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1 algorithm

#include <algorithm> #include <numeric>

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Algo	Params	Funcion
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	sort, stable_sort	f, l	ordena el intervalo
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	nth_element	f, nth, l	void ordena el n-esimo, y
			particiona el resto
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			f+n) con elem
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	lower_bound, upper_bound	f, l, elem	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	binary_search		. ,
		, ,	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	find, find_if, find_first_of	, ,	it encuentra i \in [f,l) tq. i=elem,
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2, l2)$
replace, replace_if f, l, old cambia old / pred(i) por new / pred, new reverse f, l da vuelta partition, stable_partition f, l, pred pred(i) ad, !pred(i) atras min_element, max_element f, l, [comp] it min, max de [f,l] lexicographical_compare f1,l1,f2,l2 bool con [f1,l1];[f2,l2]	count, count_if	f, l, elem/pred	cuenta elem, pred(i)
		f, l, f2, l2	. , . , . ,
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	replace, replace_if	f, l, old	cambia old / pred(i) por new
$\begin{array}{llll} & \text{partition, stable_partition} & \text{f, l, pred} & \text{pred(i) ad, !pred(i) atras} \\ & \text{min_element, max_element} & \text{f, l, [comp]} & & it \text{min, max de [f,l]} \\ & \text{lexicographical_compare} & & \text{f1,l1,f2,l2} & & bool \text{ con [f1,l1]_i[f2,l2]} \\ \end{array}$			
$\begin{array}{ll} \text{min_element, max_element} & \text{f, l, [comp]} & it \text{ min, max de [f,l]} \\ \text{lexicographical_compare} & \text{f1,l1,f2,l2} & bool \text{ con [f1,l1]_i[f2,l2]} \\ \end{array}$,	
$lexicographical_compare \qquad f1,11,f2,12 \qquad bool \ con \ [f1,11]_i[f2,l2]$	partition, stable_partition		
	•		
	$lexicographical_compare$	f1,l1,f2,l2	
	$next/prev_permutation$	f,l	deja en [f,l) la perm sig, ant
set_intersection, f1, l1, f2, l2, res [res,) la op. de conj	,	f1, l1, f2, l2, res	$[res, \ldots)$ la op. de conj
set_difference, set_union,			
set_symmetric_difference,			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		f, l, e / e /	
make_heap hace un heap de [f,l)	_		
is_heap f,l $bool$ es $[f,l)$ un heap		/	
accumulate $f,l,i,[op]$ $T = \sum /oper de [f,l)$			
inner_product $f1, 11, f2, i$ $T = i + [f1, 11) \cdot [f2,)$	inner_product		
partial_sum			
-			Pos. del primer 1 desde la derecha
			Cant. de ceros desde la izquierda.
builtin_ctz unsigned int Cant. de ceros desde la derecha.		0	Cant. de ceros desde la derecha.
builtin_popcount unsigned int Cant. de 1's en x.			
builtin_parity unsigned int 1 si x es par, 0 si es impar.			
builtin_XXXXXXII unsigned ll = pero para long long's.	_builtin_XXXXXXII	unsigned ll	= pero para long long's.

Estructuras

2.1 RMQ (static) - MODIFICAR

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL \geq ceil(logn); Usar [] para llenar arreglo y luego build().

```
1 struct RMQ{
     #define LVL 10
     tipo vec[LVL] [1<<(LVL+1)];
     tipo &operator[](int p){return vec[0][p];}
     tipo get(int i, int j) {//intervalo [i,j)
       int p = 31-__builtin_clz(j-i);
       return min(vec[p][i],vec[p][j-(1<<p)]);
7
8
     void build(int n) {//O(nlogn)
       int mp = 31-__builtin_clz(n);
10
       forn(p, mp) forn(x, n-(1<<p))
11
         vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
    }};
13
```

2.2 Segment Tree

2.2.1 Segment Tree Recursivo

- 2.2.2 ST Iterativo (Consulta en rango, modificacion a posicion)
- 2.2.3 ST Iterativo (Consulta a posicion, modificacion en rango)

```
/*Segment Tree modificar un rango, acceder a una posicion
     solo sirve cuando la operacion que realizamos es conmutativa
     por ejemplo la suma, pero no funciona con la asignacion
   //adiciona value al rango [1, r)
   void modify(int 1, int r, int value) {// rango [1, r)
    for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1) {
       if (1&1) t[1++] += value;
       if (r&1) t[--r] += value;
9
10
11
   //acceder a la posicion
   int query(int p) {
     int res = 0;
14
    for (p += n; p > 0; p >>= 1) res += t[p];
```

```
return res:
16
  }
17
   //Si necesitamos actualizar todo lo podemos hacer en O(n)
   //Y luego acceder a las hojas en O(1)
   void push() {
    for (int i = 1; i < n; ++i) {
21
      t[i<<1] += t[i];
      t[i<<1|1] += t[i];
23
       t[i] = 0;
25
26 | }
                     2.2.4 Segment Tree con Punteros
                          2.2.5 Segment Tree 2D
                     2.2.6 Segment Tree Lazy - Suma
                           Segment Tree Lazy - Pintar
                      2.2.8 Segment Tree Persistente
                           2.3 Fenwick Tree
                          2.3.1 Fenwick Tree 2D
  11 T[1025][1025];
   int n;
3
   11 query(int x, int y)
5
6
     11 \text{ res} = 0;
7
     for(int i = x; i \ge 0; i = (i & (i+1)) - 1)
8
          for(int j = y; j >= 0; j = (j & (j+1)) - 1)
9
               res += T[i][i];
10
       return res;
11
12
13
   void update(int x, int y, int val)
14
15
    for(int i = x; i < n; i = (i | (i+1)))
16
           for(int j = y; j < n; j = (j | (j+1)))
17
```

T[i][j] += val;

18

19

2.4 Union Find con rank

```
1 /*========= <Union find rangos> ========
  Complexity: O(N)
  index 0 to n - 1 warning
   Complexity O(N)
   */
5
   #define MAX INSERTE_VALOR_AQUI
   int padre[MAX];
   int rango[MAX];
   void MakeSet(int n){
      for (int i = 0; i < n; ++i) {
          padre[i] = i; rango[i] = 0; }
11
12
   int Find(int x) {
      if(x == padre[x])
          return x:
15
      return padre[x] = Find(padre[x]);
16
17
   void UnionbyRank(int x , int y){
      int xRoot = Find(x);
      int yRoot = Find(y);
20
      //el padre de ambas componentes sera el de mayor altura
21
      if(rango[xRoot] > rango[yRoot])//X tiene mas altura que Y
22
          padre[yRoot] = xRoot;
23
      }else{//Y} >= X
24
          padre[xRoot] = yRoot;
25
          if(rango[xRoot] == rango[yRoot])//si poseen la misma altura
26
              rango[yRoot]++;//incremento el rango de la nueva raiz
27
      }
28
29 }
                        2.5 BigInteger C++
1 // g++ -std=c++11 "bigint.cpp" -o run
2
   Contain a useful big int, overload all operators, including cin, cout,
   comparator, build via string (prefer this metod) or long long, for now
      this not have a
6 to_string method
7 Problem for practice: UVA 494
8
  */
```

```
9 // base and base_digits must be consistent
                                                                                                         if (carry)
                                                                                    51
   const int base = 1000000000;
                                                                                                             res.a[i] -= base;
                                                                                    52
   const int base_digits = 9;
                                                                                                    }
                                                                                    53
11
                                                                                                    return res;
12
                                                                                    54
                                                                                                }
   struct bigint {
                                                                                    55
13
                                                                                                return *this - (-v);
       vector<int> a;
14
                                                                                    56
                                                                                            }
       int sign;
                                                                                    57
15
16
                                                                                    58
       bigint():
                                                                                            bigint operator-(const bigint &v) const {
17
                                                                                    59
                                                                                                if (sign == v.sign) {
           sign(1) {
18
                                                                                    60
       }
                                                                                                    if (abs() >= v.abs()) {
19
                                                                                    61
                                                                                                         bigint res = *this;
20
                                                                                    62
                                                                                                         for (int i = 0, carry = 0; i < (int) v.a.size() || carry</pre>
       bigint(long long v) {
21
                                                                                    63
           *this = v;
                                                                                                             : ++i) {
22
       }
                                                                                                             res.a[i] -= carry + (i < (int) v.a.size() ? v.a[i] :
23
                                                                                    64
24
                                                                                                             carry = res.a[i] < 0;</pre>
       bigint(const string &s) {
25
                                                                                                             if (carry)
           read(s);
26
       }
                                                                                                                 res.a[i] += base;
                                                                                    67
27
                                                                                                        }
28
       void operator=(const bigint &v) {
                                                                                                         res.trim();
                                                                                    69
29
           sign = v.sign;
                                                                                                         return res;
30
                                                                                    70
           a = v.a;
31
                                                                                    71
       }
                                                                                                    return -(v - *this);
                                                                                    72
32
                                                                                    73
33
                                                                                                return *this + (-v);
       void operator=(long long v) {
                                                                                    74
34
           sign = 1;
                                                                                            }
                                                                                    75
35
           if (v < 0)
                                                                                    76
36
                sign = -1, v = -v;
                                                                                            void operator*=(int v) {
                                                                                    77
37
           for (; v > 0; v = v / base)
                                                                                                if (v < 0)
                                                                                    78
38
                a.push_back(v % base);
                                                                                                    sign = -sign, v = -v;
                                                                                    79
39
                                                                                                for (int i = 0, carry = 0; i < (int) a.size() || carry; ++i) {</pre>
       }
                                                                                    80
40
                                                                                                    if (i == (int) a.size())
                                                                                    81
41
       bigint operator+(const bigint &v) const {
                                                                                                         a.push_back(0);
                                                                                    82
42
                                                                                                    long long cur = a[i] * (long long) v + carry;
           if (sign == v.sign) {
                                                                                    83
43
                bigint res = v;
                                                                                                    carry = (int) (cur / base);
                                                                                    84
44
                                                                                                    a[i] = (int) (cur % base);
                                                                                    85
45
                                                                                                    //asm("divl %%ecx" : "=a"(carry), "=d"(a[i]) : "A"(cur), "c
                for (int i = 0, carry = 0; i < (int) max(a.size(), v.a.size
46
                                                                                    86
                    ()) || carry; ++i) {
                                                                                                         "(base));
                    if (i == (int) res.a.size())
47
                                                                                    87
                        res.a.push_back(0);
                                                                                                trim();
                                                                                    88
48
                    res.a[i] += carry + (i < (int) a.size() ? a[i] : 0);
                                                                                            }
                                                                                    89
49
                    carry = res.a[i] >= base;
50
                                                                                    90
```

```
bigint operator*(int v) const {
                                                                                                   if (v < 0)
91
                                                                                      132
            bigint res = *this;
                                                                                                       sign = -sign, v = -v;
                                                                                      133
92
                                                                                                   for (int i = (int) \ a.size() - 1, rem = 0; i \ge 0; --i) {
            res *= v;
93
                                                                                      134
                                                                                                       long long cur = a[i] + rem * (long long) base;
            return res;
94
                                                                                      135
        }
                                                                                                       a[i] = (int) (cur / v);
                                                                                      136
95
                                                                                                       rem = (int) (cur % v);
96
                                                                                      137
        friend pair bigint, bigint divmod(const bigint &a1, const bigint &
                                                                                                   }
                                                                                      138
97
            b1) {
                                                                                                   trim();
                                                                                      139
            int norm = base / (b1.a.back() + 1);
                                                                                              }
                                                                                      140
98
            bigint a = a1.abs() * norm;
                                                                                      141
99
            bigint b = b1.abs() * norm;
                                                                                              bigint operator/(int v) const {
                                                                                      142
100
                                                                                                   bigint res = *this;
            bigint q, r;
101
                                                                                      143
            q.a.resize(a.a.size());
                                                                                                   res /= v:
102
                                                                                      144
                                                                                                   return res;
                                                                                      145
103
            for (int i = a.a.size() - 1; i >= 0; i--) {
                                                                                              }
                                                                                      146
104
                r *= base:
                                                                                      147
105
                r += a.a[i]:
                                                                                              int operator%(int v) const {
106
                                                                                      148
                 int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b.a.size()];</pre>
                                                                                                   if (v < 0)
                                                                                      149
107
                 int s2 = r.a.size() \le b.a.size() - 1 ? 0 : r.a[b.a.size() -
                                                                                                       v = -v:
                                                                                      150
108
                                                                                                   int m = 0;
                                                                                      151
                int d = ((long long) base * s1 + s2) / b.a.back();
                                                                                                   for (int i = a.size() - 1; i >= 0; --i)
                                                                                      152
109
                                                                                                       m = (a[i] + m * (long long) base) % v;
                 r -= b * d;
110
                 while (r < 0)
                                                                                                   return m * sign;
                                                                                      154
111
                                                                                              }
                     r += b, --d;
                                                                                      155
112
                q.a[i] = d;
                                                                                      156
113
            }
                                                                                              void operator+=(const bigint &v) {
                                                                                      157
114
                                                                                                   *this = *this + v;
                                                                                      158
115
            q.sign = a1.sign * b1.sign;
                                                                                      159
116
                                                                                              void operator-=(const bigint &v) {
            r.sign = a1.sign;
                                                                                      160
117
            q.trim();
                                                                                                   *this = *this - v;
                                                                                      161
118
                                                                                              }
            r.trim();
                                                                                      162
119
            return make_pair(q, r / norm);
                                                                                              void operator*=(const bigint &v) {
                                                                                      163
120
        }
                                                                                                   *this = *this * v:
                                                                                      164
121
                                                                                              }
                                                                                      165
122
                                                                                              void operator/=(const bigint &v) {
        bigint operator/(const bigint &v) const {
                                                                                      166
123
            return divmod(*this, v).first;
                                                                                                   *this = *this / v;
                                                                                      167
124
        }
                                                                                              }
                                                                                      168
125
                                                                                      169
126
        bigint operator%(const bigint &v) const {
                                                                                              bool operator<(const bigint &v) const {</pre>
                                                                                      170
127
            return divmod(*this, v).second;
                                                                                                   if (sign != v.sign)
                                                                                      171
128
        }
                                                                                                       return sign < v.sign;</pre>
                                                                                      172
129
                                                                                                   if (a.size() != v.a.size())
                                                                                      173
130
                                                                                                       return a.size() * sign < v.a.size() * v.sign;</pre>
        void operator/=(int v) {
131
                                                                                      174
```

```
for (int i = a.size() - 1; i >= 0; i--)
                                                                                               }
175
                                                                                      218
                 if (a[i] != v.a[i])
                                                                                      219
176
                     return a[i] * sign < v.a[i] * sign;</pre>
                                                                                               long longValue() const {
                                                                                      220
177
                                                                                                   long long res = 0;
            return false;
                                                                                      221
178
        }
                                                                                                   for (int i = a.size() - 1; i >= 0; i--)
                                                                                      222
179
                                                                                                        res = res * base + a[i];
180
                                                                                      223
        bool operator>(const bigint &v) const {
                                                                                                   return res * sign;
181
                                                                                      224
            return v < *this;
                                                                                               }
                                                                                       225
182
        }
183
                                                                                       226
        bool operator<=(const bigint &v) const {</pre>
                                                                                               friend bigint gcd(const bigint &a, const bigint &b) {
                                                                                       227
184
            return !(v < *this);</pre>
                                                                                                   return b.isZero() ? a : gcd(b, a % b);
185
                                                                                      228
        }
186
                                                                                      229
        bool operator>=(const bigint &v) const {
                                                                                               friend bigint lcm(const bigint &a, const bigint &b) {
187
                                                                                      230
            return !(*this < v);</pre>
                                                                                                   return a / gcd(a, b) * b;
                                                                                      231
188
        }
                                                                                               }
                                                                                      232
189
        bool operator==(const bigint &v) const {
                                                                                       233
190
            return !(*this < v) && !(v < *this);
                                                                                               void read(const string &s) {
191
                                                                                      234
        }
                                                                                                   sign = 1;
192
                                                                                       235
        bool operator!=(const bigint &v) const {
                                                                                                   a.clear():
                                                                                      236
193
            return *this < v || v < *this;
                                                                                                   int pos = 0;
194
                                                                                                   while (pos < (int) s.size() && (s[pos] == '-' || s[pos] == '+'))
        }
                                                                                       238
195
                                                                                                        {
196
        void trim() {
                                                                                                        if (s[pos] == '-')
197
                                                                                       239
                                                                                                            sign = -sign;
            while (!a.empty() && !a.back())
198
                                                                                      240
                 a.pop_back();
                                                                                      241
199
            if (a.empty())
                                                                                      242
200
                 sign = 1;
                                                                                                   for (int i = s.size() - 1; i >= pos; i -= base_digits) {
                                                                                      243
201
        }
                                                                                                        int x = 0:
                                                                                      244
202
                                                                                                       for (int j = max(pos, i - base_digits + 1); j <= i; j++)
                                                                                      245
203
        bool isZero() const {
                                                                                                            x = x * 10 + s[j] - '0';
                                                                                      246
204
            return a.empty() || (a.size() == 1 && !a[0]);
                                                                                                        a.push_back(x);
                                                                                      247
205
        }
                                                                                      248
206
                                                                                                   trim():
                                                                                      249
207
        bigint operator-() const {
                                                                                               }
                                                                                      250
208
            bigint res = *this;
                                                                                      251
209
            res.sign = -sign;
                                                                                               friend istream& operator>>(istream &stream, bigint &v) {
                                                                                      252
210
            return res;
                                                                                                   string s;
211
                                                                                      253
        }
                                                                                                   stream >> s:
                                                                                      254
212
                                                                                                   v.read(s);
                                                                                      255
213
        bigint abs() const {
                                                                                                   return stream;
                                                                                      256
214
            bigint res = *this;
                                                                                      257
^{215}
            res.sign *= res.sign;
                                                                                      258
216
                                                                                               friend ostream& operator<<(ostream &stream, const bigint &v) {
            return res;
^{217}
                                                                                      259
```

```
if (v.sign == -1)
                                                                                       302
260
                 stream << '-';
                                                                                                    int k = n \gg 1;
                                                                                       303
261
            stream << (v.a.empty() ? 0 : v.a.back());</pre>
                                                                                                    vll a1(a.begin(), a.begin() + k);
262
                                                                                       304
            for (int i = (int) v.a.size() - 2; i >= 0; --i)
                                                                                                    vll a2(a.begin() + k, a.end());
263
                                                                                       305
                 stream << setw(base_digits) << setfill('0') << v.a[i];</pre>
                                                                                                    vll b1(b.begin(), b.begin() + k);
                                                                                       306
264
                                                                                                    vll b2(b.begin() + k, b.end());
            return stream;
265
                                                                                       307
        }
266
                                                                                       308
                                                                                                    vll a1b1 = karatsubaMultiply(a1, b1);
267
                                                                                       309
        static vector<int> convert_base(const vector<int> &a, int old_digits
                                                                                                    vll a2b2 = karatsubaMultiply(a2, b2);
268
                                                                                       310
             , int new_digits) {
                                                                                       311
            vector<long long> p(max(old_digits, new_digits) + 1);
                                                                                                    for (int i = 0; i < k; i++)
                                                                                       312
269
                                                                                                        a2[i] += a1[i];
            p[0] = 1;
270
                                                                                       313
            for (int i = 1; i < (int) p.size(); i++)</pre>
                                                                                                    for (int i = 0; i < k; i++)
271
                                                                                       314
                 p[i] = p[i - 1] * 10;
                                                                                                        b2[i] += b1[i];
272
                                                                                       315
            vector<int> res;
273
                                                                                       316
            long long cur = 0;
                                                                                                    vll r = karatsubaMultiply(a2, b2);
274
                                                                                       317
                                                                                                    for (int i = 0; i < (int) a1b1.size(); i++)</pre>
            int cur_digits = 0;
275
                                                                                       318
            for (int i = 0; i < (int) a.size(); i++) {
                                                                                                        r[i] -= a1b1[i];
276
                                                                                       319
                 cur += a[i] * p[cur_digits];
                                                                                                    for (int i = 0; i < (int) a2b2.size(); i++)</pre>
                                                                                       320
277
                 cur_digits += old_digits;
                                                                                                        r[i] -= a2b2[i];
                                                                                       321
278
                 while (cur_digits >= new_digits) {
                                                                                       322
279
                     res.push_back(int(cur % p[new_digits]));
                                                                                                    for (int i = 0; i < (int) r.size(); i++)</pre>
280
                                                                                       323
                     cur /= p[new_digits];
                                                                                                        res[i + k] += r[i];
                                                                                       324
281
                                                                                                    for (int i = 0; i < (int) a1b1.size(); i++)</pre>
                     cur_digits -= new_digits;
                                                                                       325
282
                 }
                                                                                                        res[i] += a1b1[i];
                                                                                       326
283
            }
                                                                                                    for (int i = 0; i < (int) a2b2.size(); i++)</pre>
                                                                                       327
284
            res.push_back((int) cur);
                                                                                                        res[i + n] += a2b2[i];
                                                                                       328
285
            while (!res.empty() && !res.back())
                                                                                       329
                                                                                                    return res;
286
                                                                                               }
                 res.pop_back();
                                                                                       330
287
            return res;
                                                                                       331
288
        }
                                                                                               bigint operator*(const bigint &v) const {
                                                                                       332
289
                                                                                                    vector<int> a6 = convert_base(this->a, base_digits, 6);
290
                                                                                       333
                                                                                                    vector<int> b6 = convert_base(v.a, base_digits, 6);
        typedef vector<long long> vll;
                                                                                       334
291
                                                                                                    vll a(a6.begin(), a6.end());
                                                                                       335
292
        static vll karatsubaMultiply(const vll &a, const vll &b) {
                                                                                                    vll b(b6.begin(), b6.end());
                                                                                       336
293
            int n = a.size():
                                                                                                    while (a.size() < b.size())</pre>
                                                                                       337
294
            vll res(n + n);
                                                                                                        a.push_back(0);
295
                                                                                       338
                                                                                                    while (b.size() < a.size())</pre>
            if (n <= 32) {
                                                                                       339
296
                 for (int i = 0; i < n; i++)
                                                                                                        b.push_back(0);
                                                                                       340
297
                     for (int j = 0; j < n; j++)
                                                                                                    while (a.size() & (a.size() - 1))
                                                                                       341
298
                         res[i + j] += a[i] * b[j];
                                                                                                        a.push_back(0), b.push_back(0);
299
                                                                                       342
                                                                                                    vll c = karatsubaMultiply(a, b);
                 return res;
                                                                                       343
300
            }
                                                                                                    bigint res;
301
                                                                                       344
```

```
res.sign = sign * v.sign;
345
          for (int i = 0, carry = 0; i < (int) c.size(); i++) {
346
             long long cur = c[i] + carry;
347
             res.a.push_back((int) (cur % 1000000));
348
             carry = (int) (cur / 1000000);
349
350
          res.a = convert_base(res.a, 6, base_digits);
351
          res.trim();
352
          return res;
353
354
355
356
   int main() {
357
      bigint a=0;
358
      359
      bigint b;
360
      361
      bigint n;
362
      while(cin >> n) {
363
          if(n==0){break;}
364
          a += n;
365
366
      cout<<a<<endl;
367
368
```

