lis(int n) {
12   forn(i,n) mv[i] = INF;
13   forn(i,n) mi[i] = -1; // Camino
14   forn(i,n) p [i] = -1; // Camino
15   mv[0] = -INF;
16   int res = 0;
17   forn(i,n) {
18     // Con upper_bound es maxima subsecuencia no decreciente.
19     // Con lower_bound es maxima subsecuencia creciente.
20     int me = upper_bound(mv,mv+n,v[i]) - mv;
21     p[i] = mi[me-1]; // Camino
22     mv[me] = v[i];
23     mi[me] = i; // Camino
24     if (me > res) res = me;
25   }
26   for(int a = mi[res], i = res - 1;a != -1; a = p[a], i--) // Camino
27     l[i] = a; // Indices: poniendo l[i] = v[a] quedan los valores.
28   return res;
29 }
```

3.2. Mo

```
1  // g++ -std=c++11 "mo.cpp" -o run
2  /***
3  =============== <Mo> ===================
4  Contain a sample about Mo algorithm
5  Brief explanation when use Mo:
6  Explain where and when we can use above algorithm
7
8  As mentioned, this algorithm is offline, that means we cannot use it
       when we are forced to stick to given order of queries.
9   That also means we cannot use this when there are update operations.
       Not just that, there is one important possible limitation:
10 We should be able to write the functions add and remove. There will be
       many cases where add is trivial but remove is not.
11 One such example is where we want maximum in a range. As we add elements
       , we can keep track of maximum. But when we remove elements
```

```
it is not trivial. Anyways in that case we can use a set to add elements
    , remove elements and report minimum.
In that case the add and delete operations are O(log N) (Resulting in O(
    N * Sqrt(N) * log N) algorithm).

Suggestion first use the add operation, then the erase operation
Problem for practice: DQUERY spoj
Input: N, then N elements of array M querys with a range L,R
*/
const int MAXV = 1e6 + 10;
const int N = 30010;
const int M = 200010;
int cnt[MAXV];
int v[N];

struct query{
  int l,r,pos;
  query(){}
};
int n;
query qu[M];
int ans[M];

int ret = 0;
void add(int pos){
  pos = v[pos];
  cnt[pos]++;
  if(cnt[pos] == 1){
    ret++;
  }
}
void erase(int pos){
  pos = v[pos];
  cnt[pos]--;
  if(!cnt[pos])ret--;
}
int main(){
  n = in();
  for(int i = 0; i < n;i++){
    v[i] = in();
  }
  int block = ceil(sqrt(n));
  int q = in();
  for(int i = 0; i < q;i++){
    qu[i].l = in() - 1,qu[i].r = in() - 1,qu[i].pos = i;
  }
  sort(qu,qu + q,[&](const query &a,const query &b){
    if(a.l / block != b.l / block)
      return a.l / block < b.l / block;
    return a.r < b.r;
  });
  int l = 0, r = 0;
  for(int i = 0; i < q;i++){
    int nl = qu[i].l,nr = qu[i].r;
    while(l > nl){
      add(--l);
    }
    while(r <= nr){
      add(r++);
    }
    while(l < nl){
      erase(l++);
    }
    while(r > nr + 1){
      erase(--r);
    }

    ans[qu[i].pos] = ret;
  }
  for(int i = 0; i < q;i++)printf("%d\n",ans[i]);
}
```

3.3. Ternary Search - Reales

$f[x]$ increases and then decreases, and we want the maximum value of $f[x]$.

```
double l = a, r = b;
for(int i=0; i<200; i++) {
  double l1 = (l*2+r)/3;
  double l2 = (l+2*r)/3;
  if(f(l1) > f(l2))
    r = l2;
  else
    l = l1;
}
x = l;
```

Si $f(x)$ es facil derivar se deriva $f(x) -> f'(x)$ luego se iguala $f'(x) = 0$ y se despeja x, estamos consiguiendo el punto donde la pendiente es zero (si es de varias variables se deriva parcialmente).
Si no se puede despejar x de $f'(x)$ y estamos seguros que tiene forma de parabola, se puede *aplicar binary search*, buscamos el punto donde la pendiente cambie de signo(osea sea igual a 0)

4. Strings

4.1. Manacher

```
vector<int> manacher(const string &_s) {
  int n = _s.size();
  string s(2 * n + 3, '#');
  s[0] = '%', s[s.size() - 1] = '$';//no deben estar en la cadena
  for (int i = 0; i < n; i++)
    s[(i + 1) * 2] = _s[i];

  n = s.size();
  vector<int> P(n, 0);
  int C = 0, R = 0;
  for (int i = 1; i < n - 1; i++) {
    int j = C - (i - C);
    if (R > i)
      P[i] = min(R - i, P[j]);
    while (s[i + 1 + P[i]] == s[i - 1 - P[i]])
      P[i]++;
    if (i + P[i] > R) {
      C = i;
      R = i + P[i];
    }
  }
  return P;
}
bool is_pal(const vector<int> &mnch_vec, int i, int j) {//[i, j] - i<=j
  int len = j - i + 1;
  i = (i + 1) * 2;//idx to manacher vec idx
  j = (j + 1) * 2;
  int mid = (i + j) / 2;
  return mnch_vec[mid] >= len;
}
int main() {
  string s;
  cin >> s;
  vector<int> mnch_vec= manacher(s);
  if (is_pal(mnch_vec, 2, 7)) {
    //la subcadena desde la posicion 2 a la 7 es palindrome
  }
  return 0;
}
```

4.2. Trie(estatico)

```
int trie[10000000][26];
int sig;
int root = 0;
void reset() {
    sig = -1;
    addNode();//Root
}
void addNode() {
    sig++;
    memset(trie[sig], -1, sizeof(trie[sig]));
}
void insert(const string &s) {
    int cur = root;
    for (int i = 0; i < sz(s); i++) {
    int c = s[i] - 'a';
    if (trie[cur][c] == -1) {
        addNode();
        trie[cur][c] = sig;
    }
    cur = trie[cur][c];
    }
}
```

4.3. Suffix Array O(n log n) con LCP (Kasai) O(n)

4.4. Minima rotacion lexicografica

```
/*
Rotacion Lexicografica minima MinRotLex(cadena,tamanio)
para cambiar inicio de la cadena char s[300]; int h; s+h;
retorna inicio de la rotacion minima :D
*/
int MinRotLex(const char *s, const int slen) {
```

```
   int i = 0, j = 1, k = 0, x, y, tmp;
   while(i < slen && j < slen && k < slen) {
      x = i + k;
      y = j + k;
      if(x >= slen)  x -= slen;
      if(y >= slen)  y -= slen;
      if(s[x] == s[y]) {
         k++;
      } else if(s[x] > s[y]) {
         i = j+1 > i+k+1 ? j+1 : i+k+1;
         k = 0;
         tmp = i, i = j, j = tmp;
      } else {
         j = i+1 > j+k+1 ? i+1 : j+k+1;
         k = 0;
      }
   }
   return i;
}
int main(){
  int n;
  scanf("%d",&n);getchar();
  while(n--){
    char str[1000009];
    gets(str);
    printf("%d\n",MinRotLex(str,strlen(str))+1);
  }
}
```

4.5. Matching

4.5.1. KMP

```
string T;//cadena donde buscar(where)
string P;//cadena a buscar(what)
int b[MAXLEN];//back table b[i] maximo borde de [0..i)
void kmppre(){//by gabina with love
    int i =0, j=-1; b[0]=-1;
    while(i<sz(P)){
        while(j>=0 && P[i] != P[j]) j=b[j];
        i++, j++, b[i] = j;
    }
}
void kmp(){
    int i=0, j=0;
    while(i<sz(T)){
        while(j>=0 && T[i]!=P[j]) j=b[j];
        i++, j++;
        if(j==sz(P)) printf("P is found at index %d in T\n", i-j), j=b[j
            ];
    }
}

int main(){
    cout << "T=";
    cin >> T;
    cout << "P=";
    cin.ignore();
    cin >> P;
    kmppre();
    kmp();
    return 0;
}
```

4.5.2. Z - Por aprender

4.5.3. Matching con suffix array

```
vector<int> FindOccurrences(const string& pattern, const string& text) {
  vector<int> result;
  int minIndex = 0,maxIndex = text.size();
  while(minIndex < maxIndex){
    int mid = (minIndex + maxIndex) >> 1;
    if(cmp(pattern,sa[mid]) > 0)minIndex = mid + 1;
    else maxIndex = mid;
  }
  int start = minIndex;
  maxIndex = text.size();
  while(minIndex < maxIndex){
    int mid = (minIndex + maxIndex) >> 1;
    if(cmp(pattern,sa[mid]) < 0)maxIndex = mid;
    else minIndex = mid + 1;
  }
  int end = maxIndex;
  for(int i = start;  i < end;i++){
    result.push_back(sa[i]);
  }
  return result;
```

```
}
```

4.5.4. Matching con BWT

```
map<char,int>fo;//first ocurrence
map<char,vector<int> >count;//count the i-th ocurrence of symbol
string first;//first colum of bwt
string alpha = "ACGT$";//change this
void preprocess(const string& bwt) {//recieves a BWT
  string ans = "";
  first = bwt;
  sort(first.begin(),first.end());
  for(int i = 0;first[i];i++){
    if(!fo.count(first[i]))fo[first[i]] = i;
  }
  for(char u:alpha)count[u].push_back(0);
  for(int i = 1; i <= bwt.size();i++){
    for(char u:alpha)
      count[u].push_back(count[u].back() + (bwt[i - 1] == u));
  }
}
//return the number of ocurrences of the pattern
int bwtmatch(int bot,string &pattern){
  int top = 0;
  while(top <= bot){
    if(pattern.size()){
      char letter = pattern.back();
      pattern.pop_back();
      if(count[letter][bot + 1]){
        top = fo[letter] + count[letter][top];
        bot = fo[letter] + count[letter][bot + 1] - 1;
      }
      else return 0;
    }
    else return bot - top + 1;
  }
  return 0;
}
```

4.5.5. Matching con Aho-Corasick

```

struct trie{
  map<char, trie> next;
  trie* tran[256];//transiciones del automata
  int idhoja, szhoja;//id de la hoja o 0 si no lo es
  //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
      es hoja
  trie *padre, *link, *nxthoja;
  char pch;//caracter que conecta con padre
  trie(): tran(),  idhoja(), padre(), link() {}
  void insert(const string &s, int id=1, int p=0){//id>0!!!
    if(p<sz(s)){
      trie &ch=next[s[p]];
      tran[(int)s[p]]=&ch;
      ch.padre=this, ch.pch=s[p];
      ch.insert(s, id, p+1);
    }
    else idhoja=id, szhoja=sz(s);
  }
  trie* get_link() {
    if(!link){
      if(!padre) link=this;//es la raiz
      else if(!padre->padre) link=padre;//hijo de la raiz
      else link=padre->get_link()->get_tran(pch);
    }
    return link; }
  trie* get_tran(int c) {
    if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
    return tran[c]; }
  trie *get_nxthoja(){
    if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
    return nxthoja; }
  void print(int p){
    if(idhoja) cout << "found " << idhoja << "  at position " << p-
        szhoja << endl;
    if(get_nxthoja()) get_nxthoja()->print(p); }
  void matching(const string &s, int p=0){
    print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
}tri;


int main(){
  tri=trie();//clear
  tri.insert("ho", 1);
  tri.insert("hoho", 2);
```

4.6. Suffix Automaton

