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1. algorithm

 $\# include < \! algorithm \! > \# include < \! numeric \! >$

Algo	Params	Funcion
sort, stable_sort	f, 1	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y
		particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
		f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se
		puede insertar elem para que
		quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace $resul+i=f+i \ \forall i$
find, find_if, find_first_of	f, l, elem	it encuentra i \in [f,l) tq. i=elem,
	/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2, l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca $[f2,l2) \in [f,l)$
replace, replace_if	f, l, old	cambia old / pred(i) por new
	/ pred, new	
reverse	f, 1	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	bool con [f1,l1];[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection,	f1, l1, f2, l2, res	[res,) la op. de conj
set_difference, set_union,		
set_symmetric_difference,		
push_heap, pop_heap,	f, l, e / e /	mete/saca e en heap [f,l),
make_heap		hace un heap de [f,l)
is_heap	f,l	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum /\text{oper de [f,l)}$
inner_product	f1, l1, f2, i	$T = i + [f1, 11) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum /oper de [f,f+i] \forall i \in [f,l)$
_builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
_builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
_builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
_builtin_popcount	unsigned int	Cant. de 1's en x.
_builtin_parity	unsigned int	1 si x es par, 0 si es impar.
_builtin_XXXXXXII	unsigned ll	= pero para long long's.

2. Estructuras

2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL \geq ceil(logn); Usar [] para llenar arreglo y luego build().

```
1 struct RMQ{
     #define LVL 10
     tipo vec[LVL] [1<<(LVL+1)];</pre>
     tipo &operator[](int p){return vec[0][p];}
     tipo get(int i, int j) {//intervalo [i,j)
       int p = 31-_builtin_clz(j-i);
6
       return min(vec[p][i],vec[p][j-(1<<p)]);
8
     void build(int n) {//O(nlogn)
9
       int mp = 31-__builtin_clz(n);
10
       forn(p, mp) forn(x, n-(1<<p))
11
         vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
12
13
    }};
```

2.2. Segment Tree

2.2.1. Segment Tree Recursivo

```
1 //inclusive segment tree [L,R]
2 int T[4 * N];
   void init(int node = 1,int l = 0,int r = n - 1){
     if(1 == r)T[node] = v[1];
     else{
5
       int mid = (1 + r) >> 1;
6
       init(2 * node,1,mid);
       init(2 * node + 1, mid + 1, r);
       T[node] = op(T[2 * node], T[2 * node + 1]);
9
10
11
   void update(int pos,int val,int node = 1,int l = 0,int r = n - 1){
     if(r < pos || 1 > pos)return;
     if(l == r)T[node] = val;
14
     else{
15
       int mid = (1 + r) >> 1;
16
       update(pos,val,2 * node,1,mid);
17
       update(pos, val, 2 * node + 1, mid + 1, r);
18
19
       T[node] = op(T[2 * node], T[2 * node + 1]);
```

```
20
^{21}
   int query(int x,int y,int node = 1,int l = 0,int r = n - 1){
^{22}
     if(r < x || 1 > y)return NEUTRO;
23
     if(x <= 1 && r <= y)return T[node];</pre>
     else{
25
       int mid = (1 + r) >> 1;
26
       return op(query(x,y,2 * node,1,mid),query(x,y,2 * node + 1,mid + 1,r
27
           ));
28
29
    2.2.2. ST Iterativo - (Consulta en rango, modificacion a posicion)
  //Segment tree iterative [1,r)
  int T[2 * N];
   void init(){
     for(int i = n; i < 2 * n; i++)T[i] = val[i];
    for(int i = n - 1; i \ge 1; i--)T[i] = op(T[i << 1], T[i << 1 | 1]);
5
6
   int op(int a,int b){
     //an asociative function
     return a + b;
9
10
   void update(int pos,int u){
     pos += n;
12
     for(pos >= 1; pos >= 1; pos >= 1)T[pos] = op(T[pos << 1],T[pos << 1
13
         | 1]);
14
15
   int query(int 1,int r){
16
     1 += n, r += n;
17
     int ans = NEUTRO;
18
     while(l < r){
       if(1 \& 1)ans = op(ans,T[1++]);
       if (r \& 1) ans = op(ans, T[--r]);
       1 >>= 1,r >>= 1;
22
     }
23
     return ans;
25 }
    2.2.3. ST Iterativo - (Consulta a posicion, modificacion en rango)
1 /*Segment Tree modificar un rango, acceder a una posicion
```

```
solo sirve cuando la operacion que realizamos es conmutativa
     por ejemplo la suma, pero no funciona con la asignacion
3
   */
4
   //adiciona value al rango [1, r)
   void modify(int 1, int r, int value) {// rango [1, r)
     for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
       if (l&1) t[l++] += value;
       if (r&1) t[--r] += value;
    }
10
11
   //acceder a la posicion
   int query(int p) {
     int res = 0:
14
     for (p += n; p > 0; p >>= 1) res += t[p];
     return res;
16
17
   //Si necesitamos actualizar todo lo podemos hacer en O(n)
   //Y luego acceder a las hojas en O(1)
   void push() {
    for (int i = 1; i < n; ++i) {
      t[i << 1] += t[i];
      t[i<<1|1] += t[i];
       t[i] = 0;
24
    }
25
26 }
```

2.2.4. Segment Tree con Punteros

```
1 /*La creacion y las queries son [0, n)
   [cerrado, abierto)
   Por alguna razon el destructor hace que se borre todo
   inmediatamente despues de que termina el constructor*/
   const int maxn = 1000000;
   tipo v[maxn];
   const tipo NEUTRO = PONGA_AQUI_EL_NEUTRO;
   tipo oper(tipo a, tipo b) {
     return min(a, b);
10
11
12 | struct segment_tree {
     segment_tree *L, *R;
13
     int 1, r;
14
     tipo value;
15
```

T[pos] = value;

12

```
segment_tree(): L(nullptr), R(nullptr), value(NEUTRO) {}
                                                                                           for(pos >>= 1; pos >= 1; pos >>= 1)
16
                                                                                    13
                                                                                             T[pos] = T[pos << 1] + T[pos << 1 | 1];
     segment_tree(int _l, int _r) : l(_l), r(_r) {
                                                                                    14
17
       if (1 + 1 == r) {
                                                                                         }
                                                                                    15
18
         value = v[1];
                                                                                         void update(int pos,int value){
19
                                                                                    16
       } else {
                                                                                           pos += n;
                                                                                    17
20
         int mid = (1 + r) >> 1;
                                                                                           T[pos] += value;
21
                                                                                    18
         L = new segment_tree(1, mid);
                                                                                           for(pos >>= 1; pos >= 1; pos >>= 1)
22
                                                                                    19
         R = new segment_tree(mid, r);
                                                                                             T[pos] = T[pos << 1] + T[pos << 1 | 1];
23
                                                                                    20
         value = oper(L->value, R->value);
                                                                                         }
24
                                                                                   21
                                                                                         int query(int 1,int r){
       }
                                                                                   22
25
     }
                                                                                           1 += n; r+= n;
                                                                                   23
26
     //~segment_tree() {delete L; delete R;}//NO PONER!!! ERROR!!!
                                                                                           int ans = 0;
27
                                                                                   24
     void update(int pos, int val) {
                                                                                           while(1 < r){
28
                                                                                   25
                                                                                             if(1 & 1)ans += T[1++];
       if (r \le pos || 1 > pos) return;
29
       if (1 + 1 == r) {
                                                                                             if(r \& 1)ans += T[--r];
30
         v[pos] = value = val;
                                                                                             1 >>= 1, r >>= 1;
31
       } else {
                                                                                           }
32
                                                                                    29
         L->update(pos, val);
                                                                                           return ans;
33
                                                                                    30
         R->update(pos, val);
                                                                                         }
                                                                                   31
34
         value = oper(L->value, R->value);
                                                                                       };
                                                                                    32
35
       }
                                                                                       struct st{
36
                                                                                         int n;
37
     tipo query(int a, int b) {
                                                                                         vector<segmetree>T;
38
                                                                                   35
       if (a <= 1 && r <= b) return value;
                                                                                         st(){}
                                                                                    36
39
       if (1 \ge b \mid | r \le a) return NEUTRO;
                                                                                         st(int _){
                                                                                   37
40
       return oper(L->query(a, b), R->query(a, b));
                                                                                           n = _{:}
                                                                                   38
41
     }
                                                                                           for(int i = 0; i < 2 * n; i++){
^{42}
                                                                                   39
43 | } tree;
                                                                                             T.push_back(segmetree(n));
                                                                                    40
                                                                                           }
                                                                                    41
                          2.2.5. Segment Tree 2D
                                                                                         }
                                                                                    42
                                                                                         void update(int x,int y,int val){
                                                                                    43
                                                                                           x += n:
   typedef long long 11;
                                                                                    44
                                                                                           T[x].update(y,val);
   struct segmetree{
                                                                                    45
                                                                                           segmetree ok;
                                                                                    46
     int n;
3
                                                                                           for(x >>= 1; x >= 1; x >>= 1){
                                                                                   47
     vector<ll>T;
4
                                                                                             T[x].rupdate(y,T[x << 1].query(y,y + 1));
     segmetree(){n = 0;}
5
                                                                                             T[x].update(y,T[x << 1 | 1].query(y,y + 1));
     segmetree(int _){
                                                                                    49
6
                                                                                           }
       n = _{;}
                                                                                    50
       T.resize(2 * n + 1);
                                                                                   51
8
                                                                                         11 query(int 1,int b,int r,int t){
                                                                                   52
9
                                                                                           1 += n:
     void rupdate(int pos,int value){
                                                                                   53
10
                                                                                           r += n;
       pos += n;
                                                                                   54
11
                                                                                           r++,t++;
```

55

```
ll ans = OLL:
                                                                                        shift(id, l, r);
56
       while(1 < r){
                                                                                        int mid = (1+r)/2;
57
         if(l & 1)ans += T[l++].query(b,t);
                                                                                        increase(x, y, v, id * 2, 1, mid);
58
                                                                                        increase(x, y, v, id*2+1, mid, r);
         if(r & 1)ans += T[--r].query(b,t);
                                                                                   34
59
         1 >>= 1, r >>= 1;
                                                                                        s[id] = s[id * 2] + s[id * 2 + 1];
60
       }
61
                                                                                   36
       return ans;
                                                                                      //(We should call increase(l r x))
62
                                                                                      int sum(int x,int y,int id = 1,int l = 0,int r = n){
    }
63
64 };
                                                                                       if(x \ge r \text{ or } 1 \ge y) \text{ return } 0;
                                                                                       if(x \le 1 \&\& r \le y) return s[id];
                     2.2.6. Segment Tree Lazy - Suma
                                                                                       shift(id, l, r);
                                                                                       int mid = (1+r)/2;
                                                                                        return sum(x, y, id * 2, 1, mid) +
1 //Todo es [1, r)
   void build(int id = 1,int l = 0,int r = n){
                                                                                                sum(x, y, id * 2 + 1, mid, r);
                                                                                   44
                                                                                   45 }
     if(r - 1 < 2) \{ // 1 + 1 == r \}
       s[id] = a[1];
4
                                                                                                        2.2.7. Segment Tree Lazy - Pintar
       return ;
5
6
     int mid = (1+r)/2;
                                                                                    void shift(int id){
     build(id * 2, 1, mid);
                                                                                        if(lazv[id])
     build(id * 2 + 1, mid, r);
                                                                                          lazy[2 * is] = lazy[2 * id + 1] = lazy[id];
     s[id] = s[id * 2] + s[id * 2 + 1];
                                                                                        lazy[id] = 0;
                                                                                      }
                                                                                    5
11
   //agui poner los arrays lazy and id
                                                                                      //color > 1, por que se usa el 0 para decir que no hay lazy
   void upd(int id,int 1,int r,int x){// increase all members in this
                                                                                      void upd(int x,int y,int color, int id = 0,int 1 = 0,int r = n)\{//
       interval by x
                                                                                          painting the interval [x,y) whith color "color"
     lazy[id] += x;
                                                                                        if(x \ge r \text{ or } 1 \ge y) \text{ return };
     s[id] += (r - 1) * x;
                                                                                        if(x \le 1 \&\& r \le y){
15
                                                                                          lazy[id] = color;
                                                                                   10
16
   //A function to pass the update information to its children :
                                                                                          return ;
17
                                                                                   11
   void shift(int id,int l,int r){//pass update information to the children
                                                                                        }
                                                                                   12
     int mid = (1+r)/2;
                                                                                   13
                                                                                        int mid = (1+r)/2;
19
     upd(id * 2, 1, mid, lazy[id]);
                                                                                        shift(id);
                                                                                   14
20
     upd(id * 2 + 1, mid, r, lazy[id]);
                                                                                        upd(x, y, color, 2 * id, 1, mid);
                                                                                   15
21
     lazv[id] = 0;// passing is done
                                                                                        upd(x, y, color, 2*id+1, mid, r);
22
                                                                                   16
                                                                                      }
                                                                                   17
23
    //A function to perform increase gueries :
                                                                                   18
24
   void increase(int x,int y,int v,int id = 1,int l = 0,int r = n){
                                                                                      set <int> se:
     if(x \ge r \text{ or } 1 \ge y) \text{ return };
                                                                                      void cnt(int id = 1,int l = 0,int r = n){
26
     if(x \le 1 \&\& r \le y){
                                                                                        if(lazy[id]){
                                                                                   21
27
       upd(id, 1, r, v);
                                                                                          se.insert(lazy[id]);
                                                                                   22
28
                                                                                          return : // there is no need to see the children, because all the
       return ;
                                                                                   23
29
     }
                                                                                               interval is from the same color
30
```

```
3 | void init(int n) {
24
     if(r - 1 < 2) return ;
                                                                                      tree = vector<tipo>(n, 0);
25
     int mid = (1+r)/2;
                                                                                      maxn = n;
26
     cnt(2 * id, 1, mid);
                                                                                    }
                                                                                  6
27
     cnt(2*id+1, mid, r);
                                                                                    void add(int i, tipo k) { //i valid [1, n)
28
                                                                                      for(; i < maxn; i += i&-i ) tree[i] += k;</pre>
29
                                                                                    }
                                                                                  9
                     2.2.8. Segment Tree Persistente
                                                                                 10
                                                                                    tipo get(int i){//returns sum [1, i]
int segcnt = 0;
                                                                                     tipo s = 0;
  struct segment {
2
                                                                                     for(; i > 0; i-=i&-i) s+=tree[i];
       int 1, r, lid, rid, sum;
                                                                                      return s;
   } segs[2000000];
                                                                                 15 }
   int build(int 1, int r) {
       if (1 > r) return -1;
                                                                                                            2.3.1. Fenwick Tree 2D
       int id = segcnt++;
7
       segs[id].l = 1;
8
       segs[id].r = r;
                                                                                    11 T[1025] [1025];
9
       if (1 == r) segs[id].lid = -1, segs[id].rid = -1;
                                                                                    int n;
10
       else {
11
                                                                                    11 query(int x, int y)
           int m = (1 + r) / 2;
12
           segs[id].lid = build(l , m);
                                                                                  6
13
           segs[id].rid = build(m + 1, r); }
                                                                                      11 \text{ res} = 0;
                                                                                  7
14
       segs[id].sum = 0;
                                                                                      for(int i = x; i \ge 0; i = (i & (i+1)) - 1)
15
       return id; }
                                                                                             for(int j = y; j \ge 0; j = (j & (j+1)) - 1)
16
                                                                                 9
   int update(int idx, int v, int id) {
                                                                                                 res += T[i][i];
                                                                                 10
17
       if (id == -1) return -1;
                                                                                        return res;
                                                                                 11
18
       if (idx < segs[id].l || idx > segs[id].r) return id;
                                                                                    }
                                                                                 12
19
       int nid = segcnt++;
                                                                                 13
20
       segs[nid].l = segs[id].l;
                                                                                    void update(int x, int y, int val)
21
       segs[nid].r = segs[id].r;
22
                                                                                 15
                                                                                      for(int i = x; i < n; i = (i | (i+1)))
       segs[nid].lid = update(idx, v, segs[id].lid);
                                                                                 16
23
                                                                                            for(int j = y; j < n; j = (j | (j+1)))
       segs[nid].rid = update(idx, v, segs[id].rid);
                                                                                 17
24
       segs[nid].sum = segs[id].sum + v;
                                                                                                 T[i][j] += val;
                                                                                 18
25
       return nid: }
                                                                                 19 }
26
   int query(int id, int 1, int r) {
27
                                                                                                        2.4. Union Find con rank
       if (r < segs[id].1 || segs[id].r < 1) return 0;</pre>
28
       if (1 <= segs[id].l && segs[id].r <= r) return segs[id].sum;</pre>
29
                                                                                  1 /*======== <Union find rangos> ==========
       return query(segs[id].lid, 1, r) + query(segs[id].rid, 1, r); }
30
                                                                                  2 Complexity: O(N)
                           2.3. Fenwick Tree
                                                                                  3 index 0 to n - 1 warning
                                                                                    Complexity O(N)
                                                                                    */
vector<tipo> tree;
                                                                                  5
2 | int maxn;
                                                                                    #define MAX INSERTE_VALOR_AQUI
```

58

```
7 int padre[MAX];
   int rango[MAX];
   void MakeSet(int n){
       for (int i = 0; i < n; ++i) {
           padre[i] = i; rango[i] = 0; }
11
12
   int Find(int x) {
13
       if(x == padre[x])
14
           return x;
15
       return padre[x] = Find(padre[x]);
16
17
   void UnionbyRank(int x , int y){
18
       int xRoot = Find(x):
19
       int yRoot = Find(y);
20
       //el padre de ambas componentes sera el de mayor altura
21
       if(rango[xRoot] > rango[yRoot])//X tiene mas altura que Y
22
           padre[yRoot] = xRoot;
23
       }else{//Y} >= X
24
           padre[xRoot] = yRoot;
25
           if(rango[xRoot] == rango[yRoot])//si poseen la misma altura
26
               rango[yRoot]++;//incremento el rango de la nueva raiz
27
       }
28
29 }
```