2.6 UnorderedSet

```
//Compilar: g++ --std=c++11
  struct Hash{
2
     size_t operator()(const ii &a)const{
3
       size_t s=hash<int>()(a.fst);
4
       return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
5
6
     size_t operator()(const vector<int> &v)const{
7
       size_t s=0;
8
       for(auto &e : v)
9
         s = hash < int > ()(e) + 0x9e3779b9 + (s < 6) + (s > 2);
10
       return s:
11
     }
12
13
   unordered_set<ii, Hash> s;
   unordered_map<ii, int, Hash> m;//map<key, value, hasher>
```

2.7 Ordered Set

```
1
    A brief explanation about use of a powerful library: orderd_set
    Reference link: http://codeforces.com/blog/entry/11080
    and a hash for the type pair
5
   #include <ext/pb_ds/assoc_container.hpp>
   #include <ext/pb_ds/tree_policy.hpp>
   using namespace __gnu_pbds;
   typedef tree<int,null_type,less<int>,rb_tree_tag,
       tree_order_statistics_node_update> ordered_set;
11
12 If we want to get map but not the set, as the second argument type must
       be used mapped type. Apparently,
   the tree supports the same operations as the set (at least I haven't
        any problems with them before),
   but also there are two new features - it is find_by_order() and
        order_of_key().
   The first returns an iterator to the k-th largest element (counting
        from zero), the second - the number of items
in a set that are strictly smaller than our item. Example of use:
   *
17
18 * */
```

2.8 Treap Modo Set

- 2.9 Treap Implicito(Rope)
- 2.10 Treap Toby and Bones

2.11 Convex Hull Trick Estatico

```
// g++ "convexhulltrick.cpp" -o run
/***

Contain a sample about convex hull trick optimization this recivie N
pairs:

"value of length" and a cost, we need to minimize the value of
grouping
this pairs taken the most large pair as the cost of the group

Problem for practice: aquire
```

```
//newly inserted line is now the best for that query
9
                                                                                    47
   #include <iostream>
                                                                                         if (pointer>=M.size())
                                                                                    48
   #include <vector>
                                                                                           pointer=M.size()-1;
11
                                                                                    49
   #include <algorithm>
                                                                                         //Any better line must be to the right, since query values are
                                                                                    50
   using namespace std;
                                                                                         //non-decreasing
                                                                                    51
                                                                                         while (pointer<M.size()-1&&
   int pointer; //Keeps track of the best line from previous query
                                                                                    52
                                                                                           M[pointer+1]*x+B[pointer+1]<M[pointer]*x+B[pointer])</pre>
   vector<long long> M; //Holds the slopes of the lines in the envelope
                                                                                    53
   vector<long long> B; //Holds the y-intercepts of the lines in the
                                                                                           pointer++;
                                                                                    54
       envelope
                                                                                         return M[pointer] *x+B[pointer];
                                                                                    55
   //Returns true if either line 11 or line 13 is always better than line
                                                                                    56
                                                                                       int main()
                                                                                    57
   bool bad(int 11, int 12, int 13)
                                                                                    58
                                                                                         int M,N,i;
19
                                                                                    59
                                                                                         pair<int, int> a[50000];
                                                                                    60
20
                                                                                         pair<int,int> rect[50000];
     intersection(11,12) has x-coordinate (b1-b2)/(m2-m1)
                                                                                    61
21
     intersection(11,13) has x-coordinate (b1-b3)/(m3-m1)
                                                                                         scanf("%d",&M);
22
                                                                                    62
     set the former greater than the latter, and cross-multiply to
                                                                                         for (i=0; i<M; i++)
23
                                                                                    63
     eliminate division
                                                                                           scanf("%d<sub>|</sub>%d",&a[i].first,&a[i].second);
24
                                                                                         //Sort first by height and then by width (arbitrary labels)
                                                                                    65
25
     return (B[13]-B[11])*(M[11]-M[12])<(B[12]-B[11])*(M[11]-M[13]);
                                                                                         sort(a,a+M);
26
                                                                                         for (i=0,N=0; i<M; i++)
                                                                                    67
27
    //Adds a new line (with lowest slope) to the structure
                                                                                         {
                                                                                    68
   void add(long long m,long long b)
                                                                                           /*
                                                                                    69
29
                                                                                           When we add a higher rectangle, any rectangles that are also
30
                                                                                    70
     //First, let's add it to the end
                                                                                           equally thin or thinner become irrelevant, as they are
                                                                                    71
31
     M.push_back(m);
                                                                                           completely contained within the higher one; remove as many
                                                                                    72
32
     B.push_back(b);
                                                                                           as necessary
                                                                                    73
33
     //If the penultimate is now made irrelevant between the
                                                                                           */
                                                                                    74
34
                                                                                           while (N>0&&rect[N-1].second<=a[i].second)</pre>
         antepenultimate
                                                                                    75
     //and the ultimate, remove it. Repeat as many times as necessary
                                                                                             N--:
                                                                                    76
35
     while (M.size() >= 3\&\&bad(M.size() - 3, M.size() - 2, M.size() - 1))
                                                                                           rect[N++]=a[i]; //add the new rectangle
                                                                                    77
36
                                                                                    78
37
       M.erase(M.end()-2):
                                                                                         long long cost;
                                                                                    79
38
       B.erase(B.end()-2);
                                                                                         add(rect[0].second,0);
                                                                                    80
39
                                                                                         //initially, the best line could be any of the lines in the envelope,
     }
                                                                                    81
40
                                                                                         //that is, any line with index 0 or greater, so set pointer=0
41
   //Returns the minimum y-coordinate of any intersection between a given
                                                                                         pointer=0;
                                                                                    83
       vertical
                                                                                         for (i=0: i<N: i++)
                                                                                    84
    //line and the lower envelope
                                                                                    85
   long long query(long long x)
                                                                                           cost=query(rect[i].first);
                                                                                    86
                                                                                           if (i<N)
45
                                                                                    87
                                                                                              add(rect[i+1].second,cost);
    //If we removed what was the best line for the previous query, then
                                                                                    88
                                                                                         }
         the
                                                                                    89
```

17

```
printf("%lld\n",cost);
     return 0;
91
92 }
                  2.12 Convex Hull Trick Dinamico
1 // g++ -std=c++11 "convexhulltrick_dynamic.cpp" -o run
   /***
2
    ========= <Convex hull trick dynamic version version>
   warning with the use of this, this is a black box, try to use only in an
         emergency.
  Problem for practice: aquire
   #include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
   const ll is_query = -(1LL<<62);</pre>
     struct Line {
     ll m, b;
     mutable multiset<Line>::iterator it;
13
     const Line *succ(multiset<Line>::iterator it) const;
14
     bool operator<(const Line& rhs) const {</pre>
15
       if (rhs.b != is_query) return m < rhs.m;</pre>
16
       const Line *s=succ(it):
17
       if(!s) return 0:
18
       11 x = rhs.m:
19
       return b - s->b < (s->m - m) * x;
20
21
22
   struct HullDynamic : public multiset<Line>{ // will maintain upper hull
        for maximum
     bool bad(iterator y) {
24
       iterator z = next(y);
25
       if (v == begin()) {
26
         if (z == end()) return 0;
27
         return y->m == z->m && y->b <= z->b;
28
29
       iterator x = prev(y);
30
       if (z == end()) return y \rightarrow m == x \rightarrow m \&\& y \rightarrow b <= x \rightarrow b;
31
       return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
32
33
     iterator next(iterator y){return ++y;}
```

```
iterator prev(iterator y){return --y;}
       void insert_line(ll m, ll b) {
36
       iterator y = insert((Line) { m, b });
       y->it=y;
       if (bad(y)) { erase(y); return; }
39
       while (next(y) != end() && bad(next(y))) erase(next(y));
       while (y != begin() && bad(prev(y))) erase(prev(y));
41
42
     ll eval(ll x) {
43
       Line 1 = *lower_bound((Line) { x, is_query });
       return 1.m * x + 1.b;
45
    }
46
47
   }h:
   const Line *Line::succ(multiset<Line>::iterator it) const{
49 | return (++it==h.end()? NULL : &*it);}
                            2.13 Misof Tree
http://codeforces.com/blog/entry/10493#comment-159335
3 Sirve para encontrar el i-esimo numero de un conjunto de numeros que
       vamos insertando en el arbol.
4 Sirve solo si nuestros numeros son del 0 al n-1 (pero podemos mapearlos
       antes de usarlos)
5 La idea es esta:
  Funcionamiento:
    - En el fondo sigue siendo un Segment-Tree (hacemos que 'n' sea 2^x)
    - Cada nodo guarda cuantos numeros hay en el intervalo (entonces en
         tree[1] dice cuantos numeros tenemos en total)
    - Se sigue representando los hijos del nodo 'i' con '2 * i' (izq) y '2
          * i + 1' (der);
   Query:
10
    - si kth es mas grande que todos los que tenemos(tree[1]) o es
11
         negativo entonces -1
     - siempre nos mantenemos en el nodo de la izquierda y si es necario
12
         avanzamos al de la derecha
                         'i <<= 1'
13
       - si kth es mas grande que el nodo de la izquierda(el actual) quiere
14
            decir que podemos quitarle todos esos
      numeros a nuestra busqueda 'kth - tree[i]' y buscar el nuevo kth en
15
           el arbol de la derecha
         if (kth > tree [i]) kth -= tree [i++];
16
```

- Ojo en el 'i++' ahi es donde avanzamos al nodo de la derecha

Página 12 de ??

```
- luego hace su formula rara que aun no entendi xD:
         'i - leaf + (kth > tree [i])';
19
20
   const int MaxN = 1e6;
22
   int a [MaxN], s [MaxN];
   int leaf, tree [100 + MaxN << 2];</pre>
   void bld (int n) { leaf = 1 << (32 - __builtin_clz (n)); }</pre>
   void add (int x) { for (int i = leaf + x; i; i >>= 1) ++tree [i]; }//
       Podemos insertar mas de una copia la vez tree [i] += xcopies;
   void del (int x) { for (int i = leaf + x; i; i >>= 1) --tree [i]; }//
       Podemos eliminar mas de una copia la vez tree [i] -= xcopies;
   // en "leaf + x" esta cuantas copias tenemos de "x"
   //Cuidado con intentar hacer del con mas copias de las disponibles, el
       kth() no funcionaria
   long kth (int kth, int i = -1) {
       if (kth > tree [1] || kth <= 0) return i;
     for (i = 1; i < leaf; i <<= 1) if (kth > tree [i]) kth -= tree [i++];
       return i - leaf + (kth > tree [i]);
34
35 }
```

