Bits, Bytes and Integers

Introduction to Computer Systems

Danfeng Shan
Xi’an Jiaotong University

Announcements

- **ICSServer已准备好**
  - 注意：请不要恶意**占用服务**器资源！包括不仅限于挖矿、跑大模型！

- **两个Tutorial**
  - 使用SSH登录ICSServer（胡博瑄）
  - Bash Shell（李雨轩）

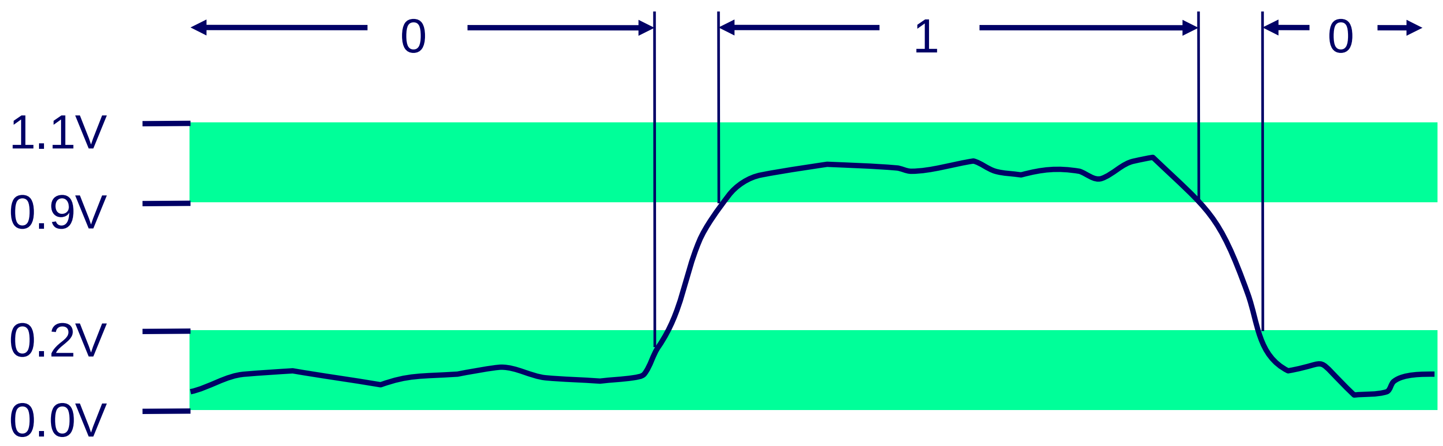
- **Lab 1 (Data Lab) 已经公布，截止时间：03.09（周日）**

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Everything is bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- Why bits? Electronic Implementation
  - Easy to store with bistable elements (双稳态器件)
  - Reliably transmitted on noisy and inaccurate wires



Encoding Byte Values

- **Byte = 8 bits**
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$\texttt{15213:}\quad \underbrace{\texttt{0011}}_{\texttt{3}}\ \underbrace{\texttt{1011}}_{\texttt{B}}\ \underbrace{\texttt{0110}}_{\texttt{6}}\ \underbrace{\texttt{1101}}_{\texttt{D}}$$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8
	"ILP32"	"LP64"

Today: Bits, Bytes, and Integers

- Representing information as bits
- **Bit-level manipulations**
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- Representations in memory, pointers, strings

Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
  - Encode “True” as 1 and “False” as 0

And

A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

A|B = 1 when either A=1 or B=1 or both

|	0	1
0	0	1
1	1	1

Not

~A = 1 when A=0

~	0	1
	1	0

Exclusive-Or (Xor)

A^B = 1 when A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any “integral” data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise

Contrast: Logic Operations in C

Contrast to Bit-Level Operators

- **Logic Operations: &&, ||, !**
  - View 0 as “False”
  - Anything nonzero as “True”
  - Always return 0 or 1
  - Early termination

Watch out for && vs. & (and || vs. |)…
Super common C programming pitfall!

Examples (char data type)

- !0x41 → 0x00
- !0x00 → 0x01
- !!0x41→ 0x01

- 0x69 && 0x55 → 0x01
- 0x69 || 0x55 → 0x01
- p && *p (avoids null pointer access)

Shift Operations

- **Left Shift: `x << y`**
  - Shift bit-vector **`x`** left **`y`** positions
    - Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift: `x >> y`**
  - Shift bit-vector **`x`** right **`y`** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

Argument **`x`**	`01100010`
`<< 3`	`00010000`
Log. `>> 2`	`00011000`
Arith. `>> 2`	`00011000`

Argument **`x`**	`10100010`
`<< 3`	`00010000`
Log. `>> 2`	`00101000`
Arith. `>> 2`	`11101000`

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed, negation
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Question?

```
int foo = -1;
unsigned bar = 1;

(foo < bar) == true ?
```

Encoding "Integers"

Unsigned

Signed (twos complement)

Given a bit vector x, w bits long…

$$\mathrm{B2U}(x) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

$$\mathrm{B2T}(x) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

Examples (w = 5)

±16	8	4	2	1
0	1	0	1	0

$0 + 8 + 0 + 2 + 0 = \quad 10$

16	8	4	2	1
1	0	1	1	0
-16	8	4	2	1

$16 + 8 + 0 + 2 + 0 = \quad 26$

$-16 + 8 + 0 + 2 + 0 = -10$

Negation: Complement & Increment

- **Negate through complement and increase**

  `~x + 1 == -x`

- **Why?**
  - `-x + x       == 0` *(by definition)*
  - `~x + x       == 1111…111 == -1`
  - `~x + x + 1 == 0`
  - `(~x+1) + x == 0`
  - `~x+1       == -x`

```
    x   1 0 0 1 1 1 0 1
+  ~x   0 1 1 0 0 0 1 0
-----------------------
   -1   1 1 1 1 1 1 1 1
```

Example: x = 15213

	Decimal	Hex	Binary
`x`	15213	3B 6D	00111011 01101101
`~x`	-15214	C4 92	11000100 10010010
`~x+1`	-15213	C4 93	11000100 10010011
`y`	-15213	C4 93	11000100 10010011