2.5. BigInteger C++

```
1 // g++ -std=c++11 "bigint.cpp" -o run
  /***
2
   Contain a useful big int, overload all operators, including cin, cout,
  comparator, build via string (prefer this metod) or long long, for now
      this not have a
  to_string method
  Problem for practice: UVA 494
  */
8
  // base and base_digits must be consistent
  const int base = 1000000000;
  const int base_digits = 9;
11
12
  struct bigint {
13
      vector<int> a;
14
      int sign;
15
16
```

```
bigint():
17
           sign(1) {
18
       }
19
20
       bigint(long long v) {
21
            *this = v;
22
       }
23
24
       bigint(const string &s) {
25
           read(s);
26
       }
27
28
       void operator=(const bigint &v) {
29
           sign = v.sign;
30
           a = v.a;
31
       }
32
33
       void operator=(long long v) {
34
           sign = 1;
35
           if (v < 0)
                sign = -1, v = -v;
37
           for (; v > 0; v = v / base)
                a.push_back(v % base);
39
       }
40
41
       bigint operator+(const bigint &v) const {
42
           if (sign == v.sign) {
43
                bigint res = v;
44
45
                for (int i = 0, carry = 0; i < (int) max(a.size(), v.a.size
46
                    ()) || carry; ++i) {
                    if (i == (int) res.a.size())
47
                        res.a.push_back(0);
48
                    res.a[i] += carry + (i < (int) a.size() ? a[i] : 0);
49
                    carry = res.a[i] >= base;
50
                    if (carry)
51
                        res.a[i] -= base;
52
                }
53
54
                return res;
55
           return *this - (-v);
56
57
```

```
bigint operator-(const bigint &v) const {
                                                                                                int norm = base / (b1.a.back() + 1);
59
                                                                                    98
           if (sign == v.sign) {
                                                                                                bigint a = a1.abs() * norm;
                                                                                    99
60
                if (abs() >= v.abs()) {
                                                                                                bigint b = b1.abs() * norm;
                                                                                    100
61
                    bigint res = *this;
                                                                                                bigint q, r;
                                                                                    101
62
                    for (int i = 0, carry = 0; i < (int) v.a.size() || carry</pre>
                                                                                                q.a.resize(a.a.size());
                                                                                    102
63
                        ; ++i) {
                                                                                    103
                                                                                                for (int i = a.a.size() - 1; i >= 0; i--) {
                        res.a[i] -= carry + (i < (int) v.a.size() ? v.a[i] :
                                                                                    104
64
                                                                                                    r *= base:
                                                                                    105
                        carry = res.a[i] < 0;</pre>
                                                                                                    r += a.a[i]:
                                                                                    106
65
                        if (carry)
                                                                                                    int s1 = r.a.size() \le b.a.size() ? 0 : r.a[b.a.size()];
                                                                                    107
66
                            res.a[i] += base;
                                                                                                    int s2 = r.a.size() \le b.a.size() - 1 ? 0 : r.a[b.a.size() -
67
                                                                                    108
68
                    res.trim():
                                                                                                    int d = ((long long) base * s1 + s2) / b.a.back();
69
                                                                                    109
                    return res:
                                                                                                    r -= b * d:
                                                                                   110
70
                }
                                                                                                    while (r < 0)
                                                                                   111
71
                return -(v - *this);
                                                                                                        r += b, --d;
                                                                                   112
72
           }
                                                                                                    q.a[i] = d;
73
                                                                                    113
           return *this + (-v);
                                                                                                }
                                                                                   114
74
       }
                                                                                   115
75
                                                                                                q.sign = a1.sign * b1.sign;
                                                                                   116
76
       void operator*=(int v) {
                                                                                                r.sign = a1.sign;
                                                                                   117
77
           if (v < 0)
                                                                                                q.trim();
78
                sign = -sign, v = -v;
                                                                                                r.trim();
                                                                                   119
79
           for (int i = 0, carry = 0; i < (int) a.size() || carry; ++i) {
                                                                                                return make_pair(q, r / norm);
                                                                                   120
80
                if (i == (int) a.size())
                                                                                            }
                                                                                    121
81
                    a.push_back(0);
                                                                                    122
82
               long long cur = a[i] * (long long) v + carry;
                                                                                   123
                                                                                            bigint operator/(const bigint &v) const {
83
                carry = (int) (cur / base);
                                                                                                return divmod(*this, v).first;
                                                                                    124
84
                a[i] = (int) (cur % base);
                                                                                            }
                                                                                    125
85
                //asm("divl %%cx" : "=a"(carry), "=d"(a[i]) : "A"(cur), "c
                                                                                    126
86
                    "(base));
                                                                                            bigint operator%(const bigint &v) const {
                                                                                    127
           }
                                                                                                return divmod(*this, v).second;
                                                                                    128
87
                                                                                            }
           trim();
88
                                                                                    129
       }
                                                                                    130
89
                                                                                            void operator/=(int v) {
                                                                                   131
90
       bigint operator*(int v) const {
                                                                                                if (v < 0)
                                                                                    132
91
           bigint res = *this;
                                                                                                    sign = -sign, v = -v;
                                                                                   133
92
                                                                                                for (int i = (int) \ a.size() - 1, rem = 0; i \ge 0; --i) {
           res *= v;
93
                                                                                    134
                                                                                                    long long cur = a[i] + rem * (long long) base;
           return res;
                                                                                   135
94
                                                                                                    a[i] = (int) (cur / v);
       }
                                                                                   136
95
                                                                                                    rem = (int) (cur % v);
                                                                                   137
96
       friend pair bigint, bigint divmod(const bigint &a1, const bigint &
                                                                                    138
97
           b1) {
                                                                                                trim();
                                                                                   139
```

```
}
                                                                                                }
140
                                                                                        183
                                                                                                bool operator<=(const bigint &v) const {</pre>
                                                                                       184
141
        bigint operator/(int v) const {
                                                                                                    return !(v < *this);</pre>
                                                                                       185
142
            bigint res = *this;
143
                                                                                        186
            res /= v;
                                                                                                bool operator>=(const bigint &v) const {
                                                                                       187
144
            return res;
                                                                                                    return !(*this < v);
145
                                                                                       188
                                                                                                }
        }
146
                                                                                       189
                                                                                                bool operator==(const bigint &v) const {
                                                                                       190
147
                                                                                                    return !(*this < v) && !(v < *this);
        int operator%(int v) const {
148
                                                                                       191
            if (v < 0)
                                                                                                }
                                                                                        192
149
                                                                                                bool operator!=(const bigint &v) const {
                 v = -v:
150
                                                                                       193
                                                                                                    return *this < v || v < *this;
            int m = 0:
151
                                                                                       194
            for (int i = a.size() - 1; i >= 0; --i)
                                                                                                }
152
                                                                                       195
                 m = (a[i] + m * (long long) base) % v;
                                                                                        196
153
            return m * sign;
                                                                                                void trim() {
                                                                                       197
154
        }
                                                                                                    while (!a.empty() && !a.back())
155
                                                                                                         a.pop_back();
156
                                                                                       199
                                                                                                    if (a.empty())
        void operator+=(const bigint &v) {
157
             *this = *this + v;
                                                                                                         sign = 1;
                                                                                       201
158
        }
                                                                                                }
                                                                                        202
159
        void operator-=(const bigint &v) {
                                                                                       203
160
             *this = *this - v;
                                                                                                bool isZero() const {
161
                                                                                       204
        }
                                                                                                    return a.empty() || (a.size() == 1 && !a[0]);
                                                                                       205
162
        void operator*=(const bigint &v) {
                                                                                                }
                                                                                       206
163
            *this = *this * v;
                                                                                       207
164
        }
                                                                                                bigint operator-() const {
                                                                                       208
165
        void operator/=(const bigint &v) {
                                                                                                    bigint res = *this;
                                                                                       209
166
             *this = *this / v;
                                                                                                    res.sign = -sign;
                                                                                       210
167
        }
                                                                                                    return res;
                                                                                       211
168
                                                                                                }
                                                                                       212
169
        bool operator<(const bigint &v) const {</pre>
                                                                                       213
170
            if (sign != v.sign)
                                                                                                bigint abs() const {
                                                                                       214
171
                 return sign < v.sign;</pre>
                                                                                                    bigint res = *this;
172
                                                                                       215
            if (a.size() != v.a.size())
                                                                                                    res.sign *= res.sign;
                                                                                       216
173
                return a.size() * sign < v.a.size() * v.sign;</pre>
                                                                                                    return res;
                                                                                       217
174
            for (int i = a.size() - 1; i >= 0; i--)
                                                                                                }
                                                                                       218
175
                 if (a[i] != v.a[i])
                                                                                       219
176
                     return a[i] * sign < v.a[i] * sign;</pre>
                                                                                                long longValue() const {
                                                                                       220
177
            return false;
                                                                                                    long long res = 0;
                                                                                       221
178
        }
                                                                                                    for (int i = a.size() - 1; i >= 0; i--)
                                                                                       222
179
                                                                                                         res = res * base + a[i];
                                                                                       223
180
        bool operator>(const bigint &v) const {
                                                                                                    return res * sign;
181
                                                                                       ^{224}
                                                                                                }
            return v < *this;
182
                                                                                       ^{225}
```

```
static vector<int> convert_base(const vector<int> &a, int old_digits
226
                                                                                     268
        friend bigint gcd(const bigint &a, const bigint &b) {
                                                                                                  , int new_digits) {
227
            return b.isZero() ? a : gcd(b, a % b);
                                                                                                  vector<long long> p(max(old_digits, new_digits) + 1);
228
                                                                                     269
        }
                                                                                                  p[0] = 1;
229
                                                                                     270
        friend bigint lcm(const bigint &a, const bigint &b) {
                                                                                                  for (int i = 1; i < (int) p.size(); i++)
                                                                                     271
230
            return a / gcd(a, b) * b;
                                                                                                      p[i] = p[i - 1] * 10;
                                                                                     272
231
        }
                                                                                                  vector<int> res;
232
                                                                                     273
                                                                                                  long long cur = 0;
233
                                                                                     274
        void read(const string &s) {
                                                                                                  int cur_digits = 0;
234
                                                                                     275
                                                                                                  for (int i = 0; i < (int) a.size(); i++) {
            sign = 1;
235
                                                                                                      cur += a[i] * p[cur_digits];
            a.clear();
236
                                                                                     277
            int pos = 0;
                                                                                                      cur_digits += old_digits;
237
                                                                                     278
            while (pos < (int) s.size() && (s[pos] == '-' || s[pos] == '+'))
                                                                                                      while (cur_digits >= new_digits) {
                                                                                     279
238
                 {
                                                                                                           res.push_back(int(cur %p[new_digits]));
                if (s[pos] == '-')
                                                                                                           cur /= p[new_digits];
                                                                                     281
239
                                                                                                           cur_digits -= new_digits;
                     sign = -sign;
240
                                                                                                      }
241
                ++pos;
                                                                                     283
                                                                                                  }
242
                                                                                     284
            for (int i = s.size() - 1; i >= pos; i -= base_digits) {
                                                                                                  res.push_back((int) cur);
                                                                                     285
243
                int x = 0;
                                                                                                  while (!res.empty() && !res.back())
                                                                                      286
244
                for (int j = max(pos, i - base_digits + 1); j <= i; j++)
                                                                                                      res.pop_back();
                                                                                     287
245
                     x = x * 10 + s[i] - '0';
                                                                                                  return res;
246
                                                                                     288
                                                                                              }
                a.push_back(x);
                                                                                     289
247
                                                                                     290
248
            trim();
                                                                                              typedef vector<long long> vll;
                                                                                     291
249
        }
                                                                                     292
250
                                                                                              static vll karatsubaMultiply(const vll &a, const vll &b) {
                                                                                     293
251
        friend istream& operator>>(istream &stream, bigint &v) {
                                                                                                  int n = a.size();
                                                                                     294
252
                                                                                                  vll res(n + n);
            string s;
                                                                                     295
253
            stream >> s;
                                                                                                  if (n <= 32) {
                                                                                     296
254
            v.read(s);
                                                                                                      for (int i = 0; i < n; i++)
255
                                                                                     297
                                                                                                           for (int j = 0; j < n; j++)
            return stream;
256
                                                                                     298
        }
                                                                                                               res[i + j] += a[i] * b[j];
                                                                                     299
257
                                                                                                      return res:
                                                                                     300
258
        friend ostream& operator<<(ostream &stream, const bigint &v) {</pre>
                                                                                                  }
                                                                                     301
259
            if (v.sign == -1)
                                                                                     302
260
                stream << '-';
                                                                                                  int k = n \gg 1;
261
                                                                                     303
            stream << (v.a.empty() ? 0 : v.a.back());
                                                                                                  vll a1(a.begin(), a.begin() + k);
                                                                                     304
262
            for (int i = (int) \ v.a.size() - 2; i >= 0; --i)
                                                                                                  vll a2(a.begin() + k, a.end());
                                                                                     305
263
                 stream << setw(base_digits) << setfill('0') << v.a[i];</pre>
                                                                                                  vll b1(b.begin(), b.begin() + k);
                                                                                     306
264
                                                                                                  vll b2(b.begin() + k, b.end());
            return stream;
265
                                                                                     307
        }
266
                                                                                     308
                                                                                                  vll a1b1 = karatsubaMultiply(a1, b1);
267
                                                                                     309
```

```
vll a2b2 = karatsubaMultiply(a2, b2);
310
311
            for (int i = 0; i < k; i++)
312
                 a2[i] += a1[i];
313
            for (int i = 0; i < k; i++)
314
                 b2[i] += b1[i];
315
316
            vll r = karatsubaMultiply(a2, b2);
317
            for (int i = 0; i < (int) a1b1.size(); i++)</pre>
318
                 r[i] = a1b1[i];
319
            for (int i = 0; i < (int) a2b2.size(); i++)</pre>
320
                 r[i] = a2b2[i];
321
322
            for (int i = 0; i < (int) r.size(); i++)
323
                 res[i + k] += r[i];
324
            for (int i = 0; i < (int) a1b1.size(); i++)
325
                 res[i] += a1b1[i];
326
            for (int i = 0; i < (int) a2b2.size(); i++)</pre>
327
                 res[i + n] += a2b2[i]:
328
            return res;
329
        }
330
331
        bigint operator*(const bigint &v) const {
332
            vector<int> a6 = convert_base(this->a, base_digits, 6);
333
            vector<int> b6 = convert_base(v.a, base_digits, 6);
334
            vll a(a6.begin(), a6.end());
335
            vll b(b6.begin(), b6.end());
336
            while (a.size() < b.size())</pre>
337
                 a.push_back(0);
338
            while (b.size() < a.size())</pre>
339
                 b.push_back(0);
340
            while (a.size() & (a.size() - 1))
341
                 a.push_back(0), b.push_back(0);
342
            vll c = karatsubaMultiply(a, b);
343
            bigint res;
344
            res.sign = sign * v.sign;
345
            for (int i = 0, carry = 0; i < (int) c.size(); i++) {
346
                 long long cur = c[i] + carry;
347
                res.a.push_back((int) (cur % 1000000));
348
                 carry = (int) (cur / 1000000);
349
350
            res.a = convert_base(res.a, 6, base_digits);
351
            res.trim();
352
```

```
353
          return res;
354
   };
355
356
   int main() {
357
       bigint a=0;
358
       359
       bigint b;
       bigint n;
       while(cin >> n) {
363
          if(n==0){break;}
364
          a += n:
365
       }
366
       cout<<a<<endl;</pre>
367
368 | }
                               UnorderedSet
                         2.6.
 1 //Compilar: g++ --std=c++11
   struct Hash{
     size_t operator()(const ii &a)const{
       size_t s=hash<int>()(a.fst);
 4
      return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
 5
    }
 6
    size_t operator()(const vector<int> &v)const{
      size_t s=0;
8
      for(auto &e : v)
9
        s = hash < int > ()(e) + 0x9e3779b9 + (s < < 6) + (s > > 2);
      return s;
11
    }
12
   };
13
   unordered_set<ii, Hash> s;
unordered_map<ii, int, Hash> m;//map<key, value, hasher>
                          2.7. Ordered Set
1 /*
   A brief explanation about use of a powerful library: orderd_set
    Reference link: http://codeforces.com/blog/entry/11080
    and a hash for the type pair
4
5
   #include <ext/pb_ds/assoc_container.hpp>
  #include <ext/pb_ds/tree_policy.hpp>
```

```
8 using namespace __gnu_pbds;
9 /*typedef tree<int,null_type,less<int>,rb_tree_tag,
       tree_order_statistics_node_update> ordered_set;
10 If we want to get map but not the set, as the second argument type must
       be used mapped type. Apparently, the tree supports the same
       operations as the set (at least I haven't any problems with them
       before), but also there are two new features - it is find_by_order
       () and order_of_key().
11 The first returns an iterator to the k-th largest element (counting from
        zero), the second - the number of items
   in a set that are strictly smaller than our item. Example of use:*/
   template <typename T>
   using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
       tree_order_statistics_node_update>;
   int main(){
     ordered_set<int> s;
16
     s.insert(1);
17
     s.insert(3);
18
     cout << *s.find_by_order(0) << endl; // print the 0-th smallest number</pre>
19
          in s(0-based)
     cout << s.order_of_key(2) << endl; // the number of elements in the s</pre>
20
         less than 2
     //find_by_order(i) devuelve iterador al i-esimo elemento
21
     //order_of_key(k): devuelve la pos del lower bound de k
22
     //Ej: 12, 100, 505, 1000, 10000.
23
    //order_of_key(10) == 0, order_of_key(100) == 1,
    //order_of_key(707) == 3, order_of_key(9999999) == 5
25
     return 0;
26
27 }
                         2.8. Treap Modo Set
```

```
struct treap {
     typedef struct _node {
2
       int x, y, cnt;
3
       _node *1, *r;
       _{\text{node}}(\text{int }_{x}): x(_{x}), y((\text{rand}() << 16) ^ rand()), cnt(1), l(nullptr)
5
            ), r(nullptr) {}
       ~_node() {delete 1; delete r;}
       void recalc() {
          cnt = 1:
8
          if (1) cnt += 1->cnt;
9
          if (r) cnt += r->cnt;
10
```

```
}
11
     } *node;
12
     treap(): root(nullptr) {}
13
     ~treap() {delete root;}
14
     node root;
15
     /*Divide el arbol que tiene como raiz a "t", guarda en
16
       L todos los nodos con key menor o igual a "x"
17
       R todos los nodos con key mayor estricto a x
18
       t es destruido/modificado
19
      */
20
     void split(node t, int x, node &L, node &R) {
21
       if (t == nullptr) {L = R = nullptr; return;}
22
       if (t->x <= x) {
23
         split(t->r, x, t->r, R);
24
         L = t:
25
       } else {
         split(t->1, x, L, t->1);
         R = t;
       }
29
       t->recalc();
30
     }
31
     /*Une los dos nodos L y R en un solo arbol y los devuelve.
       L y R son modificados
33
      */
34
     node merge(node L, node R) {
35
       if (L == nullptr) return R;
36
       if (R == nullptr) return L;
37
       if (L->y > R->y) {
        L->r = merge(L->r, R);
         L->recalc();
         return L;
41
       } else {
         R->1 = merge(L, R->1);
         R->recalc():
         return R:
       }
46
47
     /*Inserta un solo nodo con key igual a "x"*/
     void insert(int x) {
49
       //verificar que no se inserten elementos repetidos
       //pueden ocasionar que ya no cumplan la propiedad de BST
       //pueden haber keys iguales a izquierda y derecha
52
       node L, R;
53
```

```
split(root, x, L, R);
54
       root = merge(merge(L, new _node(x)), R);
55
56
     /*Borra todos los nodos con key igual a "x"
57
       No pasa nada si no hay nodos con key igual a "x"*/
58
     void erase(int x) {
59
       node L, m, R;
60
       split(root, x, L, R);
61
       split(L, x - 1, L, m);
62
       root = merge(L, R);
63
64
     int count(int x) {
65
       node L. m. R:
       split(root, x, L, R);
67
       split(L, x - 1, L, m);
68
       int ans = cnt(m);
69
       root = merge(merge(L, m), R);
70
       return ans;
71
72
     /*Borra un nodo con key igual a "x"
73
       No sucede nada si no hay nodo con ese key*/
74
     void erase_one(int x) {
75
       node L, m, R;
76
       split(root, x, L, R);
77
       split(L, x - 1, L, m);
78
       if (m)
79
         m = merge(m->1, m->r);
80
       root = merge(merge(L, m), R);
81
82
     /*k-esimo indexado desde 0
83
        devuelve INT_MAX si no hay k-esimo*/
84
     int kthElement(int k) {
85
       node cur = root:
86
       while (cur != nullptr) {
87
         int sizeLeft = cnt(cur->1);
88
         if (sizeLeft == k)
89
           return cur->x;
90
         cur = sizeLeft > k ? cur->l : cur->r:
91
         if (sizeLeft < k)</pre>
92
           k = (sizeLeft + 1);
93
       }
94
       return INT_MAX;
95
96
```

```
/*Numero de elementos con keys menores a "x"*/
97
      int less(int x) {
98
        node cur = root;
99
        int ans = 0;
100
        while (cur != nullptr) {
101
          if (cur->x >= x) {
102
             cur = cur->l;
103
          } else {
104
             ans += cnt(cur->1) + 1;
105
             cur = cur->r;
          }
107
        }
108
        return ans;
109
110
      int cnt(node t) const {
111
        return t ? t->cnt : 0;
      }
113
      int size() const {
        return cnt(root):
115
116
    /*//from e-maxx.ru son mas rapidos(no x mucho) pero menos entendibles
    //solo borra un elemento
     void erase(node &t, int x) {
       if (t\rightarrow x == x)
        t = merge(t->1, t->r);
121
        else
122
          erase(x < t->x?t->1:t->r, x);
123
        if (t)
124
          t->recalc();
125
      }
126
      void erase_one(int x) {erase(root, x);}
127
      void insert(node &t, node it) {
128
        if (t == nullptr)
129
         t = it:
130
        else if (it->y > t->y)
131
          split(t, it\rightarrow x, it\rightarrow l, it\rightarrow r), t = it;
132
133
           insert(it\rightarrow x < t\rightarrow x? t\rightarrow 1: t\rightarrow r, it);
134
        t->recalc();
135
136
      void insert(int x) {insert(root, new _node(x));}*/
137
138 };
```