2.14 SQRT Decomposition Basic

```
const int maxn = 500010;
2
  int n;
   tipo v[maxn];//vector principal
   tipo lazy [maxn];
   pair<tipo, tipo> t[maxn];//para poder reordenar los elementos
8
   int SQRT;
   int N;//nro. de buckets
11
   //Recalcula y aplica el lazy al bucket con indice idx
   //guarda la informacion necesaria del bucket en otros vectores
   //podria ser la suma del bucket, o el min/max del bucket
   void recalc(int idx) {
     int a = idx * SQRT, b = min(n, (idx + 1) * SQRT);
    for (int i = a; i < b; i++) {
17
       v[i] += lazy[idx];
18
       t[i] = make_pair(v[i], i);
19
```

```
20
     lazv[idx] = 0;
21
     sort(t + a, t + b);
22
23
24
   //adiciona delta a todos los elementos
   //en el intervalo cerrado [a, b]
   void add(int a, int b, tipo delta) {
     int idx_a = a / SQRT, idx_b = b / SQRT;
     if (idx_a == idx_b) {
    for (int i = a; i <= b; i++)
30
         v[i] += delta;
31
    recalc(idx a):
    } else {
       //head
       for (int i = a, \lim = \min(n, (idx_a + 1) * SQRT); i < \lim; i++)
         v[i] += delta;
36
       recalc(idx_a);//OJO puede ser necesario
       //bodv
38
       for (int i = idx_a + 1; i < idx_b; i++)
         lazv[i] += delta;
40
      //tail
       for (int i = idx_b * SQRT; i <= b; i++)</pre>
42
         v[i] += delta;
43
       recalc(idx_b);//OJO puede ser necesario
44
45
   }
46
47
   //tambien podria ser en un rango como en el add
   tipo query(tipo val) {
     tipo ans = 0;
50
     //recorro todos los buckets
51
     for (int idx = 0: idx < N: idx++) {
52
       int a = idx * SQRT, b = min(n, (idx + 1) * SQRT);
      //... hacer algo ...
54
55
     return ans;
56
57
   int main() {
     //leer n, q y los elementos de v
60
     SQRT = (int)sqrt(n) + 1;
61
     N = (n + SQRT - 1) / SQRT; //nro. de buckets
```

```
//construir cada bucket
for (int idx = 0; idx < N; idx++)
recalc(idx);

//resto del programa
return 0;
}
```

2.15 Nro. Elementos menores o iguales a x en O(log(n))

```
//insersion y consulta de cuantos <= en log n
   struct legset {
2
     int maxl; vector<int> c;
     int pref(int n, int l) { return (n>>(maxl-l))|(1<<l); }</pre>
4
     void ini(int ml) { maxl=ml; c=vector<int>(1<<(maxl+1)); }</pre>
5
     //inserta c copias de e, si c es negativo saca c copias
6
     void insert(int e, int q=1) { forn(l,maxl+1) c[pref(e,l)]+=q; }
     int leq(int e) {
8
       int r=0,a=1;
9
       forn(i,maxl) {
10
         a<<=1; int b=(e>>maxl-i-1)&1;
11
         if (b) r+=c[a]; a|=b;
12
       } return r + c[a]; //sin el c[a] da los estrictamente menores
13
14
     int size() { return c[1]; }
15
     int count(int e) { return c[e|(1<<maxl)]; }</pre>
17 | };
```

3 Algos

3.1 LIS en O(n log n) con Reconstruccion

```
//Para non-increasing, cambiar comparaciones y revisar busq binaria
//Given an array, paint it in the least number of colors so that each
color turns to a non-increasing subsequence.
//Solution:Min number of colors=Length of the longest increasing
subsequence
// Las lineas marcadas con // Camino no son necesarias si no se desea
reconstruir el camino.
// #define MAXN 1000000
int v[MAXN]; // INPUT del algoritmo.
int mv[MAXN];
```

```
9 int mi[MAXN] ,p[MAXN]; // Camino
  int 1[MAXN]; // Aca apareceria la maxima subsecuencia creciente(los
       indices)
   int lis(int n) {
     forn(i,n) mv[i] = INF;
    forn(i,n) mi[i] = -1; // Camino
     forn(i,n) p [i] = -1; // Camino
14
     mv[0] = -INF;
15
     int res = 0;
16
     forn(i,n) {
17
       // Con upper_bound es maxima subsecuencia no decreciente.
18
       // Con lower_bound es maxima subsecuencia creciente.
19
       int me = upper_bound(mv,mv+n,v[i]) - mv;
20
       p[i] = mi[me-1]; // Camino
21
       mv[me] = v[i];
22
       mi[me] = i; // Camino
23
       if (me > res) res = me;
24
25
     for(int a = mi[res], i = res - 1; a != -1; a = p[a], i--) // Camino
26
       l[i] = a; // Indices: poniendo l[i] = v[a] quedan los valores.
     return res;
28
29 }
```

3.2 Mo

```
// g++ -std=c++11 "mo.cpp" -o run
/***

Contain a sample about Mo algorithm
Brief explanation when use Mo:
Explain where and when we can use above algorithm

As mentioned, this algorithm is offline, that means we cannot use it when we are forced to stick to given order of queries.

That also means we cannot use this when there are update operations.
Not just that, there is one important possible limitation:
We should be able to write the functions add and remove. There will be many cases where add is trivial but remove is not.

One such example is where we want maximum in a range. As we add elements , we can keep track of maximum. But when we remove elements it is not trivial. Anyways in that case we can use a set to add elements , remove elements and report minimum.
```

```
13 In that case the add and delete operations are O(log N) (Resulting in O(
       N * Sqrt(N) * log N) algorithm).
14
   Suggestion first use the add operation, then the erase operation
   Problem for practice: DQUERY spoj
   Input: N, then N elements of array M querys with a range L,R
18
   const int MAXV = 1e6 + 10;
   const int N = 30010;
   const int M = 200010;
   int cnt[MAXV];
   int v[N];
24
   struct query{
     int 1,r,pos;
26
     query(){}
28
   int n;
   query qu[M];
   int ans[M];
32
   int ret = 0;
   void add(int pos){
34
     pos = v[pos];
35
     cnt[pos]++;
36
     if(cnt[pos] == 1){
37
       ret++;
38
39
40
   void erase(int pos){
41
     pos = v[pos];
42
     cnt[pos]--;
43
     if(!cnt[pos])ret--;
44
45
   int main(){
46
     n = in():
47
     for(int i = 0; i < n; i++){
48
       v[i] = in();
49
50
     int block = ceil(sqrt(n));
51
     int q = in();
52
     for(int i = 0; i < q; i++){
53
       qu[i].l = in() - 1, qu[i].r = in() - 1, qu[i].pos = i;
54
```

```
55
     sort(qu,qu + q,[&](const query &a,const query &b){
56
       if(a.l / block != b.l / block)
57
          return a.1 / block < b.1 / block;
58
       return a.r < b.r;
59
     });
60
     int 1 = 0, r = 0;
61
     for(int i = 0; i < q; i++){
62
       int nl = qu[i].l,nr = qu[i].r;
63
       while(l > nl){
          add(--1);
65
       }
66
       while(r <= nr){</pre>
          add(r++);
       while(1 < n1){</pre>
          erase(1++);
71
72
       while(r > nr + 1){
73
          erase(--r);
       }
75
76
       ans[qu[i].pos] = ret;
77
     for(int i = 0; i < q; i++)printf("%d\n",ans[i]);
79
80 }
```

4 Strings

4.1 Manacher

```
vector<int> manacher(const string &_s) {
    int n = _s.size();
     string s(2 * n + 3, '#');
     s[0] = \%, s[s.size() - 1] = \%, s[s.size() - 1] = \%
     for (int i = 0; i < n; i++)
      s[(i + 1) * 2] = _s[i];
6
7
    n = s.size():
    vector<int> P(n, 0);
    int C = 0, R = 0;
10
    for (int i = 1; i < n - 1; i++) {
11
      int j = C - (i - C);
12
```

```
if (R > i)
13
         P[i] = min(R - i, P[i]);
14
       while (s[i + 1 + P[i]] == s[i - 1 - P[i]])
15
         P[i]++;
16
       if (i + P[i] > R) {
17
         C = i;
18
         R = i + P[i];
19
20
     }
^{21}
     return P;
22
23
    bool is_pal(const vector<int> &mnch_vec, int i, int j) {//[i, j] - i<=j
     int len = i - i + 1:
25
     i = (i + 1) * 2://idx to manacher vec idx
     j = (j + 1) * 2;
     int mid = (i + j) / 2;
28
     return mnch_vec[mid] >= len;
29
30
   int main() {
31
     string s;
32
     cin >> s;
33
     vector<int> mnch_vec= manacher(s);
34
     if (is_pal(mnch_vec, 2, 7)) {
35
       //la subcadena desde la posicion 2 a la 7 es palindrome
36
     }
37
     return 0;
38
39 }
```

4.2 Trie - Punteros y bfs

4.3 Suffix Array O(n log n) con LCP (Kasai) O(n)

4.4 Minima rotacion lexicografica

```
/*
Rotacion Lexicografica minima MinRotLex(cadena,tamanio)
para cambiar inicio de la cadena char s[300]; int h; s+h;
retorna inicio de la rotacion minima :D

*/
int MinRotLex(const char *s, const int slen) {
   int i = 0, j = 1, k = 0, x, y, tmp;
   while(i < slen && j < slen && k < slen) {
        x = i + k;
        y = j + k;
}</pre>
```

```
if(x \ge slen) x -= slen:
11
         if(v >= slen) v -= slen;
12
         if(s[x] == s[y]) {
13
            k++;
14
         } else if(s[x] > s[y]) {
15
            i = j+1 > i+k+1 ? j+1 : i+k+1;
            k = 0;
            tmp = i, i = j, j = tmp;
         } else {
            j = i+1 > j+k+1 ? i+1 : j+k+1;
            k = 0:
21
         }
22
      }
23
      return i;
24
25
   int main(){
     int n;
     scanf("%d",&n);getchar();
     while(n--){
       char str[1000009];
       gets(str);
31
       printf("%d\n",MinRotLex(str,strlen(str))+1);
    }
33
34 }
```

4.5 Matching

4.5.1 KMP

```
string T;//cadena donde buscar(where)
string P://cadena a buscar(what)
   int b[MAXLEN];//back table b[i] maximo borde de [0..i)
   void kmppre(){//by gabina with love
       int i =0, j=-1; b[0]=-1;
       while(i<sz(P)){</pre>
6
           while(j>=0 && P[i] != P[j]) j=b[j];
           i++, j++, b[i] = j;
8
       }
9
   }
10
   void kmp(){
       int i=0, j=0;
12
       while(i<sz(T)){</pre>
13
           while(j>=0 && T[i]!=P[j]) j=b[j];
14
           i++, j++;
15
```

18

19

20

trie* get_link() {

if(!link){

15

16

root=last=cur++;

root->clear();

```
if(j==sz(P)) printf("Puis found at index %d in T\n", i-j), j=b[j
                                                                                             if(!padre) link=this;//es la raiz
16
                                                                                   21
                                                                                             else if(!padre->padre) link=padre;//hijo de la raiz
               ];
                                                                                   22
       }
                                                                                   23
17
   }
18
                                                                                   24
                                                                                           return link; }
                                                                                   25
19
   int main(){
                                                                                         trie* get_tran(int c) {
20
                                                                                   26
       cout << "T=";
21
                                                                                   27
                                                                                          return tran[c]; }
       cin >> T;
22
                                                                                   28
       cout << "P=";
                                                                                         trie *get_nxthoja(){
23
                                                                                   29
       cin.ignore();
24
       cin >> P;
                                                                                          return nxthoja; }
25
                                                                                   31
                                                                                         void print(int p){
       kmppre();
26
                                                                                   32
       kmp();
27
                                                                                   33
       return 0:
                                                                                               szhoja << endl;</pre>
28
29 }
                                                                                   34
                           4.5.2 Z - Por aprender
                      4.5.3 Matching con suffix array
                                                                                      }tri;
                                                                                   38
                         4.5.4 Matching con BWT
                                                                                      int main(){
                     4.5.5 Matching con Aho-Corasick
                                                                                        tri=trie();//clear
                                                                                        tri.insert("ho", 1);
                                                                                   42
1
                                                                                        tri.insert("hoho", 2);
   struct trie{
     map<char, trie> next;
     trie* tran[256];//transiciones del automata
4
     int idhoja, szhoja;//id de la hoja o 0 si no lo es
5
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
6
         es hoja
     trie *padre, *link, *nxthoja;
7
     char pch;//caracter que conecta con padre
8
     trie(): tran(), idhoja(), padre(), link() {}
                                                                                           int step;
                                                                                    5
9
     void insert(const string &s, int id=1, int p=0){//id>0!!!
                                                                                           void clear() {
                                                                                    6
10
       if(p<sz(s)){</pre>
                                                                                               pre=0;
                                                                                    7
11
         trie &ch=next[s[p]];
                                                                                               step=0;
12
                                                                                    8
         tran[(int)s[p]]=&ch;
                                                                                    9
13
         ch.padre=this, ch.pch=s[p];
                                                                                   10
14
         ch.insert(s, id, p+1);
                                                                                      } *root,*last;
15
       }
16
       else idhoja=id, szhoja=sz(s);
                                                                                      void init() {
17
```

```
else link=padre->get_link()->get_tran(pch);
      if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
      if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
      if(idhoja) cout << "found," << idhoja << ", at position," << p-
      if(get_nxthoja()) get_nxthoja()->print(p); }
    void matching(const string &s, int p=0){
      print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
                            Suffix Automaton
const int N = INSERTE_VALOR;//maxima longitud de la cadena
  struct State { //OJO!!! tamanio del alfabeto, si MLE -> map
      State *pre, *go[26];//se puede usar un map<char, State*> go
          memset(go,0,sizeof(go));//go.clear();
  State statePool[N * 2],*cur;
      cur=statePool:
14
```

```
17 | }
                                                                                          State *p = root;
                                                                                   59
   void Insert(int w) {
                                                                                          for(int i = 0; i < m; i++) {
                                                                                   60
18
                                                                                              int x = B[i] - a';
       State *p=last;
19
                                                                                   61
       State *np=cur++;
                                                                                               if(p->go[x]) {
20
                                                                                   62
       np->clear();
                                                                                                   len++;
21
                                                                                   63
       np->step=p->step+1;
                                                                                                   p = p - go[x];
22
                                                                                   64
       while(p&&!p->go[w])
                                                                                              } else {
23
                                                                                   65
           p->go[w]=np,p=p->pre;
                                                                                                   while (p && !p->go[x]) p = p->pre;
24
                                                                                   66
       if(p==0)
                                                                                                   if(!p) p = root, len = 0;
25
                                                                                   67
                                                                                                   else len = p->step+1, p = p->go[x];
           np->pre=root;
26
                                                                                               }
       else {
27
                                                                                   69
           State *q=p->go[w];
                                                                                               if (len > ans)
28
                                                                                   70
           if(p->step+1==q->step)
                                                                                                   ans = len, bestpos = i;
29
                                                                                   71
               np->pre=q;
                                                                                          }
                                                                                   72
30
                                                                                          //return ans; //solo el tamanio del lcs
           else {
                                                                                   73
31
                                                                                          return string(B + bestpos - ans + 1, B + bestpos + 1);
               State *nq=cur++;
32
                                                                                   74
               nq->clear();
                                                                                      }
33
                                                                                   75
               memcpy(nq->go,q->go,sizeof(q->go));//nq->go = q->go; para
34
                                                                                   76
                                                                                      /*Numero de subcadenas distintas + 1(subcadena vacia) en O(|A|)
                    mapa
               nq->step=p->step+1;
                                                                                      OJO: Por alguna razon Suffix Array es mas rapido
35
               nq->pre=q->pre;
                                                                                      Se reduce a contar el numero de paths que inician en q0 y terminan
36
                                                                                      en cualquier nodo. dp[u] = # de paths que inician en u
               q->pre=nq;
37
                                                                                      - Se debe construir el automata en el main(init y Insert's)
               np->pre=nq;
38
               while (p\&\&p->go[w]==q)
                                                                                      - Setear dp en -1
39
                   p->go[w]=nq, p=p->pre;
                                                                                      */
                                                                                   83
40
                                                                                      number dp[N * 2];
           }
41
       }
                                                                                      number num_dist_substr(State *u = root) {
42
                                                                                          if (dp[u - statePool] != -1) return dp[u - statePool];
       last=np;
43
                                                                                          number ans = 1;//el path vacio que representa este nodo
                                                                                   87
44
                                                                                          for (int v = 0; v < 26; v++)//usar for (auto) para mapa
    *####################### Suffix Automata #################*/
                                                                                   88
45
                                                                                               if (u->go[v])
                                                                                   89
46
    /*#################### Algunas aplicaciones ###############*/
                                                                                                   ans += num_dist_substr(u->go[v]);
                                                                                   90
   //Obtiene el LCSubstring de 2 cadenas en O(|A| + |B|)
                                                                                          return (dp[u - statePool] = ans);
                                                                                   91
   string lcs(char A[N], char B[N]) {
                                                                                   92
       int n.m:
                                                                                   93
50
       n = strlen(A); m = strlen(B);
                                                                                       /*Suma la longitud de todos los substrings en O(|A|)
51
       //Construccion: O(|A|)
                                                                                      - Construir el automata(init y insert's)
52
                                                                                      - Necesita el metodo num dist substr (el de arriba)
       //solo hacerlo una vez si A no cambia
53
       init();
                                                                                      - setear dp's en -1
                                                                                   97
54
                                                                                      */
       for(int i=0; i<n; i++)
                                                                                   98
55
           Insert(A[i]-'a'); //Fin construccion
                                                                                      number dp1[N * 2];
56
       //LCS: 0(|B|)
                                                                                      number sum_length_dist_substr(State *u = root) {
57
                                                                                          if (dp1[u - statePool] != -1) return dp1[u - statePool];
       int ans = 0, len = 0, bestpos = 0;
58
                                                                                  101
```

```
number ans = 0;//el path vacio que representa este nodo
102
        for (int v = 0; v < 26; v++)//usar for (auto) para mapa
103
            if (u->go[v])
104
                 ans += (num_dist_substr(u->go[v]) + sum_length_dist_substr(u
105
                     ->go[v]));
        return (dp1[u - statePool] = ans);
106
107
108
109
    Pregunta si p es subcadena de la cadena con la cual esta construida
110
    el automata.
111
    Complejidad: - Construir O(|Texto|) - solo una vez (init e insert's)
112
                  - Por Consulta O(|patron a buscar|)
113
114
    bool is_substring(char p[N]) {
115
        State *u = root;
116
        for (int i = 0; p[i]; i++) {
117
            if (!u->go.count(p[i]))//esta con map!!!
118
                 return false:
119
            u = u \rightarrow go[p[i]];//esta con map!!!
120
        }
121
        return true;
122
123 }
```