Complement & Increment Examples

$x = 0$

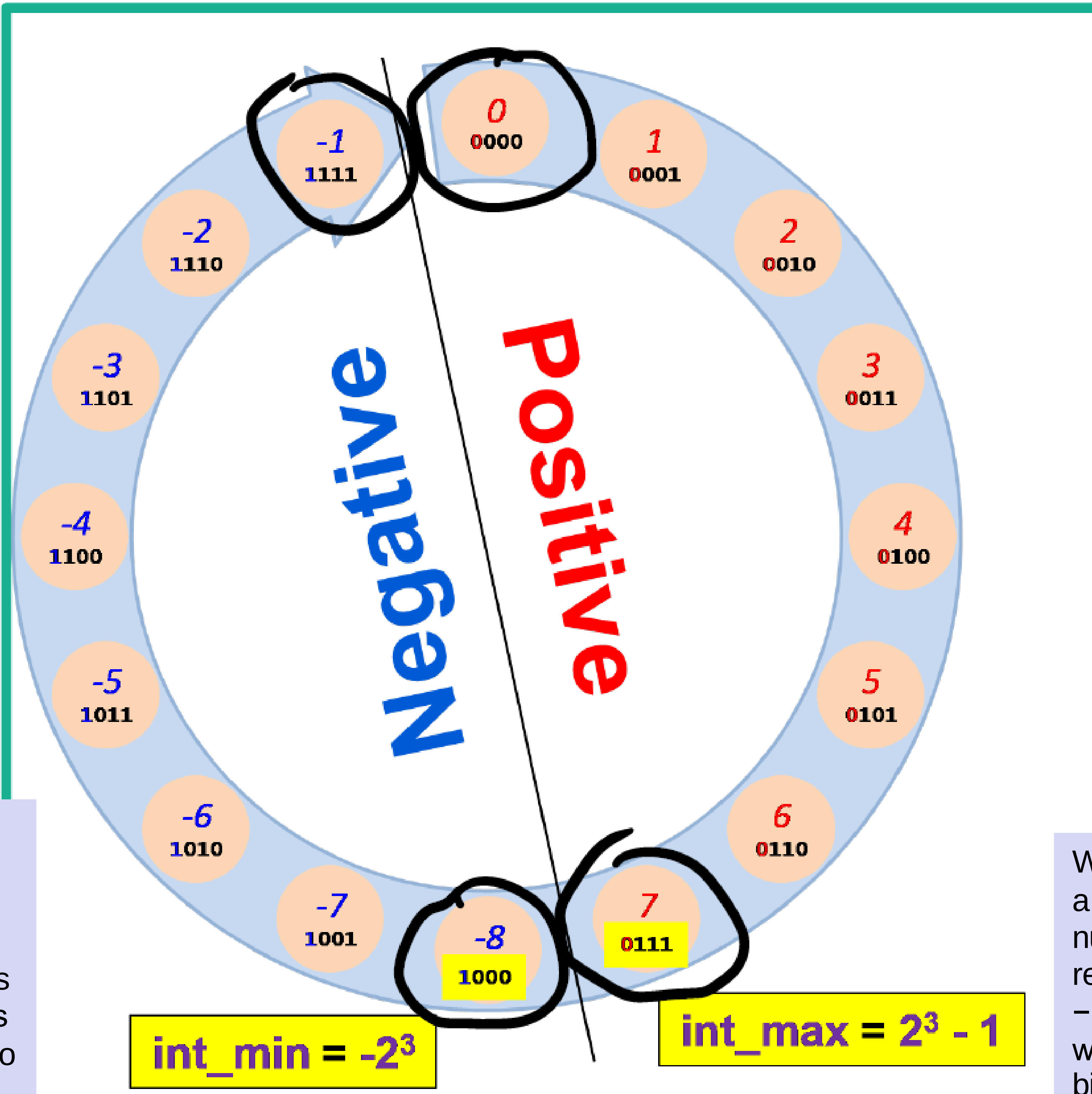
	Decimal	Hex	Binary
0	**0**	00 00	00000000 00000000
~0	**-1**	FF FF	11111111 11111111
~0+1	**0**	00 00	00000000 00000000

$x = T_{\min}$

	Decimal	Hex	Binary
x	**-32768**	80 00	10000000 00000000
~x	**32767**	7F FF	01111111 11111111
~x+1	**-32768**	80 00	10000000 00000000

Oops!
It's still
negative!

Eight negative values:
−1, − 2, …, − 8



Eight *non*-negative values:
0, 1, …, 7

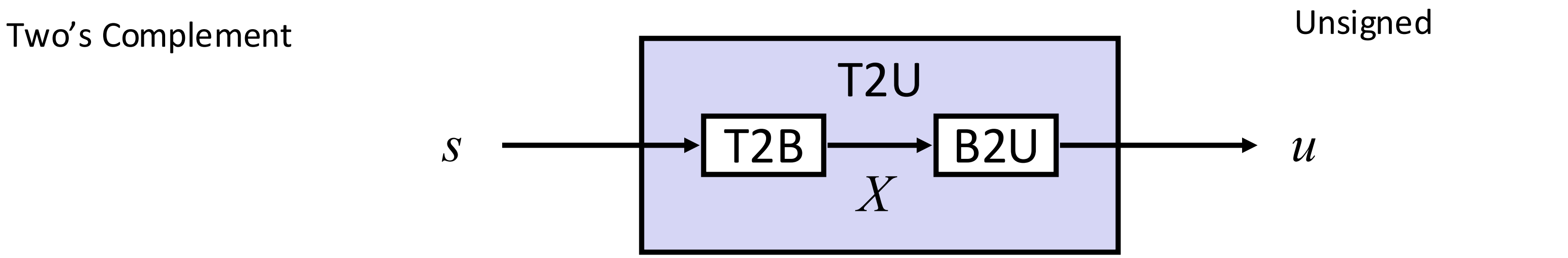
Mathematicians would prefer it if a 4-bit signed number could represent values −8…8, but that's $2^4 + 1$ values, so they won't all fit.

What if we made a 4-bit signed number only represent values −7…7? Then we wouldn't be using bit pattern `1000`…

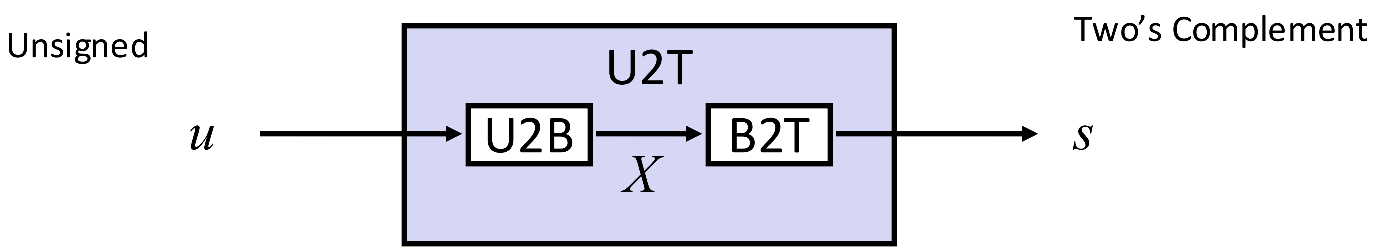
Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed, negation
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- Representations in memory, pointers, strings

Mapping Between Signed & Unsigned



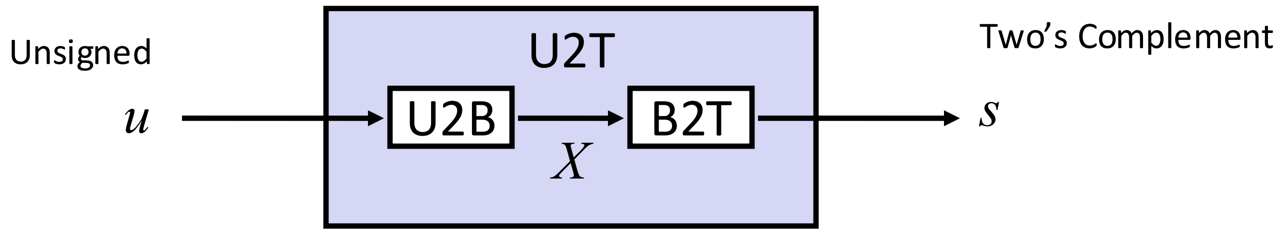
Maintain Same Bit Pattern



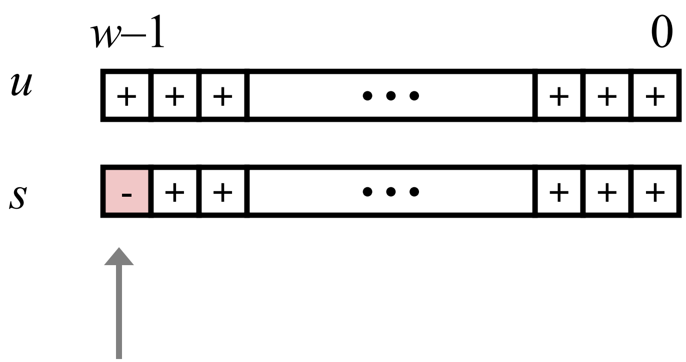
Maintain Same Bit Pattern

- **Mappings between unsigned and two's complement numbers:**
  Keep bit representations and reinterpret

