Exercícios - Discussão sobre Sistemas Lineares

① $\begin{cases} ax + 4y = 1 \\ x + 2y = b \end{cases}$ $D = \begin{vmatrix} a & 4 \\ 1 & 2 \end{vmatrix} = 2a - 4$ $\boxed{a = 2}$ $\boxed{b = 1/2}$

$DX = \begin{vmatrix} 1 & 4 \\ b & 2 \end{vmatrix} = 2 - 4b$ $X = \frac{DX}{D} = \frac{2-4b \,:2}{2a-4 \,:2} = \frac{1-2b}{a-2} = \frac{1-\frac{2}{2}}{2-2} = \frac{0}{0}$

Se: $x = \boxed{\frac{0}{0}}$ $1 - 2b =$

$2b = 1$

Então: SP.I $b = \frac{1}{2}$ (B)

② $\begin{cases} x + ky = 1 \\ kx + y = 1 - k \end{cases}$ $\begin{pmatrix} 1 & k & \vdots & 1 \\ k & 1 & \vdots & 1-k \end{pmatrix} \sim \begin{pmatrix} & & \vdots & \\ 0 & -k^2+1 & \vdots & -2k+1 \end{pmatrix}$

$y(-k^2+1) = -2k+1$ I) $-2k+1 = 0$ $-k^2+1 = 0$

$y = \frac{-2k+1}{-k+1}$ $k = \frac{1}{2}$ $k = \pm\sqrt{1}$

$k = -1, 1$

(F)

Há mais de um valor para k.

II) (F) Se $K=1$ ou $K=-1$, a solução é impossível.

III) (F) K admite mais de um valor. (D)

(3) a) $\begin{cases} x+2y+cz=1 \\ y+z=2 \\ 3x+2y+2z=-1 \end{cases}$

$3c \quad 2 \quad 0 = 3c+2$

$\det A = \begin{vmatrix} 1 & 2 & c \\ 0 & 1 & 1 \\ 3 & 2 & 2 \end{vmatrix} \begin{matrix} 1 & 2 \\ 0 & 1 \\ 3 & 2 \end{matrix} = 8-3c-2 = 6-3c$

$2 \quad 6 \quad 0 = 8$

$A = \begin{pmatrix} 1 & 2 & c \\ 0 & 1 & 1 \\ 3 & 2 & 2 \end{pmatrix}$

b) $6-3c \neq 0$ $\qquad c \neq 2$

$c = \frac{-6}{-3} = 2$

(4) $\begin{cases} x-y=k \\ 12x-ky+z=1 \\ 36x+kz=2 \end{cases}$

$0 \quad 0 \quad -12k$

$d = \begin{vmatrix} 1 & -1 & 0 \\ 12 & -k & 1 \\ 36 & 0 & k \end{vmatrix} \begin{matrix} 1 & -1 \\ 12 & -k \\ 36 & 0 \end{matrix} = -k^2-36+12k$

$-k^2 \quad -36 \quad 0$

$-k^2-36+12k \neq 0 \quad \cdot(-1)$

$k^2-12k+36 \neq 0$ $\qquad x = \frac{12 \pm 0}{2} = 6$ //

$\Delta = 144-4.1.36$ $\qquad$ (E)

$\Delta = 0 \therefore x_1 = x_2$ $\qquad \boxed{x \neq 6}$

(5) $\begin{cases} x-y+z=6 \\ 2x+y-z=-3 \\ x+2y-z=-5 \end{cases}$

$1 \quad -2 \quad 2 = 1$

$d = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix} \begin{matrix} 1 & -1 \\ 2 & 1 \\ 1 & 2 \end{matrix} = 4-1 = 3$ //

$-1 \quad 1 \quad 4 = 4$

$-5 \quad -12 \quad -3 = -20$ $\qquad -3 \quad 5 \quad -12 = -10$

$dx = \begin{vmatrix} 6 & -1 & 1 \\ -3 & 1 & -1 \\ -8 & 2 & -1 \end{vmatrix} \begin{matrix} 6 & -1 \\ -3 & 1 \\ -5 & 2 \end{matrix} = -17+20 = 3$ // $\qquad dy = \begin{vmatrix} 1 & 6 & 1 \\ 2 & -3 & -1 \\ 1 & -5 & -1 \end{vmatrix} \begin{matrix} 1 & 6 \\ 2 & -3 \\ 1 & -5 \end{matrix} = -13+10 = -3$ //