4.7 K-esima permutacion de una cadena

```
//Entrada: Una cadena cad(std::string), un long th
    //Salida : La th-esima permutacion lexicografica de cad
   string ipermutacion(string cad, long long int th){
     sort(cad.begin(), cad.end());
4
     string sol = "";
5
     int pos;
6
     for(int c = cad.size() - 1; c \ge 0; c = 0)
       pos = th / fact[c];
       th %= fact[c]:
9
       sol += cad[pos];
10
       cad.erase(cad.begin() + pos);
11
     }
12
     return sol;
13
14 | }
```

5 Geometria

5.1 Interseccion de circunferencias - Sacar de Agustin

5.2 Graham Scan

5.3 Cortar Poligono

```
//cuts polygon Q along the line ab
//stores the left side (swap a, b for the right one) in P
void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
    P.clear();
    forn(i, sz(Q)){
        double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
        if(left1>=0) P.pb(Q[i]);
        if(left1*left2<0)
        P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
    }
}</pre>
```

5.4 Interseccion de rectangulos

```
#define MAXC 2501
   struct Rect{
     int x1, y1, x2, y2;
     int color;
     int area:
     Rect(int _x1, int _y1, int _x2, int _y2){
6
       x1 = x1;
7
       y1 = _y1;
8
       x2 = x2;
9
       y2 = _y2;
10
       getArea();
11
12
     int getArea(){
13
       if(x1>=x2 || y1>=y2)return area = 0;
14
       return area = (x2-x1)*(y2-y1);
15
     }
16
   }:
17
   Rect interseccion(Rect t, Rect r){
     int x1,y1,x2,y2;
19
     x1 = max(t.x1,r.x1);
20
     y1 = max(t.y1,r.y1);
21
     x2 = min(t.x2,r.x2);
22
```

```
y2 = min(t.y2,r.y2);
     Rect res(x1,y1,x2,y2);
     return res;
25
26 | }
```

Distancia punto-recta

```
double distance_point_to_line(const point &a, const point &b, const
      point &pnt){
    double u = ((pnt.x - a.x)*(b.x - a.x) + (pnt.y - a.y)*(b.y - a.y)) /
2
        distsqr(a, b);
    point intersection;
    intersection.x = a.x + u*(b.x - a.x);
    intersection.y = a.y + u*(b.y - a.y);
    return dist(pnt, intersection);
6
7 |}
```

Distancia punto-segmento

```
1 | struct point{
     double x,y;
3
   inline double dist(const point &a, const point &b){
     return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
5
6
   inline double distsqr(const point &a, const point &b){
     return (a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y);
9
   double distance_point_to_segment(const point &a, const point &b, const
       point &pnt){
     double u = ((pnt.x - a.x)*(b.x - a.x) + (pnt.y - a.y)*(b.y - a.y)) /
11
         distsqr(a, b);
     point intersection;
12
     intersection.x = a.x + u*(b.x - a.x);
13
     intersection.y = a.y + u*(b.y - a.y);
14
15
     if (u < 0.0 | l | u > 1.0)
16
       return min(dist(a, pnt), dist(b, pnt));
17
18
     return dist(pnt, intersection);
19
20
```

5.7 Parametrizacion de rectas - Sacar de codeforces

Math

6.1 Identidades

$$\sum_{i=0}^{n} \binom{n}{i} = 2^{n}$$

$$\sum_{i=0}^{n} i \binom{n}{i} = n * 2^{n-1}$$

$$\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$$

$$\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^{2} = \frac{n(n+1)(2n+1)}{6} = \frac{n^{3}}{3} + \frac{n^{2}}{2} + \frac{n}{6}$$

$$\sum_{i=0}^{n} i (i-1) = \frac{8}{6} (\frac{n}{2}) (\frac{n}{2} + 1)(n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}$$

$$\sum_{i=0}^{n} i^{3} = \left(\frac{n(n+1)}{2}\right)^{2} = \frac{n^{4}}{4} + \frac{n^{3}}{2} + \frac{n^{2}}{4} = \left[\sum_{i=1}^{n} i\right]^{2}$$

$$\sum_{i=0}^{n} i^{4} = \frac{n(n+1)(2n+1)(3n^{2}+3n-1)}{30} = \frac{n^{5}}{5} + \frac{n^{4}}{2} + \frac{n^{3}}{3} - \frac{n}{30}$$

$$\sum_{i=0}^{n} i^{p} = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_{k}}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde) $A = I + \frac{B}{2} - 1$

6.2 Ec. Caracteristica

$$\begin{aligned} a_0T(n) + a_1T(n-1) + \ldots + a_kT(n-k) &= 0 \\ p(x) = a_0x^k + a_1x^{k-1} + \ldots + a_k \end{aligned}$$
 Sean r_1, r_2, \ldots, r_q las raíces distintas, de mult. m_1, m_2, \ldots, m_q
$$T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$
 Las constantes c_{ij} se determinan por los casos base.

6.3 Identidades de agustin y mario

6.4 Combinatorio

```
1 | forn(i, MAXN+1){//comb[i][k]=i tomados de a k
     comb[i][0]=comb[i][i]=1;
    forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
4
  ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p]
       precalculado.
    11 \text{ aux} = 1:
    while (n + k) aux = (aux * comb[n\%p][k\%p]) \%p, n/=p, k/=p;
8
    return aux;
9
```

6.5 Exp. de Numeros Mod.

```
1 | ll expmod (ll b, ll e, ll m){//0(log b)
2 | if(!e) return 1;
3 | ll q= expmod(b,e/2,m); q=(q*q)%m;
4 | return e%2? (b * q)%m : q;
5 |}
```

6.6 Exp. de Matrices y Fibonacci en log(n) - Sacar de Agustin

6.7 Gauss Jordan

```
const int N = 300;
   typedef vector<double> col;
   typedef vector<double> row;
   typedef vector<row>Matrix;
   col solution;
   int main(){
     Matrix M;
7
     M.resize(300);
     solution.resize(300);
     for(int i = 0; i < 30; i++)M[i].resize(30);
     int n;
11
     cin >> n;
12
     for(int i = 0: i < n:i++)
13
       for(int j = 0; j \le n; j++)
14
         cin >> M[i][j];
15
16
     for(int j = 0; j < n - 1; j++){
17
       int 1 = j;
18
       for(int i = j + 1; i < n; i++){
19
         if(fabs(M[i][j]) > fabs(M[l][j]))l = i;
20
^{21}
       for(int k = j; k \le n; k++){
^{22}
         swap(M[j][k],M[l][k]);
23
24
       for(int i = j + 1; i < n; i++)
25
         for(int k = n; k \ge j; k--)
26
           M[i][k] -= M[j][k] * M[i][j] / M[j][j];
27
     }
28
     double t = 0:
29
     for(int j = n - 1; j \ge 0; j--){
30
       t = 0.0:
31
       for(int k = j + 1; k < n; k++)t += M[j][k] * solution[k];
32
```

```
solution[j] = (M[j][n] - t) / M[j][j];

solution[j] = (M[j][n] - t) / M[j][j];

cout.precision(4);

for(int i = 0; i < n;i++)cout<<fixed << solution[i] << """;

return 0;

solution[j] = (M[j][n] - t) / M[j][j];

return 0;

solution[j] = (M[j][n] - t) / M[j][j];

return 0;

solution[j] = (M[j][n] - t) / M[j][j];

letter in the property of the p
```

6.8 Simplex

```
1
   #include <iostream>
   #include <iomanip>
   #include <vector>
   #include <cmath>
   #include <limits>
   using namespace std;
   typedef long double DOUBLE;
   typedef vector<DOUBLE> VD;
   typedef vector<VD> VVD;
   typedef vector<int> VI;
   const DOUBLE EPS = 1e-9:
16
   struct LPSolver {
     int m, n;
     VI B, N;
19
     VVD D;
20
21
     LPSolver(const VVD &A, const VD &b, const VD &c):
22
       m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
23
       for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[
24
           i][i];
       for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + i]
           1] = b[i]; }
       for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
       N[n] = -1; D[m + 1][n] = 1;
27
     }
28
29
     void Pivot(int r, int s) {
30
       double inv = 1.0 / D[r][s];
31
       for (int i = 0; i < m + 2; i++) if (i != r)
32
```

```
forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j
         for (int j = 0; j < n + 2; j++) if (j != s)
33
                                                                                  10
           D[i][j] -= D[r][j] * D[i][s] * inv;
                                                                                                 ];
34
       for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
                                                                                  11
                                                                                             return m; }
35
                                                                                         Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
       for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
                                                                                  12
36
       D[r][s] = inv;
                                                                                             int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
                                                                                  13
37
       swap(B[r], N[s]);
                                                                                             Mat mat(n,t);
38
                                                                                  14
                                                                                             forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
                                                                                  15
39
                                                                                             return mat;
                                                                                  16
40
     bool Simplex(int phase) {
                                                                                         double determinant(){//sacado de e maxx ru
                                                                                  17
41
       int x = phase == 1 ? m + 1 : m;
                                                                                             double det = 1;
42
       while (true) {
                                                                                             int n = sz(vec);
43
                                                                                  19
         int s = -1;
                                                                                             Mat m(*this);
44
         for (int j = 0; j \le n; j++) {
                                                                                             forn(i, n){//para cada columna
45
                                                                                  21
           if (phase == 2 && N[j] == -1) continue;
                                                                                                 int k = i:
                                                                                  22
           if (s == -1 \mid | D[x][i] < D[x][s] \mid | D[x][i] == D[x][s] && N[i] <
                                                                                                 forr(j, i+1, n)//busco la fila con mayor val abs
47
                                                                                  23
                N[s]) s = j;
                                                                                                      if(abs(m[j][i])>abs(m[k][i])) k = j;
                                                                                  24
         }
                                                                                                 if(abs(m[k][i])<1e-9) return 0;
48
                                                                                  25
         if (D[x][s] > -EPS) return true;
                                                                                                 m[i].swap(m[k]);//la swapeo
49
         int r = -1:
                                                                                                 if(i!=k) det = -det:
                                                                                  27
50
         for (int i = 0; i < m; i++) {
                                                                                                 det *= m[i][i];
51
           if (D[i][s] < EPS) continue;
                                                                                                 forr(j, i+1, n) m[i][j] /= m[i][i];
52
           if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                                                                                                 //hago 0 todas las otras filas
53
             (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B
                                                                                                 forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
                                                                                  31
54
                 [r]) r = i;
                                                                                                     forr(k, i+1, n) m[j][k]-=m[i][k]*m[j][i];
         }
                                                                                             }
                                                                                  33
55
         if (r == -1) return false;
                                                                                             return det;
                                                                                  34
56
         Pivot(r, s);
57
                                                                                  35
       }
                                                                                     };
                                                                                  36
58
     }
                                                                                  37
59
                                                                                     int n;
60
     DOUBLE Solve(VD &x) {
                                                                                     int main() {
61
                                                                                     //DETERMINANTE:
                 6.9 Matrices y determinante O(n^3)
                                                                                     //https://uva.onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&
                                                                                         page=show_problem&problem=625
                                                                                       freopen("input.in", "r", stdin);
  struct Mat {
                                                                                         ios::sync_with_stdio(0);
       vector<vector<double> > vec;
2
                                                                                         while(cin >> n && n){
       Mat(int n): vec(n, vector<double>(n) ) {}
                                                                                  44
3
                                                                                             Mat m(n):
       Mat(int n, int m): vec(n, vector<double>(m) ) {}
                                                                                             forn(i, n) forn(j, n) cin >> m[i][j];
       vector<double> &operator[](int f){return vec[f];}
5
                                                                                             cout << (11)round(m.determinant()) << endl;</pre>
       const vector<double> &operator[](int f) const {return vec[f];}
                                                                                  47
       int size() const {return sz(vec);}
                                                                                  48
                                                                                         cout << "*" << endl;
       Mat operator+(Mat &b) { ///this de n x m entonces b de n x m
                                                                                  49
                                                                                       return 0;
           Mat m(sz(b), sz(b[0]));
                                                                                  50
9
```

51 }

49

```
6.10 Teorema Chino del Resto
```

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

6.11 Criba

```
#define MAXP 100000 //no necesariamente primo
   int criba[MAXP+1]:
   void crearcriba(){
     int w[] = \{4,2,4,2,4,6,2,6\};
     for(int p=25;p<=MAXP;p+=10) criba[p]=5;</pre>
     for(int p=9;p<=MAXP;p+=6) criba[p]=3;</pre>
     for(int p=4;p<=MAXP;p+=2) criba[p]=2;</pre>
     for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])</pre>
       for(int j=p*p; j<=MAXP; j+=(p<<1)) if(!criba[j]) criba[j]=p;</pre>
10
   vector<int> primos;
   void buscarprimos(){
     crearcriba();
     forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
15
   //^{\sim} Useful for bit trick: #define SET(i) ( criba[(i)>>5]|=1<<((i)&31) ),
        #define INDEX(i) ( (criba[i>>5]>>((i)&31))&1 ), unsigned int criba[
       MAXP/32+1];
17
18
   int main() {
19
     freopen("primos", "w", stdout);
20
     buscarprimos();
```

6.12 Funciones de primos

Sea $n = \prod p_i^{k_i}$, fact(n) genera un map donde a cada p_i le asocia su k_i

```
//factoriza bien numeros hasta MAXP^2
  map<ll,ll> fact(ll n){ //0 (cant primos)
    map<11,11> ret;
    forall(p, primos){
      while(!(n%*p)){
        ret[*p]++;//divisor found
6
        n/=*p;
7
```

```
9
     if(n>1) ret[n]++;
10
     return ret;
11
12
   //factoriza bien numeros hasta MAXP
   map<11,11> fact2(11 n){ //0 (lg n)
     map<ll,ll> ret;
     while (criba[n]){
       ret[criba[n]]++;
       n/=criba[n];
18
19
     if(n>1) ret[n]++;
20
     return ret;
21
22
   //Usar asi: divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
   void divisores(const map<11,11> &f, vector<11> &divs, map<11,11>::
       iterator it, ll n=1){
       if(it==f.begin()) divs.clear();
       if(it==f.end()) { divs.pb(n); return; }
       ll p=it->fst, k=it->snd; ++it;
27
       forn(_, k+1) divisores(f, divs, it, n), n*=p;
29
   11 sumDiv (ll n){
     ll rta = 1;
31
     map<ll,ll> f=fact(n);
     forall(it, f) {
33
     11 pot = 1, aux = 0;
34
     forn(i, it->snd+1) aux += pot, pot *= it->fst;
35
     rta*=aux;
37
     return rta;
38
39
   ll eulerPhi (ll n){ // con criba: O(lg n)
     11 \text{ rta} = n;
41
     map<11,11> f=fact(n);
     forall(it, f) rta -= rta / it->first;
43
     return rta;
44
45
   11 eulerPhi2 (11 n){ // 0 (sqrt n)
     11 r = n;
     forr (i,2,n+1){
       if ((11)i*i > n) break;
```

```
if (n \% i == 0){
          while (n\%i == 0) n/=i;
51
         r = r/i; }
52
53
     if (n != 1) r= r/n;
     return r;
55
56
57
   int main() {
58
     buscarprimos();
59
     forr (x,1, 500000){
60
       cout << "x_1 = 1" << x << endl;
61
       cout << "Numero | de | factores | primos: | " << numPrimeFactors(x) << endl;</pre>
62
       cout << "Numero de distintos factores primos: " <<
            numDiffPrimeFactors(x) << endl:</pre>
       cout << "Suma, de, factores, primos:,," << sumPrimeFactors(x) << endl;
64
       cout << "Numero de divisores:" << numDiv(x) << endl;</pre>
65
       cout << "Suma, de, divisores:" << sumDiv(x) << endl;</pre>
66
       cout << "Phi de Euler: " << eulerPhi(x) << endl:
67
68
     return 0;
69
70 }
```