Relation between Signed & Unsigned



Maintain Same Bit Pattern



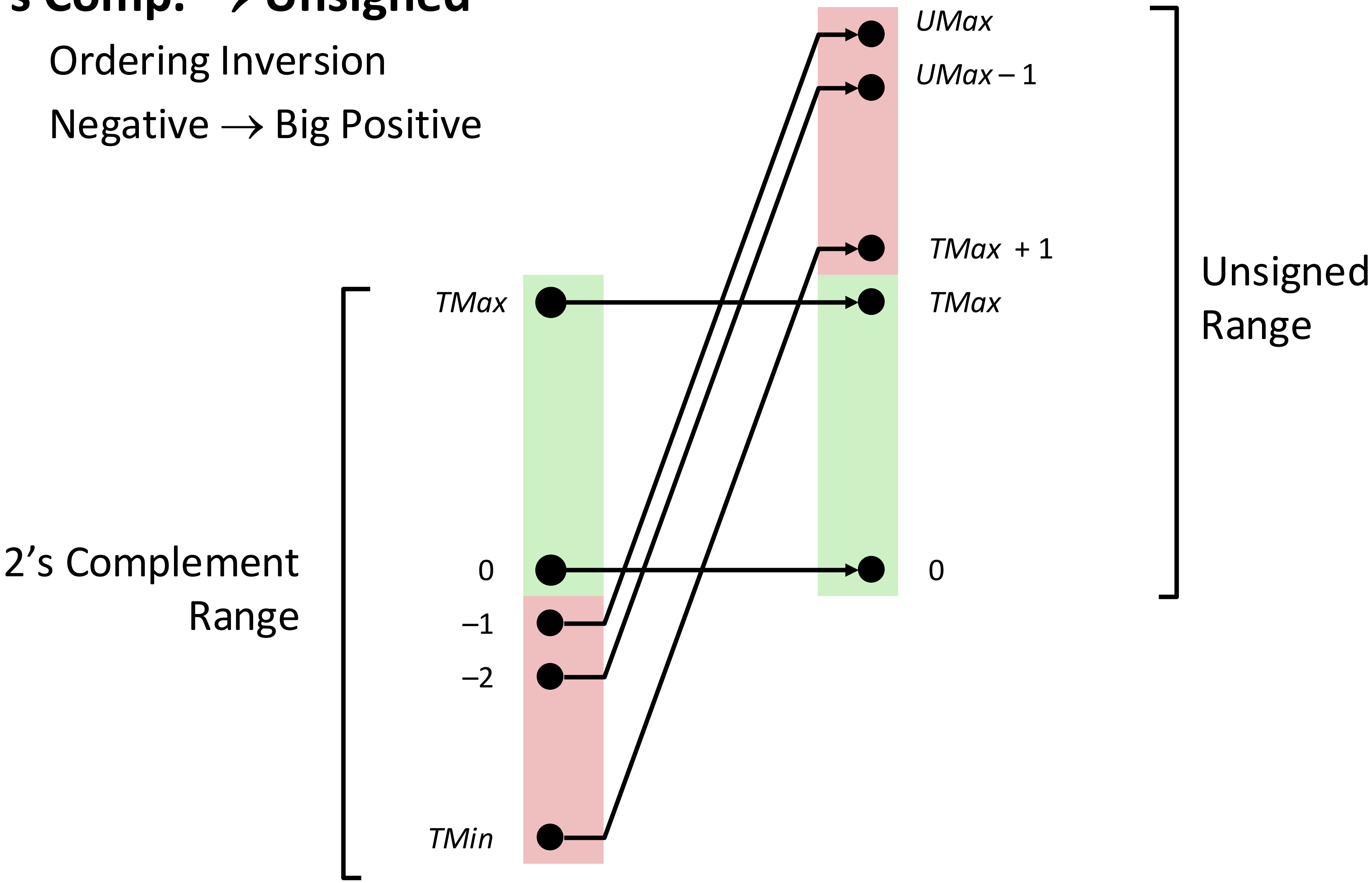
Large positive weight
becomes
Large negative weight

Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3		3
0100	4	=	4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5	± 16	11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Conversion Visualized

- **2's Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be signed integers
  - Unsigned if have "U" as suffix

    `0U, 4294967259U`

- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                      int fun(unsigned u);
uy = ty;                      uy = fun(tx);
```

Question?

example02.c

```
int foo = -1;
unsigned bar = 1;

foo < bar == true ?
```

Casting Surprises

Expression Evaluation

- If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
- Including comparison operations **<, >, ==, <=, >=**
- Examples:

Constant 1	Constant 2	Relation	Evaluation
`0`	`0U`	`==`	Unsigned
`-1`	`0`	`<`	Signed
`-1`	`0U`	`>`	Unsigned
`INT_MAX`	`INT_MIN`	`>`	Signed
`(unsigned)INT_MAX`	`INT_MIN`	`<`	Unsigned
`-1`	`-2`	`>`	Signed
`(unsigned)-1`	`-2`	`>`	Unsigned
`INT_MAX`	`((unsigned)INT_MAX) + 1`	`<`	Unsigned
`INT_MAX`	`(int)(((unsigned)INT_MAX) + 1)`	`>`	Signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

- Expression containing signed and unsigned int
  - `int` is cast to `unsigned`!!

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed, negation
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

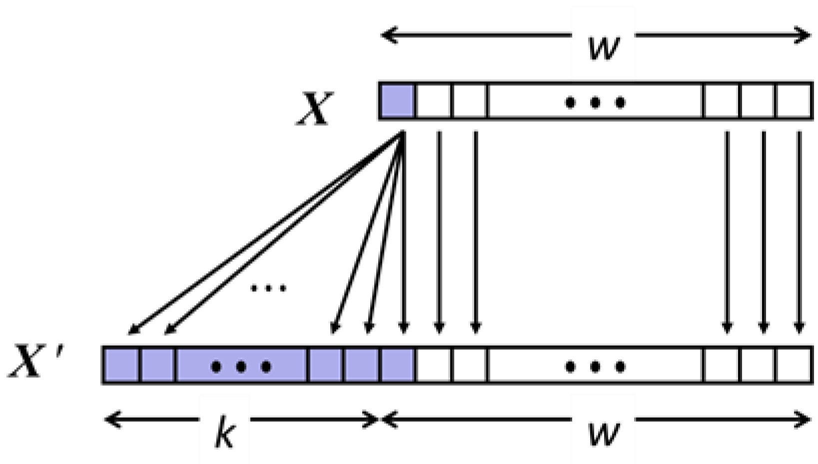
Question?

example03.c