2.9. Treap Implicito(Rope)

```
1 | struct rope {
     typedef struct _node {
2
       int value, y, cnt;
       _node *1, *r;
       _node(int _value) : value(_value), y((rand() << 16) ^ rand()), cnt</pre>
           (1), l(nullptr), r(nullptr) {}
       ~_node() {delete 1; delete r;}
       void recalc() {
         cnt = 1;
         if (1) cnt += 1->cnt;
         if (r) cnt += r->cnt;
       }
11
     } *node;
12
     rope(): root(nullptr) {}
13
     ~rope() {delete root;}
14
     node root;
15
     /*Divide el arbol que tiene como raiz a "t", guarda en
      L los primeros "x" elementos del array
      R el primer elemento de R es el elemento en posicion x(indexado desde
18
           0) del array
      t es destruido/modificado
19
      L = [0, x)
20
      R = [x, n) */
21
     void split(node t, int x, node &L, node &R) {
22
       if (t == nullptr) {L = R = nullptr; return;}
23
       int curIndex = cnt(t->1) + 1;
24
       if (curIndex <= x) {</pre>
25
         split(t->r, x - curIndex, t->r, R);
26
         L = t;
27
       } else {
28
         split(t->1, x, L, t->1);
29
         R = t;
30
       }
31
       t->recalc();
32
33
     /* Une los dos nodos L y R en un solo arbol y los devuelve.
34
        L y R son modificados */
35
     node merge(node L, node R) {
36
       if (L == nullptr) return R;
37
       if (R == nullptr) return L;
38
       if (L->y > R->y) {
39
```

```
L->r = merge(L->r, R);
         L->recalc();
41
         return L;
42
       } else {
         R->1 = merge(L, R->1);
44
         R->recalc();
         return R;
46
47
     }
48
     /*Inserta "value" en la posicion "pos"(indexado desde 0) recorre todos
          los elementos a la derecha desde la posicion pos*/
     void insert(int pos, int value) {
50
       node L, R;
       split(root, pos, L, R);//en R esta pos
       root = merge(merge(L, new _node(value)), R);
53
     /*Borra el elemento en posicion pos*/
55
     void erase(int pos) {
       node L, m, R;
57
       split(root, pos, L, R);
       split(R, 1, m, R);
59
       root = merge(L, R);
60
61
     int cnt(node t) const {
62
       return t ? t->cnt : 0;
63
    }
64
     int size() const {
65
       return cnt(root);
66
    }
67
68
69 int main() {srand(time(nullptr));return 0;}
                   2.10. Treap - Toby and Stones
const int PAINT = 0, FLIP = 1, REVERSE = 2;
  struct rope {
     typedef struct _node {
       int value, y, cnt;
4
       int negros;
5
       bool rev;
6
       bool lazy_flip;
7
       int lazy_pintar;
8
       _node *1, *r;
```

```
_node(int _value) : value(_value), y((rand() << 16) ^ rand()), cnt</pre>
                                                                                          }
10
                                                                                   51
           (1), l(nullptr), r(nullptr) {
                                                                                          string to_string() {
                                                                                   52
         negros = _value;
                                                                                   53
                                                                                             stringstream ss;
11
         rev = false;
                                                                                             ss << "value=" << value << ",_negros=" << negros << ",_cnt="<< cnt
12
                                                                                   54
                                                                                                  << "("<<rev<< "," << lazy_pintar<<","<< lazy_flip << ")";</pre>
         lazy_flip = false;
13
         lazy_pintar = -1;
                                                                                            return ss.str();
14
                                                                                   55
                                                                                          }
15
                                                                                   56
                                                                                        } *node;
        ~_node() {delete l; delete r;}
                                                                                   57
16
       void recalc() {
                                                                                        rope(): root(nullptr) {}
17
                                                                                         ~rope() {delete root;}
         cnt = 1;
18
                                                                                        node root;
         negros = value;
19
                                                                                   60
                                                                                        void push(node &t) {
         if (1) cnt += 1->cnt, negros += 1->negros;
20
                                                                                   61
                                                                                          if (t == nullptr) return;
         if (r) cnt += r->cnt, negros += r->negros;
21
                                                                                   62
       }
                                                                                          bool op1 = ((t->lazy_pintar) != -1);
                                                                                   63
22
       void push_lazy(int type, int param = -1) {
                                                                                          bool op2 = ((t->rev) || (t->lazy_flip));
23
                                                                                   64
         if (type == PAINT) {
                                                                                          //solo puede pasar uno de los 2 casos, o ninguno esta activado o
24
                                                                                   65
           rev = false;
                                                                                               solo uno de los dos
25
                                                                                          assert((!op1 and !op2) or (op1 xor op2));
           lazy_flip = false;
26
           lazy_pintar = param;
                                                                                          if (op1) {
                                                                                   67
27
           negros = param * cnt;
                                                                                            t->value = t->lazy_pintar;
28
                                                                                            if (t->1) t->1->push_lazy(PAINT, t->lazy_pintar);
29
                                                                                   69
         if (type == FLIP) {
                                                                                            if (t->r) t->r->push_lazy(PAINT, t->lazy_pintar);
30
                                                                                   70
                                                                                            t->lazy_pintar = -1;
           if (lazy_pintar != -1) {
31
                                                                                   71
             assert(rev == false && lazy_flip == false);
                                                                                   72
32
             lazy_pintar = (1 - lazy_pintar);
                                                                                           if (op2) {//no importa el orden en que se aplique estos 2
                                                                                   73
33
             negros = lazy_pintar * cnt;
                                                                                            //reverse
                                                                                   74
34
                                                                                            if (t->rev) {
           } else {
                                                                                   75
35
                                                                                               swap(t->1, t->r);
             lazy_flip ^= true;
                                                                                   76
36
                                                                                               t->rev = false;
             negros = cnt - negros;
37
                                                                                   77
           }
                                                                                               if (t->1) t->1->push_lazy(REVERSE);
38
                                                                                   78
         }
                                                                                               if (t->r) t->r->push_lazy(REVERSE);
                                                                                   79
39
         if (type == REVERSE) {
40
                                                                                   80
           if (lazy_pintar == -1) {
                                                                                            //invertir colores
                                                                                   81
41
             rev ^= true;
                                                                                            if (t->lazy_flip) {
                                                                                   82
42
           }
                                                                                               t->value = 1 - (t->value);
                                                                                   83
43
         }
                                                                                               t->lazy_flip = false;
                                                                                   84
44
       }
                                                                                               if (t->1) t->1->push_lazy(FLIP);
45
                                                                                   85
       void verify() {
                                                                                               if (t->r) t->r->push_lazy(FLIP);
                                                                                   86
46
                                                                                            }
         bool op1 = (lazy_pintar != -1);
47
                                                                                   87
                                                                                          }
         bool op2 = (rev || lazy_flip);
                                                                                   88
48
         //solo puede pasar uno de los 2 casos, o ninguno esta activado o
                                                                                   89
49
             solo uno de los dos
                                                                                        void split(node t, int x, node &L, node &R) {
                                                                                   90
         assert((!op1 and !op2) or (op1 xor op2));
                                                                                          push(t);
50
                                                                                   91
```

```
if (t == nullptr) {L = R = nullptr; return;}
92
        int curIndex = cnt(t->1) + 1;
93
        if (curIndex <= x) {</pre>
94
          split(t->r, x - curIndex, t->r, R);
95
          L = t;
96
        } else {
97
          split(t->1, x, L, t->1);
98
          R = t;
99
        }
100
        t->recalc();
101
102
      node merge(node L, node R) {
103
        push(L); push(R);
104
        if (L == nullptr) return R;
105
        if (R == nullptr) return L;
106
        if (L->y > R->y) {
107
          L->r = merge(L->r, R);
108
          L->recalc();
109
          return L:
110
        } else {
111
          R->1 = merge(L, R->1);
112
          R->recalc();
113
          return R;
114
        }
115
      }
116
      void insert(int pos, int value) {
117
        node L, R;
118
        split(root, pos, L, R);//en R esta pos
119
        root = merge(merge(L, new _node(value)), R);
120
      }
121
      int cnt(node t) {
122
        return t ? t->cnt : 0;
123
124
      int size() {
125
        return cnt(root);
126
127
      void reverse(int i, int j) {
128
        node L. m. R:
129
        split(root, i, L, R);
130
        split(R, j - i + 1, m, R);
131
        m->push_lazy(REVERSE);
132
        root = merge(merge(L, m), R);
133
134
```

```
void flip(int i, int j) {
135
        node L, m, R;
136
        split(root, i, L, R);
137
        split(R, j - i + 1, m, R);
138
        m->push_lazy(FLIP);
139
        root = merge(merge(L, m), R);
140
141
      void pintar(int i, int j, int color) {
142
        node L, m, R;
143
        split(root, i, L, R);
144
        split(R, j - i + 1, m, R);
145
        m->push_lazy(PAINT, color);
146
        root = merge(merge(L, m), R);
147
     }
148
     void query(int i, int j) {
149
       node L, m, R;
        split(root, i, L, R);
151
        split(R, j - i + 1, m, R);
        int negros = m->negros;
        int blancos = m->cnt - negros;
        printf("%\\n", negros, blancos);
        root = merge(merge(L, m), R);
156
157
     void print(node &t, string spacio, char lado = ''') {
158
        if (t == nullptr) return;
159
        cout << spacio << lado <<"_->_" << (t->to_string()) << endl;
160
        print(t->1, spacio + "\t", 'L');
161
        print(t->r, spacio + "\t", 'R');
162
163
164
     void print() {
        print(root, "");
165
     }
166
167 };
```

2.11. Convex Hull Trick Estatico

```
// g++ "convexhulltrick.cpp" -o run
/***

Contain a sample about convex hull trick optimization this recivie N
pairs:
a "value of length" and a cost, we need to minimize the value of
grouping
```

```
6 this pairs taken the most large pair as the cost of the group
   Problem for practice: aquire
9
   #include <iostream>
   #include <vector>
   #include <algorithm>
   using namespace std;
   int pointer; //Keeps track of the best line from previous query
   vector<long long> M; //Holds the slopes of the lines in the envelope
   vector<long long> B; //Holds the y-intercepts of the lines in the
       envelope
   //Returns true if either line 11 or line 13 is always better than line
   bool bad(int 11, int 12, int 13)
19
20
     intersection(11,12) has x-coordinate (b1-b2)/(m2-m1)
21
     intersection(11.13) has x-coordinate (b1-b3)/(m3-m1)
22
     set the former greater than the latter, and cross-multiply to
23
     eliminate division
24
     */
25
     return (B[13]-B[11])*(M[11]-M[12])<(B[12]-B[11])*(M[11]-M[13]);
26
27
    //Adds a new line (with lowest slope) to the structure
28
   void add(long long m,long long b)
29
30
     //First, let's add it to the end
31
     M.push_back(m);
32
     B.push_back(b);
33
     //If the penultimate is now made irrelevant between the
34
         antepenultimate
     //and the ultimate, remove it. Repeat as many times as necessary
35
     while (M.size() \ge 3\&\&bad(M.size() - 3, M.size() - 2, M.size() - 1))
36
37
       M.erase(M.end()-2);
38
       B.erase(B.end()-2);
39
     }
40
41
   //Returns the minimum y-coordinate of any intersection between a given
   //line and the lower envelope
44 long long query(long long x)
```

```
45 {
     //If we removed what was the best line for the previous query, then
46
     //newly inserted line is now the best for that query
47
     if (pointer>=M.size())
48
       pointer=M.size()-1;
49
     //Any better line must be to the right, since query values are
50
     //non-decreasing
51
     while (pointer<M.size()-1&&
52
       M[pointer+1] *x+B[pointer+1] < M[pointer] *x+B[pointer])</pre>
53
       pointer++;
54
     return M[pointer] *x+B[pointer];
55
56
   int main()
57
   {
58
     int M,N,i;
59
     pair<int, int> a[50000];
60
     pair<int,int> rect[50000];
     scanf("%d",&M);
62
     for (i=0; i<M; i++)
63
       scanf("%, %a[i].first, %a[i].second);
64
     //Sort first by height and then by width (arbitrary labels)
65
     sort(a,a+M);
66
     for (i=0,N=0; i<M; i++)</pre>
67
68
       /*
69
       When we add a higher rectangle, any rectangles that are also
70
       equally thin or thinner become irrelevant, as they are
71
       completely contained within the higher one; remove as many
72
       as necessary
73
74
       while (N>0&&rect[N-1].second<=a[i].second)
75
76
       rect[N++]=a[i]; //add the new rectangle
77
78
     long long cost;
79
     add(rect[0].second,0);
80
     //initially, the best line could be any of the lines in the envelope,
81
     //that is, any line with index 0 or greater, so set pointer=0
82
     pointer=0;
     for (i=0; i<N; i++)
84
85
       cost=query(rect[i].first);
86
```

15

```
if (i<N)
    add(rect[i+1].second,cost);

po printf("%ld\n",cost);
    return 0;
}</pre>
```

2.12. Convex Hull Trick Dinamico

```
// g++ -std=c++11 "convexhulltrick_dynamic.cpp" -o run
2
         ======= <Convex hull trick dynamic version version>
  warning with the use of this, this is a black box, try to use only in an
        emergency.
  Problem for practice: aquire
6
   #include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
   const ll is_query = -(1LL<<62);</pre>
     struct Line {
     ll m, b;
12
     mutable multiset<Line>::iterator it:
13
     const Line *succ(multiset<Line>::iterator it) const;
14
     bool operator<(const Line& rhs) const {
15
       if (rhs.b != is_query) return m < rhs.m;</pre>
16
       const Line *s=succ(it);
17
       if(!s) return 0;
18
       11 x = rhs.m;
19
       return b - s->b < (s->m - m) * x;
20
21
^{22}
   struct HullDynamic : public multiset<Line>{ // will maintain upper hull
       for maximum
     bool bad(iterator y) {
^{24}
       iterator z = next(y);
25
       if (y == begin()) {
26
         if (z == end()) return 0;
27
         return y->m == z->m && y->b <= z->b;
28
29
       iterator x = prev(y);
30
       if (z == end()) return y->m == x->m && y->b <= x->b;
31
```

```
return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
32
33
     iterator next(iterator y){return ++y;}
34
     iterator prev(iterator y){return --y;}
35
       void insert_line(ll m, ll b) {
36
       iterator y = insert((Line) { m, b });
       y->it=y;
       if (bad(y)) { erase(y); return; }
       while (next(y) != end() && bad(next(y))) erase(next(y));
       while (y != begin() && bad(prev(y))) erase(prev(y));
42
     ll eval(ll x) {
43
       Line 1 = *lower_bound((Line) { x, is_query });
       return 1.m * x + 1.b;
    }
46
   }h:
47
   const Line *Line::succ(multiset<Line>::iterator it) const{
49 | return (++it==h.end()? NULL : &*it);}
                           2.13. Misof Tree
http://codeforces.com/blog/entry/10493#comment-159335
3 Sirve para encontrar el i-esimo numero de un conjunto de numeros que
       vamos insertando en el arbol.
4 Sirve solo si nuestros numeros son del 0 al n-1 (pero podemos mapearlos
       antes de usarlos)
5 La idea es esta:
   Funcionamiento:
    - En el fondo sigue siendo un Segment-Tree (hacemos que 'n' sea 2^x)
    - Cada nodo guarda cuantos numeros hay en el intervalo (entonces en
         tree[1] dice cuantos numeros tenemos en total)
    - Se sigue representando los hijos del nodo 'i' con '2 * i' (izq) y '2
          * i + 1' (der):
   Query:
10
    - si kth es mas grande que todos los que tenemos(tree[1]) o es
11
         negativo entonces -1
    - siempre nos mantenemos en el nodo de la izquierda y si es necario
         avanzamos al de la derecha
13
       - si kth es mas grande que el nodo de la izquierda(el actual) quiere
14
            decir que podemos quitarle todos esos
```

numeros a nuestra busqueda 'kth - tree[i]' y buscar el nuevo kth en

```
el arbol de la derecha
         if (kth > tree [i]) kth -= tree [i++];
16
       - Ojo en el 'i++' ahi es donde avanzamos al nodo de la derecha
17
     - luego hace su formula rara que aun no entendi xD:
18
         'i - leaf + (kth > tree [i])';
19
20
   const int MaxN = 1e6;
21
22
   int a [MaxN], s [MaxN];
   int leaf, tree [100 + MaxN << 2];</pre>
25
   void bld (int n) { leaf = 1 << (32 - __builtin_clz (n)); }</pre>
   void add (int x) { for (int i = leaf + x; i : i >>= 1) ++tree [i]: }//
       Podemos insertar mas de una copia la vez tree [i] += xcopies;
  void del (int x) { for (int i = leaf + x; i; i >>= 1) --tree [i]; }//
       Podemos eliminar mas de una copia la vez tree [i] -= xcopies;
   // en "leaf + x" esta cuantas copias tenemos de "x"
   //Cuidado con intentar hacer del con mas copias de las disponibles, el
       kth() no funcionaria
   long kth (int kth, int i = -1) {
       if (kth > tree [1] || kth <= 0) return i;
32
     for (i = 1; i < leaf; i <<= 1) if (kth > tree [i]) kth -= tree [i++];
33
       return i - leaf + (kth > tree [i]);
34
35 }
```

2.14. SQRT Decomposition Basic

```
const int maxn = 500010;
int n;

tipo v[maxn];//vector principal

tipo lazy[maxn];
pair<tipo, tipo> t[maxn];//para poder reordenar los elementos

int SQRT;
int N;//nro. de buckets

//Recalcula y aplica el lazy al bucket con indice idx
//guarda la informacion necesaria del bucket en otros vectores
//podria ser la suma del bucket, o el min/max del bucket
void recalc(int idx) {
   int a = idx * SQRT, b = min(n, (idx + 1) * SQRT);
```

```
for (int i = a; i < b; i++) {
       v[i] += lazv[idx];
18
       t[i] = make_pair(v[i], i);
19
20
     lazv[idx] = 0;
21
     sort(t + a, t + b);
22
23
24
   //adiciona delta a todos los elementos
   //en el intervalo cerrado [a, b]
   void add(int a, int b, tipo delta) {
     int idx_a = a / SQRT, idx_b = b / SQRT;
     if (idx a == idx b) {
29
     for (int i = a; i <= b; i++)
30
         v[i] += delta;
31
      recalc(idx_a);
    } else {
       //head
       for (int i = a, \lim = \min(n, (idx a + 1) * SQRT): i < \lim: i++)
35
         v[i] += delta;
       recalc(idx_a);//OJO puede ser necesario
37
       //body
       for (int i = idx_a + 1; i < idx_b; i++)
39
         lazv[i] += delta;
       //tail
41
       for (int i = idx_b * SQRT; i \le b; i++)
         v[i] += delta;
43
       recalc(idx_b);//OJO puede ser necesario
44
45
   }
46
47
    //tambien podria ser en un rango como en el add
   tipo query(tipo val) {
     tipo ans = 0:
    //recorro todos los buckets
51
     for (int idx = 0; idx < N; idx++) {
       int a = idx * SQRT, b = min(n, (idx + 1) * SQRT);
      //... hacer algo ...
54
     }
55
     return ans;
56
57
   int main() {
    //leer n, q y los elementos de v
```

```
60
     SQRT = (int)sqrt(n) + 1;
61
     N = (n + SQRT - 1) / SQRT; //nro. de buckets
62
     //construir cada bucket
63
     for (int idx = 0; idx < N; idx++)
64
       recalc(idx);
66
     //resto del programa
67
     return 0;
  |}
69
```

2.15. Nro. Elementos menores o iguales a x en O(log(n))

```
//insersion y consulta de cuantos <= en log n
  struct legset {
2
     int maxl; vector<int> c;
     int pref(int n, int l) { return (n>(maxl-l))|(1<<l); }
     void ini(int ml) { maxl=ml; c=vector<int>(1<<(maxl+1)); }</pre>
     //inserta c copias de e, si c es negativo saca c copias
     void insert(int e, int q=1) { forn(l,maxl+1) c[pref(e,l)]+=q; }
7
     int leq(int e) {
8
       int r=0,a=1;
9
       forn(i,maxl) {
10
         a<<=1; int b=(e>>maxl-i-1)&1;
11
         if (b) r+=c[a]; a|=b;
12
       } return r + c[a]; //sin el c[a] da los estrictamente menores
13
14
     int size() { return c[1]; }
15
     int count(int e) { return c[e|(1<<maxl)]; }</pre>
17 | };
```

3. Algos

3.1. LIS en O(n log n) con Reconstruccion

```
//Para non-increasing, cambiar comparaciones y revisar busq binaria
//Given an array, paint it in the least number of colors so that each color turns to a non-increasing subsequence.
//Solution:Min number of colors=Length of the longest increasing subsequence
// Las lineas marcadas con // Camino no son necesarias si no se desea reconstruir el camino.
```

```
6 #define MAXN 1000000
  int v[MAXN]; // INPUT del algoritmo.
   int mv[MAXN];
   int mi[MAXN] ,p[MAXN]; // Camino
   int 1 [MAXN]; // Aca apareceria la maxima subsecuencia creciente(los
       indices)
int lis(int n) {
    forn(i,n) mv[i] = INF;
     forn(i,n) mi[i] = -1; // Camino
     forn(i,n) p [i] = -1; // Camino
     mv[0] = -INF;
15
     int res = 0;
16
     forn(i,n) {
17
       // Con upper_bound es maxima subsecuencia no decreciente.
       // Con lower_bound es maxima subsecuencia creciente.
       int me = upper_bound(mv,mv+n,v[i]) - mv;
       p[i] = mi[me-1]; // Camino
21
       mv[me] = v[i];
       mi[me] = i: // Camino
       if (me > res) res = me;
    }
25
     for(int a = mi[res], i = res - 1;a != -1; a = p[a], i--) // Camino
26
       1[i] = a; // Indices: poniendo 1[i] = v[a] quedan los valores.
27
     return res;
28
29 }
```

3.2. Mo

```
// g++ -std=c++11 "mo.cpp" -o run
/***

Contain a sample about Mo algorithm
Brief explanation when use Mo:
Explain where and when we can use above algorithm

As mentioned, this algorithm is offline, that means we cannot use it when we are forced to stick to given order of queries.

That also means we cannot use this when there are update operations.
Not just that, there is one important possible limitation:

We should be able to write the functions add and remove. There will be many cases where add is trivial but remove is not.

One such example is where we want maximum in a range. As we add elements , we can keep track of maximum. But when we remove elements
```

53

for(int i = 0; $i < q; i++){$

```
12 it is not trivial. Anyways in that case we can use a set to add elements
        , remove elements and report minimum.
  In that case the add and delete operations are O(log N) (Resulting in O(
       N * Sqrt(N) * log N) algorithm).
14
   Suggestion first use the add operation, then the erase operation
   Problem for practice: DQUERY spoj
   Input: N, then N elements of array M querys with a range L,R
18
   const int MAXV = 1e6 + 10;
   const int N = 30010;
   const int M = 200010;
   int cnt[MAXV]:
   int v[N];
24
   struct query{
     int 1,r,pos;
26
     query(){}
27
28
   int n;
   query qu[M];
   int ans[M];
31
32
   int ret = 0;
   void add(int pos){
34
     pos = v[pos];
35
     cnt[pos]++;
36
     if(cnt[pos] == 1){
37
       ret++;
38
     }
39
40
   void erase(int pos){
41
     pos = v[pos];
42
     cnt[pos]--;
43
     if(!cnt[pos])ret--;
44
45
   int main(){
46
     n = in():
47
     for(int i = 0; i < n; i++){
48
       v[i] = in();
49
50
     int block = ceil(sqrt(n));
51
     int q = in();
52
```

```
qu[i].1 = in() - 1, qu[i].r = in() - 1, qu[i].pos = i;
54
55
     sort(qu,qu + q,[&](const query &a,const query &b){
56
       if(a.l / block != b.l / block)
57
         return a.1 / block < b.1 / block;
58
       return a.r < b.r;
59
     });
60
     int 1 = 0, r = 0;
61
     for(int i = 0; i < q; i++){
       int nl = qu[i].1,nr = qu[i].r;
63
       while(l > nl){
64
         add(--1):
65
       }
66
       while(r <= nr){</pre>
         add(r++);
       }
69
       while(1 < n1){</pre>
         erase(1++):
71
72
       while(r > nr + 1){
73
         erase(--r);
74
       }
75
76
       ans[qu[i].pos] = ret;
77
78
     for(int i = 0; i < q; i++)printf("%\n",ans[i]);
80 }
                     3.3. Ternary Search - Reales
    f[x] increases and then decreases, and we want the maximum value of f[x].
1 //yeputons ~ 300 iteraciones es mas que suficiente
   double l = a, r = b;
   for(int i=0; i<200; i++) {
     double 11 = (1*2+r)/3;
     double 12 = (1+2*r)/3;
     if(f(11) > f(12))
       r = 12;
     else
       1 = 11:
9
   }
10
_{11} | x = 1;
```

10

13

14

15

16

11 }

Si f(x) es facil derivar se deriva f(x) - > f'(x) luego se iguala f'(x) = 0 y se despeja x, estamos consiguiendo el punto donde la pendiente es zero (si es de varias variables se deriva parcialmente).