```
/*#################### Suffix Automata ####################*/
const int N = INSERTE_VALOR;//maxima longitud de la cadena
struct State {  //OJO!!! tamanio del alfabeto, si MLE -> map
    State *pre,*go[26];//se puede usar un map<char, State*> go
    int step;
    void clear() {
        pre=0;
        step=0;
        memset(go,0,sizeof(go));//go.clear();
    }
} *root,*last;
State statePool[N * 2],*cur;
void init() {
    cur=statePool;
    root=last=cur++;
    root->clear();
}
void Insert(int w) {
    State *p=last;
    State *np=cur++;
    np->clear();
    np->step=p->step+1;
    while(p&&!p->go[w])
        p->go[w]=np,p=p->pre;
    if(p==0)
        np->pre=root;
    else {
        State *q=p->go[w];
        if(p->step+1==q->step)
            np->pre=q;
        else {
            State *nq=cur++;
            nq->clear();
            memcpy(nq->go,q->go,sizeof(q->go));//nq->go = q->go; para
                mapa
            nq->step=p->step+1;
            nq->pre=q->pre;
            q->pre=nq;
            np->pre=nq;
            while(p&&p->go[w]==q)
                p->go[w]=nq, p=p->pre;
        }
    }
    last=np;
}
/*#################### Suffix Automata ####################*/

/*#################### Algunas aplicaciones ####################*/
//Obtiene el LCSubstring de 2 cadenas en O(|A| + |B|)
string lcs(char A[N], char B[N]) {
    int n,m;
    n = strlen(A); m = strlen(B);
    //Construccion: O(|A|)
    //solo hacerlo una vez si A no cambia
    init();
    for(int i=0; i<n; i++)
        Insert(A[i]-'a'); //Fin construccion
    //LCS: O(|B|)
    int ans = 0, len = 0, bestpos = 0;
    State *p = root;
    for(int i = 0; i < m; i++) {
        int x = B[i]-'a';
        if(p->go[x]) {
            len++;
            p = p->go[x];
        } else {
            while (p && !p->go[x]) p = p->pre;
            if(!p) p = root, len = 0;
            else   len = p->step+1, p = p->go[x];
        }
        if (len > ans)
            ans = len, bestpos = i;
    }
    //return ans; //solo el tamanio del lcs
    return string(B + bestpos - ans + 1, B + bestpos + 1);
}

/*Numero de subcadenas distintas + 1(subcadena vacia) en O(|A|)
OJO: Por alguna razon Suffix Array es mas rapido
Se reduce a contar el numero de paths que inician en q0 y terminan
en cualquier nodo. dp[u] = # de paths que inician en u
- Se debe construir el automata en el main(init y Insert's)
- Setear dp en -1
*/
```

```
number dp[N * 2];
number num_dist_substr(State *u = root) {
    if (dp[u - statePool] != -1) return dp[u - statePool];
    number ans = 1;//el path vacio que representa este nodo
    for (int v = 0; v < 26; v++)//usar for (auto) para mapa
        if (u->go[v])
            ans += num_dist_substr(u->go[v]);
    return (dp[u - statePool] = ans);
}

/*Suma la longitud de todos los substrings en O(|A|)
- Construir el automata(init y insert's)
- Necesita el metodo num_dist_substr (el de arriba)
- setear dp's en -1
*/
number dp1[N * 2];
number sum_length_dist_substr(State *u = root) {
    if (dp1[u - statePool] != -1) return dp1[u - statePool];
    number ans = 0;//el path vacio que representa este nodo
    for (int v = 0; v < 26; v++)//usar for (auto) para mapa
        if (u->go[v])
            ans += (num_dist_substr(u->go[v]) + sum_length_dist_substr(u
                ->go[v]));
    return (dp1[u - statePool] = ans);
}

/*
Pregunta si p es subcadena de la cadena con la cual esta construida
el automata.
Complejidad: - Construir O(|Texto|) - solo una vez (init e insert's)
             - Por Consulta O(|patron a buscar|)
*/
bool is_substring(char p[N]) {
    State *u = root;
    for (int i = 0; p[i]; i++) {
        if (!u->go.count(p[i]))//esta con map!!!
            return false;
        u = u->go[p[i]];//esta con map!!!
    }
    return true;
}
```

4.7. K-esima permutacion de una cadena

```
//Entrada: Una cadena cad(std::string), un long th
//Salida : La th-esima permutacion lexicografica de cad
string ipermutacion(string cad, long long int th){
  sort(cad.begin(), cad.end());
  string sol = "";
  int pos;
  for(int c = cad.size() - 1; c >= 0; c--){
    pos = th / fact[c];
    th %= fact[c];
    sol += cad[pos];
    cad.erase(cad.begin() + pos);
  }
  return sol;
}
```

5. Geometria

5.1. Interseccion de circunferencias - Sacar de Agustin

5.2. Graham Scan

5.3. Cortar Poligono

```
//cuts polygon Q along the line ab
//stores the left side (swap a, b for the right one) in P
void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
  P.clear();
  forn(i, sz(Q)){
    double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
    if(left1>=0) P.pb(Q[i]);
    if(left1*left2<0)
      P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
  }
}
```

5.4. Interseccion de rectangulos

```
#define MAXC 2501
struct Rect{
  int x1,y1, x2,y2;
  int color;
  int area;
```

```
  Rect(int _x1, int _y1, int _x2, int _y2){
    x1 = _x1;
    y1 = _y1;
    x2 = _x2;
    y2 = _y2;
    getArea();
  }
  int getArea(){
    if(x1>=x2 || y1>=y2)return area = 0;
    return area = (x2-x1)*(y2-y1);
  }
};
Rect interseccion(Rect t, Rect r){
  int x1,y1,x2,y2;
  x1 = max(t.x1,r.x1);
  y1 = max(t.y1,r.y1);
  x2 = min(t.x2,r.x2);
  y2 = min(t.y2,r.y2);
  Rect res(x1,y1,x2,y2);
  return res;
}
```

5.5. Distancia punto-recta

```
double distance_point_to_line(const point &a, const point &b, const
    point &pnt){
  double u = ((pnt.x - a.x)*(b.x - a.x) + (pnt.y - a.y)*(b.y - a.y)) /
      distsqr(a, b);
  point intersection;
  intersection.x = a.x + u*(b.x - a.x);
  intersection.y = a.y + u*(b.y - a.y);
  return dist(pnt, intersection);
}
```

5.6. Distancia punto-segmento

```
struct point{
  double x,y;
};
inline double dist(const point &a, const point &b){
  return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
inline double distsqr(const point &a, const point &b){
  return (a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y);
}
double distance_point_to_segment(const point &a, const point &b, const
    point &pnt){
  double u = ((pnt.x - a.x)*(b.x - a.x) + (pnt.y - a.y)*(b.y - a.y)) /
      distsqr(a, b);
  point intersection;
  intersection.x = a.x + u*(b.x - a.x);
  intersection.y = a.y + u*(b.y - a.y);

  if (u < 0.0 || u > 1.0)
    return min(dist(a, pnt), dist(b, pnt));

  return dist(pnt, intersection);
}
```

5.7. Parametrizacion de rectas - Sacar de codeforces

6. Math

6.1. Identidades

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$
$$\sum_{i=0}^{n} i\binom{n}{i} = n * 2^{n-1}$$
$$\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$$
$$\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$
$$\sum_{i=0}^{n} i(i-1) = \frac{8}{6}(\frac{n}{2})(\frac{n}{2}+1)(n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}$$
$$\sum_{i=0}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^{n} i\right]^2$$
$$\sum_{i=0}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$
$$\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1}\binom{p}{k}(n+1)^{p-k+1}$$
$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde)

$$A = I + \frac{B}{2} - 1$$

6.2. Ec. Caracteristica

$$a_0T(n) + a_1T(n-1) + ... + a_kT(n-k) = 0$$
$$p(x) = a_0x^k + a_1x^{k-1} + ... + a_k$$

Sean $r_1, r_2, ..., r_q$ las raíces distintas, de mult. $m_1, m_2, ..., m_q$

$$T(n) = \sum_{i=1}^{q} \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$

Las constantes c_{ij} se determinan por los casos base.

6.3. Identidades de agustin y mario

6.4. Combinatorio

```
forn(i, MAXN+1){//comb[i][k]=i tomados de a k
  comb[i][0]=comb[i][i]=1;
  forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
}
ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p]
    precalculado.
  ll aux = 1;
  while (n + k) aux = (aux * comb[n%p][k%p]) %p, n/=p, k/=p;
  return aux;
}
```

6.5. Exp. de Numeros Mod.

```
ll expmod (ll b, ll e, ll m){//O(log b)
  if(!e) return 1;
  ll q= expmod(b,e/2,m); q=(q*q)%m;
  return e%2? (b * q)%m : q;
}
```

6.6. Exp. de Matrices y Fibonacci en log(n)

```
tipo mod;
struct M22 {          //|a b|
  tipo a, b, c, d; //|c d|
  M22 operator * (const M22 &p) const {
    return (M22){(a*p.a+b*p.c) % mod, (a*p.b+b*p.d) % mod, (c*p.a+d*p.c)
         % mod, (c*p.b+d*p.d) % mod};
  }
};
M22 operator ^ (const M22 &p, tipo n) {
  if(!n) return (M22){1, 0, 0, 1};//matriz identidad
  M22 q = p ^ (n/2); q = q * q;
  return (n % 2)? p * q : q;
}
//devuelve el n-esimo fibonacci (index 0)
//f0 = fibo(0), f1 = fibo(1);
tipo fibo(tipo n, tipo f0, tipo f1) {
  M22 mat=(M22){0, 1, 1, 1}^n;
  return (mat.a*f0 + mat.b*f1) % mod;
}
```

6.7. Gauss Jordan

```
const int N = 300;
typedef vector<double> col;
typedef vector<double> row;
typedef vector<row>Matrix;
col solution;
int main(){
  Matrix M;
  M.resize(300);
  solution.resize(300);
  for(int i = 0; i < 30;i++)M[i].resize(30);
  int n;
  cin >> n;
  for(int i = 0; i < n;i++)
    for(int j = 0; j <= n;j++)
      cin >> M[i][j];

  for(int j = 0; j < n - 1;j++){
    int l =j;
    for(int i = j + 1; i < n;i++){
      if(fabs(M[i][j]) > fabs(M[l][j]))l = i;
    }
    for(int k = j; k <= n;k++){
      swap(M[j][k],M[l][k]);
    }
    for(int i = j + 1; i < n;i++)
      for(int k = n; k >= j;k--)
        M[i][k] -= M[j][k] * M[i][j] / M[j][j];
  }
  double t = 0;
  for(int j = n - 1; j >= 0; j--){
    t = 0.0;
    for(int k = j + 1; k < n;k++)t += M[j][k] * solution[k];
    solution[j] = (M[j][n] - t) / M[j][j];
  }
  cout.precision(4);
  for(int i = 0; i < n;i++)cout<<fixed << solution[i] << "␣";
  return 0;
}
```

6.8. Simplex

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[
        i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n +
        1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] <
             N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B
              [r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<
```

```
          DOUBLE>::infinity();//NO SOLUTION
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j]
               < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();//
        INFINITY
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];//solution find
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE:␣" << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << "␣" << x[i];
  cerr << endl;
  return 0;
}
```