```
int x = 0x8000;
short sx = (short) x;
int y = sx;
```

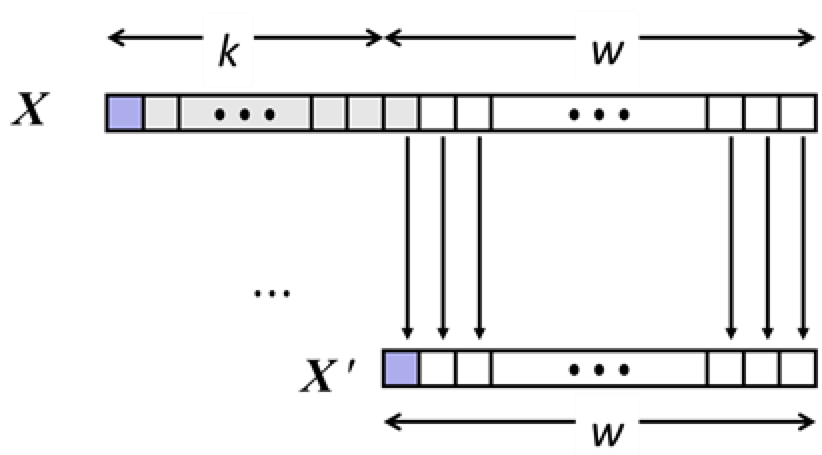
Sign Extension and Truncation

- **Sign Extension**



Make *k* copies of sign bit

- **Truncation**



Chop off *k* highest bits

Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
−10 =	1	0	1	1	0

	-32	16	8	4	2	1
−10 =	1	1	0	1	1	0

Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

Question?

example03.c

```
int x = 0x8000;
short sx = (short) x;
int y = sx;
```

Today: Bits, Bytes, and Integers

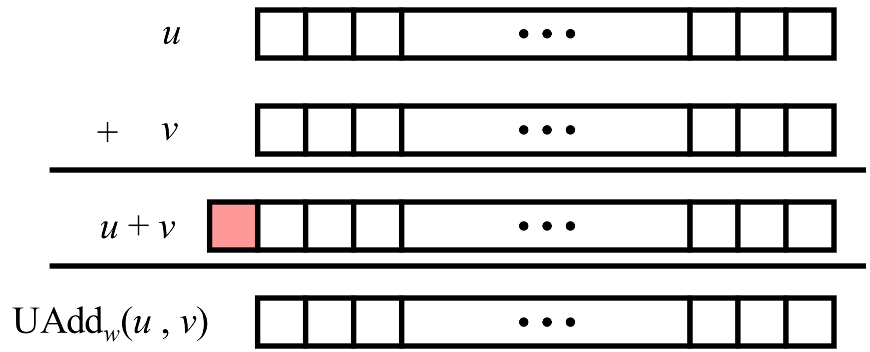
- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- **Standard Addition Function**
  - Ignores carry output

```
unsigned char     1110 1001      E9      233
                + 1101 0101    + D5    + 213
                ___________    ____    _____
                ___________    ____    _____
```

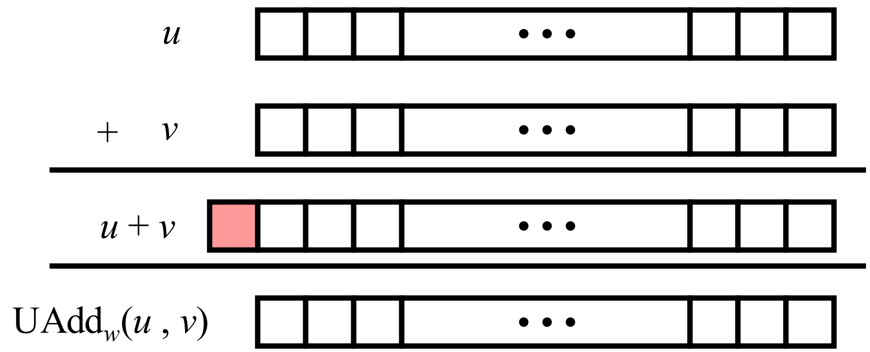
Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



Standard Addition Function

- Ignores carry output