Si no se puede despejar x de f'(x) y estamos seguros que tiene forma de parabola, se puede *aplicar binary search*, buscamos el punto donde la pendiente cambie de signo(osea sea igual a 0)

3.4. Ternary Search - Enteros

f[x] increases and then decreases, and we want the maximum value of f[x].

```
//f[x] increases and then decreases, and we want the maximum value of f[
    x].
int lo = -1, hi = n;
while (hi - lo > 1){
    int mid = (hi + lo)>>1;
    if (f(mid) > f(mid + 1))
        hi = mid;
    else
        lo = mid;
}
//lo + 1 is the answer
```

4. Strings

4.1. Manacher

```
vector<int> manacher(const string &_s) {
     int n = _s.size();
2
     string s(2 * n + 3, '#');
     s[0] = \%, s[s.size() - 1] = \%, s[s.size() - 1] = \%
     for (int i = 0; i < n; i++)
5
      s[(i + 1) * 2] = _s[i];
6
7
     n = s.size();
8
     vector<int> P(n, 0);
9
     int C = 0, R = 0:
10
    for (int i = 1; i < n - 1; i++) {
11
       int j = C - (i - C);
12
       if (R > i)
13
        P[i] = min(R - i, P[i]);
14
       while (s[i + 1 + P[i]] == s[i - 1 - P[i]])
15
```

```
P[i]++:
16
       if (i + P[i] > R) {
17
         C = i;
18
         R = i + P[i];
19
20
21
     return P;
22
23
   bool is_pal(const vector<int> &mnch_vec, int i, int j) {//[i, j] - i<=j
     int len = i - i + 1;
     i = (i + 1) * 2; //idx to manacher vec idx
     j = (j + 1) * 2;
27
28
     int mid = (i + i) / 2:
     return mnch vec[mid] >= len:
29
30
   int main() {
     string s;
     cin >> s;
    vector<int> mnch_vec= manacher(s);
     if (is_pal(mnch_vec, 2, 7)) {
       //la subcadena desde la posicion 2 a la 7 es palindrome
    }
37
     return 0;
39 }
                           4.2. Trie(estatico)
int trie[10000000] [26];
   int sig;
   int root = 0;
   void reset() {
       sig = -1;
       addNode();//Root
6
   | }
7
   void addNode() {
       sig++;
9
```

memset(trie[sig], -1, sizeof(trie[sig]));

for (int i = 0; i < sz(s); i++) {

void insert(const string &s) {

if (trie[cur][c] == -1) {

int cur = root:

int c = s[i] - 'a';

4.3. Suffix Array O(n log n) con LCP (Kasai) O(n)

4.4. Minima rotacion lexicografica

```
1
   Rotacion Lexicografica minima MinRotLex(cadena,tamanio)
   para cambiar inicio de la cadena char s[300]; int h; s+h;
   retorna inicio de la rotacion minima :D
5
   int MinRotLex(const char *s, const int slen) {
      int i = 0, j = 1, k = 0, x, y, tmp;
7
      while(i < slen && j < slen && k < slen) {</pre>
8
         x = i + k;
         y = j + k;
         if(x >= slen) x -= slen;
11
         if(y >= slen) y -= slen;
12
         if(s[x] == s[y]) {
13
            k++:
14
         } else if(s[x] > s[y]) {
15
            i = j+1 > i+k+1 ? j+1 : i+k+1;
16
            k = 0:
17
            tmp = i, i = j, j = tmp;
18
         } else {
19
            j = i+1 > j+k+1 ? i+1 : j+k+1;
20
            k = 0;
21
         }
22
23
      return i;
24
25
   int main(){
26
     int n;
27
     scanf("%d",&n);getchar();
28
     while(n--){
29
       char str[1000009];
30
       gets(str);
31
       printf("%\n",MinRotLex(str,strlen(str))+1);
32
     }
33
```

```
34 }
                             4.5. Matching
                                4.5.1. KMP
 string T;//cadena donde buscar(where)
string P;//cadena a buscar(what)
   int b[MAXLEN];//back table b[i] maximo borde de [0..i)
   void kmppre(){//by gabina with love
       int i = 0, j=-1; b[0]=-1;
       while(i<sz(P)){</pre>
           while(j>=0 && P[i] != P[j]) j=b[j];
           i++, j++, b[i] = j;
8
       }
9
10
   void kmp(){
       int i=0, j=0;
       while(i<sz(T)){</pre>
13
           while(j>=0 && T[i]!=P[j]) j=b[j];
14
15
           i++, j++;
           if(j==sz(P)) printf("Puisufounduatuindexu/duinuT\n", i-j), j=b[j
16
       }
17
   }
18
19
   int main(){
       cout << "T=";
21
       cin >> T;
22
       cout << "P=";
23
       cin.ignore();
24
       cin >> P;
25
       kmppre();
26
       kmp();
27
       return 0;
28
29 }
                          4.5.2. Z - Por aprender
                     4.5.3. Matching con suffix array
| vector<int> FindOccurrences(const string& pattern, const string& text) {
     vector<int> result;
```

int minIndex = 0,maxIndex = text.size();

while(minIndex < maxIndex){</pre>

```
int mid = (minIndex + maxIndex) >> 1;
5
       if(cmp(pattern,sa[mid]) > 0)minIndex = mid + 1;
6
       else maxIndex = mid;
7
     }
8
     int start = minIndex;
9
     maxIndex = text.size();
10
     while(minIndex < maxIndex){</pre>
11
       int mid = (minIndex + maxIndex) >> 1;
12
       if(cmp(pattern,sa[mid]) < 0)maxIndex = mid;</pre>
13
       else minIndex = mid + 1;
14
     }
15
     int end = maxIndex;
16
     for(int i = start; i < end;i++){</pre>
       result.push_back(sa[i]);
18
     }
19
     return result;
20
21 }
```

4.5.4. Matching con BWT

```
map<char,int>fo;//first ocurrence
   map<char,vector<int> >count;//count the i-th ocurrence of symbol
   string first;//first colum of bwt
   string alpha = "ACGT$";//change this
   void preprocess(const string& bwt) {//recieves a BWT
     string ans = "";
6
     first = bwt;
7
     sort(first.begin(),first.end());
8
     for(int i = 0;first[i];i++){
9
       if(!fo.count(first[i]))fo[first[i]] = i;
10
     }
11
     for(char u:alpha)count[u].push_back(0);
12
     for(int i = 1; i <= bwt.size();i++){</pre>
13
       for(char u:alpha)
14
         count[u].push_back(count[u].back() + (bwt[i - 1] == u));
15
     }
16
17
   //return the number of ocurrences of the pattern
18
   int bwtmatch(int bot,string &pattern){
19
     int top = 0;
20
     while(top <= bot){</pre>
21
       if(pattern.size()){
22
         char letter = pattern.back();
23
```

```
pattern.pop_back();
24
         if(count[letter][bot + 1]){
25
           top = fo[letter] + count[letter][top];
26
           bot = fo[letter] + count[letter][bot + 1] - 1;
27
         }
28
         else return 0;
29
30
       else return bot - top + 1;
31
32
     return 0;
33
34 }
```

```
4.5.5. Matching con Aho-Corasick
1
   struct trie{
     map<char, trie> next;
     trie* tran[256];//transiciones del automata
     int idhoja, szhoja;//id de la hoja o 0 si no lo es
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
         es hoia
     trie *padre, *link, *nxthoja;
7
     char pch;//caracter que conecta con padre
8
     trie(): tran(), idhoja(), padre(), link() {}
9
     void insert(const string &s, int id=1, int p=0){//id>0!!!
10
       if(p<sz(s)){</pre>
11
         trie &ch=next[s[p]];
12
         tran[(int)s[p]]=&ch;
13
         ch.padre=this, ch.pch=s[p];
14
         ch.insert(s, id, p+1);
15
16
       else idhoja=id, szhoja=sz(s);
17
18
     trie* get_link() {
19
       if(!link){
20
         if(!padre) link=this://es la raiz
21
         else if(!padre->padre) link=padre;//hijo de la raiz
22
         else link=padre->get_link()->get_tran(pch);
23
       }
24
       return link: }
25
     trie* get_tran(int c) {
26
       if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
27
       return tran[c]; }
28
```

while(p&&!p->go[w])

p->go[w]=np,p=p->pre;

23

24

```
trie *get_nxthoja(){
                                                                                          if(p==0)
29
                                                                                  25
       if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
                                                                                              np->pre=root;
                                                                                  26
30
       return nxthoja; }
                                                                                  27
                                                                                          else {
31
     void print(int p){
                                                                                              State *q=p->go[w];
32
                                                                                  28
       if(idhoja) cout << "found," << idhoja << ", at position," << p-
                                                                                              if(p->step+1==q->step)
33
                                                                                  29
           szhoja << endl;
                                                                                                  np->pre=q;
                                                                                   30
       if(get_nxthoja()) get_nxthoja()->print(p); }
                                                                                              else {
34
                                                                                  31
     void matching(const string &s, int p=0){
                                                                                                  State *nq=cur++;
35
                                                                                   32
       print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
                                                                                                  nq->clear();
36
                                                                                   33
                                                                                                  memcpv(nq->go,q->go,sizeof(q->go));//nq->go = q->go; para
   }tri;
37
                                                                                   34
                                                                                                      mapa
38
                                                                                                  nq->step=p->step+1;
39
                                                                                  35
   int main(){
                                                                                                  nq->pre=q->pre;
                                                                                   36
     tri=trie();//clear
                                                                                                  q->pre=nq;
                                                                                  37
     tri.insert("ho", 1);
                                                                                                  np->pre=nq;
                                                                                  38
     tri.insert("hoho", 2);
                                                                                                  while (p\&\&p->go[w]==q)
                                                                                   39
                                                                                                      p->go[w]=nq, p=p->pre;
                                                                                   40
                               Suffix Automaton
                                                                                              }
                                                                                   41
                                                                                          }
                                                                                   42
                                                                                          last=np;
    /*################################*/
                                                                                   43
   const int N = INSERTE_VALOR; //maxima longitud de la cadena
                                                                                   44
                                                                                      /*##################### Suffix Automata ###############/
   struct State { //OJO!!! tamanio del alfabeto, si MLE -> map
       State *pre, *go[26];//se puede usar un map<char, State*> go
                                                                                   46
                                                                                      /*################### Algunas aplicaciones ##############*/
       int step:
5
                                                                                      //Obtiene el LCSubstring de 2 cadenas en O(|A| + |B|)
       void clear() {
6
                                                                                      string lcs(char A[N], char B[N]) {
           pre=0;
                                                                                          int n,m;
           step=0;
                                                                                   50
8
                                                                                          n = strlen(A); m = strlen(B);
           memset(go,0,sizeof(go));//go.clear();
                                                                                  51
9
                                                                                          //Construccion: O(|A|)
       }
10
                                                                                          //solo hacerlo una vez si A no cambia
   } *root,*last;
11
                                                                                          init();
   State statePool[N * 2],*cur;
                                                                                  54
                                                                                          for(int i=0; i<n; i++)</pre>
   void init() {
13
                                                                                              Insert(A[i]-'a'); //Fin construccion
       cur=statePool;
                                                                                  56
14
                                                                                          //LCS: 0(|B|)
       root=last=cur++;
                                                                                  57
15
                                                                                          int ans = 0, len = 0, bestpos = 0;
                                                                                  58
       root->clear();
16
                                                                                          State *p = root;
   }
                                                                                  59
17
                                                                                          for(int i = 0; i < m; i++) {</pre>
   void Insert(int w) {
                                                                                  60
18
                                                                                              int x = B[i] - a':
       State *p=last:
                                                                                  61
19
                                                                                              if(p->go[x]) {
       State *np=cur++;
                                                                                  62
20
                                                                                                  len++;
       np->clear();
                                                                                  63
21
                                                                                                  p = p - go[x];
       np->step=p->step+1;
                                                                                  64
22
```

65

66

} else {

while (p && !p->go[x]) p = p->pre;

```
if(!p) p = root, len = 0;
67
                else len = p->step+1, p = p->go[x];
68
           }
69
           if (len > ans)
70
                ans = len, bestpos = i;
71
72
       //return ans; //solo el tamanio del lcs
73
       return string(B + bestpos - ans + 1, B + bestpos + 1);
74
75
76
    /*Numero de subcadenas distintas + 1(subcadena vacia) en O(|A|)
    OJO: Por alguna razon Suffix Array es mas rapido
    Se reduce a contar el numero de paths que inician en q0 y terminan
    en cualquier nodo. dp[u] = # de paths que inician en u
    - Se debe construir el automata en el main(init y Insert's)
81
     Setear dp en -1
83
   number dp[N * 2];
   number num dist substr(State *u = root) {
85
       if (dp[u - statePool] != -1) return dp[u - statePool];
86
       number ans = 1;//el path vacio que representa este nodo
87
       for (int v = 0; v < 26; v++)//usar for (auto) para mapa
88
           if (u->go[v])
89
                ans += num_dist_substr(u->go[v]);
90
       return (dp[u - statePool] = ans);
91
92
93
    /*Suma la longitud de todos los substrings en O(|A|)
94
    - Construir el automata(init y insert's)
95
    - Necesita el metodo num_dist_substr (el de arriba)
96
     setear dp's en -1
97
98
   number dp1[N * 2];
99
   number sum_length_dist_substr(State *u = root) {
100
       if (dp1[u - statePool] != -1) return dp1[u - statePool];
101
       number ans = 0;//el path vacio que representa este nodo
102
       for (int v = 0; v < 26; v++)//usar for (auto) para mapa
103
           if (u->go[v])
104
                ans += (num_dist_substr(u->go[v]) + sum_length_dist_substr(u
105
                    ->go[v]));
       return (dp1[u - statePool] = ans);
106
107
108
```

```
109
   Pregunta si p es subcadena de la cadena con la cual esta construida
110
    el automata.
111
    Complejidad: - Construir O(|Texto|) - solo una vez (init e insert's)
                  - Por Consulta O(|patron a buscar|)
113
114
   bool is_substring(char p[N]) {
115
        State *u = root;
116
        for (int i = 0; p[i]; i++) {
117
            if (!u->go.count(p[i]))//esta con map!!!
                 return false;
119
            u = u \rightarrow go[p[i]];//esta con map!!!
120
        }
121
        return true;
122
123 }
```

4.7. K-esima permutacion de una cadena

```
1 //Entrada: Una cadena cad(std::string), un long th
   //Salida : La th-esima permutacion lexicografica de cad
   string ipermutacion(string cad, long long int th){
     sort(cad.begin(), cad.end());
4
     string sol = "";
5
     int pos;
6
     for(int c = cad.size() - 1; c >= 0; c--){
       pos = th / fact[c];
       th %= fact[c];
9
       sol += cad[pos];
10
       cad.erase(cad.begin() + pos);
11
12
     return sol;
13
14 }
```

5. Geometria

5.1. Graham Scan

5.2. Cortar Poligono

```
//cuts polygon Q along the line ab
//stores the left side (swap a, b for the right one) in P
void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
P.clear();
forn(i, sz(Q)){
```

```
double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%z(Q)]-a);
if(left1>=0) P.pb(Q[i]);
if(left1*left2<0)
P.pb(inter(line(Q[i], Q[(i+1)%z(Q)]), line(a, b)));
}
</pre>
```

5.3. Interseccion de rectangulos

```
#define MAXC 2501
   struct Rect{
2
     int x1,y1, x2,y2;
     int color;
4
     int area;
     Rect(int _x1, int _y1, int _x2, int _y2){
       x1 = x1:
       v1 = _v1;
       x2 = _x2;
       y2 = _y2;
10
       getArea();
11
12
     int getArea(){
13
       if(x1>=x2 || y1>=y2)return area = 0;
14
       return area = (x2-x1)*(y2-y1);
15
16
17
   Rect interseccion(Rect t, Rect r){
     int x1, v1, x2, v2;
     x1 = max(t.x1,r.x1);
     y1 = max(t.y1,r.y1);
     x2 = min(t.x2,r.x2);
     y2 = min(t.y2,r.y2);
     Rect res(x1, y1, x2, y2);
     return res;
25
26 | }
```

5.4. Distancia punto-recta

```
double distance_point_to_line(const point &a, const point &b, const
    point &pnt){
    double u = ((pnt.x - a.x)*(b.x - a.x) + (pnt.y - a.y)*(b.y - a.y)) /
        distsqr(a, b);
    point intersection;
    intersection.x = a.x + u*(b.x - a.x);
```

```
intersection.y = a.y + u*(b.y - a.y);
return dist(pnt, intersection);
}
```

5.5. Distancia punto-segmento

```
struct point{
     double x,y;
   };
3
   inline double dist(const point &a, const point &b){
     return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
   inline double distsqr(const point &a, const point &b){
     return (a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y);
   double distance_point_to_segment(const point &a, const point &b, const
       point &pnt){
     double u = ((pnt.x - a.x)*(b.x - a.x) + (pnt.y - a.y)*(b.y - a.y)) /
         distsqr(a, b);
     point intersection;
     intersection.x = a.x + u*(b.x - a.x);
     intersection.y = a.y + u*(b.y - a.y);
15
     if (u < 0.0 | | u > 1.0)
16
       return min(dist(a, pnt), dist(b, pnt));
17
18
     return dist(pnt, intersection);
19
20 }
```

5.6. Parametrizacion de rectas - Sacar de codeforces

6. Math

6.1. Identidades

```
\sum_{i=0}^{n} {n \choose i} = 2^n
\sum_{i=0}^{n} i {n \choose i} = n * 2^{n-1}
\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}
\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}
\sum_{i=0}^{n} i(i-1) = \frac{8}{6} (\frac{n}{2})(\frac{n}{2} + 1)(n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}
\sum_{i=0}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^{n} i\right]^2
```

```
\sum_{i=0}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}r = e - v + k + 1
```

Teorema de Pick: (Area, puntos interiores y puntos en el borde) $A = I + \frac{B}{2} - 1$

6.2. Ec. Caracteristica

```
\begin{aligned} a_0T(n) + a_1T(n-1) + \ldots + a_kT(n-k) &= 0 \\ p(x) = a_0x^k + a_1x^{k-1} + \ldots + a_k \end{aligned} Sean r_1, r_2, \ldots, r_q las raíces distintas, de mult. m_1, m_2, \ldots, m_q T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n Las constantes c_{ij} se determinan por los casos base.
```

6.3. Identidades de agustin y mario

6.4. Combinatorio

6.5. Exp. de Numeros Mod.

6.6. Exp. de Matrices y Fibonacci en log(n)

```
tipo mod;
tipo a, b, c, d; //|c d|
M22 operator * (const M22 &p) const {
```

```
return (M22){(a*p.a+b*p.c) % mod, (a*p.b+b*p.d) % mod, (c*p.a+d*p.c)
            % mod, (c*p.b+d*p.d) % mod};
    }
6
7
  };
  M22 operator ^ (const M22 &p, tipo n) {
    if(!n) return (M22){1, 0, 0, 1};//matriz identidad
    M22 q = p (n/2); q = q * q;
     return (n %2)? p * q : q;
11
12
   //devuelve el n-esimo fibonacci (index 0)
   //f0 = fibo(0), f1 = fibo(1);
   tipo fibo(tipo n, tipo f0, tipo f1) {
    M22 \text{ mat}=(M22)\{0, 1, 1, 1\}^n;
    return (mat.a*f0 + mat.b*f1) % mod;
18 }
```

6.7. Gauss Jordan

```
_{1} | const int N = 300:
   typedef vector<double> col;
   typedef vector<double> row;
   typedef vector<row>Matrix;
   col solution;
   int main(){
     Matrix M:
     M.resize(300):
     solution.resize(300);
     for(int i = 0; i < 30;i++)M[i].resize(30);
10
     int n;
11
     cin >> n;
12
     for(int i = 0; i < n; i++)
13
       for(int j = 0; j \le n; j++)
14
          cin >> M[i][j];
15
16
     for(int j = 0; j < n - 1; j++){
17
18
       int 1 = i;
       for(int i = j + 1; i < n; i++){
19
          if(fabs(M[i][j]) > fabs(M[l][j]))l = i;
20
21
       for(int k = j; k \le n; k++){
22
          swap(M[j][k],M[l][k]);
23
24
       for(int i = j + 1; i < n; i++)
25
```

```
for(int k = n; k \ge j;k--)
26
           M[i][k] -= M[j][k] * M[i][j] / M[j][j];
27
     }
28
     double t = 0;
29
     for(int j = n - 1; j \ge 0; j--){
30
       t = 0.0;
31
       for(int k = j + 1; k < n; k++)t += M[j][k] * solution[k];
32
       solution[j] = (M[j][n] - t) / M[j][j];
33
     }
34
     cout.precision(4);
35
     for(int i = 0; i < n;i++)cout<<fixed << solution[i] << "";</pre>
     return 0;
37
38 | }
```