6.13 Phollard's Rho (rolando)

```
1 | ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}
2
   11 mulmod (11 a, 11 b, 11 c) { //returns (a*b)%c, and minimize overfloor
    11 x = 0, y = a\%c;
     while (b > 0){
      if (b \% 2 == 1) x = (x+y) \% c;
       y = (y*2) \% c;
       b /= 2;
9
     return x % c;
10
11
12
   ll expmod (ll b, ll e, ll m){//0(\log b)}
     if(!e) return 1:
     ll q = \exp(b, e/2, m); q = \min(q, q, m);
15
    return e%2? mulmod(b,q,m) : q;
16
17 }
```

```
18
   bool es_primo_prob (ll n, int a)
20
     if (n == a) return true;
21
     11 s = 0, d = n-1;
     while (d \% 2 == 0) s++, d/=2;
24
    ll x = expmod(a,d,n);
25
     if ((x == 1) \mid | (x+1 == n)) return true;
    forn (i, s-1){
28
      x = mulmod(x, x, n);
      if (x == 1) return false:
       if (x+1 == n) return true:
31
32
     return false;
33
   }
34
35
   bool rabin (ll n){ //devuelve true si n es primo
     if (n == 1) return false;
     const int ar[] = \{2,3,5,7,11,13,17,19,23\};
    forn (j,9)
      if (!es_primo_prob(n,ar[j]))
         return false;
41
     return true;
42
43
44
   ll rho(ll n){
       if( (n \& 1) == 0 ) return 2;
       11 x = 2 , y = 2 , d = 1;
       ll c = rand() % n + 1;
48
       while( d == 1 ){
           x = (mulmod(x, x, n) + c)%n;
           y = (mulmod(y, y, n) + c)%n;
51
           y = (mulmod(y, y, n) + c)%n;
           if(x - y \ge 0) d = gcd(x - y, n);
           else d = gcd(y - x, n);
54
55
       return d==n? rho(n):d;
57
   map<ll,ll> prim;
void factRho (ll n){ //O (lg n)^3. un solo numero
```

forn(i, n){

```
if (n == 1) return:
61
     if (rabin(n)){
62
       prim[n]++;
63
       return;
64
65
     11 factor = rho(n);
66
     factRho(factor);
67
     factRho(n/factor);
68
  |}
69
                                6.14 GCD
1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
                         6.15 Extended Euclid
  void extendedEuclid (ll a, ll b) { \frac{1}{a} * x + b * y = d
     if (!b) { x = 1; y = 0; d = a; return;}
2
     extendedEuclid (b, a%b);
3
     11 x1 = y;
4
     11 y1 = x - (a/b) * y;
     x = x1; y = y1;
6
7
  |}
                                6.16 LCM
1 | tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
                              6.17 Inversos
  #define MAXMOD 15485867
   11 inv[MAXMOD];//inv[i]*i=1 mod MOD
   void calc(int p){\frac{}{0}}
     inv[1]=1;
4
     forr(i, 2, p) inv[i] = p-((p/i)*inv[p\%i])\%p;
5
6
   int inverso(int x){\frac{1}{\log x}}
     return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
     return expmod(x, MOD-2);//si mod es primo
9
10 }
                              6.18 Simpson
  double integral (double a, double b, int n=10000) {//O(n), n=cantdiv
     double area=0, h=(b-a)/n, fa=f(a), fb;
```

```
fb=f(a+h*(i+1));
4
       area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
5
6
     return area*h/6.;}
                              6.19 Fraction
   tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
   struct frac{
     tipo p,q:
     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
     void norm(){
5
       tipo a = mcd(p,q);
6
       if(a) p/=a, q/=a;
7
       else q=1;
8
       if (q<0) q=-q, p=-p;}
9
     frac operator+(const frac& o){
10
       tipo a = mcd(q, o.q);
11
       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
12
     frac operator-(const frac& o){
13
       tipo a = mcd(q, o.q);
14
       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
15
     frac operator*(frac o){
16
       tipo a = mcd(q, o.p), b = mcd(o.q, p);
17
       return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
18
     frac operator/(frac o){
19
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
20
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
21
     bool operator<(const frac &o) const{return p*o.q < o.p*q;}</pre>
22
     bool operator==(frac o){return p==o.p&&q==o.q;}
23
24 };
                             6.20 Polinomio
           int m = sz(c), n = sz(o.c);
1
           vector<tipo> res(max(m,n));
2
           forn(i, m) res[i] += c[i];
3
           forn(i, n) res[i] += o.c[i];
4
           return poly(res); }
5
       poly operator*(const tipo cons) const {
6
       vector<tipo> res(sz(c));
7
           forn(i, sz(c)) res[i]=c[i]*cons;
8
           return poly(res);
9
```

```
poly operator*(const poly &o) const {
                                                                                         S = S + Li * y[i]; }
10
                                                                                 51
           int m = sz(c), n = sz(o.c);
                                                                                      return S;
                                                                                 52
11
           vector<tipo> res(m+n-1);
                                                                                    }
                                                                                 53
12
           forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
13
                                                                                 54
           return poly(res);
                                                                                 55 int main(){
14
     tipo eval(tipo v) {
                                                                                      return 0;
15
                                                                                 57 }
       tipo sum = 0;
16
       dforn(i, sz(c)) sum=sum*v + c[i];
17
                                                                                                             6.21 Ec. Lineales
       return sum: }
18
       //poly contains only a vector<int> c (the coeficients)
19
     //the following function generates the roots of the polynomial
                                                                                  | bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
20
    //it can be easily modified to return float roots
                                                                                      int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
21
     set<tipo> roots(){
                                                                                      vector<int> p; forn(i,m) p.push_back(i);
22
       set<tipo> roots;
                                                                                      forn(i, rw) {
23
       tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
                                                                                        int uc=i, uf=i;
24
                                                                                  5
       vector<tipo> ps,qs;
                                                                                        forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;
25
                                                                                  6
       forr(p,1,sqrt(a0)+1) if (a0\%p==0) ps.pb(p),ps.pb(a0/p);
26
                                                                                             uc=c;}
       forr(q,1,sqrt(an)+1) if (an\%q==0) qs.pb(q),qs.pb(an/q);
                                                                                        if (feq(a[uf][uc], 0)) { rw = i; break; }
27
                                                                                  7
       forall(pt.ps)
                                                                                        forn(j, n) swap(a[j][i], a[j][uc]);
28
        forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
                                                                                        swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
29
           tipo root = abs((*pt) / (*qt));
                                                                                         tipo inv = 1 / a[i][i]; //aca divide
30
           if (eval(root)==0) roots.insert(root);
                                                                                        forr(j, i+1, n) {
31
         }
                                                                                          tipo v = a[j][i] * inv;
32
       return roots; }
                                                                                          forr(k, i, m) a[j][k]-=v * a[i][k];
33
                                                                                          y[j] -= v*y[i];
34
                                                                                 14
   pair<poly,tipo> ruffini(const poly p, tipo r) {
                                                                                 15
     int n = sz(p.c) - 1;
                                                                                      } // rw = rango(a), aca la matriz esta triangulada
36
                                                                                 16
     vector<tipo> b(n);
                                                                                      forr(i, rw, n) if (!feq(y[i],0)) return false; // checkeo de
37
                                                                                 17
     b[n-1] = p.c[n];
                                                                                           compatibilidad
38
     dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
                                                                                      x = vector < tipo > (m, 0);
39
                                                                                 18
     tipo resto = p.c[0] + r*b[0];
                                                                                      dforn(i, rw){
40
                                                                                 19
     poly result(b);
41
                                                                                        tipo s = v[i];
                                                                                 20
     return make_pair(result,resto);
                                                                                        forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
42
                                                                                 21
                                                                                        x[p[i]] = s / a[i][i]; //aca divide
43
                                                                                 22
   poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
                                                                                      }
44
                                                                                 23
       poly A; A.c.pb(1);
                                                                                      ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
45
                                                                                 24
       forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux;
46
                                                                                      forn(k, m-rw) {
                                                                                 25
            }
                                                                                         ev[k][p[k+rw]] = 1;
                                                                                 26
     poly S; S.c.pb(0);
                                                                                         dforn(i, rw){
47
                                                                                 27
     forn(i,sz(x)) { poly Li;
                                                                                           tipo s = -a[i][k+rw];
48
                                                                                 28
       Li = ruffini(A,x[i]).fst;
                                                                                          forr(j, i+1, rw) s -= a[i][j]*ev[k][p[i]];
49
                                                                                 29
       Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
                                                                                           ev[k][p[i]] = s / a[i][i]; //aca divide
50
                                                                                 30
           coefficients instead of 1.0 to avoid using double
                                                                                 31
```

```
}
                                                                                            }
32
                                                                                     2
     return true;
33
                                                                                     3
34 }
                                                                                            Complex w,wn;
                                                                                     4
                                                                                     5
                              6.22 Karatsuba
                                                                                            for(int i = 1; i \le p; ++i){
                                                                                     6
                                                                                                int M = (1 << i), K = (M >> 1);
   |void addTo(vector<int>& a, const vector<int>& b, int k){
                                                                                                wn = Complex(cos(s*2.0*M_PI/(double)M), sin(s*2.0*M_PI/(double)M
       if (a.size() < b.size() + k) a.resize(b.size() + k);</pre>
2
                                                                                                    ));
       for (int i = 0; i < b.size(); i++) a[i+k] += b[i];
3
4
                                                                                                for(int i = 0; i < n; i += M){
   void subFrom(vector<int>& a, const vector<int>& b){
5
                                                                                                    w = Complex(1.0, 0.0);
                                                                                    11
       for (int i = 0; i < b.size(); i++) a[i] -= b[i];
6
                                                                                                    for(int 1 = j; 1 < K + j; ++1){
                                                                                    12
7
                                                                                                        Complex t = w;
                                                                                    13
   // a = a + b
                                                                                                        t *= A[1 + K]:
   void addTo(vector<int>& a, const vector<int>& b){
                                                                                                        Complex u = A[1];
                                                                                    15
       addTo(a, b, 0);
10
                                                                                                        A[1] += t;
11
                                                                                                        u -= t;
                                                                                    17
   vector<int> karatsuba(const vector<int>& a. const vector<int>& b)
12
                                                                                                        A[1 + K] = u;
13
                                                                                                        w *= wn:
                                                                                    19
       int alen = a.size();
14
                                                                                                    }
       int blen = b.size();
15
                                                                                                }
                                                                                    21
       if (alen == 0 || blen == 0) return vector<int>();
16
                                                                                            }
                                                                                    22
       if (alen < blen) return karatsuba(b, a);</pre>
17
                                                                                    23
       if (alen < 50) return multiply(a, b);
18
                                                                                            if(s==-1){
19
                                                                                                for(int i = 0; i < n; ++i)
                                                                                    25
       int half = alen / 2;
20
                                                                                                    A[i] /= (double)n;
                                                                                    26
       vector<int> a0(a.begin(), a.begin() + half);
21
                                                                                            }
                                                                                    27
       vector<int> a1(a.begin() + half, a.end());
22
                                                                                    28
       vector<int> b0(b.begin(), b.begin() + min<int>(blen, half));
23
                                                                                    29
       vector<int> b1(b.begin() + min<int>(blen, half), b.end());
24
                                                                                        vector<Complex> FFT_Multiply(vector<Complex> &P, vector<Complex> &Q){
25
                                                                                            int n = P.size()+Q.size();
                                                                                    31
       vector<int> z0 = karatsuba(a0, b0);
26
                                                                                            while(n!=lowbit(n)) n += lowbit(n);
                                                                                    32
       vector<int> z2 = karatsuba(a1, b1);
27
                                                                                    33
       addTo(a0, a1);
28
                                                                                            P.resize(n.0):
                                                                                    34
       addTo(b0, b1);
29
                                                                                            Q.resize(n,0);
       vector<int> z1 = karatsuba(a0, b0);
30
                                                                                    36
       subFrom(z1, z0);
31
                                                                                            FFT(P,1);
                                                                                    37
       subFrom(z1, z2);
32
                                                                                            FFT(Q,1);
33
                                                                                    39
       vector<int> res;
34
                                                                                            vector<Complex> R;
                                                                                    40
                                                                                            for(int i=0;i<n;i++) R.push_back(P[i]*Q[i]);</pre>
                                 6.23 FFT
                                                                                    41
                                                                                    42
                                                                                            FFT(R,-1);
           A[i] = a[rev];
                                                                                    43
1
```

```
return R;

return R;

// Para multiplicacion de enteros grandes
const long long B = 100000;
const int D = 5;
```

6.24 Tablas y cotas (Primos, Divisores, Factoriales, etc)

Factoriales		
0! = 1	11! = 39.916.800	
1! = 1	$12! = 479.001.600 \ (\in \mathtt{int})$	
2! = 2	13! = 6.227.020.800	
3! = 6	14! = 87.178.291.200	
4! = 24	15! = 1.307.674.368.000	
5! = 120	16! = 20.922.789.888.000	
6! = 720	17! = 355.687.428.096.000	
7! = 5.040	18! = 6.402.373.705.728.000	
8! = 40.320	19! = 121.645.100.408.832.000	
9! = 362.880	$20! = 2.432.902.008.176.640.000 (\in tint)$	
10! = 3.628.800	21! = 51.090.942.171.709.400.000	
$\max \text{ signed tint} = 9.223.372.036.854.775.807$		
max unsigned tint = $18.446.744.073.709.551.615$		

Primos

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 $113\ 127\ 131\ 137\ 139\ 149\ 151\ 157\ 163\ 167\ 173\ 179\ 181\ 191\ 193\ 197\ 199\ 211\ 223\ 227$ 229 233 239 241 251 257 263 269 271 277 281 283 293 307 311 313 317 331 337 347 349 353 359 367 373 379 383 389 397 401 409 419 421 431 433 439 443 449 457 461 $463\ 467\ 479\ 487\ 491\ 499\ 503\ 509\ 521\ 523\ 541\ 547\ 557\ 563\ 569\ 571\ 577\ 587\ 593\ 599$ $601\ 607\ 613\ 617\ 619\ 631\ 641\ 643\ 647\ 653\ 659\ 661\ 673\ 677\ 683\ 691\ 701\ 709\ 719\ 727$ $733\ 739\ 743\ 751\ 757\ 761\ 769\ 773\ 787\ 797\ 809\ 811\ 821\ 823\ 827\ 829\ 839\ 853\ 857\ 859$ 863 877 881 883 887 907 911 919 929 937 941 947 953 967 971 977 983 991 997 1009 1013 1019 1021 1031 1033 1039 1049 1051 1061 1063 1069 1087 1091 1093 1097 1103 $1109\ 1117\ 1123\ 1129\ 1151\ 1153\ 1163\ 1171\ 1181\ 1187\ 1193\ 1201\ 1213\ 1217\ 1223\ 1229$ 1231 1237 1249 1259 1277 1279 1283 1289 1291 1297 1301 1303 1307 1319 1321 1327 1361 1367 1373 1381 1399 1409 1423 1427 1429 1433 1439 1447 1451 1453 1459 1471 1481 1483 1487 1489 1493 1499 1511 1523 1531 1543 1549 1553 1559 1567 1571 1579 $1583\ 1597\ 1601\ 1607\ 1609\ 1613\ 1619\ 1621\ 1627\ 1637\ 1657\ 1663\ 1667\ 1669\ 1693\ 1697$ $1699\ 1709\ 1721\ 1723\ 1733\ 1741\ 1747\ 1753\ 1759\ 1777\ 1783\ 1787\ 1789\ 1801\ 1811\ 1823$ $1831\ 1847\ 1861\ 1867\ 1871\ 1873\ 1877\ 1879\ 1889\ 1901\ 1907\ 1913\ 1931\ 1933\ 1949\ 1951$ 1973 1979 1987 1993 1997 1999 2003 2011 2017 2027 2029 2039 2053 2063 2069 2081

Primos cercanos a 10^n

 $\begin{array}{c} 9941\ 9949\ 9967\ 9973\ 10007\ 10009\ 10037\ 10039\ 10061\ 10067\ 10069\ 10079 \\ 99961\ 99971\ 99989\ 99991\ 100003\ 100003\ 1000033\ 1000037\ 1000039 \\ 9999943\ 9999971\ 99999991\ 10000019\ 10000079\ 10000103\ 10000121 \\ 99999941\ 99999959\ 9999971\ 99999989\ 100000007\ 100000037\ 100000039\ 100000049 \\ 99999893\ 99999929\ 99999937\ 1000000007\ 1000000009\ 1000000021\ 1000000033 \end{array}$