6.9. Matrices y determinante $O(n^3)$

```
struct Mat {
    vector<vector<double> > vec;
    Mat(int n): vec(n, vector<double>(n) ) {}
    Mat(int n, int m): vec(n, vector<double>(m) ) {}
    vector<double> &operator[](int f){return vec[f];}
    const vector<double> &operator[](int f) const {return vec[f];}
    int size() const {return sz(vec);}
    Mat operator+(Mat &b) { ///this de n x m entonces b de n x m
        Mat m(sz(b),sz(b[0]));
        forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j
            ];
        return m;    }
    Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
        int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
        Mat mat(n,t);
        forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
        return mat;    }
    double determinant(){//sacado de e maxx ru
        double det = 1;
        int n = sz(vec);
        Mat m(*this);
        forn(i, n){//para cada columna
            int k = i;
            forr(j, i+1, n)//busco la fila con mayor val abs
                if(abs(m[j][i])>abs(m[k][i])) k = j;
            if(abs(m[k][i])<1e-9) return 0;
            m[i].swap(m[k]);//la swapeo
            if(i!=k) det = -det;
            det *= m[i][i];
            forr(j, i+1, n) m[i][j] /= m[i][i];
            //hago 0 todas las otras filas
            forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
                forr(k, i+1, n) m[j][k]-=m[i][k]*m[j][i];
        }
        return det;
    }
};
```

```cpp
int n;
int main() {
//DETERMINANTE:
//https://uva.onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&page=show_problem&problem=625
  freopen("input.in", "r", stdin);
    ios::sync_with_stdio(0);
    while(cin >> n && n){
        Mat m(n);
        forn(i, n) forn(j, n) cin >> m[i][j];
        cout << (ll)round(m.determinant()) << endl;
    }
    cout  << "*" << endl;
  return 0;
}
```

6.10. Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

6.11. Criba

```cpp
#define MAXP 100000 //no necesariamente primo
int criba[MAXP+1];
void crearcriba(){
  int w[] = {4,2,4,2,4,6,2,6};
  for(int p=25;p<=MAXP;p+=10) criba[p]=5;
  for(int p=9;p<=MAXP;p+=6) criba[p]=3;
  for(int p=4;p<=MAXP;p+=2) criba[p]=2;
  for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])
    for(int j=p*p;j<=MAXP;j+=(p<<1)) if(!criba[j]) criba[j]=p;
}
vector<int> primos;
void buscarprimos(){
  crearcriba();
  forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
}
//~ Useful for bit trick: #define SET(i) ( criba[(i)>>5]|=1<<((i)&31) ), #define INDEX(i) ( (criba[i>>5]>>((i)&31))&1 ), unsigned int criba[MAXP/32+1];


int main() {
  freopen("primos", "w", stdout);
  buscarprimos();
```

6.12. Funciones de primos

Sea $n = \prod p_i^{k_i}$, fact(n) genera un map donde a cada p_i le asocia su k_i

```cpp
//factoriza bien numeros hasta MAXP^2
map<ll,ll> fact(ll n){ //O (cant primos)
  map<ll,ll> ret;
  forall(p, primos){
    while(!(n%*p)){
      ret[*p]++;//divisor found
      n/=*p;
    }
  }
  if(n>1) ret[n]++;
  return ret;
}
//factoriza bien numeros hasta MAXP
map<ll,ll> fact2(ll n){ //O (lg n)
  map<ll,ll> ret;
  while (criba[n]){
    ret[criba[n]]++;
    n/=criba[n];
  }
  if(n>1) ret[n]++;
  return ret;
}
//Usar asi:  divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
void divisores(const map<ll,ll> &f, vector<ll> &divs, map<ll,ll>::iterator it, ll n=1){
    if(it==f.begin()) divs.clear();
    if(it==f.end()) { divs.pb(n);  return; }
    ll p=it->fst, k=it->snd; ++it;
    forn(_, k+1) divisores(f, divs, it, n), n*=p;
}
ll sumDiv (ll n){
  ll rta = 1;
  map<ll,ll> f=fact(n);
  forall(it, f) {
```

```
  ll pot = 1, aux = 0;
  forn(i, it->snd+1) aux += pot, pot *= it->fst;
  rta*=aux;
  }
  return rta;
}
ll eulerPhi (ll n){ // con criba: O(lg n)
  ll rta = n;
  map<ll,ll> f=fact(n);
  forall(it, f) rta -= rta / it->first;
  return rta;
}
ll eulerPhi2 (ll n){ // O (sqrt n)
  ll r = n;
  forr (i,2,n+1){
    if ((ll)i*i > n) break;
    if (n % i == 0){
      while (n%i == 0) n/=i;
      r -= r/i; }
  }
  if (n != 1) r-= r/n;
  return r;
}

int main() {
  buscarprimos();
  forr (x,1, 500000){
    cout << "x = " << x << endl;
    cout << "Numero de factores primos: " << numPrimeFactors(x) << endl;
    cout << "Numero de distintos factores primos: " <<
        numDiffPrimeFactors(x) << endl;
    cout << "Suma de factores primos: " << sumPrimeFactors(x) << endl;
    cout << "Numero de divisores: " << numDiv(x) << endl;
    cout << "Suma de divisores: " << sumDiv(x) << endl;
    cout << "Phi de Euler: " << eulerPhi(x) << endl;
  }
  return 0;
}
```

6.13. Phollard's Rho (rolando)

```
ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}

ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c, and minimize overfloor
  ll x = 0, y = a%c;
  while (b > 0){
    if (b % 2 == 1) x = (x+y) % c;
    y = (y*2) % c;
    b /= 2;
  }
  return x % c;
}

ll expmod (ll b, ll e, ll m){//O(log b)
  if(!e) return 1;
  ll q= expmod(b,e/2,m); q=mulmod(q,q,m);
  return e%2? mulmod(b,q,m) : q;
}

bool es_primo_prob (ll n, int a)
{
  if (n == a) return true;
  ll s = 0,d = n-1;
  while (d % 2 == 0) s++,d/=2;

  ll x = expmod(a,d,n);
  if ((x == 1) || (x+1 == n)) return true;

  forn (i, s-1){
    x = mulmod(x, x, n);
    if (x == 1) return false;
    if (x+1 == n) return true;
  }
  return false;
}

bool rabin (ll n){ //devuelve true si n es primo
  if (n == 1) return false;
  const int ar[] = {2,3,5,7,11,13,17,19,23};
  forn (j,9)
    if (!es_primo_prob(n,ar[j]))
      return false;
  return true;
}

```

```
ll rho(ll n){
    if( (n & 1) == 0 ) return 2;
    ll x = 2 , y = 2 , d = 1;
    ll c = rand() % n + 1;
    while( d == 1 ){
        x = (mulmod( x , x , n ) + c)%n;
        y = (mulmod( y , y , n ) + c)%n;
        y = (mulmod( y , y , n ) + c)%n;
        if( x - y >= 0 ) d = gcd( x - y , n );
        else d = gcd( y - x , n );
    }
    return d==n? rho(n):d;
}

map<ll,ll> prim;
void factRho (ll n){ //O (lg n)^3. un solo numero
  if (n == 1) return;
  if (rabin(n)){
    prim[n]++;
    return;
  }
  ll factor = rho(n);
  factRho(factor);
  factRho(n/factor);
}
```

6.14. GCD

```
tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
```

6.15. Extended Euclid

```
void extendedEuclid (ll a, ll b){ //a * x + b * y = d
  if (!b) { x = 1; y = 0; d = a; return;}
  extendedEuclid (b, a%b);
  ll x1 = y;
  ll y1 = x - (a/b) * y;
  x = x1; y = y1;
}
```

6.16. LCM

```
tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
```

6.17. Inversos

```
#define MAXMOD 15485867
ll inv[MAXMOD];//inv[i]*i=1 mod MOD
void calc(int p){//O(p)
  inv[1]=1;
  forr(i, 2, p) inv[i]= p-((p/i)*inv[p%i])%p;
}
int inverso(int x){//O(log x)
  return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
  return expmod(x, MOD-2);//si mod es primo
}
```

6.18. Simpson

```
double integral(double a, double b, int n=10000) {//O(n), n=cantdiv
  double area=0, h=(b-a)/n, fa=f(a), fb;
  forn(i, n){
    fb=f(a+h*(i+1));
    area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
  }
  return area*h/6.;}
```

6.19. Fraction

```
tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
struct frac{
  tipo p,q;
  frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
  void norm(){
    tipo a = mcd(p,q);
    if(a) p/=a, q/=a;
    else q=1;
    if (q<0) q=-q, p=-p;}
  frac operator+(const frac& o){
    tipo a = mcd(q,o.q);
    return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
  frac operator-(const frac& o){
    tipo a = mcd(q,o.q);
    return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
  frac operator*(frac o){
    tipo a = mcd(q,o.p), b = mcd(o.q,p);
    return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
  frac operator/(frac o){
```

```
    tipo a = mcd(q,o.q), b = mcd(o.p,p);
    return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
  bool operator<(const frac &o) const{return p*o.q < o.p*q;}
  bool operator==(frac o){return p==o.p&&q==o.q;}
};
```

6.20. Polinomio

```
        int m = sz(c), n = sz(o.c);
        vector<tipo> res(max(m,n));
        forn(i, m) res[i] += c[i];
        forn(i, n) res[i] += o.c[i];
        return poly(res);    }
    poly operator*(const tipo cons) const {
    vector<tipo> res(sz(c));
        forn(i, sz(c)) res[i]=c[i]*cons;
        return poly(res);    }
    poly operator*(const poly &o) const {
        int m = sz(c), n = sz(o.c);
        vector<tipo> res(m+n-1);
        forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
        return poly(res);    }
  tipo eval(tipo v) {
    tipo sum = 0;
    dforn(i, sz(c)) sum=sum*v + c[i];
    return sum; }
    //poly contains only a vector<int> c (the coeficients)
  //the following function generates the roots of the polynomial
//it can be easily modified to return float roots
  set<tipo> roots(){
    set<tipo> roots;
    tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
    vector<tipo> ps,qs;
    forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
    forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs.pb(an/q);
    forall(pt,ps)
      forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
        tipo root = abs((*pt) / (*qt));
        if (eval(root)==0) roots.insert(root);
      }
    return roots; }
};
pair<poly,tipo> ruffini(const poly p, tipo r) {
  int n = sz(p.c) - 1 ;
  vector<tipo> b(n);
  b[n-1] = p.c[n];
  dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
  tipo resto = p.c[0] + r*b[0];
  poly result(b);
  return make_pair(result,resto);
}
poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
    poly A; A.c.pb(1);
    forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux;
        }
  poly S; S.c.pb(0);
  forn(i,sz(x)) { poly Li;
    Li = ruffini(A,x[i]).fst;
    Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
        coefficients instead of 1.0 to avoid using double
    S = S + Li * y[i];  }
  return S;
}

int  main(){
  return 0;
}
```