6.8. Tridiangonal

```
//TRIDIAGONAL SOLVER
   // solve ai*x_i-1 + bi*x_i + ci*x_i+1 = d_i
   //a_0 = 0 c_n-1 = 0
   /*
4
     b :subdiagonal
     a :diagonal
     c :super diagonal
   //the answer is in D
   #define MAXN 5000
   long double A[MAXN], B[MAXN], C[MAXN], D[MAXN], X[MAXN];
   void solve(int n) {
       C[0] /= B[0]; D[0] /= B[0];
13
       for(int i = 1; i < n - 1; i++) C[i] /= B[i] - A[i] *C[i-1];
14
       for(int i = 1; i < n; i++)D[i] = (D[i] - A[i] * D[i-1]) / (B[i] - A[i])
15
           ] * C[i-1]);
       X[n-1] = D[n-1];
16
       for (int i = n-2; i >= 0; i--) X[i] = D[i] - C[i] * X[i+1];
17
18 }
```

6.9. Simplex

```
1 // Two-phase simplex algorithm for solving linear programs of the form
 1//
2
 1//
         maximize
                      c^T x
  //
         subject to Ax <= b
 1//
                      x >= 0
6 //
```

```
7 // INPUT: A -- an m x n matrix
             b -- an m-dimensional vector
8
9 //
             c -- an n-dimensional vector
10 //
             x -- a vector where the optimal solution will be stored
  //
11
   // OUTPUT: value of the optimal solution (infinity if unbounded
              above, nan if infeasible)
   //
13
   //
14
  // To use this code, create an LPSolver object with A, b, and c as
   // arguments. Then, call Solve(x).
17
   #include <iostream>
   #include <iomanip>
   #include <vector>
   #include <cmath>
   #include <limits>
   using namespace std;
25
   typedef long double DOUBLE;
   typedef vector<DOUBLE> VD;
   typedef vector<VD> VVD;
   typedef vector<int> VI;
   const DOUBLE EPS = 1e-9;
31
32
   struct LPSolver {
     int m, n;
     VI B, N;
35
     VVD D;
36
37
38
     LPSolver(const VVD &A, const VD &b, const VD &c):
       m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2))  {
39
       for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[
40
           i][i];
       for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + i]
41
           1] = b[i]: }
       for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
       N[n] = -1; D[m + 1][n] = 1;
43
44
45
     void Pivot(int r, int s) {
46
       double inv = 1.0 / D[r][s];
47
```

```
for (int i = 0; i < m + 2; i++) if (i != r)
                                                                                              Pivot(i, s);
         for (int j = 0; j < n + 2; j++) if (j != s)
                                                                                            }
                                                                                   88
49
           D[i][j] -= D[r][j] * D[i][s] * inv;
                                                                                          }
                                                                                   89
50
       for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
                                                                                          if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();//
                                                                                   90
51
       for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
                                                                                              INFINITY
52
       D[r][s] = inv;
                                                                                          x = VD(n);
53
       swap(B[r], N[s]);
                                                                                          for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
54
                                                                                          return D[m][n + 1];//solution find
55
56
                                                                                   94
     bool Simplex(int phase) {
                                                                                      };
57
       int x = phase == 1 ? m + 1 : m;
58
                                                                                      int main() {
       while (true) {
59
         int s = -1:
         for (int j = 0; j \le n; j++) {
                                                                                       const int m = 4:
           if (phase == 2 && N[j] == -1) continue;
                                                                                        const int n = 3;
           if (s == -1 \mid | D[x][i] < D[x][s] \mid | D[x][i] == D[x][s] && N[i] <
                                                                                        DOUBLE _A[m][n] = {
63
                                                                                         \{6, -1, 0\},\
                N[s]) s = j;
         }
                                                                                         \{-1, -5, 0\},\
64
         if (D[x][s] > -EPS) return true:
                                                                                         \{1, 5, 1\},\
65
         int r = -1;
                                                                                         \{-1, -5, -1\}
66
         for (int i = 0; i < m; i++) {
                                                                                       };
                                                                                  106
67
           if (D[i][s] < EPS) continue;
                                                                                        DOUBLE _b[m] = \{ 10, -4, 5, -5 \};
68
           if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                                                                                        DOUBLE _{c}[n] = \{ 1, -1, 0 \};
                                                                                  108
69
             (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B
                                                                                  109
70
                  [r]) r = i;
                                                                                        VVD A(m);
                                                                                  110
         }
                                                                                        VD b(_b, _b + m);
                                                                                  111
71
         if (r == -1) return false;
                                                                                        VD c(_c, _c + n);
72
                                                                                  112
         Pivot(r, s);
                                                                                        for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);
                                                                                  113
73
       }
                                                                                  114
74
     }
                                                                                  115
                                                                                        LPSolver solver(A, b, c);
75
                                                                                        VD x;
                                                                                  116
76
     DOUBLE Solve(VD &x) {
                                                                                        DOUBLE value = solver.Solve(x);
                                                                                  117
77
       int r = 0:
                                                                                  118
78
       for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
                                                                                        cerr << "VALUE: | " << value << endl; // VALUE: 1.29032
                                                                                  119
79
       if (D[r][n + 1] < -EPS) {
                                                                                        cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
                                                                                  120
80
         Pivot(r. n):
                                                                                        for (size_t i = 0; i < x.size(); i++) cerr << "" << x[i];
                                                                                  121
81
         if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<
                                                                                        cerr << endl:</pre>
                                                                                  122
82
             DOUBLE>::infinitv()://NO SOLUTION
                                                                                  123
                                                                                        return 0:
                                                                                  124 }
         for (int i = 0; i < m; i++) if (B[i] == -1) {
83
           int s = -1;
84
                                                                                                   6.10. Matrices y determinante O(n^3)
           for (int j = 0; j \le n; j++)
85
             if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j]
86
                  < N[s]) s = j;
                                                                                   struct Mat {
```

```
vector<vector<double> > vec:
2
       Mat(int n): vec(n, vector<double>(n) ) {}
3
       Mat(int n, int m): vec(n, vector<double>(m) ) {}
4
       vector<double> &operator[](int f){return vec[f];}
5
       const vector<double> &operator[](int f) const {return vec[f];}
6
       int size() const {return sz(vec);}
       Mat operator+(Mat &b) { ///this de n x m entonces b de n x m
8
           Mat m(sz(b),sz(b[0]));
9
           forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j
10
               ];
           return m:
11
       Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
12
           int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
13
           Mat mat(n,t);
14
           forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
15
           return mat:
16
       double determinant(){//sacado de e maxx ru
17
           double det = 1;
18
           int n = sz(vec):
19
           Mat m(*this);
20
           forn(i, n){//para cada columna
21
               int k = i;
22
               forr(j, i+1, n)//busco la fila con mayor val abs
23
                   if(abs(m[j][i])>abs(m[k][i])) k = j;
24
               if(abs(m[k][i])<1e-9) return 0;
25
               m[i].swap(m[k]);//la swapeo
26
               if(i!=k) det = -det;
27
               det *= m[i][i];
28
               forr(j, i+1, n) m[i][j] /= m[i][i];
29
               //hago 0 todas las otras filas
30
               forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
31
                   forr(k, i+1, n) m[j][k]-=m[i][k]*m[j][i];
32
           }
33
           return det:
34
       }
35
   };
36
37
   int n:
   int main() {
   //DETERMINANTE:
   //https://uva.onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&
       page=show_problem&problem=625
    freopen("input.in", "r", stdin);
```

6.11. Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

6.12. Criba

```
#define MAXP 100000 //no necesariamente primo
int criba[MAXP+1];
   void crearcriba(){
     int w[] = \{4,2,4,2,4,6,2,6\};
     for(int p=25;p<=MAXP;p+=10) criba[p]=5;</pre>
     for(int p=9;p<=MAXP;p+=6) criba[p]=3;</pre>
     for(int p=4;p<=MAXP;p+=2) criba[p]=2;</pre>
7
     for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])</pre>
       for(int j=p*p;j<=MAXP;j+=(p<<1)) if(!criba[j]) criba[j]=p;</pre>
9
10
   vector<int> primos;
   void buscarprimos(){
12
     crearcriba();
13
     forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
14
15
   //^{\sim} Useful for bit trick: #define SET(i) ( criba[(i)>>5]|=1<<((i)&31) ),
        #define INDEX(i) ( (criba[i>>5]>>((i)&31))&1 ), unsigned int criba[
       MAXP/32+1];
17
18
   int main() {
     freopen("primos", "w", stdout);
20
     buscarprimos();
21
```

6.13. Funciones de primos

```
Sea n = \prod p_i^{k_i}, fact(n) genera un map donde a cada p_i le asocia su k_i
```

```
//factoriza bien numeros hasta MAXP^2
  map<ll,ll> fact(ll n){ //0 (cant primos)
2
     map<ll,ll> ret;
     forall(p, primos){
       while(!(n %*p)){
         ret[*p]++;//divisor found
         n/=*p;
       }
8
     if(n>1) ret[n]++;
     return ret;
11
12
    //factoriza bien numeros hasta MAXP
13
   map<11,11> fact2(11 n){ //0 (1g n)}
     map<ll,ll> ret;
15
     while (criba[n]){
       ret[criba[n]]++;
       n/=criba[n];
18
19
     if(n>1) ret[n]++;
20
     return ret:
21
22
   //Usar asi: divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
   void divisores(const map<11,11> &f, vector<11> &divs, map<11,11>::
       iterator it, ll n=1){
       if(it==f.begin()) divs.clear();
25
       if(it==f.end()) { divs.pb(n); return; }
26
       ll p=it->fst, k=it->snd; ++it;
27
       forn(_, k+1) divisores(f, divs, it, n), n*=p;
28
29
   11 sumDiv (ll n){
30
     ll rta = 1;
31
     map<ll,ll> f=fact(n);
32
     forall(it, f) {
33
     11 \text{ pot} = 1, \text{ aux} = 0;
34
     forn(i, it->snd+1) aux += pot, pot *= it->fst;
35
     rta*=aux;
36
     }
37
     return rta;
38
  |}
39
```

```
40 | ll eulerPhi (ll n){ // con criba: O(lg n)
     11 \text{ rta} = n;
41
     map<11,11> f=fact(n);
42
     forall(it, f) rta -= rta / it->first;
43
     return rta;
44
45
   11 eulerPhi2 (11 n){ // 0 (sqrt n)
46
     11 r = n;
    forr (i,2,n+1){
    if ((11)i*i > n) break;
    if (n \% i == 0){
         while (n\% == 0) n/=i;
         r = r/i:
     }
53
     if (n != 1) r= r/n;
     return r;
   }
56
57
   int main() {
     buscarprimos();
     forr (x,1, 500000){
60
       cout << "x_=_" << x << endl;
61
       cout << "Numero_de_factores_primos:_" << numPrimeFactors(x) << endl;</pre>
62
       cout << "Numero de distintos factores primos: " <<
63
            numDiffPrimeFactors(x) << endl;</pre>
       cout << "Suma_de_factores_primos:_" << sumPrimeFactors(x) << endl;</pre>
64
       cout << "Numero_de_divisores:_" << numDiv(x) << endl;</pre>
65
       cout << "Suma_de_divisores:__" << sumDiv(x) << endl;</pre>
       cout << "Phi_de_Euler:_" << eulerPhi(x) << endl;</pre>
67
     }
68
     return 0;
69
70 }
```

6.14. Phollard's Rho (rolando)

```
b /= 2:
                                                                                            y = (mulmod(y, y, n) + c) n;
                                                                                 51
                                                                                            y = (mulmod(y, y, n) + c) n;
9
                                                                                            if(x - y \ge 0) d = gcd(x - y, n);
     return x % c;
10
                                                                                            else d = gcd(y - x, n);
11
                                                                                        }
                                                                                 55
12
                                                                                        return d==n? rho(n):d;
   ll expmod (ll b, ll e, ll m){\frac{1}{0}}
                                                                                 56
                                                                                    }
     if(!e) return 1;
                                                                                 57
     11 q = expmod(b, e/2, m); q = mulmod(q, q, m);
                                                                                 58
    return e %2? mulmod(b,q,m) : q;
                                                                                    map<ll,ll> prim;
16
                                                                                    void factRho (ll n){ //O (lg n)^3. un solo numero
17
                                                                                      if (n == 1) return;
18
   bool es_primo_prob (ll n, int a)
                                                                                     if (rabin(n)){
                                                                                       prim[n]++;
20
     if (n == a) return true:
                                                                                       return;
21
     11 s = 0, d = n-1;
     while (d \% 2 == 0) s++, d/=2;
                                                                                     11 factor = rho(n);
23
                                                                                     factRho(factor):
24
     11 x = expmod(a,d,n);
                                                                                      factRho(n/factor);
25
                                                                                69 }
     if ((x == 1) \mid | (x+1 == n)) return true:
26
27
                                                                                                                6.15. GCD
     forn (i, s-1){
28
      x = mulmod(x, x, n);
29
                                                                                 tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
       if (x == 1) return false;
30
       if (x+1 == n) return true;
31
                                                                                                         6.16. Extended Euclid
     }
32
     return false;
33
                                                                                 void extendedEuclid (ll a, ll b) \{ //a * x + b * y = d \}
34
                                                                                     if (!b) { x = 1; y = 0; d = a; return;}
35
                                                                                     extendedEuclid (b, a%);
   bool rabin (ll n){ //devuelve true si n es primo
36
                                                                                     11 x1 = y;
     if (n == 1) return false;
37
                                                                                     11 y1 = x - (a/b) * y;
     const int ar[] = \{2,3,5,7,11,13,17,19,23\};
38
                                                                                      x = x1; y = y1;
     forn (j,9)
39
                                                                                 7 | }
       if (!es_primo_prob(n,ar[j]))
40
        return false:
                                                                                                                6.17. LCM
41
     return true;
42
43
                                                                                 1 | tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
44
                                                                                                              6.18. Inversos
   ll rho(ll n){
       if( (n & 1) == 0 ) return 2;
46
      11 x = 2, y = 2, d = 1;
                                                                                 1 #define MAXMOD 15485867
47
      ll c = rand() % n + 1;
                                                                                 1 ll inv[MAXMOD];//inv[i]*i=1 mod MOD
48
       while(d == 1){
                                                                                 3 void calc(int p){//O(p)
49
           x = (mulmod(x, x, n) + c) n;
50
                                                                                     inv[1]=1;
```

```
forr(i, 2, p) inv[i] = p-((p/i)*inv[p%i]) %p;

forr(i, 2, p) inv[i] = p-((p/i)*inv[p%i]) %p;

int inverso(int x){//0(log x)

return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)

return expmod(x, MOD-2);//si mod es primo

}
```

6.19. Simpson

```
double integral(double a, double b, int n=10000) {//O(n), n=cantdiv
double area=0, h=(b-a)/n, fa=f(a), fb;
forn(i, n){
   fb=f(a+h*(i+1));
   area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
}
return area*h/6.:}
```

6.20. Fraction

```
tipo mcd(tipo a, tipo b){return a?mcd(b%, a):b;}
   struct frac{
     tipo p,q;
3
     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
     void norm(){
       tipo a = mcd(p,q):
6
       if(a) p/=a, q/=a;
       else q=1;
8
       if (q<0) q=-q, p=-p;}
9
     frac operator+(const frac& o){
10
       tipo a = mcd(q, o.q);
11
       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
12
     frac operator-(const frac& o){
13
       tipo a = mcd(q, o.q);
14
       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
15
     frac operator*(frac o){
16
       tipo a = mcd(q, o.p), b = mcd(o.q, p);
17
       return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
18
     frac operator/(frac o){
19
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
20
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
21
     bool operator<(const frac &o) const{return p*o.q < o.p*q;}</pre>
22
     bool operator==(frac o){return p==o.p&kq==o.q;}
23
24 | };
```

6.21. Polinomio

```
int m = sz(c), n = sz(o.c);
           vector<tipo> res(max(m,n));
2
           forn(i, m) res[i] += c[i];
3
           forn(i, n) res[i] += o.c[i];
4
           return poly(res); }
5
       poly operator*(const tipo cons) const {
6
       vector<tipo> res(sz(c));
7
           forn(i, sz(c)) res[i]=c[i]*cons;
8
           return poly(res); }
9
       poly operator*(const poly &o) const {
10
           int m = sz(c), n = sz(o.c);
11
           vector<tipo> res(m+n-1);
12
           forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
13
           return poly(res);
14
     tipo eval(tipo v) {
15
       tipo sum = 0;
16
       dforn(i, sz(c)) sum=sum*v + c[i];
       return sum: }
       //poly contains only a vector<int> c (the coeficients)
     //the following function generates the roots of the polynomial
    //it can be easily modified to return float roots
     set<tipo> roots(){
       set<tipo> roots;
23
       tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
24
       vector<tipo> ps,qs;
25
       forr(p,1,sqrt(a0)+1) if (a0\%p==0) ps.pb(p),ps.pb(a0/p);
       forr(q,1,sqrt(an)+1) if (an)q==0) qs.pb(q),qs.pb(an/q);
27
       forall(pt,ps)
28
         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
29
           tipo root = abs((*pt) / (*qt));
30
           if (eval(root)==0) roots.insert(root);
31
32
       return roots; }
33
   };
34
   pair<poly,tipo> ruffini(const poly p, tipo r) {
35
     int n = sz(p.c) - 1;
36
     vector<tipo> b(n);
37
     b[n-1] = p.c[n];
38
     dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
39
     tipo resto = p.c[0] + r*b[0];
40
     poly result(b);
41
```

```
return make_pair(result,resto);
42
43
                 poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
44
                                     poly A; A.c.pb(1);
45
                                     forn(i,sz(x)) \{ poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux; aux.c.pb(-x[i]), aux.c.pb(
46
                                                                }
                            poly S; S.c.pb(0);
47
                           forn(i,sz(x)) { poly Li;
48
                                    Li = ruffini(A,x[i]).fst;
49
                                     Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
50
                                                           coefficients instead of 1.0 to avoid using double
                                     S = S + Li * y[i]; }
51
                           return S;
52
53
54
                int main(){
                          return 0;
56
57 }
```

6.22. Ec. Lineales

```
bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
     int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
     vector<int> p; forn(i,m) p.push_back(i);
3
     forn(i, rw) {
4
       int uc=i. uf=i:
       forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;
6
           uc=c:}
       if (feq(a[uf][uc], 0)) { rw = i; break; }
7
       forn(j, n) swap(a[j][i], a[j][uc]);
8
       swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
9
       tipo inv = 1 / a[i][i]; //aca divide
10
       forr(j, i+1, n) {
11
         tipo v = a[j][i] * inv;
12
        forr(k, i, m) a[j][k]-=v * a[i][k];
13
         y[j] -= v*y[i];
14
15
     } // rw = rango(a), aca la matriz esta triangulada
16
     forr(i, rw, n) if (!feg(y[i],0)) return false; // checkeo de
17
         compatibilidad
     x = vector < tipo > (m, 0);
18
     dforn(i, rw){
19
       tipo s = y[i];
20
```

```
forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
21
       x[p[i]] = s / a[i][i]; //aca divide
22
23
     ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
24
     forn(k, m-rw) {
25
       ev[k][p[k+rw]] = 1;
26
       dforn(i, rw){
27
         tipo s = -a[i][k+rw];
28
         forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
         ev[k][p[i]] = s / a[i][i]; //aca divide
       }
31
    }
32
     return true;
33
34 }
```

6.23. Karatsuba

```
// g++ -std=c++11 "karatsuba.cpp" -o hld
2
   ======== <karatsuba> ===========
   Complexity: O(N^1.7)
   Call to karatsuba function paramter two vectors
   * INPUT: two vectors A,B cointains the coeficients of the polynomail
   * OUTPUT a vector coitains the coeficients of A * B
   */
10
   int p,k;
11
   vector<int> b,r;
13
   void trim(vector<int>& a){
       while (a.size() > 0 \&\& a.back() == 0) a.pop_back();
15
16
17
   vector<int> multiply(const vector<int>& a, const vector<int>& b){
       vector<int> c(a.size() + b.size() + 1, 0);
19
       for (int i = 0; i < a.size(); i++) {
20
           for (int j = 0; j < b.size(); j++) {
21
               c[i+j] += a[i] * b[j];
22
           }
23
       }
24
       trim(c);
25
       return c;
26
```

```
27 | }
   // a = a + b*(10^k)
   void addTo(vector<int>& a, const vector<int>& b, int k){
29
       if (a.size() < b.size() + k) a.resize(b.size() + k);</pre>
30
       for (int i = 0; i < b.size(); i++) a[i+k] += b[i];
31
32
   void subFrom(vector<int>& a, const vector<int>& b){
       for (int i = 0; i < b.size(); i++) a[i] -= b[i];
35
    // a = a + b
   void addTo(vector<int>& a, const vector<int>& b){
37
       addTo(a, b, 0);
39
   vector<int> karatsuba(const vector<int>& a, const vector<int>& b)
41
       int alen = a.size();
42
       int blen = b.size():
43
       if (alen == 0 || blen == 0) return vector<int>();
44
       if (alen < blen) return karatsuba(b, a):
45
       if (alen < 50) return multiply(a, b);
46
47
       int half = alen / 2;
48
       vector<int> a0(a.begin(), a.begin() + half);
49
       vector<int> a1(a.begin() + half, a.end());
50
       vector<int> b0(b.begin(), b.begin() + min<int>(blen, half));
51
       vector<int> b1(b.begin() + min<int>(blen, half), b.end());
52
53
       vector<int> z0 = karatsuba(a0, b0);
54
       vector<int> z2 = karatsuba(a1, b1);
55
       addTo(a0, a1);
56
       addTo(b0, b1);
57
       vector<int> z1 = karatsuba(a0, b0);
58
       subFrom(z1, z0):
59
       subFrom(z1, z2);
60
61
       vector<int> res:
62
       addTo(res, z0);
63
       addTo(res, z1, half);
64
       addTo(res, z2, half + half);
65
66
       trim(res);
67
       return res;
68
69 }
```