Cantidad de primos menores que 10^n

```
\pi(10^1) = 4; \pi(10^2) = 25; \pi(10^3) = 168; \pi(10^4) = 1229; \pi(10^5) = 9592

\pi(10^6) = 78.498; \pi(10^7) = 664.579; \pi(10^8) = 5.761.455; \pi(10^9) = 50.847.534

\pi(10^{10}) = 455.052,511; \pi(10^{11}) = 4.118.054.813; \pi(10^{12}) = 37.607.912.018
```

Divisores

```
Cantidad de divisores (\sigma_0) para algunos n/\neg \exists n' < n, \sigma_0(n') \geqslant \sigma_0(n)
       \sigma_0(60) = 12; \sigma_0(120) = 16; \sigma_0(180) = 18; \sigma_0(240) = 20; \sigma_0(360) = 24
    \sigma_0(720) = 30; \sigma_0(840) = 32; \sigma_0(1260) = 36; \sigma_0(1680) = 40; \sigma_0(10080) = 72
        \sigma_0(15120) = 80; \sigma_0(50400) = 108; \sigma_0(83160) = 128; \sigma_0(110880) = 144
    \sigma_0(498960) = 200; \sigma_0(554400) = 216; \sigma_0(1081080) = 256; \sigma_0(1441440) = 288
                            \sigma_0(4324320) = 384 : \sigma_0(8648640) = 448
             Suma de divisores (\sigma_1) para algunos n/\neg \exists n' < n, \sigma_1(n') \ge \sigma_1(n)
    \sigma_1(96) = 252; \sigma_1(108) = 280; \sigma_1(120) = 360; \sigma_1(144) = 403; \sigma_1(168) = 480
        \sigma_1(960) = 3048; \sigma_1(1008) = 3224; \sigma_1(1080) = 3600; \sigma_1(1200) = 3844
     \sigma_1(4620) = 16128; \sigma_1(4680) = 16380; \sigma_1(5040) = 19344; \sigma_1(5760) = 19890
   \sigma_1(8820) = 31122; \sigma_1(9240) = 34560; \sigma_1(10080) = 39312; \sigma_1(10920) = 40320
\sigma_1(32760) = 131040; \sigma_1(35280) = 137826; \sigma_1(36960) = 145152; \sigma_1(37800) = 148800
\sigma_1(60480) = 243840; \sigma_1(64680) = 246240; \sigma_1(65520) = 270816; \sigma_1(70560) = 280098
            \sigma_1(95760) = 386880; \sigma_1(98280) = 403200; \sigma_1(100800) = 409448
        \sigma_1(491400) = 2083200; \sigma_1(498960) = 2160576; \sigma_1(514080) = 2177280
        \sigma_1(982800) = 4305280; \sigma_1(997920) = 4390848; \sigma_1(1048320) = 4464096
     \sigma_1(4979520) = 22189440; \sigma_1(4989600) = 22686048; \sigma_1(5045040) = 23154768
    \sigma_1(9896040) = 44323200 : \sigma_1(9959040) = 44553600 : \sigma_1(9979200) = 45732192
```

7 Grafos

7.1 Bellman-Ford

```
vector<ii> G[MAX_N];//ady. list with pairs (weight, dst)
int dist[MAX_N];
void bford(int src){//O(VE)
dist[src]=0;
forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
```

```
dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);

bool hasNegCycle(){
  forn(j, N) if(dist[j]!=INF) forall(it, G[j])
  if(dist[it->snd]>dist[j]+it->fst) return true;

//inside if: all points reachable from it->snd will have -INF distance
  (do bfs)
  return false;
}
```

7.2 dijkstra grafos densos

7.3 2 SAT definitivamente no con Tarjan

```
1 // g++ -std=c++11 "twosat.cpp" -o run
   /***
    Complexity: O(N)
  Input: number of variables, then number of clause clauses in format (u
  if u,v > 0 then is equivalent to u,v
  if u,v < 0 then is equivalent to u, v
  Output: UNSATISFIABLE can't find a solution
  SATISFIABLE if exist a solution then print the assignment of all
      variables (negative for xi = false)
10
  Examples:
11
  Input:
^{12}
  3 3
13
  1 -3
   -1 2
15
   -2 -3
16
  Output
17
  SATISFIABLE
18
  1 2 -3
19
20
  Input
^{21}
  1 2
22
  1 1
  -1 -1
  Output
  UNSATISFIABLE
26
27 */
```

```
28 #include <bits/stdc++.h>
   using namespace std;
   vector<int>G[2][2000010],G2[2000010];
   int n, m;
   int scc[2000010];
   bool vis[2000010];
   vector<int>comp[2000010];
   int assign[2000010];
   int cc = 0;
   stack<int>st;
   vector<int>sta;
   void dfs(int u,int type){
    if(scc[u] != -1)return;
     scc[u] = cc;
    for(int v:G[type][u]){
       dfs(v,type);
     }
44
     if(!type)st.push(u);
45
46
   void topo(int u){
     if(vis[u])return;
48
     vis[u] = true;
49
     for(int v:G2[u])topo(v);
50
     sta.push_back(u);
51
52
   void buildGraphWitouthLoop(){
     for(int i = 0; i < 2 * n; i++){
54
       for(int j = 0; j < G[0][i].size(); j++){</pre>
55
         if(scc[i] != scc[G[0][i][j]])
56
           G2[scc[i]].push_back(scc[G[0][i][j]]);
57
       }
58
     }
59
60
   int main() {
       ios::sync_with_stdio(false);cin.tie(0);
       cin >> n >> m;
      for(int i = 0, u, v; i < m; i++){
       cin >> u >> v:
       int uu = (u > 0?(u - 1) * 2:(-u - 1) * 2 + 1);
       int vv = (v > 0?(v - 1) * 2:(-v - 1) * 2 + 1);
    // cout << uu << " " << (uu ^ 1) << "\n";
       G[0][uu ^ 1].push_back(vv);
       G[0][vv ^ 1].push_back(uu);
70
```

```
G[1][vv].push_back(uu ^ 1);
71
        G[1][uu].push_back(vv ^ 1);
72
 73
      memset(scc,-1,sizeof scc);
 74
      for(int i = 0; i < 2 * n; i++){
 75
        if(scc[i] == -1)dfs(i,0);
76
      }
77
      memset(scc,-1,sizeof scc);
78
      while(!st.empty()){
79
        int u = st.top();st.pop();
80
        if(scc[u] == -1){
 81
          dfs(u,1);
 82
          cc++;
 83
        }
 84
      }
85
      bool unsat = false;
86
      for(int i = 0; i < 2 * n; i++){
87
        if(scc[i] == scc[i ^ 1])unsat = true;
88
        comp[scc[i]].push_back(i);
 89
      }
90
      if(unsat){
91
        return cout << "UNSATISFIABLE",0;</pre>
92
93
      cout << "SATISFIABLE\n";</pre>
94
      buildGraphWitouthLoop();
95
      for(int i = 0; i < 2 * n; i++){
96
        if(!vis[i])topo(i);
97
98
      for(int u:sta){//inverse of topological sort
99
        for(int v:comp[u]){//transitivite Skew-Symmetry
100
          if(!assign[v]){
101
             assign[v] = 1;
102
             assign[v ^ 1] = -1;
103
          }
104
        }
105
106
      for(int i = 0, j = 1; i < 2 * n; i += 2, j++){
107
        cout << (j) * (assign[i]) << "";
108
      }
109
        return 0;
110
111 }
```

7.4 Prim

7.5 Articulataion Points (desgraciadamente tarjan)

```
1 // g++ -std=c++11 "articulationpointsandbridges.cpp" -o run
2
   /***
   ========= <Articulation points and bridges c++ version>
   Given a graph return a vector of paris with the bridges and a bool array
  true if the node is an articulation point
   * false otherwise
   Graph nodes: 0 to N - 1
   */
   using namespace std;
   vector<int>G[10010];
   int low[10010],num[10010],parent[10010],cc;
   //cc is my timer
int art[10010];//bool for detect art point, int for detect how many
       nodes are connected to my articulation point
int root,rC;
   int n;
   vector<pair<int,int> >bridges;
   void dfs(int u){
     low[u]=num[u]=cc++:
     for(int v:G[u]){
19
       if(num[v]==-1){
20
         parent[v]=u;
21
         if(u==root)rC++;
22
         dfs(v);
23
         if(low[v]>=num[u])art[u]++;//is a articulation point
24
         if(low[v]>num[u])bridges.push_back({u,v});//this is a bridge
25
         low[u]=min(low[u],low[v]);
26
27
       else if(v!=parent[u]){
28
           low[u]=min(low[u],num[v]);
29
30
     }
31
32
   void init(){
     bridges.clear();
34
     for(int i=0;i<n;i++){
35
       art[i]=low[i]=0;
36
```

```
num[i]=parent[i]=-1;
37
       G[i].clear();
38
     }
39
     cc=0;
40
41
    void callARTBRID(){
     for(int i=0;i<n;i++){
43
       if(num[i]==-1){
44
         root=i,rC=0;dfs(i);
45
          art[root]=(rC>1);
46
       }
47
     }
48
  |}
49
```

- 7.6 componentes biconexas y puentes (block cut tree)
 - 7.7 LCA saltitos potencias de 2
 - 7.8 LCA sparse table query O(1)

7.9 HLD

```
// g++ -std=c++11 "hld.cpp" -o hld
2
   /***
3
   Complexity: O(N*log (N))
  Given a tree and associative operation in the paths of this tree ask for
      many querys, and updates
  in nodes or edges
   Input of this example:
   N number of nodes, then N elements values in each node
   then n - 1 conections
   Q querys if T == 1 query on the path u,v
  else update node U with value val.
12
13
   Example problems: Spoj QTREE1 to QTREE6, toby and tree UVA
14
15
16
  #include <bits/stdc++.h>
  using namespace std;
18
   const int maxn = 1e5;
  const int NEUTRO = 0; // a null value for my ST
  int vec[maxn];
```

```
vector<int>G[maxn]; //the graph
   //int idx[maxn]; // case with value in the edge
   int op(int u,int v){// an operation for my path (using ST)
    //return __gcd(u,v);
     //return max(u,v);
26
     return u + v;
28
   int n;
29
   //ask to Branimir for information about this
   struct SegmentTree{
     int T[2*maxn];
32
     void init(){
33
       memset(T,0,sizeof T);
34
35
     void set(int pos,int val){
36
       pos += n;
37
       T[pos] = val;
38
       for(pos >>= 1; pos > 0; pos >>=1){
39
         T[pos] = op(T[pos << 1], T[(pos << 1)|1]);
40
41
     }
42
     int query(int 1,int r){
43
       1 += n;
44
       r += n;
45
       int ans = NEUTRO;
46
       while (1 < r)
47
         if (1 \& 1) ans = op(ans, T[1++]);
48
         if (r \& 1) ans = op( ans, T[--r] );
49
         1 >>= 1;
50
         r >>= 1;
51
52
       return ans;
53
     }
54
55
   };
   struct hld{
     int ncad; // store actual number of chain
57
     int root; // the root of a tree generally 0 or 1
58
     int pos; // pos of node in chain
59
60
     int sz[maxn]; // store the subsize of subtrees
61
     int depth[maxn]; //depth of the node, useful for LCA via HLD
62
     int parent[maxn]; // useful for LCA
63
     int where[maxn]; // where chain is the node?
64
```

```
//int edgepos[maxn]; // if the value is on the edge: stored in a node
65
      int chainIdx[maxn]; // position in the chain of the node
                                                                                       108
66
      int head[maxn]; // the head of the i-th chain
                                                                                       109
67
      //int val[maxn]; // if the value is on the edge
                                                                                       110
68
      SegmentTree tree; // this ST allow operations in the path
                                                                                       111
69
                                                                                       112
70
      void init(){//settings value, and process de HLD
                                                                                                    sc = v:
                                                                                       113
71
        root = 0;
                                                                                                  }
                                                                                       114
72
        ncad = 0;
                                                                                                }
                                                                                       115
73
        pos = 0;
                                                                                       116
74
        for(int i = 0; i \le n; i++){
75
                                                                                       117
          where [i] = head[i] = -1;
76
                                                                                       118
        }
77
                                                                                       119
        depth[root] = 0;
                                                                                                    ncad++:
                                                                                       120
78
        dfs(root , -1);
                                                                                       121
79
        descompose(root);
                                                                                       122
80
                                                                                               }
        tree.init();
81
                                                                                       123
        /* case with values in edges
82
                                                                                       124
        for(int i=0:i<n:i++){
                                                                                       125
83
          tree.set(i,val[i]);
                                                                                       126
84
        }
                                                                                       127
85
        */
86
                                                                                       128
      }
                                                                                       129
87
88
                                                                                       130
89
      ///init descomposition
                                                                                       131
90
      void dfs(int u,int pu){
                                                                                       132
91
        sz[u] = 1; //init the sz of this subtree
                                                                                       133
92
        parent[u] = pu; // assign the parent
                                                                                       134
93
        for(int i = 0; i < G[u].size(); i++){</pre>
                                                                                       135
94
          int v = G[u][i];
                                                                                       136
95
          if ( v == pu )continue;
                                                                                             }
                                                                                       137
96
          //edgepos[idx[u][i]] = v;
                                                                                       138
97
          depth[v] = depth[u] + 1;
                                                                                       139
98
          dfs(v,u);
                                                                                       140
99
          sz[u] += sz[v];
                                                                                       141
100
        }
                                                                                       142
101
      }
                                                                                                while(true){
                                                                                       143
102
      //descompose graph in HLD descomposition
                                                                                       144
103
      void descompose(int u){
                                                                                       145
104
        if( head[ncad] == -1)head[ncad] = u; // the head of ncad is u
                                                                                       146
105
        where[u] = ncad; // assign where tu node
106
        //val[pos] = cost; cost another parameter in descompose for graphs
107
                                                                                       147
```

```
with values in edges
  chainIdx[u] = pos++; //assing pos to this node
  int maxi = -1, sc = -1; //finding a special child
  for(int v:G[u]){
    if( sz[v] > maxi && where[v] == -1){
      maxi = sz[v];
  if(sc != -1)descompose(sc);
  //light nodes here:
  for(int v:G[u]){
    if(where[v] == -1){}
      descompose(v);
///end descomposition
int lca(int u,int v){
  while(where[u]!=where[v]){
    if(depth[ head[ where[u] ] ] > depth[ head[ where[v] ] ])u =
        parent[ head[ where[u] ] ];
    else v = parent[ head[ where[v] ] ];
  return depth[u] < depth[v] ? u:v;</pre>
void update(int u, int val){
  tree.set(chainIdx[u],val);
int query(int u,int v){
 // if ( u == v) return NEUTRO; value in edges
  int vChain = where[v];
  int ans = NEUTRO;
    int uChain = where[u];
    if(uChain == vChain){
      // return op(ans, tree.query( chainIdx[v] + 1, chainIdx[u] + 1)
          ); value in edges
      return op(ans, tree.query( chainIdx[v], chainIdx[u] + 1) );
```

```
}
                                                                                            }
148
                                                                                    191
          int hu = head[uChain];
                                                                                    192
                                                                                         }
149
          ans = op( ans, tree.query(chainIdx[hu], chainIdx[u] + 1) );
                                                                                    193 }
150
          u = parent[hu];
151
                                                                                                         7.10 centroid descomposition
        }
152
      }
153
                                                                                                                  7.11 euler cycle
154
      int Q(int u,int v){
155
                                                                                     int n,m,ars[MAXE], eq;
        int L = lca(u,v);
156
                                                                                     vector<int> G[MAXN];//fill G,n,m,ars,eq
        return op( query(u,L) , query(v,L) );
157
                                                                                     3 | list<int> path:
     }
158
                                                                                       int used[MAXN]:
    }HLD;
159
                                                                                       bool usede[MAXE]:
    int main(){
160
                                                                                        queue<list<int>::iterator> q;
      //ios::sync_with_stdio(false);cin.tie(0);
161
                                                                                       int get(int v){
      while(cin >> n){
162
                                                                                          while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;</pre>
        for(int i = 0; i < n; i++)G[i].clear();</pre>
163
                                                                                         return used[v]:
        for(int i = 0; i < n; i++){
164
                                                                                    10
          cin >> vec[i];
165
                                                                                       void explore(int v, int r, list<int>::iterator it){
                                                                                    11
        }
166
                                                                                         int ar=G[v][get(v)]; int u=v^ars[ar];
        for(int i = 1, u,v ; i < n; i++){
167
                                                                                         usede[ar]=true;
                                                                                    13
          cin >> u >> v;
168
                                                                                         list<int>::iterator it2=path.insert(it, u);
                                                                                    14
          G[u].push_back(v);
169
                                                                                         if(u!=r) explore(u, r, it2);
                                                                                    15
          G[v].push_back(u);
170
                                                                                         if(get(v)<sz(G[v])) q.push(it);</pre>
          /* case with value in edges
171
                                                                                    17
          G[u].push_back(make_pair(v,w));
172
                                                                                        void euler(){
          idx[u].push_back(i-1);
173
                                                                                         zero(used), zero(usede);
          G[v].push_back(make_pair(u,w));
174
                                                                                         path.clear();
                                                                                    20
          idx[v].push_back(i-1);
175
                                                                                         q=queue<list<int>::iterator>();
                                                                                    21
176
                                                                                         path.push_back(0); q.push(path.begin());
                                                                                    22
           */
177
                                                                                         while(sz(a)){
                                                                                    23
        }
178
                                                                                            list<int>::iterator it=q.front(); q.pop();
        HLD.init();
179
                                                                                            if(used[*it] < sz(G[*it])) explore(*it, *it, it);</pre>
                                                                                    25
       for(int i = 0; i < n; i++){
180
                                                                                    26
          HLD.update(i, vec[i]);
181
                                                                                         reverse(path.begin(), path.end());
                                                                                    27
        }
182
                                                                                    28
        int question;
183
                                                                                       void addEdge(int u, int v){
        cin >> question;
184
                                                                                         G[u].pb(eq), G[v].pb(eq);
       for(int i = 0, t, u ,v; i < question; i++){
185
                                                                                         ars[eq++]=u^v;
                                                                                    31
          cin >> t >> u >> v;
186
                                                                                    32 }
          if(t == 1){
187
            cout << HLD.Q(u,v) << "\n";
                                                                                                     7.12 diámetro y centro de un árbol
188
189
          else HLD.update(u,v);
                                                                                     1 /***
190
```

```
======== <Diameter and center of a tree> ==============
   //Problem: Given a tree get the center (or centers)
  /* the nodes in the tree that minimize the length of the longest path
       from it to any other node.
  * *Finding tree centers:
    * If diameter length is even, then we have one tree center. If odd,
        then we have 2 centers.
    * E.g. 1-2-3-4-5 -> center is 3
    * E.g. 1-2-3-4-5-6 \rightarrow center is 3, 4
    * On other side, we can get the worst nodes through the center nodes.
    * A worst node is one that is an end of a diameter, so it has the worst
         tree height
   Input:
   * No
   Output:
13
   dfs: calculate the diameter of the tree
   * maxi stores the diameter
   findingCenters() return the centers
   Nodes in graph 1 to N careful with this
   Complexity: O(N)
19
20
21
22
   vector<int>G[5010];
23
   int maxi=-1,far;
   int n;
25
   int pre[5010];
   int Queue[5010];
27
28
   void dfs(int path,int u,int parent){
29
     pre[u]=parent;
30
     if(path>=maxi){
31
       maxi=path;
32
       far=u:
33
     }
34
     for(int v:G[u]){
35
       if(parent!=v){
36
         dfs(path+1,v,u);//path + w if the graph as weighted
37
       }
38
     }
39
40
  | pair<int,int> findingCenters(){
```

```
maxi=-1:
42
     dfs(0,1,-1);
43
     dfs(0,far,-1);
44
     int t=far,L=0;
45
     while(t!=-1){
46
       Queue[L]=t;
47
       t=pre[t];
48
       ++L;
49
     }
50
     int a=-1,b=-1;
51
     if(L&1){
52
       a=Queue[L/2];
53
     }
54
     else{
55
       a=min(Queue[L/2-1],Queue[L/2]),b=max(Queue[L/2-1],Queue[L/2]);
57
     return {a,b};
59 }
```