6.21. Ec. Lineales

```
bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
  int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
  vector<int> p; forn(i,m) p.push_back(i);
  forn(i, rw) {
    int uc=i, uf=i;
    forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;
        uc=c;}
    if (feq(a[uf][uc], 0)) { rw = i; break; }
    forn(j, n) swap(a[j][i], a[j][uc]);
    swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
    tipo inv = 1 / a[i][i]; //aca divide
    forr(j, i+1, n) {
      tipo v = a[j][i] * inv;
      forr(k, i, m) a[j][k]-=v * a[i][k];
      y[j] -= v*y[i];
    }
```

```
  } // rw = rango(a), aca la matriz esta triangulada
  forr(i, rw, n) if (!feq(y[i],0)) return false; // checkeo de
      compatibilidad
  x = vector<tipo>(m, 0);
  dforn(i, rw){
    tipo s = y[i];
    forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
    x[p[i]] = s / a[i][i]; //aca divide
  }
  ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
  forn(k, m-rw) {
    ev[k][p[k+rw]] = 1;
    dforn(i, rw){
      tipo s = -a[i][k+rw];
      forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
      ev[k][p[i]] = s / a[i][i]; //aca divide
    }
  }
  return true;
}
```

6.22. Karatsuba

```
// g++ -std=c++11 "karatsuba.cpp" -o hld

/***
=============== <karatsuba> ==================
Complexity: O(N^1.7)
Call to karatsuba function paramter two vectors
* INPUT: two vectors A,B cointains the coeficients of the polynomail
* OUTPUT a vector coitains the coeficients of A * B
*/

int p,k;
vector<int> b,r;

void trim(vector<int>& a){
    while (a.size() > 0 && a.back() == 0) a.pop_back();
}

vector<int> multiply(const vector<int>& a, const vector<int>& b){
    vector<int> c(a.size() + b.size() + 1, 0);
    for (int i = 0; i < a.size(); i++) {
        for (int j = 0; j < b.size(); j++) {
            c[i+j] += a[i] * b[j];
        }
    }
    trim(c);
    return c;
}
// a = a + b*(10^k)
void addTo(vector<int>& a, const vector<int>& b, int k){
    if (a.size() < b.size() + k) a.resize(b.size() + k);
    for (int i = 0; i < b.size(); i++) a[i+k] += b[i];
}
void subFrom(vector<int>& a, const vector<int>& b){
    for (int i = 0; i < b.size(); i++) a[i] -= b[i];
}
// a = a + b
void addTo(vector<int>& a, const vector<int>& b){
    addTo(a, b, 0);
}
vector<int> karatsuba(const vector<int>& a, const vector<int>& b)
{
    int alen = a.size();
    int blen = b.size();
    if (alen == 0 || blen == 0) return vector<int>();
    if (alen < blen) return karatsuba(b, a);
    if (alen < 50) return multiply(a, b);

    int half = alen / 2;
    vector<int> a0(a.begin(), a.begin() + half);
    vector<int> a1(a.begin() + half, a.end());
    vector<int> b0(b.begin(), b.begin() + min<int>(blen, half));
    vector<int> b1(b.begin() + min<int>(blen, half), b.end());

    vector<int> z0 = karatsuba(a0, b0);
    vector<int> z2 = karatsuba(a1, b1);
    addTo(a0, a1);
    addTo(b0, b1);
    vector<int> z1 = karatsuba(a0, b0);
    subFrom(z1, z0);
    subFrom(z1, z2);

    vector<int> res;
    addTo(res, z0);
```

```
    addTo(res, z1, half);
    addTo(res, z2, half + half);

    trim(res);
    return res;
}
```

6.23. FFT

```
#define lowbit(x) (((x) ^ (x-1)) & (x))
typedef complex<long double> Complex;

void FFT(vector<Complex> &A, int s){
    int n = A.size();
    int p = __builtin_ctz(n);

    vector<Complex> a = A;

    for(int i = 0;i<n;++i){
        int rev = 0;
        for(int j = 0;j<p;++j){
            rev <<= 1;
            rev |= ((i >> j) & 1);
        }
        A[i] = a[rev];
    }

    Complex w,wn;

    for(int i = 1;i<=p;++i){
        int M = (1<<i), K = (M>>1);
        wn = Complex(cos(s*2.0*M_PI/(double)M), sin(s*2.0*M_PI/(double)M
            ));

        for(int j = 0;j<n;j += M){
            w = Complex(1.0, 0.0);
            for(int l = j;l<K+j;++l){
                Complex t = w;
                t *= A[l + K];
                Complex u = A[l];
                A[l] += t;
                u -= t;
                A[l + K] = u;
                w *= wn;
            }
        }
    }

    if(s==-1){
        for(int i = 0;i<n;++i)
            A[i] /= (double)n;
    }
}

vector<Complex> FFT_Multiply(vector<Complex> &P, vector<Complex> &Q){
    int n = P.size()+Q.size();
    while(n!=lowbit(n)) n += lowbit(n);

    P.resize(n,0);
    Q.resize(n,0);

    FFT(P,1);
    FFT(Q,1);

    vector<Complex> R;
    for(int i=0;i<n;i++) R.push_back(P[i]*Q[i]);

    FFT(R,-1);

    return R;
}

// Para multiplicacion de enteros grandes
const long long B = 100000;
const int D = 5;
```

6.24. Tablas y cotas (Primos, Divisores, Factoriales, etc)

Factoriales

$0! = 1$	$11! = 39.916.800$
$1! = 1$	$12! = 479.001.600$ ($\in$ `int`)
$2! = 2$	$13! = 6.227.020.800$
$3! = 6$	$14! = 87.178.291.200$
$4! = 24$	$15! = 1.307.674.368.000$
$5! = 120$	$16! = 20.922.789.888.000$
$6! = 720$	$17! = 355.687.428.096.000$
$7! = 5.040$	$18! = 6.402.373.705.728.000$
$8! = 40.320$	$19! = 121.645.100.408.832.000$
$9! = 362.880$	$20! = 2.432.902.008.176.640.000$ ($\in$ `tint`)
$10! = 3.628.800$	$21! = 51.090.942.171.709.400.000$

max signed tint = 9.223.372.036.854.775.807
max unsigned tint = 18.446.744.073.709.551.615

Primos

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109
113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199 211 223 227
229 233 239 241 251 257 263 269 271 277 281 283 293 307 311 313 317 331 337 347
349 353 359 367 373 379 383 389 397 401 409 419 421 431 433 439 443 449 457 461
463 467 479 487 491 499 503 509 521 523 541 547 557 563 569 571 577 587 593 599
601 607 613 617 619 631 641 643 647 653 659 661 673 677 683 691 701 709 719 727
733 739 743 751 757 761 769 773 787 797 809 811 821 823 827 829 839 853 857 859
863 877 881 883 887 907 911 919 929 937 941 947 953 967 971 977 983 991 997 1009
1013 1019 1021 1031 1033 1039 1049 1051 1061 1063 1069 1087 1091 1093 1097 1103
1109 1117 1123 1129 1151 1153 1163 1171 1181 1187 1193 1201 1213 1217 1223 1229
1231 1237 1249 1259 1277 1279 1283 1289 1291 1297 1301 1303 1307 1319 1321 1327
1361 1367 1373 1381 1399 1409 1423 1427 1429 1433 1439 1447 1451 1453 1459 1471
1481 1483 1487 1489 1493 1499 1511 1523 1531 1543 1549 1553 1559 1567 1571 1579
1583 1597 1601 1607 1609 1613 1619 1621 1627 1637 1657 1663 1667 1669 1693 1697
1699 1709 1721 1723 1733 1741 1747 1753 1759 1777 1783 1787 1789 1801 1811 1823
1831 1847 1861 1867 1871 1873 1877 1879 1889 1901 1907 1913 1931 1933 1949 1951
1973 1979 1987 1993 1997 1999 2003 2011 2017 2027 2029 2039 2053 2063 2069 2081

Primos cercanos a 10^n

9941 9949 9967 9973 10007 10009 10037 10039 10061 10067 10069 10079
99961 99971 99989 99991 100003 100019 100043 100049 100057 100069
999959 999961 999979 999983 1000003 1000033 1000037 1000039
9999943 9999971 9999973 9999991 10000019 10000079 10000103 10000121
99999941 99999959 99999971 99999989 100000007 100000037 100000039 100000049
999999893 999999929 999999937 1000000007 1000000009 1000000021 1000000033

Cantidad de primos menores que 10^n

$\pi(10^1) = 4$; $\pi(10^2) = 25$; $\pi(10^3) = 168$; $\pi(10^4) = 1229$; $\pi(10^5) = 9592$
$\pi(10^6) = 78.498$; $\pi(10^7) = 664.579$; $\pi(10^8) = 5.761.455$; $\pi(10^9) = 50.847.534$
$\pi(10^{10}) = 455.052{,}511$; $\pi(10^{11}) = 4.118.054.813$; $\pi(10^{12}) = 37.607.912.018$

Divisores

Cantidad de divisores (σ_0) para *algunos* $n / \neg\exists n' < n, \sigma_0(n') \geqslant \sigma_0(n)$
$\sigma_0(60) = 12$; $\sigma_0(120) = 16$; $\sigma_0(180) = 18$; $\sigma_0(240) = 20$; $\sigma_0(360) = 24$
$\sigma_0(720) = 30$; $\sigma_0(840) = 32$; $\sigma_0(1260) = 36$; $\sigma_0(1680) = 40$; $\sigma_0(10080) = 72$
$\sigma_0(15120) = 80$; $\sigma_0(50400) = 108$; $\sigma_0(83160) = 128$; $\sigma_0(110880) = 144$
$\sigma_0(498960) = 200$; $\sigma_0(554400) = 216$; $\sigma_0(1081080) = 256$; $\sigma_0(1441440) = 288$
$\sigma_0(4324320) = 384$; $\sigma_0(8648640) = 448$
Suma de divisores (σ_1) para *algunos* $n / \neg\exists n' < n, \sigma_1(n') \geqslant \sigma_1(n)$
$\sigma_1(96) = 252$; $\sigma_1(108) = 280$; $\sigma_1(120) = 360$; $\sigma_1(144) = 403$; $\sigma_1(168) = 480$
$\sigma_1(960) = 3048$; $\sigma_1(1008) = 3224$; $\sigma_1(1080) = 3600$; $\sigma_1(1200) = 3844$
$\sigma_1(4620) = 16128$; $\sigma_1(4680) = 16380$; $\sigma_1(5040) = 19344$; $\sigma_1(5760) = 19890$
$\sigma_1(8820) = 31122$; $\sigma_1(9240) = 34560$; $\sigma_1(10080) = 39312$; $\sigma_1(10920) = 40320$
$\sigma_1(32760) = 131040$; $\sigma_1(35280) = 137826$; $\sigma_1(36960) = 145152$; $\sigma_1(37800) = 148800$
$\sigma_1(60480) = 243840$; $\sigma_1(64680) = 246240$; $\sigma_1(65520) = 270816$; $\sigma_1(70560) = 280098$
$\sigma_1(95760) = 386880$; $\sigma_1(98280) = 403200$; $\sigma_1(100800) = 409448$
$\sigma_1(491400) = 2083200$; $\sigma_1(498960) = 2160576$; $\sigma_1(514080) = 2177280$
$\sigma_1(982800) = 4305280$; $\sigma_1(997920) = 4390848$; $\sigma_1(1048320) = 4464096$
$\sigma_1(4979520) = 22189440$; $\sigma_1(4989600) = 22686048$; $\sigma_1(5045040) = 23154768$
$\sigma_1(9896040) = 44323200$; $\sigma_1(9959040) = 44553600$; $\sigma_1(9979200) = 45732192$

