Exercícios

① $\binom{8}{3} = \frac{8!}{3!\,(8-3)!} = \frac{8.7.6.5!}{3!\;5!} = \frac{336}{6} = \boxed{56}$ (B)

② $\binom{200}{198} = \frac{200\cdot 199 . 198!}{198!\,(200-198)!} = \frac{200\cdot 199}{2} = \boxed{19900}$ (A)

③ $\binom{n-1}{2} = \binom{n+1}{4}$

$\binom{n-1}{2} = \frac{(n-1)\cdot(n-2)\cdot(n-3)!}{2!\,((n-1)-2)} = \frac{(n-1)(n-2)\cancel{(n-3)}}{2!\,\cancel{(n-3)}} = \frac{n^2-2n-n+2}{2}$

$\frac{n^2-2n-n+2}{2} = 0 \rightarrow 0{,}5n^2 - 1{,}5n + 1 = 0$

$\Delta = 2{,}25 + 4 \cdot 0{,}5 \cdot 1$

$\Delta = 0{,}25$

$n_1 = \frac{1{,}5+0{,}5}{2 \cdot 0{,}5} = \boxed{2}$ $\quad n_2 = \frac{1{,}5-0{,}5}{2 \cdot 0{,}5} = \boxed{1}$

$\binom{n-1}{2} = \binom{n+1}{4} \rightarrow \frac{(n-1)!}{2!\,(n-3)} = \frac{(n+1)\cdot n(n-1)!}{4!\,(n-3)}$

$\frac{1}{2} = \frac{n^2+n}{24} \rightarrow 2n^2 + 2n - 24 = 0 \;:2$

$n^2 + n - 12 = 0$

$3 + -4 = -1$

$3 \cdot -4 = -12$

$n_2 = -4$ → não convém

$n_1 = \boxed{3}$

$V = \{1, 2, 3\}$

④ $\binom{20}{13} + \binom{20}{14} = \binom{21}{14}$

$\binom{n}{k} + \binom{n}{k+1} \rightarrow \binom{n+1}{k+1}$

$\binom{21}{14} = \binom{21}{7}$

$14 + 7 = 21$

↓

complementar

C

⑤ $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{n}$

Soma na linha $n = 2^n$

$$\left.\begin{array}{lll} 1 & = 1 & \Rightarrow 2^0 \\ 11 & = 2 & \Rightarrow 2^1 \\ 121 & = 4 & \Rightarrow 2^2 \\ 1331 & = 8 & \Rightarrow 2^3 \end{array}\right] 2^n$$

⑥

a) $\sum_{p=0}^{10}\binom{10}{p} \Rightarrow 2^n \Rightarrow 2^{10} = 1024$

b) $\sum_{p=0}^{9}\binom{10}{p} \Rightarrow 2^{n+1} - \binom{n+1}{n+1} \Rightarrow 2^{10} - \binom{10}{10} = 1024-1 = 1023$

c) $\sum_{p=2}^{9}\binom{9}{p} = 2^n - \binom{n}{p-2} - \binom{n}{p-1} \Rightarrow 2^9 - \underbrace{\binom{9}{0}}_{1} - \underbrace{\binom{9}{1}}_{9} = 502$

d) $\sum_{p=4}^{10}\binom{p}{4} = \binom{11}{5} = \frac{11.10.9.8.7.6!}{5!(11-5)!} = \frac{55440}{720} = 462 \Rightarrow \binom{n+1}{k+1}$

e) $\sum_{p=5}^{10}\binom{p}{5} = \binom{11}{6} = \frac{11.10.9.8.7.6}{6!(11-6)} = \frac{55440}{120} = 462 \Rightarrow \binom{n+1}{k+1}$

⑦ $\sum_{k=0}^{m}\binom{m}{k} = 512 \Rightarrow \binom{m}{0}+\binom{m}{1}+\dots\binom{m}{k} \Rightarrow$ soma na linha m

$2^m = 512 \Rightarrow 2^m = 2^9 \Rightarrow \boxed{m = 9}$

E