```
unsigned char      1110 1001       E9        233
               +   1101 0101     + D5      + 213
                 1 1011 1110      1BE        446
                   1011 1110       BE        190
```

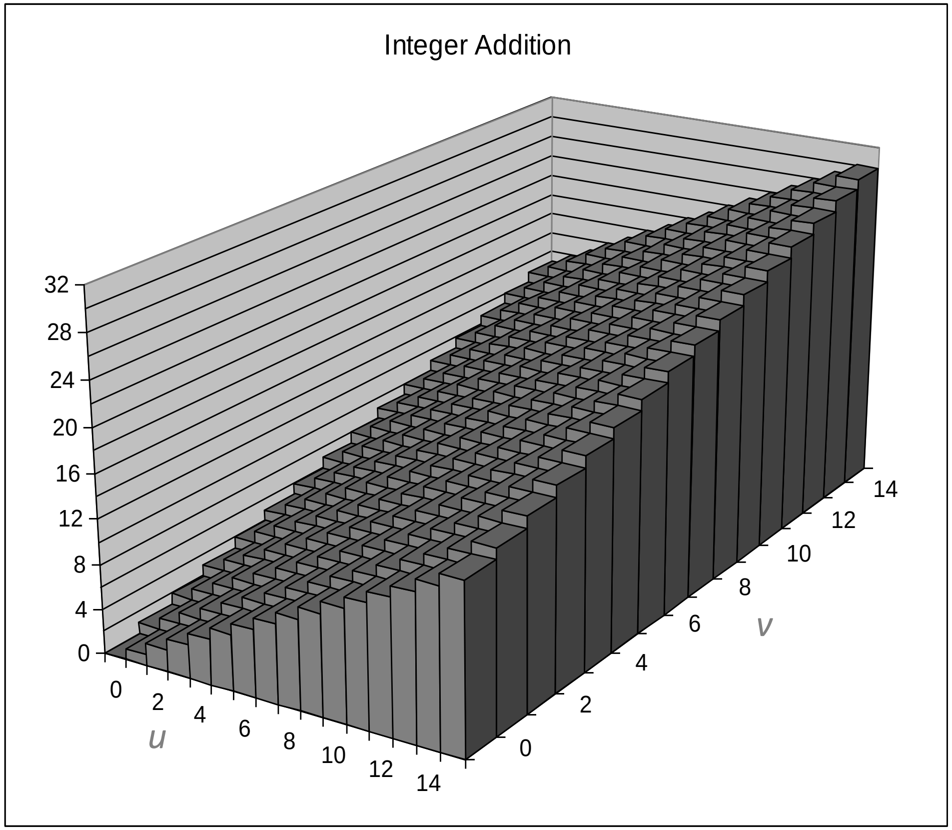
Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Visualizing (Mathematical) Integer Addition

Integer Addition

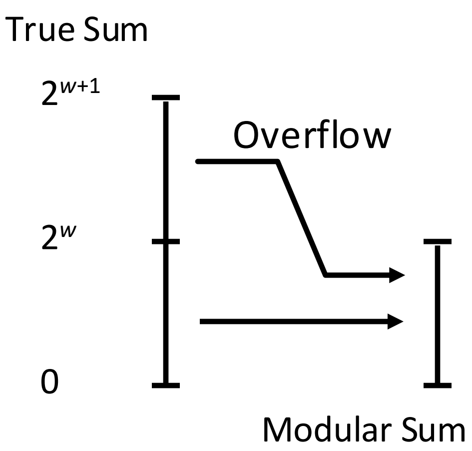
- 4-bit integers *u*, *v*
- Compute true sum $\text{Add}_4(u, v)$
- Values increase linearly with *u* and *v*
- Forms planar surface

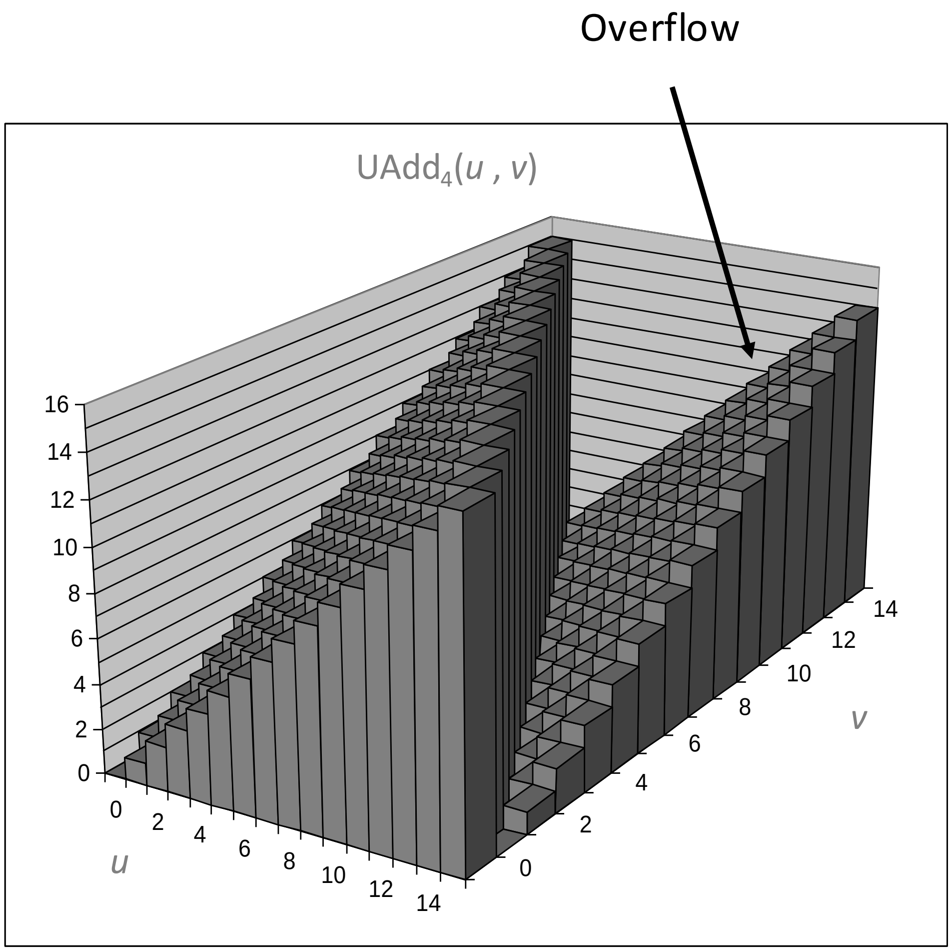
$\text{Add}_4(u, v)$



Visualizing Unsigned Addition

- **Wraps Around**
  - If true sum ≥ 2^w
  - At most once



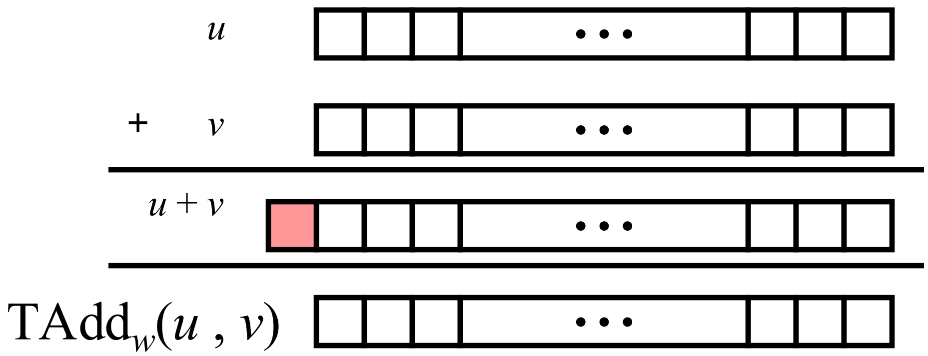


Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

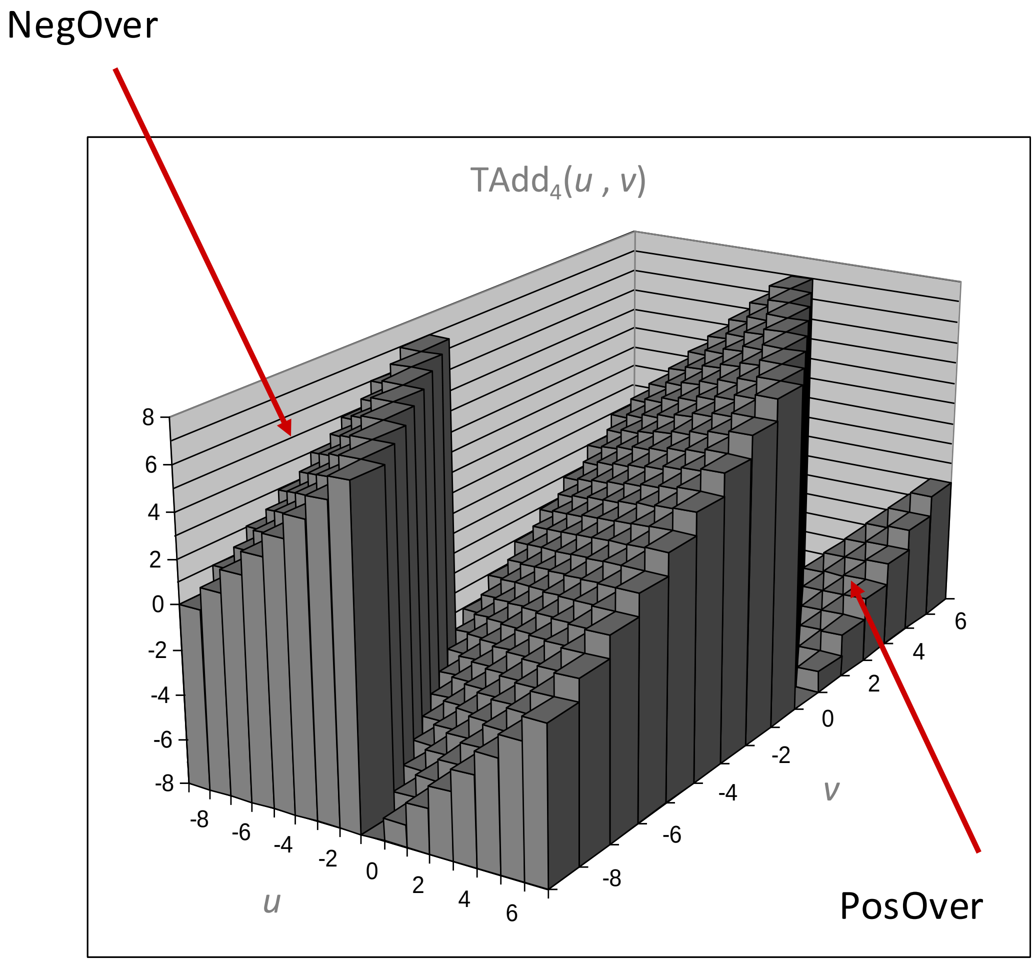
```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