7. Grafos

7.1. Bellman-Ford

```
1  int negative_cycle(vector<vector<int> > &G, vector<vector<int> > &cost)
       {
2    //write your code here
3    bool nc = false;
4    int n = G.size();
5    vector<int>dist(n,INT_MAX / 2);
6    dist[0] = 0;
7    for(int i = 0; i < n - 1; i++)
8    for(int u = 0; u < n;u++)
9      for(int j = 0; j < G[u].size();j++){
10       int v = G[u][j];
11       int w = cost[u][j];
12       dist[v] = min(dist[v], dist[u] + w);
13     }
```

```
  for(int u = 0; u < n;u++){
  for(int j = 0; j < G[u].size();j++){
    int v = G[u][j];
    int w = cost[u][j];
    if(dist[v] > dist[u] + w)nc = true;
  }
  }
  return nc;
}
```

7.2. dijkstra grafos densos

7.3. 2 SAT definitivamente no con Tarjan

```
// g++ -std=c++11 "twosat.cpp" -o run
/***
=============== <Two Sat> ==================
Complexity: O(N)
Input: number of variables, then number of clause clauses in format (u
    or v)
if u,v > 0 then is equivalent to u,v
if u,v < 0 then is equivalent to  u ,  v
Output: UNSATISFIABLE can't find a solution
SATISFIABLE if exist a solution then print the assignment of all
    variables (negative for xi = false)

Examples:
Input:
3 3
1 -3
-1 2
-2 -3
Output
SATISFIABLE
1 2 -3
*
Input
1 2
1 1
-1 -1
Output
UNSATISFIABLE
*/
#include <bits/stdc++.h>
using namespace std;
vector<int>G[2][2000010],G2[2000010];
int n, m;
int scc[2000010];
bool vis[2000010];
vector<int>comp[2000010];
int assign[2000010];
int cc = 0;
stack<int>st;
vector<int>sta;
void dfs(int u,int type){
  if(scc[u] != -1)return;
  scc[u] = cc;
  for(int v:G[type][u]){
    dfs(v,type);
  }
  if(!type)st.push(u);
}
void topo(int u){
  if(vis[u])return;
  vis[u] = true;
  for(int v:G2[u])topo(v);
  sta.push_back(u);
}
void buildGraphWitouthLoop(){
  for(int i = 0;i < 2 * n;i++){
    for(int j = 0;j < G[0][i].size();j++){
      if(scc[i] != scc[G[0][i][j]])
        G2[scc[i]].push_back(scc[G[0][i][j]]);
    }
  }
}
int main() {
    ios::sync_with_stdio(false);cin.tie(0);
    cin >> n >> m;
    for(int i = 0,u,v; i < m;i++){
    cin >> u >> v;
    int uu = (u > 0?(u - 1) * 2:(-u - 1) * 2 + 1);
    int vv = (v > 0?(v - 1) * 2:(-v - 1) * 2 + 1);
  //  cout << uu << " " << (uu ^ 1) << "\n";
    G[0][uu ^ 1].push_back(vv);
    G[0][vv ^ 1].push_back(uu);
    G[1][vv].push_back(uu ^ 1);
```

```
    G[1][uu].push_back(vv ^ 1);
  }
  memset(scc,-1,sizeof scc);
  for(int i = 0; i < 2 * n;i++){
    if(scc[i] == -1)dfs(i,0);
  }
  memset(scc,-1,sizeof scc);
  while(!st.empty()){
    int u = st.top();st.pop();
    if(scc[u] == -1){
      dfs(u,1);
      cc++;
    }
  }
  bool unsat = false;
  for(int i = 0; i < 2 * n;i++){
    if(scc[i] == scc[i ^ 1])unsat = true;
    comp[scc[i]].push_back(i);
  }
  if(unsat){
    return cout << "UNSATISFIABLE",0;
  }
  cout << "SATISFIABLE\n";
  buildGraphWitouthLoop();
  for(int i = 0; i < 2 * n;i++){
    if(!vis[i])topo(i);
  }
  for(int u:sta){//inverse of topological sort
    for(int v:comp[u]){//transitivite Skew-Symmetry
      if(!assign[v]){
        assign[v] = 1;
        assign[v ^ 1] = -1;
      }
    }
  }
  for(int i = 0,j = 1; i < 2 * n; i += 2,j++){
    cout << (j) * (assign[i]) << "␣";
  }
    return 0;
}
```

7.4. Prim

7.5. Articulataion Points (desgraciadamente tarjan)

```
// g++ -std=c++11 "articulationpointsandbridges.cpp" -o run
/***
=============== <Articulation points and bridges c++ version>
    ==================
Given a graph return a vector of paris with the bridges and a bool array
     art[]
true if the node is an articulation point
* false otherwise
Graph nodes: 0 to N - 1
*/
using namespace std;
vector<int>G[10010];
int low[10010],num[10010],parent[10010],cc;
//cc is my timer
int art[10010];//bool for detect art point, int for detect how many
    nodes are connected to my articulation point
int root,rC;
int n;
vector<pair<int,int> >bridges;
void dfs(int u){
  low[u]=num[u]=cc++;
  for(int v:G[u]){
    if(num[v]==-1){
      parent[v]=u;
      if(u==root)rC++;
      dfs(v);
      if(low[v]>=num[u])art[u]++;//is a articulation point
      if(low[v]>num[u])bridges.push_back({u,v});//this is a bridge
      low[u]=min(low[u],low[v]);
    }
    else if(v!=parent[u]){
        low[u]=min(low[u],num[v]);
    }
  }
}
void init(){
  bridges.clear();
  for(int i=0;i<n;i++){
    art[i]=low[i]=0;
```

```
    num[i]=parent[i]=-1;
    G[i].clear();
  }
  cc=0;
}
void callARTBRID(){
  for(int i=0;i<n;i++){
    if(num[i]==-1){
      root=i,rC=0;dfs(i);
      art[root]=(rC>1);
    }
  }
}
```

7.6. componentes biconexas y puentes (block cut tree)

```
int V;
vector<int> G[MAXN];
int dfn[MAXN],low[MAXN];
vector< vector<int> > C;
stack< pair<int, int> > stk;
void cache_bc(int x, int y){
    vector<int> com;
    int tx,ty;
    do{
        tx = stk.top().first, ty = stk.top().second;
        stk.pop();
        com.push_back(tx), com.push_back(ty);
    }while(tx!=x || ty!=y);
    C.push_back(com);
}

void DFS(int cur, int prev, int number){
    dfn[cur] = low[cur] = number;
    for(int i = G[cur].size()-1;i>=0;--i){
        int next = G[cur][i];
        if(next==prev) continue;
        if(dfn[next]==-1){
            stk.push(make_pair(cur,next));
            DFS(next,cur,number+1);
            low[cur] = min(low[cur], low[next]);
            if(low[next]>=dfn[cur]) cache_bc(cur,next);
        }else low[cur] = min(low[cur],dfn[next]);
    }
}

void biconn_comp(){
    memset(dfn,-1,sizeof(dfn));
    C.clear();
    DFS(0,0,0);
    int comp = C.size();
    printf("%d\n",comp);
    for(int i = 0;i < comp;++i){
        sort(C[i].begin(),C[i].end());
        C[i].erase(unique(C[i].begin(),C[i].end()),C[i].end());
        int m = C[i].size();
        for(int j = 0;j < m;++j) printf("%d ",1 + C[i][j]);
        printf("\n");
    }
}
```