7.13 algoritmo hungaro

7.14 union find dinámico

```
#include <bits/stdc++.h>
   using namespace std;
   #define dprint(v) cerr << #v"=" << v << endl //;)
   #define forr(i,a,b) for(int i=(a); i<(b); i++)</pre>
   #define forn(i,n) forr(i,0,n)
   #define dforn(i,n) for(int i=n-1; i>=0; i--)
   #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
   #define sz(c) ((int)c.size())
   #define zero(v) memset(v, 0, sizeof(v))
   #define pb push_back
   #define fst first
   #define snd second
   #define mkp make_pair
   typedef long long 11;
   typedef pair<int,int> ii;
16
   struct UnionFind {
17
       int n, comp;
18
       vector<int> pre,si,c;
19
       UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
20
           forn(i,n) pre[i] = i; }
21
```

```
int find(int u){return u==pre[u]?u:find(pre[u]);}
                                                                                           void go(int 1, int r) {
22
                                                                                   63
       bool merge(int u, int v) {
                                                                                               if(l+1==r){
                                                                                   64
23
                                                                                                   if (q[1].type == QUERY)//Aqui responder la query usando el
           if((u=find(u))==(v=find(v))) return false;
                                                                                    65
^{24}
           if(si[u]<si[v]) swap(u, v);</pre>
25
           si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
                                                                                                        res.pb(dsu.comp);//aqui query=cantidad de componentes
                                                                                   66
26
           return true;
                                                                                                            conexas
27
       }
                                                                                                    return;
28
                                                                                   67
       int snap(){return sz(c);}
                                                                                               }
29
                                                                                    68
       void rollback(int snap){
                                                                                               int s=dsu.snap(), m = (l+r) / 2;
30
                                                                                    69
           while(sz(c)>snap){
                                                                                               forr(i,m,r) if(match[i]!=-1 && match[i]<1) dsu.merge(q[i].u, q[i</pre>
31
                                                                                    70
               int v = c.back(); c.pop_back();
                                                                                                    ].v);
32
               si[pre[v]] -= si[v], pre[v] = v, comp++;
                                                                                               go(1,m);
33
                                                                                   71
                                                                                               dsu.rollback(s):
                                                                                   72
34
       }
                                                                                               s = dsu.snap();
                                                                                   73
35
                                                                                               forr(i,1,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[
                                                                                   74
36
   enum {ADD,DEL,QUERY};
                                                                                                    il.v):
   struct Query {int type,u,v;};
                                                                                               go(m,r);
                                                                                   75
   struct DynCon {
                                                                                               dsu.rollback(s);
39
                                                                                    76
                                                                                           }
       vector<Query> q;
                                                                                    77
40
       UnionFind dsu;
                                                                                       }dc;
41
       vector<int> match,res;
                                                                                    79
42
       map<ii,int> last;//se puede no usar cuando hay identificador para
                                                                                       // Problema ejemplo: http://codeforces.com/gym/100551/problem/A
43
           cada arista (mejora poco)
                                                                                   81
       DynCon(int n=0):dsu(n){}
                                                                                       int n,k;
44
       void add(int u, int v) {
                                                                                    83
45
                                                                                       int main() {
           if(u>v) swap(u,v);
46
                                                                                           //~ freopen("in", "r", stdin);
           q.pb((Query){ADD, u, v}), match.pb(-1);
                                                                                    85
47
           last[ii(u,v)] = sz(q)-1;
                                                                                           freopen("connect.in", "r", stdin);
                                                                                    86
48
       }
                                                                                           freopen("connect.out", "w", stdout);
                                                                                   87
49
       void remove(int u, int v) {
                                                                                           ios::sync_with_stdio(0);
                                                                                    88
50
           if(u>v) swap(u,v);
                                                                                           while(cin \gg n \gg k){
                                                                                    89
51
           q.pb((Query){DEL, u, v});
                                                                                           dc=DynCon(n);
52
                                                                                   90
           int prev = last[ii(u,v)];
                                                                                           forn(_,k) { string ord; cin >> ord;
                                                                                   91
53
                                                                                             if (ord=="?") {
           match[prev] = sz(q)-1;
                                                                                   92
54
           match.pb(prev);
                                                                                               dc.query();
                                                                                   93
55
                                                                                             } else if (ord=="+") { int a,b; cin>>a>>b; a--;b--;
                                                                                   94
56
       void query() {//podria pasarle un puntero donde guardar la respuesta
                                                                                               dc.add(a,b);
57
                                                                                   95
           q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
                                                                                             } else if (ord=="-") { int a.b: cin>>a>>b: a--:b--:
                                                                                   96
58
       void process() {
                                                                                               dc.remove(a,b);
                                                                                   97
59
           forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
                                                                                             } else assert(false);
60
                 sz(q);
                                                                                   99
           go(0,sz(q));
                                                                                               if(!k) continue;//k==0 WTF
                                                                                   100
61
       }
                                                                                               dc.process();
62
                                                                                   101
```

1 // g++ -std=c++11 "erdosgalloi.cpp" -o run

7.15 truquitos estúpidos por ejemplo second MST es con LCA7.16 erdos galloi

```
/***
2
    Given the grades of each node of a graph return if this form a valid
  includes: algorithm, functional, numeric, forn
   // Receives a sorted degree sequence (non ascending)
   O(NlgN)
7
8
   bool isGraphicSequence(const vector<int> &seq) // O(n lg n)
11
     vector<int> sum;
12
     int n = seq.size();
13
14
     if (n == 1 \&\& seq[0] != 0) return false;
15
16
     sum.reserve(n + 1);
17
     sum.push_back(0);
18
     for (int i = 0; i < n; ++i) sum.push_back(sum[i] + seq[i]);</pre>
19
     if ((sum[n] \& 1) == 1) return false;
20
21
     for (long long k = 1; k \le n - 1 \&\& seq[k - 1] >= k; ++k) {
22
       int j = distance(seq.begin(), upper_bound(seq.begin() + k, seq.end()
23
           , k,
                                                greater<int>())) +
24
               1;
^{25}
       long long left = sum[k];
26
       long long right = k * (k - 1) + (j - k - 1) * k + (sum[n] - sum[j - k])
27
          1]):
28
      if (left > right) return false;
29
     }
30
31
     return true;
32
```

```
33 }
```

- 7.17 grafo funcional hallar k-esimo partiendo de un nodo 7.18 konig
 - 7.19 min-vertex cover bipartitos
 - 7.20 max-flow (min cost versión)

```
1 // g++ -std=c++11 "maxflowmincost.cpp" -o run
          ======= <Max flow-min cost c++ version> ==========
   Given a grapth with edges with a capacity C and weight D
   * compute the max-flow min cost
  Edmond karps idea
  * Complexity O(v *E*log(v))
  Problem for practice: Dijkstra Dijkstra uva
   */
   #define REP(i,j,k) for(int (i)=(j);(i)<(k);++(i))
   #define MP make_pair
   using namespace std;
14
   #define MAXN 500
   #define MAXM MAXN * 5
   typedef vector<int> VI;
   typedef long long 11;
   const int INF = 1E9; // $infinity$: be careful to make this big enough
  int S; // source
  int T; // sink
  int FN; // number of nodes
   int FM; // number of edges (initialize this to 0)
24 // ra[a]: edges connected to a (NO MATTER WHICH WAY!!!); clear this in
       the beginning
25 VI ra[MAXN];
   int kend[MAXM], cap[MAXM], cost[MAXM]; // size: TWICE the number of
27
28 // Adds an edge from a to b with capacity c and cost d and returns the
       number of the new edge
29
30 | int addedge(int a, int b, int c, int d) {
```

```
int i = 2*FM;
                                                                                           }
31
                                                                                    73
     kend[i] = b;
                                                                                           totf += minflow;
                                                                                   74
32
     cap[i] = c;
                                                                                           totw += minflow*dst[T];
                                                                                    75
33
     cost[i] = d;
34
                                                                                    76
     ra[a].push_back(i);
                                                                                         return make_pair(totf, totw);
     kend[i+1] = a;
36
                                                                                    78
     cap[i+1] = 0;
                                                                                       void init(){
37
     cost[i+1] = -d;
                                                                                         FN=4*n+15;//make this big n=number of nodes of the graph
38
     ra[b].push_back(i+1);
                                                                                         FM=0;
39
     FM++;
                                                                                         S=0,T=n+1;
40
                                                                                        for(int i=0;i<FN;i++)ra[i].clear();//clear the graph be careful</pre>
     return i;
41
                                                                                    84 }
42
   int n:
43
                                                                                                       7.21 max-flow corto con matriz
   int dst[MAXM], pre[MAXM], pret[MAXM];
   //finding the shortest path via fanding duan, also it works with bellman
                                                                                    1 // g++ "maxflowMVEK.cpp" -o run
    //or dijkstra (careful of negative cycles)
                                                                                       /***
   bool spfa(){
                                                                                       ========= <Max Flow with matriz Edmonds karp c++ version>
     REP(i,0,FN) dst[i] = INF;
48
     dst[S] = 0;
                                                                                       //Given a graph with capacitys find the max-flow
49
     queue<int> que; que.push(S);
50
     while(!que.empty()){
51
                                                                                       Nodes indexed 1 to N
       int x = que.front(); que.pop();
                                                                                       * Complexity O(N *E)
52
       for (int t : ra[x]){
                                                                                       Problem for practice: UVA 820
53
         int y = kend[t], nw = dst[x] + cost[t];
                                                                                       */
54
                                                                                    9
         if(cap[t] > 0 && nw<dst[y]){</pre>
                                                                                       #define N 500
55
           dst[y] = nw; pre[y] = x; pret[y] = t; que.push(y);
                                                                                       int cap[N][N], pre[N], n;
56
         }
                                                                                       int s://source
57
       }
                                                                                       int t://destination
58
                                                                                       bool bfs() {
59
     return dst[T]!=INF;
                                                                                           queue<int>q;
60
                                                                                    15
61
                                                                                           q.push(s);
                                                                                    16
    // returns the maximum flow and the minimum cost for this flow
                                                                                           memset(pre,-1,sizeof pre);
62
                                                                                    17
    pair<ll.ll> solve(){
                                                                                           pre[s]=s;
                                                                                    18
     11 \text{ totw} = 0, \text{ totf} = 0;
                                                                                           while(!q.empty()){
64
                                                                                    19
     while(spfa()){
                                                                                               int u=q.front();q.pop();
65
                                                                                   20
       int minflow = INF;
66
                                                                                               if(u==t)return true;
                                                                                   21
       for (int x = T; x!=S; x = pre[x]){
                                                                                               for(int i=1:i \le n:i++){//nodes 1 to n
67
                                                                                   22
         minflow = min(minflow, cap[pret[x]]);
                                                                                                    if(pre[i] == -1&&cap[u][i])pre[i] = u,q.push(i);
68
                                                                                   23
                                                                                               }
69
                                                                                   24
       for (int x = T; x!=S; x = pre[x]){
70
                                                                                   25
         cap[pret[x]] -= minflow;
                                                                                           return false;
71
                                                                                   26
         cap[pret[x]^1] += minflow;
                                                                                   27 | }
72
```

```
28
   int maxFlow() {
29
       int mf=0,f,v;//max flow, flow for a path, the vertex
30
       while(bfs()){//while encountered a path source to destination
31
           v=t;//min
32
           f=INT_MAX;//make this big enough
33
           while(pre[v]!=v){f=min(f,cap[pre[v]][v]),v=pre[v];}//finding the
34
                min capacity
           v=t;mf+=f;
35
           while(pre[v]!=v){cap[pre[v]][v]-=f,cap[v][pre[v]]+=f,v=pre[v];}
36
               //update the flow
       }
37
       return mf;
38
39
   void init(){
     memset(cap,0,sizeof cap);
     //cap[u][v]+=capacidad,cap[v][u]+=capacidad
42
43 }
                       7.22 max-flow sin matriz
   // g++ -std=c++11 "maxflowNMEK.cpp" -o run
```

```
2
   ============== <Max Flow with-out matriz Edmonds karp c++ version>
       _____
   //Given a graph with capacitys find the max-flow
5
   Nodes indexed 1 to N
   * Complexity O(N *E)
   Problem for practice: UVA 820
   * Input N number of nodes,
   * M edges conections
   * compute the flow with source 1 and sink N
11
12
   using namespace std;
13
   const int N = 110;
   const int M = 10010 * 2;
   vector<int>G[N]:
   int kend[M], cap[M], cost[M];
  int edge = 0;
18
   int s,t;
   void add(int u,int v,int c){
     int forward = edge * 2, backward = edge * 2 + 1;
21
```

```
kend[forward] = v:
22
     cap[forward] = c;
23
     G[u].push_back(forward);
24
     kend[backward] = u;
25
     cap[backward] = 0;
26
     G[v].push_back(backward);
27
     edge++;
28
29
   int vis[M],pre[M],pret[M];
   bool bfs(){
     for(int i = 0; i <= 100;i++)vis[i] = false;</pre>
32
     vis[s] = true;
33
     queue<int>q;
34
     q.push(s);
35
     while(!q.empty()){
36
       int u = q.front();q.pop();
       for(int edge:G[u]){
38
         int v = kend[edge];
         if(cap[edge] > 0 && !vis[v]){
           vis[v] = true;
           pre[v] = u;
42
            pret[v] = edge;//the edge store the information
            q.push(v);
44
45
46
47
     return vis[t];
48
49
   int max_flow(){
     int totf = OLL;
     while(bfs()){
52
       int minflow = INT_MAX;
       for(int x = t; x != s; x = pre[x]){
54
          minflow = min(minflow,cap[pret[x]]);
55
56
       for(int x = t; x != s; x = pre[x]){
57
          cap[pret[x]] -= minflow;
          cap[pret[x] ^ 1] += minflow;
59
60
       totf += minflow;
61
62
     return totf;
63
64 | }
```

```
65 | int main(){
                                                                                         int qt=0; q[qt++]=src;
                                                                                  22
                                                                                         for(int qh=0; qh<qt; qh++){</pre>
     int n,m;
                                                                                  23
66
     scanf("%d",&n,&m);
                                                                                             int u =q[qh];
67
                                                                                  24
     for(int i = 0, u, v, ca; i < m; i++){}
                                                                                             forall(e, G[u]){
68
                                                                                  25
       scanf("%d_{\sqcup}%d_{\sqcup}%d",&u,&v,&ca);
                                                                                                  int v=e->to;
69
                                                                                  26
                                                                                                 if(dist[v]<0 \&\& e->f < e->cap)
       add(u,v,ca);
70
     }
                                                                                                      dist[v]=dist[u]+1, q[qt++]=v;
71
                                                                                  28
                                                                                             }
     s = 1, t = n;
72
                                                                                  29
     printf("%lld\n",max_flow());
                                                                                         }
                                                                                  30
74 | }
                                                                                         return dist[dst]>=0;
                                                                                  32
                                7.23 Dinic
                                                                                     ll dinic_dfs(int u, ll f){
                                                                                         if(u==dst) return f:
                                                                                  34
                                                                                         for(int &i=work[u]; i<sz(G[u]); i++){</pre>
                                                                                             Edge &e = G[u][i];
  const int MAX = 300;
                                                                                             if(e.cap<=e.f) continue;</pre>
  // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v
                                                                                             int v=e.to;
       ]==-1 (del lado del dst)
                                                                                  38
                                                                                             if(dist[v]==dist[u]+1){
4 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los
       conjuntos mas proximos a src y dst respectivamente):
                                                                                                      11 df=dinic_dfs(v, min(f, e.cap-e.f));
                                                                                  40
                                                                                                      if(df>0){
5 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices
                                                                                  41
       de V2 con it->f>0, es arista del Matching
                                                                                                              e.f+=df, G[v][e.rev].f-= df;
                                                                                  42
                                                                                                              return df; }
6 // Min Vertex Cover: vertices de V1 con dist[v] ==-1 + vertices de V2 con
                                                                                  43
                                                                                             }
        dist[v]>0
                                                                                  44
7 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex
                                                                                  45
                                                                                         return 0;
                                                                                  46
s // Max Clique: construir la red de G complemento (debe ser bipartito!) y
                                                                                  47
                                                                                     11 maxFlow(int _src, int _dst){
        encontrar un Max Independet Set
                                                                                  48
                                                                                         src=_src, dst=_dst;
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices
                                                                                  49
       no cubierto hasta el momento, tomar cualquier arista de el
                                                                                         11 result=0;
                                                                                  50
                                                                                         while(dinic_bfs()){
  int nodes, src, dst;
                                                                                  51
                                                                                             fill(work, work+nodes, 0);
  int dist[MAX], q[MAX], work[MAX];
                                                                                  52
                                                                                             while(ll delta=dinic_dfs(src,INF))
  struct Edge {
                                                                                  53
12
                                                                                                  result+=delta:
       int to, rev;
                                                                                  54
13
                                                                                         }
                                                                                  55
       ll f, cap;
14
                                                                                         // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
       Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(
15
                                                                                             forman el min-cut
           cap) {}
                                                                                         return result; }
                                                                                  57
16
   vector<Edge> G[MAX];
                                                                                                7.24 máximo emparejamiento bipartito
   void addEdge(int s, int t, ll cap){
18
       G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
19
                                                                                  1 // g++ -std=c "bipartitematching.cpp" -o run
           0)):}
  bool dinic_bfs(){
                                                                                  2 /***
                                                                                     ======== <MCBM max cardinality bipartite matching c++ version>
       fill(dist, dist+nodes, -1), dist[src]=0;
21
```

```
_____
   Return the bipartite matching of a Graph
   * Format of nodes: 1 to N
6
   const int N = 100010;
   vector<int>G[N];
   bool v[N];//for the greedy speed up
   int match[N];
   bool vis[N];
   int n,m;
   //calling aumenting path
   bool aug(int u){
       if(vis[u])return false;
16
       vis[u]=true:
17
       for(int i=0;i<(int)G[u].size();++i){</pre>
18
       int r=G[u][i];
19
           if(match[r]==-1||aug(match[r])){
20
                match[r]=u;match[u]=r;return true;
21
           }
22
       }
23
       return 0;
24
25
26
    //findging all augmenting path's
27
   int solve(){
28
      bool check=true;
29
      while(check){
30
           check=false;
31
           memset(vis,0,sizeof vis);
32
           for(int i=1;i<=n;++i){</pre>
33
         if(!v[i]&&match[i]==-1){
34
           bool op=aug(i);
35
           check | = op;
36
           mc+=op;
37
         }
38
       }
39
40
       return mc;
41
^{42}
   void init(){
43
     memset(v,0,sizeof v);
44
     memset(vis,false,sizeof vis);
45
```

```
mc=0:
46
     memset(match,-1,sizeof match);
47
       for(int i=0;i<=n;i++)G[i].clear();</pre>
48
49
   void greedySpeedUp(){
50
     //greedy optimization, match with the first not matched
     for(int i=1;i<=n;++i){
             for(int j=0; j<(int)G[i].size();++j){</pre>
                 if(match[G[i][j]]==-1){
54
             match[G[i][j]]=i,match[i]=G[i][j],mc++,v[i]=true;break;
         }
56
57
58
59 }
```