6.24. FFT

```
1 | #define lowbit(x) (((x) ^ (x-1)) & (x))
   typedef complex<long double> Complex;
   void FFT(vector<Complex> &A, int s){
        int n = A.size();
        int p = __builtin_ctz(n);
        vector<Complex> a = A;
8
9
        for(int i = 0; i < n; ++i){
10
            int rev = 0;
11
            for(int j = 0; j < p; ++j){
12
                rev <<= 1;
                rev |= ((i >> j) \& 1);
15
            A[i] = a[rev];
16
17
18
       Complex w,wn;
19
20
        for(int i = 1; i \le p; ++i){
21
            int M = (1 << i), K = (M >> 1);
            wn = Complex(cos(s*2.0*M_PI/(double)M), sin(s*2.0*M_PI/(double)M
23
                ));
24
            for(int j = 0; j < n; j += M){
25
                w = Complex(1.0, 0.0);
26
                for(int l = j; 1 < K + j; ++1){
27
                     Complex t = w;
28
                     t *= A[1 + K];
29
                     Complex u = A[1];
30
                     A[1] += t;
31
                     u -= t;
32
                     A[1 + K] = u;
33
                     w = wn;
34
35
            }
36
        }
37
38
39
        if(s==-1){
            for(int i = 0; i < n; ++i)
40
```

```
A[i] /= (double)n:
41
       }
42
   }
43
44
   vector<Complex> FFT_Multiply(vector<Complex> &P, vector<Complex> &Q){
45
       int n = P.size()+Q.size();
46
       while(n!=lowbit(n)) n += lowbit(n);
47
48
       P.resize(n,0);
49
       Q.resize(n,0);
50
51
       FFT(P,1);
52
       FFT(Q,1);
53
54
       vector<Complex> R;
55
       for(int i=0;i<n;i++) R.push_back(P[i]*Q[i]);</pre>
56
57
       FFT(R,-1);
58
59
       return R;
60
61
62
    // Para multiplicacion de enteros grandes
   const long long B = 100000;
   const int D = 5;
```

6.25. Tablas y cotas (Primos, Divisores, Factoriales, etc)

```
Factoriales
0! = 1
                  11! = 39.916.800
1! = 1
                   12! = 479.001.600 \ (\in int)
2! = 2
                   13! = 6.227.020.800
3! = 6
                  14! = 87.178.291.200
4! = 24
                   15! = 1.307.674.368.000
5! = 120
                  16! = 20.922.789.888.000
6! = 720
                   17! = 355.687.428.096.000
7! = 5.040
                  18! = 6.402.373.705.728.000
8! = 40.320
                  19! = 121.645.100.408.832.000
9! = 362.880
                  20! = 2.432.902.008.176.640.000 \ (\in \text{tint})
10! = 3.628.800 \mid 21! = 51.090.942.171.709.400.000
       \max \text{ signed tint} = 9.223.372.036.854.775.807
     max unsigned tint = 18.446.744.073.709.551.615
```

Primos

 $2\ 3\ 5\ 7\ 11\ 13\ 17\ 19\ 23\ 29\ 31\ 37\ 41\ 43\ 47\ 53\ 59\ 61\ 67\ 71\ 73\ 79\ 83\ 89\ 97\ 101\ 103\ 107\ 109$ 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199 211 223 227 $229\ 233\ 239\ 241\ 251\ 257\ 263\ 269\ 271\ 277\ 281\ 283\ 293\ 307\ 311\ 313\ 317\ 331\ 337\ 347$ $349\ 353\ 359\ 367\ 373\ 379\ 383\ 389\ 397\ 401\ 409\ 419\ 421\ 431\ 433\ 439\ 443\ 449\ 457\ 461$ $463\ 467\ 479\ 487\ 491\ 499\ 503\ 509\ 521\ 523\ 541\ 547\ 557\ 563\ 569\ 571\ 577\ 587\ 593\ 599$ $601\ 607\ 613\ 617\ 619\ 631\ 641\ 643\ 647\ 653\ 659\ 661\ 673\ 677\ 683\ 691\ 701\ 709\ 719\ 727$ $733\ 739\ 743\ 751\ 757\ 761\ 769\ 773\ 787\ 797\ 809\ 811\ 821\ 823\ 827\ 829\ 839\ 853\ 857\ 859$ 863 877 881 883 887 907 911 919 929 937 941 947 953 967 971 977 983 991 997 1009 $1013\ 1019\ 1021\ 1031\ 1033\ 1039\ 1049\ 1051\ 1061\ 1063\ 1069\ 1087\ 1091\ 1093\ 1097\ 1103$ $1109\ 1117\ 1123\ 1129\ 1151\ 1153\ 1163\ 1171\ 1181\ 1187\ 1193\ 1201\ 1213\ 1217\ 1223\ 1229$ $1231\ 1237\ 1249\ 1259\ 1277\ 1279\ 1283\ 1289\ 1291\ 1297\ 1301\ 1303\ 1307\ 1319\ 1321\ 1327$ 1361 1367 1373 1381 1399 1409 1423 1427 1429 1433 1439 1447 1451 1453 1459 1471 1481 1483 1487 1489 1493 1499 1511 1523 1531 1543 1549 1553 1559 1567 1571 1579 1583 1597 1601 1607 1609 1613 1619 1621 1627 1637 1657 1663 1667 1669 1693 1697 $1699\ 1709\ 1721\ 1723\ 1733\ 1741\ 1747\ 1753\ 1759\ 1777\ 1783\ 1787\ 1789\ 1801\ 1811\ 1823$ $1831\ 1847\ 1861\ 1867\ 1871\ 1873\ 1877\ 1879\ 1889\ 1901\ 1907\ 1913\ 1931\ 1933\ 1949\ 1951$ 1973 1979 1987 1993 1997 1999 2003 2011 2017 2027 2029 2039 2053 2063 2069 2081

Primos cercanos a 10^n

 $\begin{array}{c} 9941\ 9949\ 9967\ 9973\ 10007\ 10009\ 10037\ 10039\ 10061\ 10067\ 10069\ 10079 \\ 99961\ 99971\ 99989\ 99991\ 100003\ 100003\ 1000033\ 1000037\ 1000039 \\ 9999943\ 9999971\ 99999991\ 10000019\ 10000079\ 10000103\ 10000121 \\ 99999941\ 99999959\ 9999971\ 99999989\ 100000007\ 100000037\ 100000039\ 100000049 \\ 99999893\ 99999929\ 99999937\ 1000000007\ 1000000009\ 1000000021\ 1000000033 \end{array}$

Cantidad de primos menores que 10^n

```
\pi(10^1) = 4; \pi(10^2) = 25; \pi(10^3) = 168; \pi(10^4) = 1229; \pi(10^5) = 9592

\pi(10^6) = 78.498; \pi(10^7) = 664.579; \pi(10^8) = 5.761.455; \pi(10^9) = 50.847.534

\pi(10^{10}) = 455.052,511; \pi(10^{11}) = 4.118.054.813; \pi(10^{12}) = 37.607.912.018
```

Divisores

```
Cantidad de divisores (\sigma_0) para algunos\ n/\neg \exists n' < n, \sigma_0(n') \geqslant \sigma_0(n)

\sigma_0(60) = 12; \sigma_0(120) = 16; \sigma_0(180) = 18; \sigma_0(240) = 20; \sigma_0(360) = 24

\sigma_0(720) = 30; \sigma_0(840) = 32; \sigma_0(1260) = 36; \sigma_0(1680) = 40; \sigma_0(10080) = 72

\sigma_0(15120) = 80; \sigma_0(50400) = 108; \sigma_0(83160) = 128; \sigma_0(110880) = 144

\sigma_0(498960) = 200; \sigma_0(554400) = 216; \sigma_0(1081080) = 256; \sigma_0(1441440) = 288

\sigma_0(4324320) = 384; \sigma_0(8648640) = 448

Suma de divisores (\sigma_1) para algunos\ n/\neg \exists n' < n, \sigma_1(n') \geqslant \sigma_1(n)

\sigma_1(96) = 252; \sigma_1(108) = 280; \sigma_1(120) = 360; \sigma_1(144) = 403; \sigma_1(168) = 480

\sigma_1(960) = 3048; \sigma_1(1008) = 3224; \sigma_1(1080) = 3600; \sigma_1(1200) = 3844

\sigma_1(4620) = 16128; \sigma_1(4680) = 16380; \sigma_1(5040) = 19344; \sigma_1(5760) = 19890
```

```
\begin{array}{c} \sigma_1(8820) = 31122 \ ; \ \sigma_1(9240) = 34560 \ ; \ \sigma_1(10080) = 39312 \ ; \ \sigma_1(10920) = 40320 \\ \sigma_1(32760) = 131040 \ ; \ \sigma_1(35280) = 137826 \ ; \ \sigma_1(36960) = 145152 \ ; \ \sigma_1(37800) = 148800 \\ \sigma_1(60480) = 243840 \ ; \ \sigma_1(64680) = 246240 \ ; \ \sigma_1(65520) = 270816 \ ; \ \sigma_1(70560) = 280098 \\ \sigma_1(95760) = 386880 \ ; \ \sigma_1(98280) = 403200 \ ; \ \sigma_1(100800) = 409448 \\ \sigma_1(491400) = 2083200 \ ; \ \sigma_1(498960) = 2160576 \ ; \ \sigma_1(514080) = 2177280 \\ \sigma_1(982800) = 4305280 \ ; \ \sigma_1(997920) = 4390848 \ ; \ \sigma_1(1048320) = 4464096 \\ \sigma_1(4979520) = 22189440 \ ; \ \sigma_1(4989600) = 22686048 \ ; \ \sigma_1(5045040) = 23154768 \\ \sigma_1(9896040) = 44323200 \ ; \ \sigma_1(9959040) = 44553600 \ ; \ \sigma_1(9979200) = 45732192 \end{array}
```

7. Grafos

7.1. Bellman-Ford

```
int negative_cycle(vector<vector<int> > &G, vector<vector<int> > &cost)
     //write your code here
2
     bool nc = false;
     int n = G.size();
     vector<int>dist(n,INT_MAX / 2);
     dist[0] = 0;
     for(int i = 0; i < n - 1; i++)
     for(int u = 0; u < n; u++)
       for(int j = 0; j < G[u].size(); j++){
         int v = G[u][i]:
         int w = cost[u][j];
         dist[v] = min(dist[v], dist[u] + w);
12
13
     for(int u = 0; u < n; u++){
14
     for(int j = 0; j < G[u].size();j++){</pre>
15
       int v = G[u][i];
16
       int w = cost[u][j];
17
       if(dist[v] > dist[u] + w)nc = true;
18
     }
19
     return nc;
21
```

7.2. dijkstra grafos densos

7.3. 2 SAT definitivamente no con Tarjan

```
3 | ========== <Two Sat> ==========
   Complexity: O(N)
5 | Input: number of variables, then number of clause clauses in format (u
_{6} if u,v > 0 then is equivalent to u,v
  if u,v < 0 then is equivalent to u, v
   Output: UNSATISFIABLE can't find a solution
   SATISFIABLE if exist a solution then print the assignment of all
       variables (negative for xi = false)
10
   Examples:
   Input:
   3 3
   1 -3
   -1 2
   -2 -3
   Output
   SATISFIABLE
   1 2 -3
19
   *
   Input
   1 2
   1 1
   -1 -1
   Output
   UNSATISFIABLE
27
   #include <bits/stdc++.h>
   using namespace std;
   vector<int>G[2][2000010],G2[2000010];
   int n, m;
31
   int scc[2000010];
   bool vis[2000010];
   vector<int>comp[2000010];
  int assign[2000010];
   int cc = 0;
   stack<int>st;
   vector<int>sta:
   void dfs(int u,int type){
    if(scc[u] != -1)return;
     scc[u] = cc;
    for(int v:G[type][u]){
42
       dfs(v,type);
43
```

```
44
     if(!type)st.push(u);
45
46
   void topo(int u){
47
     if(vis[u])return;
48
     vis[u] = true;
49
     for(int v:G2[u])topo(v);
     sta.push_back(u);
51
52
    void buildGraphWitouthLoop(){
53
     for(int i = 0; i < 2 * n; i++){
54
       for(int j = 0; j < G[0][i].size(); j++){
55
         if(scc[i] != scc[G[0][i][j]])
56
           G2[scc[i]].push_back(scc[G[0][i][j]]);
57
       }
58
     }
59
60
   int main() {
       ios::sync_with_stdio(false);cin.tie(0);
62
       cin >> n >> m;
63
       for(int i = 0, u, v; i < m; i++){
64
       cin >> u >> v;
65
       int uu = (u > 0?(u - 1) * 2:(-u - 1) * 2 + 1);
66
       int vv = (v > 0?(v - 1) * 2:(-v - 1) * 2 + 1);
67
     // cout << uu << " " << (uu ^ 1) << "\n";
68
       G[0][uu ^ 1].push_back(vv);
69
       G[0][vv ^ 1].push_back(uu);
70
       G[1][vv].push_back(uu ^ 1);
71
       G[1] [uu] .push_back(vv ^ 1);
72
     }
73
     memset(scc,-1,sizeof scc);
74
     for(int i = 0; i < 2 * n; i++){
75
       if(scc[i] == -1)dfs(i,0);
76
     }
77
     memset(scc,-1,sizeof scc);
78
     while(!st.empty()){
79
       int u = st.top();st.pop();
80
       if(scc[u] == -1){
81
         dfs(u,1);
82
         cc++;
83
       }
84
85
     bool unsat = false;
```

```
for(int i = 0: i < 2 * n:i++){
        if(scc[i] == scc[i ^ 1])unsat = true;
 88
        comp[scc[i]].push_back(i);
 89
 90
      if(unsat){
91
        return cout << "UNSATISFIABLE",0;</pre>
 92
 93
      cout << "SATISFIABLE\n";</pre>
 94
      buildGraphWitouthLoop();
 95
      for(int i = 0; i < 2 * n; i++){
        if(!vis[i])topo(i);
97
 98
      for(int u:sta){//inverse of topological sort
99
        for(int v:comp[u]){//transitivite Skew-Symmetry
100
          if(!assign[v]){
101
             assign[v] = 1;
102
             assign[v ^ 1] = -1;
        }
105
      }
106
      for(int i = 0, j = 1; i < 2 * n; i += 2, j++){
107
        cout << (j) * (assign[i]) << "";
108
      }
109
110
        return 0;
111 }
```

7.4. Prim

7.5. Articulataion Points (desgraciadamente tarjan)

```
int art[10010];//bool for detect art point, int for detect how many
                                                                                    3 int dfn[MAXN],low[MAXN];
       nodes are connected to my articulation point
                                                                                       vector< vector<int> > C;
                                                                                       stack< pair<int, int> > stk;
   int root,rC;
14
   int n;
                                                                                       void cache_bc(int x, int y){
15
                                                                                           vector<int> com;
   vector<pair<int,int> >bridges;
   void dfs(int u){
                                                                                           int tx,ty;
     low[u]=num[u]=cc++;
                                                                                           do{
18
     for(int v:G[u]){
                                                                                               tx = stk.top().first, ty = stk.top().second;
19
                                                                                    10
       if(num[v]==-1){
                                                                                               stk.pop();
                                                                                    11
20
         parent[v]=u;
                                                                                               com.push_back(tx), com.push_back(ty);
21
                                                                                    12
         if(u==root)rC++;
                                                                                           }while(tx!=x || ty!=y);
                                                                                    13
22
                                                                                           C.push_back(com);
         dfs(v);
23
                                                                                    14
         if(low[v]>=num[u])art[u]++;//is a articulation point
                                                                                       }
                                                                                    15
24
         if(low[v]>num[u])bridges.push_back({u,v});//this is a bridge
25
                                                                                    16
         low[u]=min(low[u],low[v]);
                                                                                       void DFS(int cur, int prev, int number){
                                                                                    17
26
       }
                                                                                           dfn[cur] = low[cur] = number;
27
       else if(v!=parent[u]){
                                                                                           for(int i = G[cur].size()-1;i>=0;--i){
28
                                                                                    19
           low[u]=min(low[u],num[v]);
                                                                                               int next = G[cur][i];
29
                                                                                    20
       }
                                                                                               if(next==prev) continue;
                                                                                   21
30
     }
                                                                                               if(dfn[next] ==-1){
31
                                                                                                    stk.push(make_pair(cur,next));
                                                                                    23
32
   void init(){
                                                                                                    DFS(next,cur,number+1);
33
                                                                                    24
     bridges.clear();
                                                                                                    low[cur] = min(low[cur], low[next]);
34
                                                                                    25
                                                                                                    if(low[next]>=dfn[cur]) cache_bc(cur,next);
     for(int i=0;i<n;i++){</pre>
35
                                                                                    26
       art[i]=low[i]=0;
                                                                                               }else low[cur] = min(low[cur],dfn[next]);
                                                                                   27
36
       num[i]=parent[i]=-1;
                                                                                           }
                                                                                    28
37
       G[i].clear();
                                                                                       }
                                                                                    29
38
     }
                                                                                    30
39
                                                                                       void biconn_comp(){
     cc=0;
                                                                                   31
40
                                                                                           memset(dfn,-1,sizeof(dfn));
                                                                                    32
41
   void callARTBRID(){
                                                                                           C.clear();
42
                                                                                    33
     for(int i=0;i<n;i++){</pre>
                                                                                           DFS(0,0,0);
43
                                                                                    34
       if(num[i]==-1){
                                                                                           int comp = C.size();
                                                                                    35
44
         root=i,rC=0;dfs(i);
                                                                                           printf("%d\n",comp);
                                                                                    36
45
                                                                                           for(int i = 0; i < comp; ++i){
         art[root]=(rC>1);
                                                                                    37
46
       }
                                                                                               sort(C[i].begin(),C[i].end());
                                                                                    38
47
     }
                                                                                               C[i].erase(unique(C[i].begin(),C[i].end()),C[i].end());
48
                                                                                    39
                                                                                               int m = C[i].size();
49
                                                                                    40
                                                                                               for(int j = 0; j < m; ++j) printf(" "du", 1 + C[i][j]);</pre>
                                                                                    41
           componentes biconexas y puentes (block cut tree)
                                                                                               printf("\n");
                                                                                    42
                                                                                           }
                                                                                    43
                                                                                    44 }
1 | int V;
vector<int> G[MAXN];
```

7.7. LCA saltitos potencias de 2

7.8. LCA sparse table query O(1)

7.9. HLD

```
1 // g++ -std=c++11 "hld.cpp" -o hld
2
   /***
3
    Complexity: O(N*log (N))
   Given a tree and asociative operation in the paths of this tree ask for
       many querys, and updates
  in nodes or edges
   Input of this example:
   N number of nodes, then N elements values in each node
   then n - 1 conections
   Q querys if T == 1 query on the path u, v
   else update node U with value val.
   Example problems: Spoj QTREE1 to QTREE6, toby and tree UVA
15
16
   #include <bits/stdc++.h>
   using namespace std;
   const int maxn = 1e5;
   const int NEUTRO = 0; // a null value for my ST
   int vec[maxn];
21
   vector<int>G[maxn]; //the graph
   //int idx[maxn]; // case with value in the edge
   int op(int u,int v){// an operation for my path (using ST)
    //return __gcd(u,v);
25
    //return max(u,v);
26
    return u + v;
27
28
   int n;
29
   //ask to Branimir for information about this
   struct SegmentTree{
31
     int T[2*maxn];
32
    void init(){
33
       memset(T,0,sizeof T);
34
    }
35
     void set(int pos,int val){
36
```

```
pos += n;
       T[pos] = val;
38
       for(pos >>= 1; pos > 0; pos >>=1){
39
         T[pos] = op(T[pos << 1], T[(pos << 1)|1]);
40
       }
41
     }
42
     int query(int 1,int r){
43
       l += n;
       r += n;
45
       int ans = NEUTRO;
       while (1 < r)
47
         if (1 \& 1) ans = op(ans, T[1++]);
         if (r \& 1) ans = op(ans, T[--r]);
49
         1 >>= 1;
         r >>= 1:
51
       }
52
53
       return ans;
     }
54
   }:
55
   struct hld{
     int ncad; // store actual number of chain
57
     int root; // the root of a tree generally 0 or 1
58
     int pos; // pos of node in chain
59
60
     int sz[maxn]; // store the subsize of subtrees
61
     int depth[maxn]; //depth of the node, useful for LCA via HLD
62
     int parent[maxn]; // useful for LCA
63
     int where[maxn]; // where chain is the node?
64
     //int edgepos[maxn]; // if the value is on the edge: stored in a node
65
     int chainIdx[maxn]; // position in the chain of the node
66
     int head[maxn]; // the head of the i-th chain
67
     //int val[maxn]; // if the value is on the edge
68
     SegmentTree tree; // this ST allow operations in the path
69
70
     void init(){//settings value, and process de HLD
71
       root = 0:
72
       ncad = 0;
73
       pos = 0:
74
       for(int i = 0; i \le n; i++){
         where [i] = head[i] = -1;
76
77
       depth[root] = 0;
78
```

dfs(root , -1);