7.7. LCA saltitos potencias de 2

7.8. LCA sparse table query O(1)

7.9. HLD

```
// g++ -std=c++11 "hld.cpp" -o hld

/***
=============== <HLD> ==================
Complexity: O(N*log (N))
Given a tree and asociative operation in the paths of this tree ask for
    many querys, and updates
in nodes or edges
Input of this example:
N number of nodes, then N elements values in each node
then n - 1 conections
Q querys if T == 1 query on the path u,v
else update node U with value val.

Example problems: Spoj QTREE1 to QTREE6, toby and tree UVA
*/

#include <bits/stdc++.h>
using namespace std;
const int maxn = 1e5;
```

```
const int NEUTRO = 0; // a null value for my ST
int vec[maxn];
vector<int>G[maxn]; //the graph
//int idx[maxn]; // case with value in the edge
int op(int u,int v){// an operation for my path (using ST)
  //return __gcd(u,v);
  //return max(u,v);
  return u + v;
}
int n;
//ask to Branimir for information about this
struct SegmentTree{
  int T[2*maxn];
  void init(){
    memset(T,0,sizeof T);
  }
  void set(int pos,int val){
    pos += n;
    T[pos] = val;
    for(pos >>= 1; pos > 0; pos >>=1){
      T[pos] = op( T[pos << 1] , T[(pos << 1)|1] );
    }
  }
  int query(int l,int r){
    l += n;
    r += n;
    int ans = NEUTRO;
    while( l < r){
      if ( l & 1 ) ans = op(ans, T[l++] );
      if (r & 1 ) ans = op( ans, T[--r] );
      l >>= 1;
      r >>= 1;
    }
    return ans;
  }
};
struct hld{
  int ncad; // store actual number of chain
  int root; // the root of a tree generally 0 or 1
  int pos; // pos of node in chain

  int sz[maxn]; // store the subsize of subtrees
  int depth[maxn]; //depth of the node, useful for LCA via HLD
  int parent[maxn]; // useful for LCA
  int where[maxn]; // where chain is the node?
  //int edgepos[maxn]; // if the value is on the edge: stored in a node
  int chainIdx[maxn]; // position in the chain of the node
  int head[maxn]; // the head of the i-th chain
  //int val[maxn]; // if the value is on the edge
  SegmentTree tree; // this ST allow operations in the path

  void init(){//settings value, and process de HLD
    root = 0;
    ncad = 0;
    pos = 0;
    for(int i = 0; i <=n; i++){
      where[i] = head[i] = -1;
    }
    depth[root] = 0;
    dfs(root , -1);
    descompose(root);
    tree.init();
    /* case with values in edges
    for(int i=0;i<n;i++){
      tree.set(i,val[i]);
    }
    */
  }


  ///init descomposition
  void dfs(int u,int pu){
    sz[u] = 1; //init the sz of this subtree
    parent[u] = pu; // assign the parent
    for(int i = 0; i < G[u].size(); i++){
      int v = G[u][i];
      if ( v == pu )continue;
      //edgepos[idx[u][i]] = v;
      depth[v] = depth[u] + 1;
      dfs(v,u);
      sz[u] += sz[v];
    }
  }
  //descompose graph in HLD descomposition
  void descompose(int u){
    if( head[ncad] == -1)head[ncad] = u; // the head of ncad is u
```

```
    where[u] = ncad; // assign where tu node
    //val[pos] = cost; cost another parameter in descompose for graphs
        with values in edges
    chainIdx[u] = pos++; //assing pos to this node
    int maxi = -1, sc = -1; //finding a special child
    for(int v:G[u]){
      if( sz[v] > maxi && where[v] == -1){
        maxi = sz[v];
        sc = v;
      }
    }
    if(sc != -1)descompose(sc);
    //light nodes here:
    for(int v:G[u]){
      if(where[v] == -1){
        ncad++;
        descompose(v);
      }
    }
  }
  ///end descomposition

  int lca(int u,int v){
    while(where[u]!=where[v]){
      if(depth[ head[ where[u] ] ] > depth[ head[ where[v] ] ])u =
          parent[ head[ where[u] ] ];
      else v = parent[ head[ where[v] ] ];
    }
    return depth[u] < depth[v] ? u:v;
  }

  void update(int u, int val){
    tree.set(chainIdx[u],val);
  }

  int query(int u,int v){
    // if ( u == v) return NEUTRO; value in edges
    int vChain = where[v];
    int ans = NEUTRO;
    while(true){
      int uChain = where[u];
      if(uChain == vChain){
        // return op(ans, tree.query( chainIdx[v] + 1, chainIdx[u] + 1)
            ); value in edges
        return op(ans, tree.query( chainIdx[v], chainIdx[u] + 1) );
      }
      int hu = head[uChain];
      ans = op( ans, tree.query(chainIdx[hu], chainIdx[u] + 1) );
      u = parent[hu];
    }
  }

  int Q(int u,int v){
    int L = lca(u,v);
    return op( query(u,L) , query(v,L) );
  }
}HLD;
int main(){
  //ios::sync_with_stdio(false);cin.tie(0);
  while(cin >> n){
    for(int i = 0; i < n; i++)G[i].clear();
    for(int i = 0; i < n; i++){
      cin >> vec[i];
    }
    for(int i = 1, u,v ; i < n; i++){
      cin >> u >> v;
      G[u].push_back(v);
      G[v].push_back(u);
      /* case with value in edges
       G[u].push_back(make_pair(v,w));
      idx[u].push_back(i-1);
      G[v].push_back(make_pair(u,w));
      idx[v].push_back(i-1);

       */
    }
    HLD.init();
    for(int i = 0; i < n; i++){
      HLD.update(i, vec[i]);
    }
    int question;
    cin >> question;
    for(int i = 0, t, u ,v; i < question; i++){
      cin >> t >> u >> v;
      if( t == 1){
        cout << HLD.Q(u,v) << "\n";
```

```
      }
      else HLD.update(u,v);
    }
  }
}
```

7.10. centroid descomposition

7.11. euler cycle

```
int n,m,ars[MAXE], eq;
vector<int> G[MAXN];//fill G,n,m,ars,eq
list<int> path;
int used[MAXN];
bool usede[MAXE];
queue<list<int>::iterator> q;
int get(int v){
  while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;
  return used[v];
}
void explore(int v, int r, list<int>::iterator it){
  int ar=G[v][get(v)]; int u=v^ars[ar];
  usede[ar]=true;
  list<int>::iterator it2=path.insert(it, u);
  if(u!=r) explore(u, r, it2);
  if(get(v)<sz(G[v])) q.push(it);
}
void euler(){
  zero(used), zero(usede);
  path.clear();
  q=queue<list<int>::iterator>();
  path.push_back(0); q.push(path.begin());
  while(sz(q)){
    list<int>::iterator it=q.front(); q.pop();
    if(used[*it]<sz(G[*it])) explore(*it, *it, it);
  }
  reverse(path.begin(), path.end());
}
void addEdge(int u, int v){
  G[u].pb(eq), G[v].pb(eq);
  ars[eq++]=u^v;
}
```

7.12. diámetro y centro de un árbol

```
/***
=============== <Diameter and center of a tree> ==================
//Problem: Given a tree get the center (or centers)
/* the nodes in the tree that minimize the length of the longest path
    from it to any other node.
* *Finding tree centers:
 * If diameter length is even, then we have one tree center. If odd,
     then we have 2 centers.
 * E.g. 1-2-3-4-5   -> center is 3
 * E.g. 1-2-3-4-5-6 -> center is 3, 4
 * On other side, we can get the worst nodes through the center nodes.
 * A worst node is one that is an end of a diameter, so it has the worst
     tree height
Input:
* No
Output:
* No
dfs: calculate the diameter of the tree
* maxi stores the diameter
findingCenters() return the centers
Nodes in graph 1 to N careful with this
Complexity: O(N)
*/


vector<int>G[5010];
int maxi=-1,far;
int n;
int pre[5010];
int Queue[5010];

void dfs(int path,int u,int parent){
  pre[u]=parent;
  if(path>=maxi){
    maxi=path;
    far=u;
  }
  for(int v:G[u]){
    if(parent!=v){
      dfs(path+1,v,u);//path + w if the graph as weighted
    }
```

```
  }
}
pair<int,int> findingCenters(){
  maxi=-1;
  dfs(0,1,-1);
  dfs(0,far,-1);
  int t=far,L=0;
  while(t!=-1){
    Queue[L]=t;
    t=pre[t];
    ++L;
  }
  int a=-1,b=-1;
  if(L&1){
    a=Queue[L/2];
  }
  else{
    a=min(Queue[L/2-1],Queue[L/2]),b=max(Queue[L/2-1],Queue[L/2]);
  }
  return {a,b};
}
```

7.13. algoritmo hungaro

7.14. union find dinámico

```
#include <bits/stdc++.h>
using namespace std;
#define dprint(v) cerr << #v"=" << v << endl //;)
#define forr(i,a,b) for(int i=(a); i<(b); i++)
#define forn(i,n) forr(i,0,n)
#define dforn(i,n) for(int i=n-1; i>=0; i--)
#define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
#define sz(c) ((int)c.size())
#define zero(v) memset(v, 0, sizeof(v))
#define pb push_back
#define fst first
#define snd second
#define mkp make_pair
typedef long long ll;
typedef pair<int,int> ii;

struct UnionFind {
    int n, comp;
    vector<int> pre,si,c;
    UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
        forn(i,n) pre[i] = i; }
    int find(int u){return u==pre[u]?u:find(pre[u]);}
    bool merge(int u, int v) {
        if((u=find(u))==(v=find(v))) return false;
        if(si[u]<si[v]) swap(u, v);
        si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
        return true;
    }
    int snap(){return sz(c);}
    void rollback(int snap){
        while(sz(c)>snap){
            int v = c.back(); c.pop_back();
            si[pre[v]] -= si[v], pre[v] = v, comp++;
        }
    }
};
enum {ADD,DEL,QUERY};
struct Query {int type,u,v;};
struct DynCon {
    vector<Query> q;
    UnionFind dsu;
    vector<int> match,res;
    map<ii,int> last;//se puede no usar cuando hay identificador para
        cada arista (mejora poco)
    DynCon(int n=0):dsu(n){}
    void add(int u, int v) {
        if(u>v) swap(u,v);
        q.pb((Query){ADD, u, v}), match.pb(-1);
        last[ii(u,v)] = sz(q)-1;
    }
    void remove(int u, int v) {
        if(u>v) swap(u,v);
        q.pb((Query){DEL, u, v});
        int prev = last[ii(u,v)];
        match[prev] = sz(q)-1;
        match.pb(prev);
    }
    void query() {//podria pasarle un puntero donde guardar la respuesta
        q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
    void process() {
        forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
```

```
            sz(q);
        go(0,sz(q));
    }
    void go(int l, int r) {
        if(l+1==r){
            if (q[l].type == QUERY)//Aqui responder la query usando el dsu!
                res.pb(dsu.comp);//aqui query=cantidad de componentes conexas
            return;
        }
        int s=dsu.snap(), m = (l+r) / 2;
        forr(i,m,r) if(match[i]!=-1 && match[i]<l) dsu.merge(q[i].u, q[i].v);
        go(l,m);
        dsu.rollback(s);
        s = dsu.snap();
        forr(i,l,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[i].v);
        go(m,r);
        dsu.rollback(s);
    }
}dc;

// Problema ejemplo: http://codeforces.com/gym/100551/problem/A

int n,k;

int main() {
    //~ freopen("in", "r", stdin);
    freopen("connect.in", "r", stdin);
    freopen("connect.out", "w", stdout);
    ios::sync_with_stdio(0);
    while(cin >> n >> k){
    dc=DynCon(n);
    forn(_,k) { string ord; cin >> ord;
      if (ord=="?") {
        dc.query();
      } else if (ord=="+") { int a,b; cin>>a>>b; a--;b--;
        dc.add(a,b);
      } else if (ord=="-") { int a,b; cin>>a>>b; a--;b--;
        dc.remove(a,b);
      } else assert(false);
    }
        if(!k) continue;//k==0 WTF
        dc.process();
        forn(i,sz(dc.res)) cout << dc.res[i] << endl;
    }
    return 0;
}
```

7.15. truquitos estúpidos por ejemplo second MST es con LCA

7.16. erdos galloi

```
// g++ -std=c++11 "erdosgalloi.cpp" -o run
/***
=============== <Erdosgalloi c++ version> ==================
Given the grades of each node of a graph return if this form a valid graph
includes: algorithm, functional, numeric, forn
// Receives a sorted degree sequence (non ascending)
O(NlgN)

*/
bool isGraphicSequence(const vector<int> &seq)  // O(n lg n)
{
  vector<int> sum;
  int n = seq.size();

  if (n == 1 && seq[0] != 0) return false;

  sum.reserve(n + 1);
  sum.push_back(0);
  for (int i = 0; i < n; ++i) sum.push_back(sum[i] + seq[i]);
  if ((sum[n] & 1) == 1) return false;

  for (long long k = 1; k <= n - 1 && seq[k - 1] >= k; ++k) {
    int j = distance(seq.begin(), upper_bound(seq.begin() + k, seq.end(), k,
                                              greater<int>())) + 1;
    long long left = sum[k];
    long long right = k * (k - 1) + (j - k - 1) * k + (sum[n] - sum[j - 1]);
```

```
28
29      if (left > right) return false;
30    }
31
32    return true;
33  }
```