1110 1001	E9	-23
+ 1101 0101	+ D5	+ -43
1 1011 1110	1BE	-66
1011 1110	BE	-66

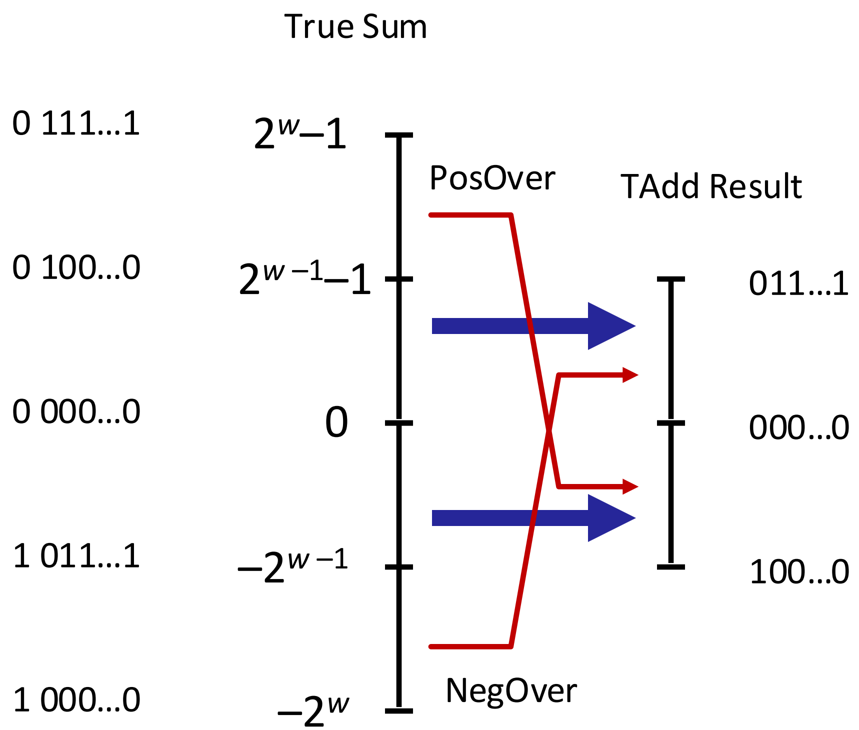
Visualizing 2's Complement Addition

- **Values**
  - 4-bit two's comp.
  - Range from -8 to +7
- **Wraps Around**
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



TAdd Overflow

- **Functionality**
  - True sum requires w+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Shifting

- **Left Shift: `x << y`**
  - Shift bit-vector `x` left `y` positions
  - Throw away extra bits on left
  - Fill with 0's on right
  - Equivalent to multiplying by 2^y
- **Right Shift: `x >> y`**
  - Shift bit-vector `x` right `y` positions
  - Throw away extra bits on right
  - Two kinds:
    - "Logical": Fill with 0's on left
    - "Arithmetic": Replicate most significant bit on left
  - *Almost* equivalent to dividing by 2^y
- **Undefined Behavior (in C)**
  - Shift amount < 0 or ≥ word size

Argument `x`	01100010
<< 3	00010000
Logical >> 2	00011000
Arithmetic >> 2	00011000

Argument `x`	10100010
<< 3	00010000
Logical >> 2	00101000
Arithmetic >> 2	11101000

Multiplication

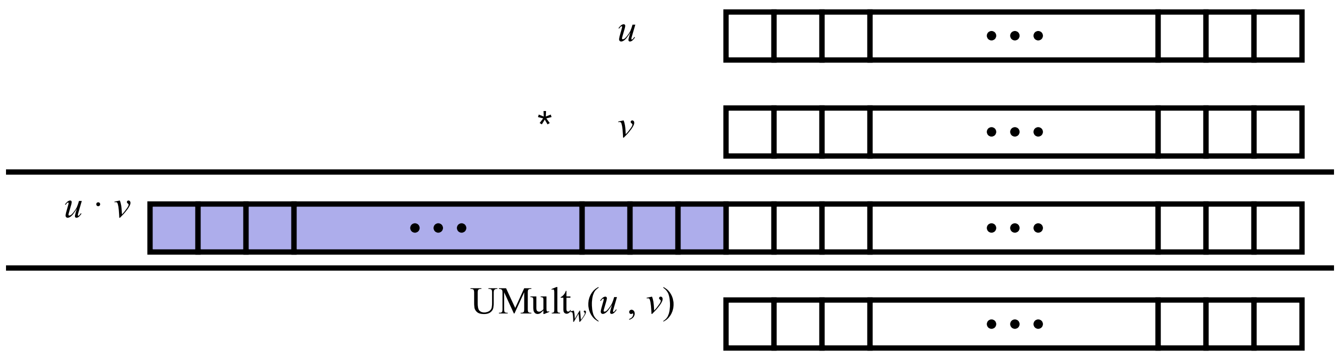
- **Goal: Computing Product of *w*-bit numbers *x*, *y***
  - Either signed or unsigned
- **But, exact results can be bigger than *w* bits**
  - Unsigned: up to 2*w* bits
    - Result range: $0 \leq x * y \leq (2^{w} - 1)^{2} = 2^{2w} - 2^{w+1} + 1$
  - Two's complement min (negative): Up to 2*w*-1 bits
    - Result range: $x * y \geq (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
  - Two's complement max (positive): Up to 2*w* bits, but only for $(TMin_{w})^{2}$
    - Result range: $x * y \leq (-2^{w-1})^{2} = 2^{2w-2}$
- **So, maintaining exact results…**
  - would need to keep expanding word size with each product computed
  - is done in software, if needed

Unsigned Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



Standard Multiplication Function

- Ignores high order *w* bits

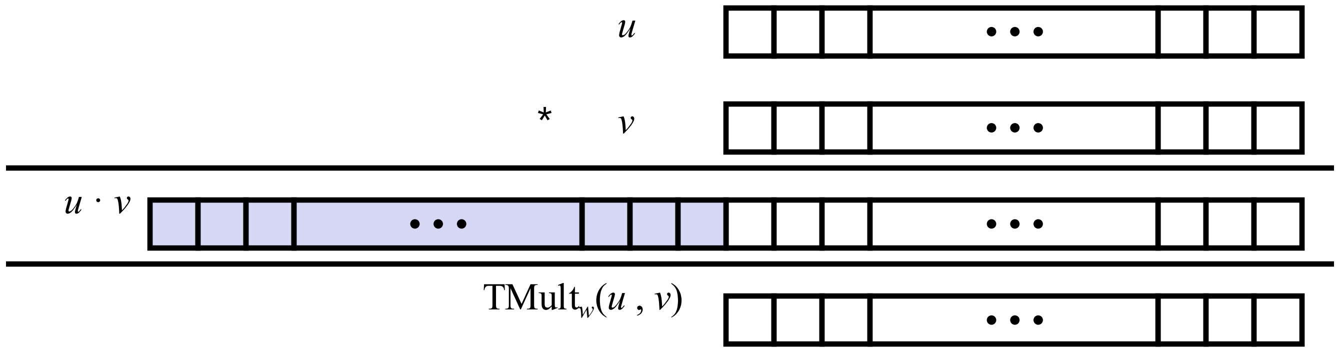
```
          1110 1001      E9       233
*         1101 0101   *  D5   *   213
 1100 0001 1101 1101    C1DD    49629
          1101 1101      DD       221
```

Signed Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



Standard Multiplication Function

- Ignores high order *w* bits
- Some of which are different for signed vs. unsigned multiplication
- Lower bits are the same