79

```
descompose(root);
                                                                                                }
80
                                                                                      122
        tree.init();
                                                                                      123
81
        /* case with values in edges
                                                                                           }
82
                                                                                      124
        for(int i=0;i<n;i++){</pre>
                                                                                           ///end descomposition
83
                                                                                      125
          tree.set(i,val[i]);
84
                                                                                      126
        }
                                                                                            int lca(int u,int v){
85
                                                                                      127
        */
                                                                                              while(where[u]!=where[v]){
86
                                                                                      128
      }
                                                                                                if(depth[ head[ where[u] ] ] > depth[ head[ where[v] ] ])u =
87
                                                                                      129
                                                                                                    parent[ head[ where[u] ] ];
88
                                                                                                else v = parent[ head[ where[v] ] ];
89
                                                                                      130
      ///init descomposition
90
                                                                                      131
      void dfs(int u,int pu){
                                                                                              return depth[u] < depth[v] ? u:v;</pre>
91
                                                                                      132
        sz[u] = 1; //init the sz of this subtree
                                                                                           }
92
                                                                                      133
        parent[u] = pu; // assign the parent
                                                                                      134
93
        for(int i = 0; i < G[u].size(); i++){</pre>
                                                                                            void update(int u, int val){
                                                                                      135
94
          int v = G[u][i];
                                                                                              tree.set(chainIdx[u],val);
95
                                                                                      136
          if ( v == pu )continue;
                                                                                           }
96
                                                                                      137
          //edgepos[idx[u][i]] = v;
97
                                                                                      138
          depth[v] = depth[u] + 1;
                                                                                            int query(int u,int v){
                                                                                      139
98
                                                                                              // if ( u == v) return NEUTRO; value in edges
          dfs(v,u);
                                                                                      140
99
          sz[u] += sz[v];
                                                                                              int vChain = where[v];
                                                                                      141
100
                                                                                              int ans = NEUTRO;
101
                                                                                      142
      }
                                                                                              while(true){
                                                                                      143
102
                                                                                                int uChain = where[u];
      //descompose graph in HLD descomposition
                                                                                      144
103
      void descompose(int u){
                                                                                                if(uChain == vChain){
                                                                                      145
104
        if( head[ncad] == -1)head[ncad] = u; // the head of ncad is u
                                                                                                  // return op(ans, tree.query( chainIdx[v] + 1, chainIdx[u] + 1)
                                                                                      146
105
        where[u] = ncad; // assign where tu node
                                                                                                       ); value in edges
106
        //val[pos] = cost; cost another parameter in descompose for graphs
                                                                                                  return op(ans, tree.query( chainIdx[v], chainIdx[u] + 1) );
                                                                                      147
107
            with values in edges
                                                                                      148
        chainIdx[u] = pos++; //assing pos to this node
                                                                                      149
                                                                                                int hu = head[uChain];
108
        int maxi = -1, sc = -1; //finding a special child
                                                                                                ans = op( ans, tree.query(chainIdx[hu], chainIdx[u] + 1) );
                                                                                      150
109
        for(int v:G[u]){
                                                                                                u = parent[hu];
                                                                                      151
110
          if (sz[v] > maxi && where[v] == -1){
                                                                                              }
                                                                                      152
111
            \max i = sz[v]:
                                                                                           }
                                                                                      153
112
            sc = v;
                                                                                     154
113
          }
                                                                                            int Q(int u,int v){
                                                                                      155
114
        }
                                                                                              int L = lca(u,v);
                                                                                      156
115
        if(sc != -1)descompose(sc);
                                                                                              return op( query(u,L) , query(v,L) );
                                                                                      157
116
        //light nodes here:
                                                                                           }
                                                                                      158
117
        for(int v:G[u]){
                                                                                          }HLD;
                                                                                      159
118
          if(where[v] == -1){
                                                                                          int main(){
                                                                                      160
119
                                                                                           //ios::sync_with_stdio(false);cin.tie(0);
            ncad++;
120
                                                                                           while(cin >> n){
            descompose(v);
121
                                                                                     162
```

```
for(int i = 0; i < n; i++)G[i].clear();
163
        for(int i = 0; i < n; i++){
164
          cin >> vec[i];
165
166
        for(int i = 1, u,v ; i < n; i++){
167
          cin >> u >> v;
168
          G[u].push_back(v);
169
          G[v].push_back(u);
170
          /* case with value in edges
171
           G[u].push_back(make_pair(v,w));
172
          idx[u].push_back(i-1);
173
          G[v].push_back(make_pair(u,w));
174
          idx[v].push_back(i-1);
175
176
177
        }
178
        HLD.init();
179
        for(int i = 0; i < n; i++){
180
          HLD.update(i, vec[i]);
181
        }
182
        int question;
183
        cin >> question;
184
        for(int i = 0, t, u ,v; i < question; i++){
185
          cin >> t >> u >> v;
186
          if(t == 1){
187
            cout << HLD.Q(u,v) << "\n";
188
          }
189
          else HLD.update(u,v);
190
191
     }
192
193
```

7.10. centroid descomposition

7.11. euler cycle

```
int n,m,ars[MAXE], eq;
vector<int> G[MAXN];//fill G,n,m,ars,eq

list<int> path;
int used[MAXN];

bool usede[MAXE];
queue<list<int>::iterator> q;
int get(int v){
    while(used[v]<sz(G[v]) && usede[G[v][used[v]]]) used[v]++;</pre>
```

```
return used[v];
9
10
   void explore(int v, int r, list<int>::iterator it){
11
     int ar=G[v][get(v)]; int u=v^ars[ar];
     usede[ar]=true;
13
     list<int>::iterator it2=path.insert(it, u);
14
     if(u!=r) explore(u, r, it2);
15
     if(get(v)<sz(G[v])) q.push(it);</pre>
16
17
   void euler(){
     zero(used), zero(usede);
19
     path.clear();
20
     q=queue<list<int>::iterator>();
21
     path.push_back(0); q.push(path.begin());
     while(sz(q)){
23
       list<int>::iterator it=q.front(); q.pop();
24
       if(used[*it] < sz(G[*it])) explore(*it, *it, it);</pre>
25
26
     reverse(path.begin(), path.end());
27
28
   void addEdge(int u, int v){
     G[u].pb(eq), G[v].pb(eq);
     ars[eq++]=u^v;
31
32 }
```

7.12. diámetro y centro de un árbol

```
====== <Diameter and center of a tree> =========
   //Problem: Given a tree get the center (or centers)
  /* the nodes in the tree that minimize the length of the longest path
       from it to any other node.
* *Finding tree centers:
   * If diameter length is even, then we have one tree center. If odd,
        then we have 2 centers.
   * E.g. 1-2-3-4-5 -> center is 3
    * E.g. 1-2-3-4-5-6 \rightarrow center is 3, 4
    * On other side, we can get the worst nodes through the center nodes.
    * A worst node is one that is an end of a diameter, so it has the worst
        tree height
11 Input:
  * No
12
13 Output:
```

```
14
   dfs: calculate the diameter of the tree
   * maxi stores the diameter
   findingCenters() return the centers
   Nodes in graph 1 to N careful with this
   Complexity: O(N)
20
21
^{22}
   vector<int>G[5010];
   int maxi=-1,far;
   int n;
   int pre[5010];
   int Queue[5010];
28
   void dfs(int path,int u,int parent){
     pre[u]=parent;
30
     if(path>=maxi){
31
       maxi=path;
32
       far=u;
33
     }
34
     for(int v:G[u]){
35
       if(parent!=v){
36
         dfs(path+1,v,u);//path + w if the graph as weighted
37
38
39
40
   pair<int,int> findingCenters(){
41
     maxi=-1;
42
     dfs(0,1,-1);
43
     dfs(0,far,-1);
44
     int t=far,L=0;
45
     while(t!=-1){
46
       Queue[L]=t:
47
       t=pre[t];
48
       ++L;
49
     }
50
     int a=-1.b=-1:
51
     if(L&1){
52
       a=Queue[L/2];
53
     }
54
     else{
55
       a=min(Queue[L/2-1],Queue[L/2]),b=max(Queue[L/2-1],Queue[L/2]);
56
```

```
57 | }
58 | return {a,b};
59 |
```

7.13. algoritmo hungaro

7.14. union find dinámico

```
#include <bits/stdc++.h>
   using namespace std;
   | #define dprint(v) cerr << #v"=" << v << endl //;)
   #define forr(i,a,b) for(int i=(a); i<(b); i++)</pre>
   #define forn(i,n) forr(i,0,n)
   #define dforn(i,n) for(int i=n-1; i>=0; i--)
   #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
   #define sz(c) ((int)c.size())
   #define zero(v) memset(v, 0, sizeof(v))
   #define pb push_back
   #define fst first
   #define snd second
   #define mkp make_pair
   typedef long long 11;
   typedef pair<int,int> ii;
16
   struct UnionFind {
       int n, comp;
18
       vector<int> pre,si,c;
19
       UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
20
           forn(i,n) pre[i] = i; }
21
       int find(int u){return u==pre[u]?u:find(pre[u]);}
22
       bool merge(int u, int v) {
23
           if((u=find(u))==(v=find(v))) return false;
24
           if(si[u]<si[v]) swap(u, v);</pre>
25
           si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
26
           return true;
27
       }
28
       int snap(){return sz(c);}
29
       void rollback(int snap){
30
           while(sz(c)>snap){
31
               int v = c.back(); c.pop_back();
32
               si[pre[v]] -= si[v], pre[v] = v, comp++;
33
34
35
36 };
```

```
enum {ADD,DEL,QUERY};
   struct Query {int type,u,v;};
   struct DynCon {
39
       vector<Query> q;
40
       UnionFind dsu;
41
       vector<int> match,res;
42
       map<ii,int> last;//se puede no usar cuando hay identificador para
43
           cada arista (mejora poco)
       DynCon(int n=0):dsu(n){}
44
       void add(int u, int v) {
45
           if(u>v) swap(u,v);
46
           q.pb((Query){ADD, u, v}), match.pb(-1);
47
           last[ii(u,v)] = sz(q)-1;
48
       }
49
       void remove(int u, int v) {
50
           if(u>v) swap(u,v);
51
           q.pb((Query){DEL, u, v});
52
           int prev = last[ii(u,v)];
53
           match[prev] = sz(q)-1;
54
           match.pb(prev);
55
       }
56
       void query() {//podria pasarle un puntero donde guardar la respuesta
57
           q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
58
       void process() {
59
           forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
60
                sz(q);
           go(0,sz(q));
61
       }
62
       void go(int 1, int r) {
63
           if(l+1==r){
64
               if (q[1].type == QUERY)//Aqui responder la query usando el
65
                    res.pb(dsu.comp);//aqui query=cantidad de componentes
66
                        conexas
               return;
67
68
           int s=dsu.snap(), m = (l+r) / 2;
69
           forr(i,m,r) if(match[i]!=-1 && match[i]<1) dsu.merge(q[i].u, q[i</pre>
70
               ].v);
           go(1,m);
71
           dsu.rollback(s);
72
           s = dsu.snap();
73
           forr(i,1,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[
74
```

```
i].v);
            go(m,r);
75
            dsu.rollback(s);
76
77
   }dc;
78
79
   // Problema ejemplo: http://codeforces.com/gym/100551/problem/A
81
   int n,k;
82
   int main() {
84
       //~ freopen("in", "r", stdin);
85
       freopen("connect.in", "r", stdin);
86
       freopen("connect.out", "w", stdout);
87
       ios::sync_with_stdio(0);
       while(cin >> n >> k){
       dc=DynCon(n);
       forn(_,k) { string ord; cin >> ord;
         if (ord=="?") {
92
            dc.query();
         } else if (ord=="+") { int a,b; cin>>a>>b; a--;b--;
94
            dc.add(a,b);
95
         } else if (ord=="-") { int a,b; cin>>a>>b; a--;b--;
96
            dc.remove(a,b);
97
         } else assert(false);
98
       }
99
            if(!k) continue;//k==0 WTF
100
            dc.process();
101
            forn(i,sz(dc.res)) cout << dc.res[i] << endl;</pre>
102
103
       }
       return 0;
104
105 }
```

7.15. truquitos estúpidos por ejemplo second MST es con LCA

7.16. erdos galloi

```
// g++ -std=c++11 "erdosgalloi.cpp" -o run
/***
Given the grades of each node of a graph return if this form a valid graph
```

```
5 includes: algorithm, functional, numeric, forn
   // Receives a sorted degree sequence (non ascending)
   O(NlgN)
8
   bool isGraphicSequence(const vector<int> &seq) // O(n lg n)
11
     vector<int> sum;
^{12}
     int n = seq.size();
13
14
     if (n == 1 \&\& seq[0] != 0) return false;
15
16
     sum.reserve(n + 1);
17
     sum.push_back(0);
18
     for (int i = 0; i < n; ++i) sum.push_back(sum[i] + seq[i]);
19
     if ((sum[n] & 1) == 1) return false;
20
21
     for (long long k = 1; k \le n - 1 \&\& seq[k - 1] >= k; ++k) {
22
       int j = distance(seq.begin(), upper_bound(seq.begin() + k, seq.end()
23
                                                    greater<int>())) +
24
                1;
25
       long long left = sum[k];
26
       long long right = k * (k - 1) + (j - k - 1) * k + (sum[n] - sum[j - k])
27
           1]);
28
       if (left > right) return false;
29
30
31
     return true;
32
33 | }
```

7.17. grafo funcional hallar k-esimo partiendo de un nodo $7.18. \ \ \, \text{konig}$

- 7.19. min-vertex cover bipartitos
- 7.20. max-flow (min cost versión)

```
// g++ -std=c++11 "maxflowmincost.cpp" -o run
/***
Given a grapth with edges with a capacity C and weight D
```

```
* compute the max-flow min cost
  Edmond karps idea
  * Complexity O(v *E*log(v))
  Problem for practice: Dijkstra Dijkstra uva
   */
   #define REP(i,j,k) for(int (i)=(j);(i)<(k);++(i))
   #define MP make_pair
   using namespace std;
   #define MAXN 500
   #define MAXM MAXN * 5
   typedef vector<int> VI;
   typedef long long 11;
   const int INF = 1E9; // $infinity$: be careful to make this big enough
20 | int S; // source
  int T; // sink
  int FN: // number of nodes
   int FM; // number of edges (initialize this to 0)
24 // ra[a]: edges connected to a (NO MATTER WHICH WAY!!!); clear this in
       the beginning
  VI ra[MAXN];
   int kend[MAXM], cap[MAXM], cost[MAXM]; // size: TWICE the number of
27
   // Adds an edge from a to b with capacity c and cost d and returns the
       number of the new edge
   int addedge(int a, int b, int c, int d) {
     int i = 2*FM:
31
     kend[i] = b;
     cap[i] = c;
     cost[i] = d:
34
     ra[a].push_back(i);
35
     kend[i+1] = a;
     cap[i+1] = 0;
37
     cost[i+1] = -d:
    ra[b].push_back(i+1);
39
     FM++;
     return i;
41
42
43 | int n;
```

```
int dst[MAXM], pre[MAXM], pret[MAXM];
  //finding the shortest path via fanding duan, also it works with bellman
   //or dijkstra (careful of negative cycles)
   bool spfa(){
     REP(i,0,FN) dst[i] = INF;
48
     dst[S] = 0;
     queue<int> que; que.push(S);
50
     while(!que.empty()){
51
       int x = que.front(); que.pop();
52
       for (int t : ra[x]){
53
         int y = kend[t], nw = dst[x] + cost[t];
54
         if(cap[t] > 0 && nw<dst[y]){
55
           dst[y] = nw; pre[y] = x; pret[y] = t; que.push(y);
56
         }
57
       }
58
59
     return dst[T]!=INF;
60
61
    // returns the maximum flow and the minimum cost for this flow
   pair<11,11> solve(){
63
     11 \text{ totw} = 0, \text{ totf} = 0;
64
     while(spfa()){
65
       int minflow = INF;
66
       for (int x = T; x!=S; x = pre[x]){
67
         minflow = min(minflow, cap[pret[x]]);
68
       }
69
       for (int x = T; x!=S; x = pre[x]){
70
         cap[pret[x]] -= minflow;
71
         cap[pret[x]^1] += minflow;
72
       }
73
       totf += minflow;
74
       totw += minflow*dst[T];
75
76
     return make_pair(totf, totw);
77
78
   void init(){
79
     FN=4*n+15;//make this big n=number of nodes of the graph
80
     FM=0;
81
     S=0,T=n+1;
     for(int i=0;i<FN;i++)ra[i].clear();//clear the graph be careful</pre>
83
84 | }
```

7.21. max-flow corto con matriz

```
1 // g++ "maxflowMVEK.cpp" -o run
   /***
2
   ======== <Max Flow with matriz Edmonds karp c++ version>
       _____
   //Given a graph with capacitys find the max-flow
   Nodes indexed 1 to N
   * Complexity O(N *E)
  Problem for practice: UVA 820
   */
9
   #define N 500
   int cap[N][N], pre[N], n;
   int s;//source
   int t;//destination
   bool bfs() {
       queue<int>q;
15
       q.push(s);
       memset(pre,-1,sizeof pre);
       pre[s]=s;
18
       while(!q.empty()){
19
           int u=q.front();q.pop();
20
           if(u==t)return true:
21
           for(int i=1:i \le n:i++){//nodes 1 to n
               if(pre[i] == -1&&cap[u][i])pre[i] = u,q.push(i);
23
           }
24
25
       return false;
26
   }
27
28
   int maxFlow() {
29
       int mf=0,f,v;//max flow, flow for a path, the vertex
30
       while(bfs()){//while encountered a path source to destination
31
           v=t;//min
32
           f=INT_MAX://make this big enough
33
           while(pre[v]!=v){f=min(f,cap[pre[v]][v]),v=pre[v];}//finding the
34
                min capacity
           v=t:mf+=f:
35
           while(pre[v]!=v){cap[pre[v]][v]-=f,cap[v][pre[v]]+=f,v=pre[v];}
36
               //update the flow
       }
37
       return mf;
38
```

34

```
39
   void init(){
40
     memset(cap,0,sizeof cap);
41
    //cap[u][v]+=capacidad,cap[v][u]+=capacidad
  |}
43
                     7.22. max-flow sin matriz
1 // g++ -std=c++11 "maxflowNMEK.cpp" -o run
```

```
/***
2
      ========== <Max Flow with-out matriz Edmonds karp c++ version>
   //Given a graph with capacitys find the max-flow
   Nodes indexed 1 to N
   * Complexity O(N *E)
   Problem for practice: UVA 820
   * Input N number of nodes,
   * M edges conections
   * compute the flow with source 1 and sink N
12
   using namespace std;
   const int N = 110;
   const int M = 10010 * 2:
   vector<int>G[N]:
   int kend[M], cap[M], cost[M];
   int edge = 0;
   int s,t;
19
   void add(int u,int v,int c){
     int forward = edge * 2, backward = edge * 2 + 1;
21
     kend[forward] = v;
22
     cap[forward] = c;
23
     G[u].push_back(forward);
24
     kend[backward] = u;
25
     cap[backward] = 0;
26
     G[v].push_back(backward);
27
     edge++;
28
29
   int vis[M],pre[M],pret[M];
   bool bfs(){
31
     for(int i = 0; i <= 100;i++)vis[i] = false;</pre>
32
     vis[s] = true;
33
     queue<int>q;
```

```
q.push(s);
 35
                                                   while(!q.empty()){
36
                                                                     int u = q.front();q.pop();
37
                                                                     for(int edge:G[u]){
 38
                                                                                        int v = kend[edge];
 39
                                                                                      if(cap[edge] > 0 && !vis[v]){
                                                                                                           vis[v] = true;
                                                                                                           pre[v] = u;
                                                                                                             pret[v] = edge;//the edge store the information
  43
                                                                                                             q.push(v);
  45
  46
  47
                                                  return vis[t];
  48
  49
                                int max_flow(){
                                                   int totf = OLL;
                                                  while(bfs()){
                                                                     int minflow = INT MAX:
 53
                                                                     for(int x = t; x != s; x = pre[x]){
                                                                                      minflow = min(minflow,cap[pret[x]]);
 55
  56
                                                                     for(int x = t; x != s; x = pre[x]){
57
                                                                                        cap[pret[x]] -= minflow;
                                                                                         cap[pret[x] ^ 1] += minflow;
 59
 60
                                                                      totf += minflow;
61
 62
                                                  return totf;
 63
  64
                                int main(){
 65
                                                int n,m;
                                                scanf(" \frac{1}{2} \frac{1}
                                               for(int i = 0,u,v,ca; i < m;i++){
                                                                   scanf(" \frac{1}{2} \frac{1}
                                                                     add(u,v,ca);
 70
                                          }
71
                                               s = 1. t = n:
                                             printf(" %ld\n", max_flow());
 74 | }
```