7.17. grafo funcional hallar k-esimo partiendo de un nodo

7.18. konig

7.19. min-vertex cover bipartitos

7.20. max-flow (min cost versión)

```
1  // g++ -std=c++11 "maxflowmincost.cpp" -o run
2  /***
3  =============== <Max flow-min cost c++ version> ==================
4  Given a grapth with edges with a capacity C and weight D
5  * compute the max-flow min cost
6  Edmond karps idea
7  * Complexity O( v  *E*log(v))
8  Problem for practice: Dijkstra Dijkstra uva
9  */
10 #define REP(i,j,k) for(int (i)=(j);(i)<(k);++(i))
11 #define MP make_pair
12
13 using namespace std;
14
15 #define MAXN 500
16 #define MAXM MAXN * 5
17 typedef vector<int> VI;
18 typedef long long ll;
19 const int INF = 1E9; // $infinity$: be careful to make this big enough
       !!!
20 int S; // source
21 int T; // sink
22 int FN; // number of nodes
23 int FM; // number of edges (initialize this to 0)
24 // ra[a]: edges connected to a (NO MATTER WHICH WAY!!!); clear this in
       the beginning
25 VI ra[MAXN];
26 int kend[MAXM], cap[MAXM], cost[MAXM]; // size: TWICE the number of
       edges
27
28 // Adds an edge from a to b with capacity c and cost d and returns the
       number of the new edge
29
30 int addedge(int a, int b, int c, int d) {
31   int i = 2*FM;
32   kend[i] = b;
33   cap[i] = c;
34   cost[i] = d;
35   ra[a].push_back(i);
36   kend[i+1] = a;
37   cap[i+1] = 0;
38   cost[i+1] = -d;
39   ra[b].push_back(i+1);
40   FM++;
41   return i;
42 }
43 int n;
44 int dst[MAXM], pre[MAXM], pret[MAXM];
45 //finding the shortest path via fanding duan, also it works with bellman
       ford
46 //or dijkstra (careful of negative cycles)
47 bool spfa(){
48   REP(i,0,FN) dst[i] = INF;
49   dst[S] = 0;
50   queue<int> que; que.push(S);
51   while(!que.empty()){
52     int x = que.front(); que.pop();
53     for (int t : ra[x]){
54       int y = kend[t], nw = dst[x] + cost[t];
55       if(cap[t] > 0 && nw<dst[y]){
56         dst[y] = nw; pre[y] = x; pret[y] = t; que.push(y);
57       }
58     }
59   }
60   return dst[T]!=INF;
61 }
62 // returns the maximum flow and the minimum cost for this flow
63 pair<ll,ll> solve(){
64   ll totw = 0, totf = 0;
65   while(spfa()){
66     int minflow = INF;
67     for (int x = T; x!=S; x = pre[x]){
```

```
      minflow = min(minflow, cap[pret[x]]);
    }
    for (int x = T; x!=S; x = pre[x]){
      cap[pret[x]] -= minflow;
      cap[pret[x]^1] += minflow;
    }
    totf += minflow;
    totw += minflow*dst[T];
  }
  return make_pair(totf, totw);
}
void init(){
  FN=4*n+15;//make this big n=number of nodes of the graph
  FM=0;
  S=0,T=n+1;
  for(int i=0;i<FN;i++)ra[i].clear();//clear the graph be careful
}
```

7.21. max-flow corto con matriz

```
// g++  "maxflowMVEK.cpp" -o run
/***
================ <Max Flow with matriz Edmonds karp c++ version>
    ===================
//Given a graph with capacitys find the max-flow

Nodes indexed 1 to N
* Complexity O(N  *E)
Problem for practice: UVA 820
*/
#define N 500
int cap[N][N], pre[N], n;
int s;//source
int t;//destination
bool bfs() {
    queue<int>q;
    q.push(s);
    memset(pre,-1,sizeof pre);
    pre[s]=s;
    while(!q.empty()){
        int u=q.front();q.pop();
        if(u==t)return true;
        for(int i=1;i<=n;i++){//nodes 1 to n
            if(pre[i]==-1&&cap[u][i])pre[i]=u,q.push(i);
        }
    }
    return false;
}

int maxFlow() {
    int mf=0,f,v;//max flow, flow for a path, the vertex
    while(bfs()){//while encountered a path  source to destination
        v=t;//min
        f=INT_MAX;//make this big enough
        while(pre[v]!=v){f=min(f,cap[pre[v]][v]),v=pre[v];}//finding the
            min capacity
        v=t;mf+=f;
        while(pre[v]!=v){cap[pre[v]][v]-=f,cap[v][pre[v]]+=f,v=pre[v];}
            //update the flow
    }
    return mf;
}
void init(){
  memset(cap,0,sizeof cap);
  //cap[u][v]+=capacidad,cap[v][u]+=capacidad
}
```

7.22. max-flow sin matriz

```
// g++ -std=c++11 "maxflowNMEK.cpp" -o run
/***
================ <Max Flow with-out matriz Edmonds karp c++ version>
    ===================
//Given a graph with capacitys find the max-flow

Nodes indexed 1 to N
* Complexity O(N  *E)
Problem for practice: UVA 820
* Input N number of nodes,
* M edges conections
* compute the flow with source 1 and sink N
*/
using namespace std;
const int N = 110;
const int M = 10010 * 2;
vector<int>G[N];
```

```
int kend[M], cap[M], cost[M];
int edge = 0;
int s,t;
void add(int u,int v,int c){
  int forward = edge * 2, backward = edge * 2 + 1;
  kend[forward] = v;
  cap[forward] = c;
  G[u].push_back(forward);
  kend[backward] = u;
  cap[backward] = 0;
  G[v].push_back(backward);
  edge++;
}
int vis[M],pre[M],pret[M];
bool bfs(){
  for(int i = 0; i <= 100;i++)vis[i] = false;
  vis[s] = true;
  queue<int>q;
  q.push(s);
  while(!q.empty()){
    int u = q.front();q.pop();
    for(int edge:G[u]){
      int v = kend[edge];
      if(cap[edge] > 0 && !vis[v]){
        vis[v] = true;
        pre[v] = u;
        pret[v] = edge;//the edge store the information
        q.push(v);
      }
    }
  }
  return vis[t];
}
int max_flow(){
  int totf = 0LL;
  while(bfs()){
    int minflow = INT_MAX;
    for(int x = t; x != s; x = pre[x]){
      minflow = min(minflow,cap[pret[x]]);
    }
    for(int x = t; x != s; x = pre[x]){
      cap[pret[x]] -= minflow;
      cap[pret[x] ^ 1] += minflow;
    }
    totf += minflow;
  }
  return totf;
}
int main(){
  int n,m;
  scanf("%d %d",&n,&m);
  for(int i = 0,u,v,ca; i < m;i++){
    scanf("%d %d %d",&u,&v,&ca);
    add(u,v,ca);
  }
  s = 1, t = n;
  printf("%lld\n",max_flow());
}
```

7.23. Dinic

```

const int MAX = 300;
// Corte minimo: vertices con dist[v]>=0 (del lado de src) VS.  dist[v]==-1 (del lado del dst)
// Para el caso de la red de Bipartite Matching (Sean V1 y V2 los conjuntos mas proximos a src y dst respectivamente):
// Reconstruir matching: para todo v1 en V1 ver las aristas a vertices de V2 con it->f>0, es arista del Matching
// Min Vertex Cover: vertices de V1 con dist[v]==-1 + vertices de V2 con dist[v]>0
// Max Independent Set: tomar los vertices NO tomados por el Min Vertex Cover
// Max Clique: construir la red de G complemento (debe ser bipartito!) y encontrar un Max Independet Set
// Min Edge Cover: tomar las aristas del matching + para todo vertices no cubierto hasta el momento, tomar cualquier arista de el
int nodes, src, dst;
int dist[MAX], q[MAX], work[MAX];
struct Edge {
    int to, rev;
    ll f, cap;
    Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(cap) {}
};
vector<Edge> G[MAX];
```

```
18 void addEdge(int s, int t, ll cap){
19     G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
           0));}
20 bool dinic_bfs(){
21     fill(dist, dist+nodes, -1), dist[src]=0;
22     int qt=0; q[qt++]=src;
23     for(int qh=0; qh<qt; qh++){
24         int u =q[qh];
25         forall(e, G[u]){
26             int v=e->to;
27             if(dist[v]<0 && e->f < e->cap)
28                 dist[v]=dist[u]+1, q[qt++]=v;
29         }
30     }
31     return dist[dst]>=0;
32 }
33 ll dinic_dfs(int u, ll f){
34     if(u==dst) return f;
35     for(int &i=work[u]; i<sz(G[u]); i++){
36         Edge &e = G[u][i];
37         if(e.cap<=e.f) continue;
38         int v=e.to;
39         if(dist[v]==dist[u]+1){
40                 ll df=dinic_dfs(v, min(f, e.cap-e.f));
41                 if(df>0){
42                         e.f+=df, G[v][e.rev].f-= df;
43                         return df;  }
44         }
45     }
46     return 0;
47 }
48 ll maxFlow(int _src, int _dst){
49     src=_src, dst=_dst;
50     ll result=0;
51     while(dinic_bfs()){
52         fill(work, work+nodes, 0);
53         while(ll delta=dinic_dfs(src,INF))
54             result+=delta;
55     }
56     // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
           forman el min-cut
57     return result; }
```

7.24. máximo emparejamiento bipartito

```
1  // g++ -std=c "bipartitematching.cpp" -o run
2  /***
3  =============== <MCBM max cardinality bipartite matching c++ version>
       ==================
4  Return the bipartite matching of a Graph
5  * Format of nodes: 1 to N
6  */
7
8  const int N = 100010;
9  vector<int>G[N];
10 bool v[N];//for the greedy speed up
11 int match[N];
12 bool vis[N];
13 int n,m;
14 //calling aumenting path
15 bool aug(int u){
16     if(vis[u])return false;
17     vis[u]=true;
18     for(int i=0;i<(int)G[u].size();++i){
19     int r=G[u][i];
20         if(match[r]==-1||aug(match[r])){
21             match[r]=u;match[u]=r;return true;
22         }
23     }
24     return 0;
25 }
26 int mc;
27 //findging all augmenting path's
28 int solve(){
29    bool check=true;
30    while(check){
31         check=false;
32         memset(vis,0,sizeof vis);
33         for(int i=1;i<=n;++i){
34       if(!v[i]&&match[i]==-1){
35         bool op=aug(i);
36         check|=op;
37         mc+=op;
38       }
39     }
40     }
```

```
    return mc;
}
void init(){
  memset(v,0,sizeof v);
  memset(vis,false,sizeof vis);
  mc=0;
  memset(match,-1,sizeof match);
    for(int i=0;i<=n;i++)G[i].clear();
}
void greedySpeedUp(){
  //greedy optimization, match with the first not matched
  for(int i=1;i<=n;++i){
        for(int j=0;j<(int)G[i].size();++j){
            if(match[G[i][j]]==-1){
        match[G[i][j]]=i,match[i]=G[i][j],mc++,v[i]=true;break;
     }
       }
    }
}
```