	1110 1001	E9	-23
*	1101 0101	* D5	* -43
	0000 0011 1101 1101	03DD	989
	1101 1101	DD	-35

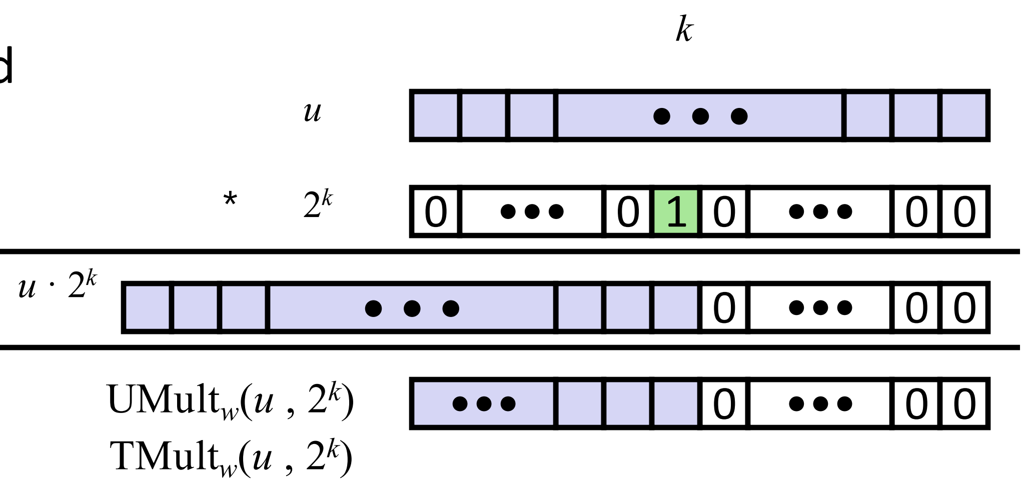
Power-of-2 Multiply with Shift

- **Operation**
  - `u << k` gives `u * `2^k
  - Both signed and unsigned

Operands: *w* bits

True Product: *w*+*k* bits

Discard *k* bits: *w* bits

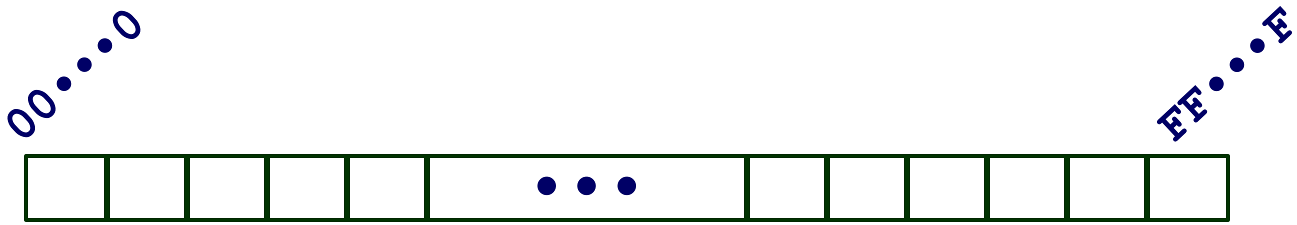


- **Examples**
  - `u << 3        ==   u * 8`
  - `(u << 5) - (u << 3)     ==    u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Byte-Oriented Memory Organization



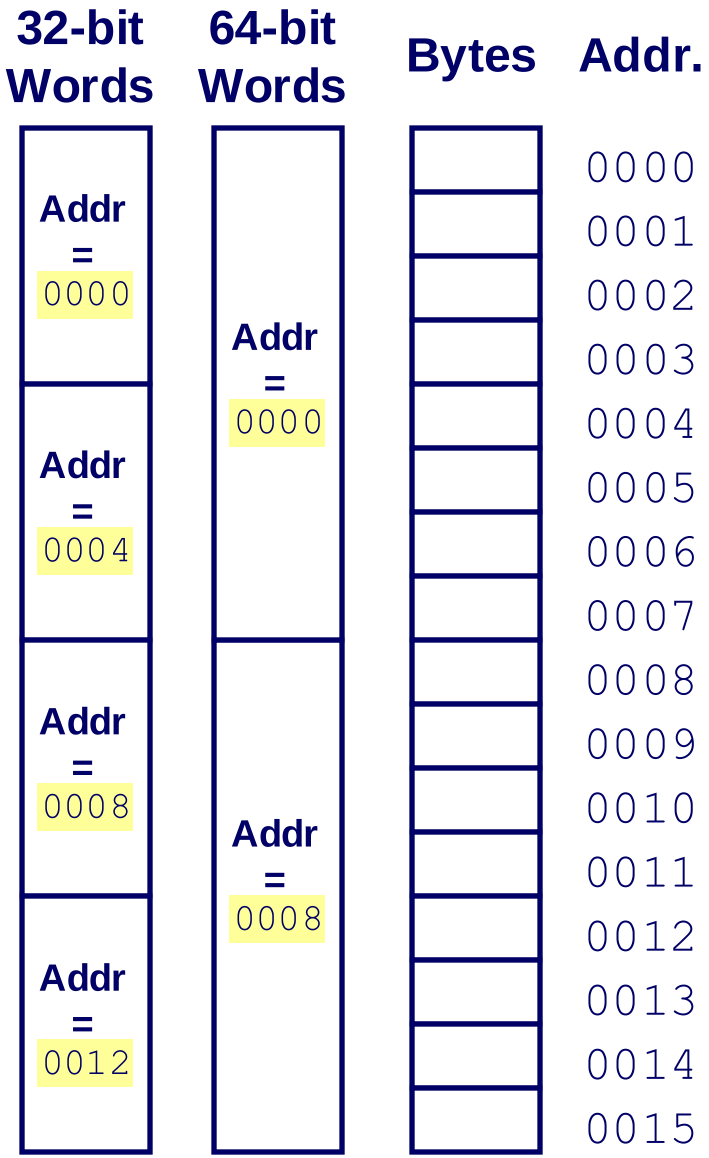
- **Programs refer to data by address**
  - Imagine all of RAM as an enormous array of bytes
  - An address is an index into that array
    - A pointer variable stores an address
- **System provides a private *address space* to each "process"**
  - A process is an instance of a program, being executed
  - An address space is one of those enormous arrays of bytes
  - Each program can see only its own code and data within its enormous array
  - We'll come back to this later ("virtual memory" classes)

Machine Words

- **Any given computer has a “Word Size”**
  - Nominal size of integer-valued data
    - and of addresses
  - Historically, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB (2^{32} bytes)
  - Currently, machines have 64-bit word size
    - Potentially, could have 16 EB (exabytes) of addressable memory
    - That’s 18.4×10^{18} bytes
  - Machines still support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

Addresses *Always* Specify Byte Locations

- Address of a word is address of the first byte in the word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

Question?

example04.c

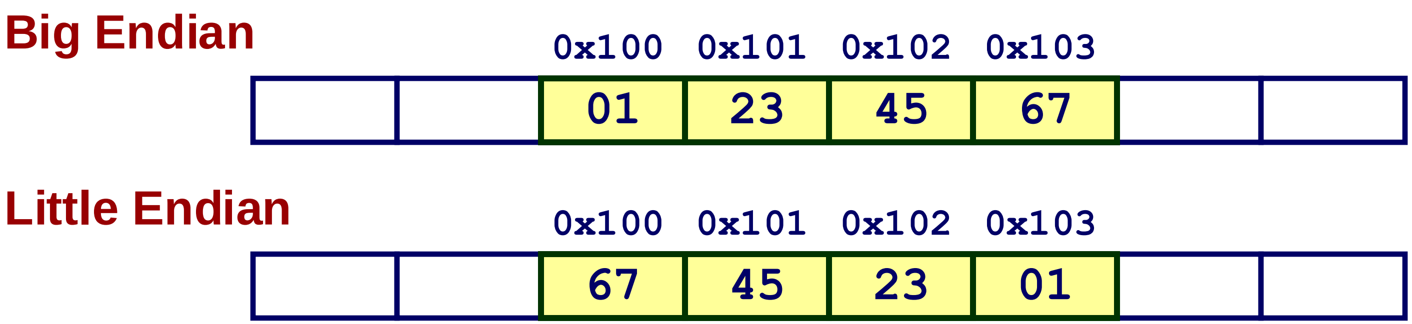
```
struct foo {
   char mem1[3];  // 3 bytes
   int mem2;      // 4 bytes
   char mem3;     // 1 byte
};
sizeof(struct foo) = ?
```