- 7.25 max-independent set en bipartitos
- 7.26 min-path cover (ver tópicos raros de halim)
 - 7.27 min-cost arborescence
- 7.28 lema de diapositivas de nico de grafos funcionales
 - 7.29 minimax y maximini con kruskal y dijkstra

```
1 // g++ -std=c++11 "maximini.cpp" -o run
  /***
   Given a weighted graph return the maximini (the maximun of the minimum)
  or the minimax (the minimum of the maximum) in the path a,b
6
  Minimax as definded as: finding the minimum of maximum edge weight among
       all posible paths
  * between two verrtices a to b, the cost for a path from a to b is
      determined by maximum edge
9 * weight along this path. Among all these possible paths from a to b,
      pick the one with the minimum
  * ax-edge-weight
  * Complexity O(E*log(E) + V + E)
12
  Problem for practice: UVA 534,544
  */
14
15 | int n;
pair<int,pair<int,int> >Edges[20000];
```

```
int t:
17
   map<string,int>mp;
   int parent[210];
   pair<int,int>child[210];
   bool vis[210];
   vector<pair<int,int> >G[210];
23
   int find(int u){return u==parent[u]?u:parent[u]=find(parent[u]);}
   void Union(int u,int v){
25
     int pu=find(u),pv=find(v);
26
     if(pu!=pv){
27
       parent[pv]=pu;
28
     }
29
30
   int mst(int a,int b){
31
     sort(Edges,Edges+t);
32
     reverse(Edges, Edges+t);//don't reverse for the minimax
33
     for(int i=0;i<=200;i++)parent[i] = i;</pre>
34
     int w,u,v, maximini = 1e8, minimax = 0;
35
     for(int i=0;i<t;i++){</pre>
36
       tie(w,u,v) = make_tuple(Edges[i].first, Edges[i].second.first, Edges
37
            [i].second.second);
       if(find(u) != find(v)){
38
         Union(u,v);
39
         G[u].push_back({v,w});
40
         G[v].push_back({u,w});
41
       }
42
     }
43
     queue<int>q;
44
     q.push(a);
45
     vis[a]=true;
46
     while(!q.empty()){
47
       int u = q.front();q.pop();
48
       //if(u==1)break:
49
       for(pair<int,double>node: G[u]){
50
         if(!vis[node.first]){
51
           vis[node.first] = true;
52
           q.push(node.first);
53
           //maximini=max(maximini,node.second);
54
            child[node.first].first = u;
55
            child[node.first].second = node.second;
56
57
       }
58
```

```
for (int t = b;t != -1;t = child[t].first){
    //cout<<t<" "<<child[t].second<<"\n";
    //minimax=max(minimax,child[t].second);
    maximini = min(maximini,child[t].second);
}
return maximini;
}</pre>
```

int survivor(int n, int m){

8 Teoria de juegos

- 8.1 Teorema fundamental de los juegos optimos
 - 8.2 Como calcular grundy
 - 9 Probabilidad
 - 9.1 Formulas clave
 - 10 Otros/utilitarios

10.1 josephus

```
int s = 0;
      for (int i=1; i <= n; ++i) s = (s+m)\%i;
      return (s+1);
4
5 }
                         10.2 josephus k = 2
   //////JAVA
       /**
2
3
      * Oparam n the number of people standing in the circle
4
      * Oreturn the safe position who will survive the execution
      * f(N) = 2L + 1 where N = 2^M + L and 0 \le L \le 2^M
6
     public int getSafePosition(int n) {
8
      // find value of L for the equation
9
       int valueOfL = n - Integer.highestOneBit(n);
10
       int safePosition = 2 * valueOfL + 1;
11
       return safePosition:
12
    }
13
```

for(int sbm=bm; sbm; sbm=(sbm-1)&bm)

17

ss<<s:

while(ss>>s){

10.3 poker

10.4 iterar subconjuntos

```
10.5 como reconstruir una DP (normal)
```

```
1 /*
2 You just need to revisit your steps in the DP. In case of 0-1 knapsack,
       lets say the original DP function was solve, and the function
       reconstruct will give you the actual solution (I'm writing the code
       in C++):
3
   int solve(int pos, int capacity){
       if(pos == no_of_objects) return 0;
5
       if(memo[pos][capacity] != -1) return memo[pos][capacity];
6
       int r1 = solve(pos + 1, capacity); //dont take
7
       int r2 = 0;
8
       if(weight[pos] <= capacity){</pre>
9
           r2 = solve(pos + 1, capacity - weight[pos]) + profit[pos]; //
10
               take
11
       return memo[pos][capacity] = max(r1, r2);
12
13
   void reconstruct(int pos, int capacity){
14
       if(pos == no_of_objects) return; //you have completed reconstruction
15
       int r1 = memo[pos + 1][capacity]; //dont take
16
       int r2 = 0;
17
       if(weight[pos] <= capacity)r2 = memo[pos + 1][capacity - weight[pos</pre>
18
           ]] + profit[pos]; //take
       if(r1 > r2) {reconstruct(pos + 1, capacity);}
19
       else{
20
           cout << "Take object " << pos << endl;</pre>
21
           reconstruct(pos + 1, capacity - weight[pos]) + profit[pos];
22
       }
23
^{24}
   After executing reconstruct, it will print all those objects that give
       you the optimal solution. As you can see, at most no_of_objects
       calls will be made in the reconstruct function.
26 | Similarly, you can reconstruct the solution of any DP greedily.
```

10.6 muajaja con j

```
#include <signal.h>
   void divzero(int p){
     while(true);}
   void segm(int p){
     exit(0);}
   //in main
   signal(SIGFPE, divzero);
 8 | signal(SIGSEGV, segm);
                             Expandir pila
#include <sys/resource.h>
2 rlimit rl;
   getrlimit(RLIMIT_STACK, &rl);
  rl.rlim_cur=1024L*1024L*256L;//256mb
5 setrlimit(RLIMIT_STACK, &rl);
                 10.7 comparar doubles for noobs
const double EPS = 1e-9;
_2 | x == y <=> fabs(x-y) < EPS
_3 | x > y <=> x > y + EPS
_4 | x >= y <=> x > y - EPS
                                infix to postfix
                         10.8
 1 //infix to postfix with shunting yard, Halim interpretation
   //plus eval function given a postfix return the result of the operation
   //format: string like (xox (xox)) o=operation x=value
   string s;
   bool isOperator(string u){
     return u=="+"||u=="-"||u=="*"||u=="/";
6
7
   bool precede(string u){
     if(u=="*"||u=="/")return true;
     return false;
10
11
   void solve(){
12
     getline(cin,s);
13
     stack<string>st;
14
     vector<string>v;
15
     stringstream ss;
16
```

```
if(isOperator(s)){
19
                                                                                      61
         while(!st.empty()&&isOperator(st.top())&&precede(st.top())>=
                                                                                      62
20
              precede(s)){
                                                                                      63
           v.push_back(st.top());st.pop();
                                                                                      64
21
         }
                                                                                      65 }
22
          st.push(s);
23
       }
24
       else{
25
          if(s=="("){
26
             st.push(s);
27
         }
28
         else{
29
           if(s==")"){
30
              while(!st.empty()&&st.top()!="("){
31
                v.push_back(st.top());st.pop();
32
33
              if(!st.empty()&&st.top()=="(")st.pop();
34
                                                                                       9
           }
35
                                                                                      10
            else {
36
              v.push_back(s);
                                                                                      11
37
38
39
       }
40
41
     while(!st.empty()){
                                                                                      15
42
       v.push_back(st.top());st.pop();
                                                                                      16
43
                                                                                      17
     }
44
     stack<double>stans;
45
     double x;
46
                                                                                      20
     for(string eva:v){
47
       if(!isOperator(eva)){
                                                                                      21
48
         stringstream nu;
49
         nu<<eva;
50
         nu>>x:
51
         stans.push(x);
52
       }
53
       else{
54
         double a=stans.top();stans.pop();
55
         double b=stans.top();stans.pop();
56
          if(eva=="*")b*=a;
57
                                                                                      31 }
         if(eva=="/")b/=a;
58
         if(eva=="+")b+=a;
59
         if(eva=="-")b-=a;
60
```

```
stans.push(b);
   }
    cout<<fixed<<stans.top()<<"\n";</pre>
                      10.9 numeros romanos
#include <bits/stdc++.h>
 using namespace std;
 map<int,string>cvt;
  string aromano(int n){
    cvt[1000] = "M";cvt[900] = "CM",cvt[500] = "D", cvt[400] = "CD";
    cvt[100] = "C";cvt[90] = "XC"; cvt[50] = "L";
    cvt[40] = "XL";cvt[10] = "X";cvt[9] = "IX";cvt[5] = "V"; cvt[4] = "IV"
    cvt[1] = "I";
    string ans = "";
   for(map<int,string>::reverse_iterator it = cvt.rbegin();it != cvt.rend
        ();it++)
      while(n >= it->first){
        ans += it->second;
       n -= it->first;
      }
   return ans;
  map<string,int>crn;
  int anumero(string R){
   map<char, int> crn;
    crn['I'] = 1; crn['V'] = 5; crn['X'] = 10; crn['L'] = 50;
    crn['C'] = 100; crn['D'] = 500; crn['M'] = 1000;
    int value = 0:
   for (int i = 0; R[i]; i++)
     if (i + 1 < R.size() && crn[R[i]] < crn[R[i+1]]) {</pre>
       value += crn[R[i+1]] - crn[R[i]];
        i++;
      else value += crn[R[i]];
    return value;
                    10.10 get k-th permutacion
```

```
vector<int>v;
   //finding the number of permutation 0....n-1
   int main()
3
   {
4
       string s;
5
       while(getline(cin,s)){
6
           stringstream ss;
7
           ss<<s;
8
           int pos=0,u;
9
           v.clear();
10
           while(ss>>u){
11
               v.push_back(u-1);
12
13
           vector<int>le(v.size(),0);
14
           for(int i=0;i<v.size();i++){</pre>
15
               for(int j=i+1; j<v.size(); j++){</pre>
16
                   if(v[i]>v[j])le[i]++;
17
               }
18
           }
19
           long long ans=OLL,fact=OLL,por=1LL;
20
           for(int i=le.size()-1;i>=0;i--){
21
               if(fact!=OLL)por*=fact;
22
               fact++;
23
               ans=ans+por*le[i];
^{24}
           }
25
           cout << ans+1 << "\n";
26
       }
27
       return 0;
28
29 }
                         10.11 sliding window
                  10.12 permutaciones de un dado
   // izquierda, derecha, arriba, al frente, abajo, atras
```

```
2
   int p[][6] = {
3
       \{0,1,2,3,4,5\},
4
       \{0,1,3,4,5,2\},\
5
       \{0,1,4,5,2,3\},\
6
       \{0,1,5,2,3,4\},
       \{1,0,2,5,4,3\},
8
       {1,0,3,2,5,4},
9
       \{1,0,4,3,2,5\},\
10
```

```
{1,0,5,4,3,2},
11
       \{2,4,5,1,3,0\},\
12
13
      \{2,4,1,3,0,5\},\
      \{2,4,3,0,5,1\},\
14
      \{2,4,0,5,1,3\},
15
      {3,5,2,1,4,0},
      {3,5,1,4,0,2},
      {3,5,4,0,2,1},
      {3,5,0,2,1,4},
      {4,2,5,0,3,1},
      {4,2,0,3,1,5},
21
      {4,2,3,1,5,0},
      {4,2,1,5,0,3},
23
      {5,3,2,0,4,1},
      {5,3,0,4,1,2},
25
      {5,3,4,1,2,0},
26
      {5,3,1,2,0,4}
27
28 };
                        10.13 ternary search
                      10.14 liebre y el tortugo
                       10.15 como usar printf
                              10.16 java
                            10.17 python
                           10.18 template
                           10.19 file setup
1 //tambien se pueden usar comas: {a, x, m, l}
touch {a..l}.in; tee {a..l}.cpp < template.cpp
```