7.25. max-independent set en bipartitos

7.26. min-path cover (ver tópicos raros de halim)

7.27. min-cost arborescence

7.28. lema de diapositivas de nico de grafos funcionales

7.29. minimax y maximini con kruskal y dijkstra

```
// g++ -std=c++11 "maximini.cpp" -o run
/***
=============== <maximini c++ version> ==================
Given a weighted graph return the maximini (the maximun of the minimum)
or the minimax (the minimum of the maximum) in the path a,b
*
Minimax as definded as: finding the minimum of maximum edge weight among
     all posible paths
* between two verrtices a to b, the cost for a path fron a to b is
    determined by maximum edge
* weight along this path. Among all these possible paths from a to b,
    pick the one with the minimum
* ax-edge-weight
* Complexity O(E*log(E) + V + E)
*
Problem for practice: UVA 534,544
*/
int n;
pair<int,pair<int,int> >Edges[20000];
int t;
map<string,int>mp;
int parent[210];
pair<int,int>child[210];
bool vis[210];
vector<pair<int,int> >G[210];

int find(int u){return u==parent[u]?u:parent[u]=find(parent[u]);}
void Union(int u,int v){
  int pu=find(u),pv=find(v);
  if(pu!=pv){
    parent[pv]=pu;
  }
}
int mst(int a,int b){
  sort(Edges,Edges+t);
  reverse(Edges,Edges+t);//don't reverse for the minimax
  for(int i=0;i<=200;i++)parent[i] = i;
  int w,u,v, maximini = 1e8, minimax = 0;
  for(int i=0;i<t;i++){
    tie(w,u,v) = make_tuple(Edges[i].first, Edges[i].second.first, Edges
        [i].second.second);
    if(find(u) != find(v)){
      Union(u,v);
      G[u].push_back({v,w});
      G[v].push_back({u,w});
    }
  }
  queue<int>q;
  q.push(a);
  vis[a]=true;
  while(!q.empty()){
    int u = q.front();q.pop();
    //if(u==1)break;
    for(pair<int,double>node: G[u]){
      if(!vis[node.first]){
        vis[node.first] = true;
        q.push(node.first);
```

```
        //maximini=max(maximini,node.second);
        child[node.first].first = u;
        child[node.first].second = node.second;
      }
    }
  }
  for(int t = b;t != -1;t = child[t].first){
    //cout<<t<<" "<<child[t].second<<"\n";
    //minimax=max(minimax,child[t].second);
    maximini = min(maximini,child[t].second);
  }
  return maximini;
}
```

8. Teoria de juegos

8.1. Teorema fundamental de los juegos optimos

```
boolean isWinning(position pos) {
    moves[] = possible positions to which I can move from the position
        pos;
    for (all x in moves)
        if (!isWinning(x)) return true;

    return false;
}
```

8.2. Como calcular grundy

```
int grundyNumber(position pos) {
    moves[] = possible positions to which I can move from pos
    set s;
    for (all x in moves) insert into s grundyNumber(x);
    //return the smallest non-negative integer not in the set s;
    int ret=0;
    while (s.contains(ret)) ret++;
    return ret;
}
```

9. Probabilidad

9.1. Formulas clave

10. Otros/utilitarios

10.1. josephus

```
int survivor(int n, int m){
   int s = 0;
   for (int i=1;i<=n;++i) s = (s+m)%i;
   return (s+1);
}
```

10.2. josephus k = 2

```
//////////JAVA
    /**
   *
   * @param n the number of people standing in the circle
   * @return the safe position who will survive the execution
   *   f(N) = 2L + 1 where N =2^M + L and 0 <= L < 2^M
   */
  public int getSafePosition(int n) {
    // find value of L for the equation
    int valueOfL = n - Integer.highestOneBit(n);
    int safePosition = 2 * valueOfL  + 1;
    return safePosition;
  }
```

10.3. poker

10.4. iterar subconjuntos

```
for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
```

10.5. como reconstruir una DP (normal)

```
/*
You just need to revisit your steps in the DP. In case of 0-1 knapsack,
    lets say the original DP function was solve, and the function
    reconstruct will give you the actual solution (I'm writing the code
    in C++):
*/
```

```
int solve(int pos, int capacity){
    if(pos == no_of_objects) return 0;
    if(memo[pos][capacity] != -1) return memo[pos][capacity];
    int r1 = solve(pos + 1, capacity); //dont take
    int r2 = 0;
    if(weight[pos] <= capacity){
        r2 = solve(pos + 1, capacity - weight[pos]) + profit[pos]; //
            take
    }
    return memo[pos][capacity] = max(r1, r2);
}
void reconstruct(int pos, int capacity){
    if(pos == no_of_objects) return; //you have completed reconstruction
    int r1 = memo[pos + 1][capacity]; //dont take
    int r2 = 0;
    if(weight[pos] <= capacity)r2 = memo[pos + 1][capacity - weight[pos
        ]] + profit[pos]; //take
    if(r1 > r2) {reconstruct(pos + 1, capacity);}
    else{
        cout << "Take␣object␣" << pos << endl;
        reconstruct(pos + 1, capacity - weight[pos]) + profit[pos];
    }
}
After executing reconstruct, it will print all those objects that give
    you the optimal solution. As you can see, at most no_of_objects
    calls will be made in the reconstruct function.
Similarly, you can reconstruct the solution of any DP greedily.
```

10.6. muajaja con j

```
#include <signal.h>
void divzero(int p){
  while(true);}
void segm(int p){
  exit(0);}
//in main
signal(SIGFPE, divzero);
signal(SIGSEGV, segm);
```

Expandir pila

```
#include <sys/resource.h>
rlimit rl;
getrlimit(RLIMIT_STACK, &rl);
rl.rlim_cur=1024L*1024L*256L;//256mb
setrlimit(RLIMIT_STACK, &rl);
```

10.7. comparar doubles for noobs

```
const double EPS = 1e-9;
x == y  <=> fabs(x-y) < EPS
x >  y  <=> x > y + EPS
x >= y  <=> x > y - EPS
```

10.8. infix to postfix

```
//infix to postfix with shunting yard, Halim interpretation
//plus eval function given a postfix return the result of the operation
//format: string like  ( x o x  ( x o x ) ) o=operation x=value
string s;
bool isOperator(string u){
  return u=="+"||u=="-"||u=="*"||u=="/";
}
bool precede(string u){
  if(u=="*"||u=="/")return true;
  return false;
}
void solve(){
  getline(cin,s);
  stack<string>st;
  vector<string>v;
  stringstream ss;
  ss<<s;
  while(ss>>s){
    if(isOperator(s)){
      while(!st.empty()&&isOperator(st.top())&&precede(st.top())>=
          precede(s)){
        v.push_back(st.top());st.pop();
      }
      st.push(s);
    }
    else{
       if(s=="("){
         st.push(s);
      }
      else{
        if(s==")"){
          while(!st.empty()&&st.top()!="("){
```

```
            v.push_back(st.top());st.pop();
          }
          if(!st.empty()&&st.top()=="(")st.pop();
        }
        else {
          v.push_back(s);
        }
      }
    }
  }
  while(!st.empty()){
    v.push_back(st.top());st.pop();
  }
  stack<double>stans;
  double x;
  for(string eva:v){
    if(!isOperator(eva)){
      stringstream nu;
      nu<<eva;
      nu>>x;
      stans.push(x);
    }
    else{
      double a=stans.top();stans.pop();
      double b=stans.top();stans.pop();
      if(eva=="*")b*=a;
      if(eva=="/")b/=a;
      if(eva=="+")b+=a;
      if(eva=="-")b-=a;
      stans.push(b);
    }
  }
  cout<<fixed<<stans.top()<<"\n";
}
```

10.9. numeros romanos

```
#include <bits/stdc++.h>
using namespace std;
map<int,string>cvt;

string aromano(int n){
  cvt[1000] = "M";cvt[900] = "CM",cvt[500] = "D", cvt[400] = "CD";
  cvt[100] = "C";cvt[90] = "XC"; cvt[50] = "L";
  cvt[40] = "XL";cvt[10] = "X";cvt[9] = "IX";cvt[5] = "V"; cvt[4] = "IV"
      ;
  cvt[1] = "I";
  string ans = "";
  for(map<int,string>::reverse_iterator it = cvt.rbegin();it != cvt.rend
      ();it++)
    while(n >= it->first){
      ans += it->second;
      n -= it->first;
    }
  return ans;
}
map<string,int>crn;
int anumero(string R){
  map<char, int> crn;
  crn['I'] = 1;   crn['V'] = 5;   crn['X'] = 10;   crn['L'] = 50;
  crn['C'] = 100; crn['D'] = 500; crn['M'] = 1000;
  int value = 0;
  for (int i = 0; R[i]; i++)
    if (i + 1 < R.size() && crn[R[i]] < crn[R[i+1]]) {
      value += crn[R[i+1]] - crn[R[i]];
      i++;
    }
    else value += crn[R[i]];
  return value;
}
```

10.10. get k-th permutacion

```
vector<int>v;
//finding the number of permutation 0.....n-1
int main()
{
    string s;
    while(getline(cin,s)){
        stringstream ss;
        ss<<s;
        int pos=0,u;
        v.clear();
        while(ss>>u){
            v.push_back(u-1);
        }
```

```
        vector<int>le(v.size(),0);
        for(int i=0;i<v.size();i++){
            for(int j=i+1;j<v.size();j++){
                if(v[i]>v[j])le[i]++;
            }
        }
        long long ans=0LL,fact=0LL,por=1LL;
        for(int i=le.size()-1;i>=0;i--){
            if(fact!=0LL)por*=fact;
            fact++;
            ans=ans+por*le[i];
        }
        cout<<ans+1<<"\n";
    }
    return 0;
}
```

10.11. sliding window

10.12. permutaciones de un dado

```
// izquierda, derecha, arriba, al frente, abajo, atras

int p[][6] = {
    {0,1,2,3,4,5},
    {0,1,3,4,5,2},
    {0,1,4,5,2,3},
    {0,1,5,2,3,4},
    {1,0,2,5,4,3},
    {1,0,3,2,5,4},
    {1,0,4,3,2,5},
    {1,0,5,4,3,2},
    {2,4,5,1,3,0},
    {2,4,1,3,0,5},
    {2,4,3,0,5,1},
    {2,4,0,5,1,3},
    {3,5,2,1,4,0},
    {3,5,1,4,0,2},
    {3,5,4,0,2,1},
    {3,5,0,2,1,4},
    {4,2,5,0,3,1},
    {4,2,0,3,1,5},
    {4,2,3,1,5,0},
    {4,2,1,5,0,3},
    {5,3,2,0,4,1},
    {5,3,0,4,1,2},
    {5,3,4,1,2,0},
    {5,3,1,2,0,4}
};
```

10.13. ternary search

10.14. liebre y el tortugo

10.15. como usar printf

10.16. java

10.17. python

10.18. template

10.19. file setup

```
//tambien se pueden usar comas: {a, x, m, l}
touch {a..l}.in; tee {a..l}.cpp < template.cpp
```