Byte Ordering

- **So, how are the bytes within a multi-byte word ordered in memory?**
- **Conventions**
  - Big Endian: Sun, PPC Mac, *network packet headers*
    - Least significant byte has highest address
  - Little Endian: *x86*, ARM processors running Android, iOS, and Windows
    - Least significant byte has lowest address

Byte Ordering Example

- **Example**
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100



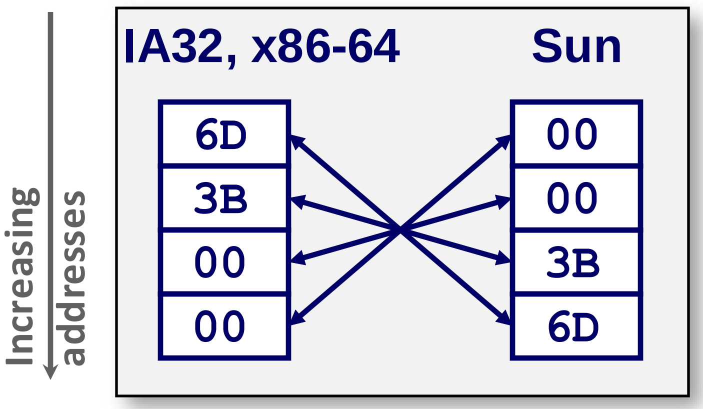
Representing Integers

Decimal: 15213

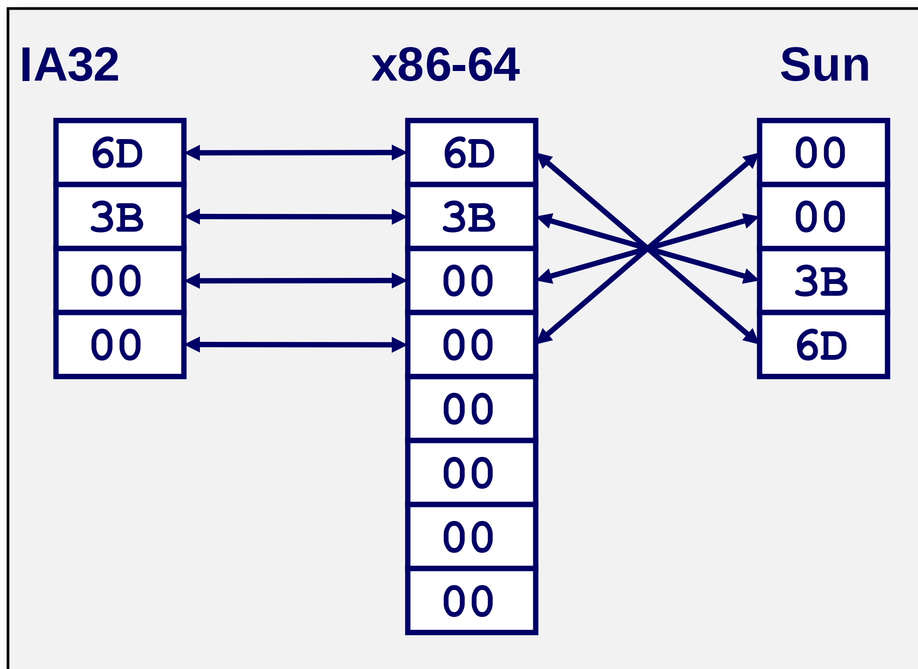
Binary: 0011 1011 0110 1101

Hex: 3 B 6 D

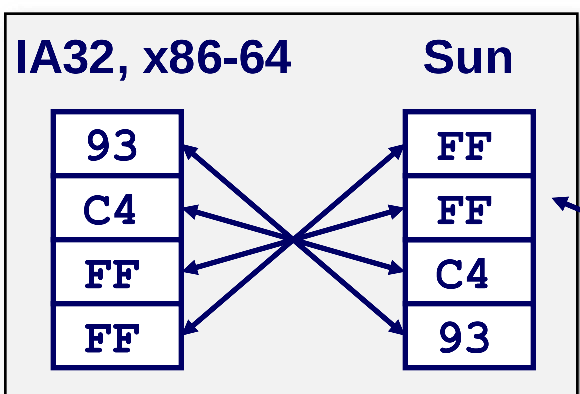
`int A = 15213;`



`long int C = 15213;`



`int B = -15213;`



Two's complement representation

Examining Data Representations

- **Code to Print Byte Representation of Data**
  - Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
  size_t i;
  for (i = 0; i < len; i++)
    printf("%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}
```

Printf directives:

%p:	Print pointer
%x:	Print Hexadecimal

`show_bytes` Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc    6d
0x7fffb7f71dbd    3b
0x7fffb7f71dbe    00
0x7fffb7f71dbf    00
```

Representing Pointers

```
int B = -15213;
int *P = &B;
```

Sun	IA32	x86-64
EF	AC	3C
FF	28	1B
FB	F5	FE
2C	FF	82
		FD
		7F
		00
		00

Different compilers & machines assign different locations to objects

Even get different results each time run program

Representing Strings

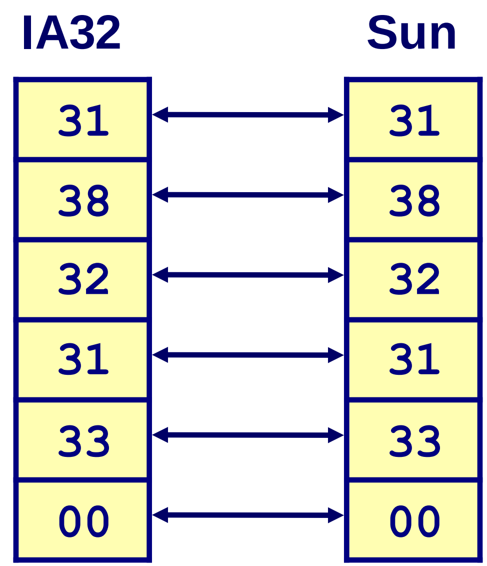
```
char S[6] = "18213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character "0" has code 0x30
    - Digit *i* has code 0x30+*i*
- String should be null-terminated
  - Final character = 0

Compatibility

- Byte ordering not an issue



Representing x86 machine code

- **x86 machine code is a sequence of *bytes***
  - Grouped into variable-length instructions, which look like strings…
  - But they contain embedded little-endian numbers…
- **Example Fragment**

Address	Instruction Code	Assembly Rendition
`8048365:`	`5b`	`pop %ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add $0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl $0x0,0x28(%ebx)`

- **Deciphering Numbers**
  - Value: `0x12ab`
  - Pad to 32 bits: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse: `ab 12 00 00`