7.23. Dinic

```
for(int &i=work[u]; i<sz(G[u]); i++){</pre>
1
                                                                                 35
  const int MAX = 300;
                                                                                            Edge &e = G[u][i];
                                                                                 36
  // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v
                                                                                            if(e.cap<=e.f) continue;</pre>
                                                                                 37
       ]==-1 (del lado del dst)
                                                                                            int v=e.to;
                                                                                 38
4 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los
                                                                                            if(dist[v]==dist[u]+1){
                                                                                 39
       conjuntos mas proximos a src y dst respectivamente):
                                                                                                     11 df=dinic_dfs(v, min(f, e.cap-e.f));
                                                                                 40
5 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices
                                                                                                     if(df>0){
                                                                                 41
       de V2 con it->f>0, es arista del Matching
                                                                                                             e.f+=df, G[v][e.rev].f-= df;
                                                                                 42
6 // Min Vertex Cover: vertices de V1 con dist[v] ==-1 + vertices de V2 con
                                                                                                             return df; }
                                                                                 43
        dist[v]>0
                                                                                 44
7 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex
                                                                                 45
                                                                                        return 0;
                                                                                 46
                                                                                    }
8 // Max Clique: construir la red de G complemento (debe ser bipartito!) y
                                                                                 47
        encontrar un Max Independet Set
                                                                                    ll maxFlow(int _src, int _dst){
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices
                                                                                        src=_src, dst=_dst;
                                                                                 49
       no cubierto hasta el momento, tomar cualquier arista de el
                                                                                        11 result=0;
                                                                                 50
  int nodes, src, dst;
                                                                                        while(dinic_bfs()){
                                                                                 51
   int dist[MAX], q[MAX], work[MAX];
                                                                                            fill(work, work+nodes, 0);
  struct Edge {
                                                                                            while(ll delta=dinic_dfs(src,INF))
                                                                                 53
       int to, rev;
                                                                                                result+=delta;
                                                                                 54
13
                                                                                        }
       ll f, cap;
                                                                                 55
14
       Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(
                                                                                        // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
15
                                                                                 56
           cap) {}
                                                                                            forman el min-cut
                                                                                        return result; }
                                                                                 57
16
   vector<Edge> G[MAX];
17
                                                                                               7.24. máximo emparejamiento bipartito
   void addEdge(int s, int t, ll cap){
       G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
19
           0));}
                                                                                 1 // g++ -std=c "bipartitematching.cpp" -o run
   bool dinic_bfs(){
                                                                                  2
20
       fill(dist, dist+nodes, -1), dist[src]=0;
21
                                                                                    ====== <MCBM max cardinality bipartite matching c++ version>
       int qt=0; q[qt++]=src;
22
       for(int qh=0; qh<qt; qh++){</pre>
                                                                                    Return the bipartite matching of a Graph
23
           int u =q[qh];
                                                                                    * Format of nodes: 1 to N
24
           forall(e, G[u]){
                                                                                    */
25
                                                                                  6
               int v=e->to;
26
               if(dist[v]<0 && e->f < e->cap)
                                                                                    const int N = 100010;
27
                   dist[v]=dist[u]+1, q[qt++]=v;
28
                                                                                    vector<int>G[N];
           }
                                                                                    bool v[N];//for the greedy speed up
29
       }
                                                                                 int match[N];
30
       return dist[dst]>=0;
                                                                                 12 bool vis[N];
31
32
                                                                                 13 int n,m;
   ll dinic_dfs(int u, ll f){
                                                                                 14 //calling aumenting path
33
       if(u==dst) return f;
34
                                                                                 15 bool aug(int u){
```

```
59 }
       if(vis[u])return false;
16
       vis[u]=true;
17
                                                                                              7.25. max-independent set en bipartitos
      for(int i=0;i<(int)G[u].size();++i){</pre>
18
       int r=G[u][i];
19
                                                                                         7.26. min-path cover (ver tópicos raros de halim)
          if(match[r]==-1||aug(match[r])){
20
               match[r]=u;match[u]=r;return true;
21
                                                                                                      7.27. min-cost arborescence
           }
22
       }
                                                                                     7.28. lema de diapositivas de nico de grafos funcionales
23
       return 0;
^{24}
                                                                                         7.29. minimax y maximini con kruskal y dijkstra
25
26
   //findging all augmenting path's
                                                                                 1 // g++ -std=c++11 "maximini.cpp" -o run
   int solve(){
      bool check=true:
                                                                                     ------ <maximini c++ version> ===========
29
      while(check){
                                                                                   Given a weighted graph return the maximini (the maximum of the minimum)
30
           check=false:
                                                                                    or the minimax (the minimum of the maximum) in the path a,b
31
           memset(vis,0,sizeof vis);
32
                                                                                 6
           for(int i=1;i<=n;++i){
                                                                                   Minimax as definded as: finding the minimum of maximum edge weight among
33
         if(!v[i]&&match[i]==-1){
                                                                                         all posible paths
34
           bool op=aug(i);
                                                                                 * between two verrtices a to b, the cost for a path from a to b is
35
           check | = op;
                                                                                        determined by maximum edge
36
           mc+=op;
                                                                                 9 * weight along this path. Among all these possible paths from a to b,
37
                                                                                        pick the one with the minimum
38
       }
                                                                                    * ax-edge-weight
39
                                                                                    * Complexity O(E*log(E) + V + E)
40
       return mc;
41
                                                                                    Problem for practice: UVA 534,544
42
   void init(){
                                                                                    */
                                                                                 14
     memset(v,0,sizeof v);
                                                                                    int n;
44
     memset(vis,false,sizeof vis);
                                                                                    pair<int,pair<int,int> >Edges[20000];
45
     mc=0:
                                                                                    int t;
46
                                                                                 17
     memset(match,-1,sizeof match);
47
                                                                                    map<string,int>mp;
       for(int i=0;i<=n;i++)G[i].clear();</pre>
                                                                                    int parent[210];
48
                                                                                    pair<int,int>child[210];
49
   void greedySpeedUp(){
                                                                                    bool vis[210];
50
     //greedy optimization, match with the first not matched
                                                                                    vector<pair<int,int> >G[210];
51
     for(int i=1;i<=n;++i){</pre>
52
                                                                                23
            for(int j=0;j<(int)G[i].size();++j){</pre>
                                                                                    int find(int u){return u==parent[u]?u:parent[u]=find(parent[u]);}
53
                if(match[G[i][j]]==-1){
                                                                                    void Union(int u.int v){
54
            match[G[i][j]]=i,match[i]=G[i][j],mc++,v[i]=true;break;
                                                                                     int pu=find(u),pv=find(v);
55
                                                                                26
         }
                                                                                      if(pu!=pv){
56
                                                                                27
57
                                                                                        parent[pv]=pu;
                                                                                28
58
                                                                                29
```

```
30 }
   int mst(int a,int b){
31
     sort(Edges,Edges+t);
32
     reverse(Edges,Edges+t);//don't reverse for the minimax
33
     for(int i=0;i<=200;i++)parent[i] = i;</pre>
34
     int w,u,v, maximini = 1e8, minimax = 0;
35
     for(int i=0;i<t;i++){</pre>
36
       tie(w,u,v) = make_tuple(Edges[i].first, Edges[i].second.first, Edges
37
            [i].second.second);
       if(find(u) != find(v)){
38
         Union(u,v);
39
         G[u].push_back(\{v,w\});
40
         G[v].push_back(\{u,w\});
41
       }
42
     }
43
     queue<int>q;
44
     q.push(a);
45
     vis[a]=true;
46
     while(!q.empty()){
47
       int u = q.front();q.pop();
48
       //if(u==1)break;
49
       for(pair<int,double>node: G[u]){
50
         if(!vis[node.first]){
51
            vis[node.first] = true;
52
           q.push(node.first);
53
            //maximini=max(maximini,node.second);
54
            child[node.first].first = u;
55
            child[node.first].second = node.second;
56
         }
57
       }
58
59
     for(int t = b;t != -1;t = child[t].first){
60
       //cout<<t<" "<<child[t].second<<"\n":</pre>
61
       //minimax=max(minimax.child[t].second):
62
       maximini = min(maximini,child[t].second);
63
64
     return maximini;
65
66
```

8. Teoria de juegos

8.1. Teorema fundamental de los juegos optimos

```
boolean isWinning(position pos) {
    moves[] = possible positions to which I can move from the position
    pos;
    for (all x in moves)
        if (!isWinning(x)) return true;

    return false;
}

8.2. Como calcular grundy

int grundyNumber(position pos) {
```

```
int grundyNumber(position pos) {
   moves[] = possible positions to which I can move from pos
   set s;
   for (all x in moves) insert into s grundyNumber(x);
   //return the smallest non-negative integer not in the set s;
   int ret=0;
   while (s.contains(ret)) ret++;
   return ret;
}
```

9. Algunas formulas de probabilidad

Importante

Verificar la independencia de eventos.

9.1. Regla general de la probabilidad

$$P(E) = \frac{Nro.casos favorables}{Nro.casos posibles}$$

9.2. Teorema de bayes (Probabilidad condicional)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

9.3. Regla de la suma

Se suman 2 probabilidades cuando los eventos no pueden ocurrir al mismo tiempo "Pasa el evento X o pasa el evento Y"

$$P(A) \cup P(B) = P(A) + P(B)$$

Regla de la suma con eventos compatibles e incompatibles (que pueden pasar al mismo tiempo o no)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

9.4. Regla de la multiplicacion

La misma idea de arriba, "Pasa el evento X y pasa el evento Y"

$$P(A \cap B) = P(A) * P(B)$$

9.5. Esperanza matematica

$$E[X] = \sum_{i=0}^{n} X * P(X)$$

En el caso continuo reemplazar sumatoria por integral

9.6. Ley de la esperanza total

Esto se usa cuando quieres calcular el valor en el que pasa el evento X ej. calcular el dia esperado en el que muere el i-esimo pez

$$E(X) = E(E(X|Y))$$

9.7. Esperanza de variables independientes

$$E(X * Y) = E(X) * E(Y)$$

9.8. Varianza

$$\sigma^2 = E(X^2) - (E(X))^2$$

9.9. Distribucion Binomial

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Ejemplo. Supongamos que se lanza un dado (con 6 caras) 51 veces y queremos conocer la probabilidad de que el número 3 salga 20 veces.

$$f(X = 20) = {51 \choose 20} \frac{1}{6}^{20} * (1 - \frac{1}{6})^{51 - 20}$$

Esperanza:

$$E(x) = n * p$$

Varianza:

$$V(x) = np * (1 - p)$$

10. Otros/utilitarios

10.1. josephus

```
int survivor(int n, int m){
int s = 0;
for (int i=1;i<=n;++i) s = (s+m) %i;
return (s+1);
}</pre>
```

10.2. josephus k = 2

```
public int getSafePosition(int n) {
      // find value of L for the equation
9
      int valueOfL = n - Integer.highestOneBit(n);
10
      int safePosition = 2 * valueOfL + 1;
11
      return safePosition;
12
13
                             10.3. poker
                     10.4. iterar subconjuntos
```

```
for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
```

10.5. como reconstruir una DP (normal)

```
1 /*
  You just need to revisit your steps in the DP. In case of 0-1 knapsack,
       lets say the original DP function was solve, and the function
       reconstruct will give you the actual solution (I'm writing the code
       in C++):
3
   int solve(int pos, int capacity){
       if(pos == no_of_objects) return 0;
5
       if(memo[pos][capacity] != -1) return memo[pos][capacity];
6
       int r1 = solve(pos + 1, capacity); //dont take
7
       int r2 = 0:
8
       if(weight[pos] <= capacity){</pre>
9
           r2 = solve(pos + 1, capacity - weight[pos]) + profit[pos]; //
10
               take
11
       return memo[pos][capacity] = max(r1, r2);
12
13
   void reconstruct(int pos, int capacity){
14
       if(pos == no_of_objects) return; //you have completed reconstruction
15
       int r1 = memo[pos + 1][capacity]; //dont take
16
       int r2 = 0;
17
       if(weight[pos] <= capacity)r2 = memo[pos + 1][capacity - weight[pos</pre>
18
           ]] + profit[pos]; //take
       if(r1 > r2) {reconstruct(pos + 1, capacity);}
19
       else{
20
           cout << "Take object " << pos << endl;</pre>
21
           reconstruct(pos + 1, capacity - weight[pos]) + profit[pos];
22
       }
23
24 | }
```

```
25 After executing reconstruct, it will print all those objects that give
       you the optimal solution. As you can see, at most no_of_objects
       calls will be made in the reconstruct function.
```

26 Similarly, you can reconstruct the solution of any DP greedily.

10.6. muajaja con j

```
1 | #include <signal.h>
void divzero(int p){
    while(true);}
  void segm(int p){
    exit(0):}
  //in main
  signal(SIGFPE, divzero);
8 signal(SIGSEGV, segm);
```

Expandir pila

```
#include <sys/resource.h>
2 rlimit rl;
  getrlimit(RLIMIT_STACK, &rl);
4 | rl.rlim_cur=1024L*1024L*256L;//256mb
5 setrlimit(RLIMIT_STACK, &rl);
```

10.7. comparar doubles for noobs

```
const double EPS = 1e-9;
_2 | x == v <=> fabs(x-v) < EPS
_3 | x > y <=> x > y + EPS
_4 | x >= y <=> x > y - EPS
```

10.8. infix to postfix

```
1 //infix to postfix with shunting yard, Halim interpretation
2 //plus eval function given a postfix return the result of the operation
3 //format: string like (x o x (x o x)) o=operation x=value
   string s;
4
   bool isOperator(string u){
     return u=="+"||u=="-"||u=="*"||u=="/":
6
7
   bool precede(string u){
    if(u=="*"||u=="/")return true;
     return false:
10
11 }
```

```
void solve(){
                                                                                           else{
                                                                                    54
                                                                                             double a=stans.top();stans.pop();
     getline(cin,s);
                                                                                   55
13
     stack<string>st;
                                                                                             double b=stans.top();stans.pop();
14
                                                                                    56
     vector<string>v;
                                                                                             if(eva=="*")b*=a;
15
                                                                                             if(eva=="/")b/=a;
     stringstream ss;
16
                                                                                    58
                                                                                             if(eva=="+")b+=a;
     ss<<s;
17
                                                                                    59
                                                                                             if(eva=="-")b-=a;
     while(ss>>s){
18
                                                                                   60
       if(isOperator(s)){
                                                                                             stans.push(b);
19
                                                                                   61
         while(!st.empty()&&isOperator(st.top())&&precede(st.top())>=
                                                                                           }
20
                                                                                   62
             precede(s)){
                                                                                         }
                                                                                    63
           v.push_back(st.top());st.pop();
                                                                                         cout<<fixed<<stans.top()<<"\n";</pre>
21
                                                                                    64
                                                                                    65 }
22
         st.push(s);
23
                                                                                                            10.9. numeros romanos
       }
24
       else{
25
          if(s=="("){
                                                                                    | #include <bits/stdc++.h>
26
            st.push(s);
27
                                                                                       using namespace std;
         }
28
                                                                                       map<int,string>cvt;
         else{
29
           if(s==")"){
                                                                                       string aromano(int n){
30
             while(!st.empty()&&st.top()!="("){
                                                                                         cvt[1000] = "M";cvt[900] = "CM",cvt[500] = "D", cvt[400] = "CD";
31
               v.push_back(st.top());st.pop();
                                                                                         cvt[100] = "C";cvt[90] = "XC"; cvt[50] = "L";
32
                                                                                         cvt[40] = "XL";cvt[10] = "X";cvt[9] = "IX";cvt[5] = "V"; cvt[4] = "IV"
33
             if(!st.empty()&&st.top()=="(")st.pop();
34
                                                                                         cvt[1] = "I":
35
           else {
                                                                                         string ans = "";
36
                                                                                   10
             v.push_back(s);
                                                                                         for(map<int,string>::reverse_iterator it = cvt.rbegin();it != cvt.rend
37
                                                                                   11
                                                                                             ();it++)
38
         }
                                                                                           while(n >= it->first){
39
                                                                                   12
       }
40
                                                                                             ans += it->second;
                                                                                   13
                                                                                             n -= it->first;
41
                                                                                   14
     while(!st.empty()){
                                                                                           }
42
                                                                                   15
       v.push_back(st.top());st.pop();
43
                                                                                         return ans;
                                                                                    16
     }
44
                                                                                   17
     stack<double>stans:
                                                                                       map<string,int>crn;
45
     double x;
                                                                                       int anumero(string R){
46
                                                                                   19
     for(string eva:v){
47
                                                                                         map<char, int> crn;
                                                                                   20
       if(!isOperator(eva)){
                                                                                         crn['I'] = 1; crn['V'] = 5; crn['X'] = 10; crn['L'] = 50;
48
                                                                                   21
         stringstream nu;
                                                                                         crn['C'] = 100; crn['D'] = 500; crn['M'] = 1000;
49
         nu<<eva;
                                                                                         int value = 0;
50
         nu>>x;
                                                                                        for (int i = 0; R[i]; i++)
51
                                                                                   24
         stans.push(x);
                                                                                           if (i + 1 < R.size() && crn[R[i]] < crn[R[i+1]]) {</pre>
52
                                                                                   25
53
                                                                                             value += crn[R[i+1]] - crn[R[i]];
                                                                                   26
```

```
i++;
                                                                                  3 | int p[][6] = {
27
                                                                                         \{0,1,2,3,4,5\},
28
                                                                                  4
       else value += crn[R[i]];
                                                                                  5
                                                                                         \{0,1,3,4,5,2\},\
29
     return value;
                                                                                         \{0,1,4,5,2,3\},\
                                                                                  6
30
                                                                                         \{0,1,5,2,3,4\},
  |}
31
                                                                                         \{1,0,2,5,4,3\},
                     10.10. get k-th permutacion
                                                                                         \{1,0,3,2,5,4\},
                                                                                         \{1,0,4,3,2,5\},
   vector<int>v;
                                                                                         \{1,0,5,4,3,2\},
   //finding the number of permutation 0....n-1
                                                                                         {2,4,5,1,3,0},
   int main()
                                                                                         \{2,4,1,3,0,5\},\
                                                                                  13
   {
4
                                                                                         \{2,4,3,0,5,1\},\
       string s;
5
                                                                                         \{2,4,0,5,1,3\},
       while(getline(cin,s)){
6
                                                                                         {3,5,2,1,4,0},
           stringstream ss;
7
                                                                                         {3,5,1,4,0,2},
           ss<<s;
8
                                                                                         {3,5,4,0,2,1},
           int pos=0,u;
9
                                                                                         {3,5,0,2,1,4},
                                                                                  19
           v.clear();
10
                                                                                         {4,2,5,0,3,1},
           while(ss>>u){
11
                                                                                         {4,2,0,3,1,5},
                                                                                 21
               v.push_back(u-1);
12
                                                                                         {4,2,3,1,5,0},
           }
13
                                                                                         {4,2,1,5,0,3},
                                                                                  23
           vector<int>le(v.size(),0);
14
                                                                                         {5,3,2,0,4,1},
           for(int i=0;i<v.size();i++){</pre>
15
                                                                                         {5,3,0,4,1,2},
                                                                                  25
               for(int j=i+1; j<v.size(); j++){</pre>
16
                                                                                         {5,3,4,1,2,0},
                                                                                  26
                   if(v[i]>v[j])le[i]++;
17
                                                                                         {5,3,1,2,0,4}
                                                                                  27
               }
18
                                                                                 28 };
           }
19
                                                                                                           10.13. ternary search
           long long ans=OLL,fact=OLL,por=1LL;
20
           for(int i=le.size()-1;i>=0;i--){
21
                                                                                                         10.14. liebre y el tortugo
               if(fact!=OLL)por*=fact;
22
               fact++;
23
                                                                                                          10.15. como usar printf
               ans=ans+por*le[i];
^{24}
                                                                                                                 10.16. java
           }
25
           cout << ans+1 << "\n";
26
                                                                                                                10.17. python
       }
27
       return 0;
28
                                                                                                              10.18. template
29 }
                                                                                  1 | #include <bits/stdc++.h>
                         10.11. sliding window
                                                                                     using namespace std;
                 10.12. permutaciones de un dado
                                                                                  3
                                                                                     /* no sirve en todos los jueces probar primero
                                                                                     int in() {
  // izquierda, derecha, arriba, al frente, abajo, atras
                                                                                  5
                                                                                         int num, c;
2
```

```
while((c = getchar_unlocked()) < '-');</pre>
       num = c - '0';
8
       while((c = getchar_unlocked()) >= '0') {
9
           num = (num << 3) + (num << 1) + (c-'0');
10
       }
11
       return num;
12
13
14
   int in(){int r=0,c;for(c=getchar();c<=32;c=getchar());if(c=='-') return</pre>
       -in();for(;c>32;r=(r<<1)+(r<<3)+c-'0',c=getchar());return r;}
   #define DBG(x) cout << #x << ": " << x << "\n";</pre>
   #define eb emplace_back
   #define pb push_back
   #define mp make_pair
   #define mt make_tuple
  int main(){
    return 0;
24 }
                             10.19. file setup
1 //tambien se pueden usar comas: {a, x, m, l}
```

touch {a..l}.in; tee {a..l}.cpp < template.cpp