$-6 \quad -5 \quad -6 = -17$ $\qquad 3 \quad -6 \quad -10 = -13$

$X = \frac{dx}{d} = \frac{3}{3} = 1$ $\quad Y = \frac{dY}{d} = \frac{-3}{3} = -1$ $\quad x - y + z = 6$

$1 + 1 + z = 6$

$z = 6 - 2$

$z = 4$

B

$X \cdot Y \cdot Z = 1 \cdot (-1) \cdot 4 = \boxed{-4}$ S.P.D

6

$$\begin{cases} x + y + z = k \\ kx + y + z = 1 \\ x + y - z = k \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & k \\ k & 1 & 1 & 1 \\ 1 & 1 & -1 & k \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 2 & -k+k \\ -1+k & 0 & 2 & -k+1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1-k & 0 & 0 & k-1 \end{array}\right)$$

$(1-k)x = k-1$

$x = \frac{k-1}{1-k}$

$-k + 1 = 0$

$k = 1$

$x = \frac{1-1}{1-1} = \frac{0}{0}$

S.P.I

D

7

$$\begin{cases} x + y + z = 1 \\ mx - 2y + 4z = 5 \\ m^2x + 4y + 16z = 25 \end{cases}$$

$$\begin{array}{ccc} -2m^2 & 16 & 16m \end{array}$$

$$d = \left|\begin{array}{ccc} 1 & 1 & 1 \\ m & -2 & 4 \\ m^2 & 4 & 16 \end{array}\right| \begin{array}{cc} 1 & 1 \\ m & -2 \\ m^2 & 4 \end{array} =$$

$$\begin{array}{ccc} -32 & 4m^2 & 4m \end{array}$$

$4m^2 + 4m - 32 + 2m^2 - 16m - 16 = 0$

$6m^2 - 12m - 48 = 0 \quad : 6$

$m^2 - 2m - 8 = 0$

$\Delta = 4 - 4 \cdot 1 \cdot (-8)$

$\Delta = 36$

$m_1 = \frac{2+6}{2} = 4$

$m_2 = \frac{2-6}{2} = -2$

$m_1 + m_2 = 4 - 2 = \boxed{2}$

B

Exercícios - Sistemas Homogêneos

1) $\begin{cases} x+7y=kx \\ 7x+y=ky \end{cases}$ $\begin{array}{c} -7 \\ 1 \end{array}\left(\begin{array}{cc|c} 1 & 7 & k \\ 7 & 1 & k \end{array}\right) \sim \left(\begin{array}{cc|c} & & \\ 0 & -48 & k-6k \end{array}\right)$

$-48y=-6k$

$y=\dfrac{-6k}{-48} \begin{array}{c} :6 \\ :6 \end{array} = \dfrac{-k}{-8} \cdot(-1)$

$y=\dfrac{k}{8}$

$k=8y$

$x+7y=kx$

$x+7y=8yx$

$\dfrac{x}{x}=8y-7y$

$y=1$

$k=8y$

$k=8\cdot1$

$\boxed{k=8}$

E

2) $\begin{cases} 3x+4y-z=0 \\ 2x-y+3z=0 \\ x+y=0 \end{cases}$ $\begin{array}{c} \\ \\ -3\;-2 \end{array}\left(\begin{array}{ccc|c} 3 & 4 & -1 & 0 \\ 2 & -1 & 3 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ & & & \end{array}\right)$

$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ & & & \\ & & & \end{array}\right)$

$x=0$

$y=0$

$z=0$

$x+y=0$

$0+y=0$

$y=\dfrac{0}{0}$ S.P.I

$V=\{0,0,0\ldots\}$

D

3) $\begin{cases} x+y+z=0 \\ kx+3y+4z=0 \\ x+ky+3z=0 \end{cases}$

$3 \quad 4k \quad 3k = 7k+3$

$d=\left|\begin{array}{ccc} 1 & 1 & k \\ k & 3 & 4 \\ 1 & k & 3 \end{array}\right|\begin{array}{cc} 1 & 1 \\ k & 3 \\ 1 & k \end{array} = k^2+13-7k-3=k^2-7k+10$

$9 \quad 4 \quad k^2 = k^2+13$

$k^2-7k+10=0$

$\Delta=49-4\cdot1\cdot10$

$\Delta=9$

$k_1=\dfrac{7+9}{2}=8$ //

$k_2=\dfrac{7-9}{2}=-1$ //

$k_1+k_2=8-1=\boxed{7} \to$ S.P.I

D

④ $\begin{cases} x + kz = 0 \\ kx + y = 0 \\ x + ky = 0 \end{cases}$

$$\begin{array}{ccccc} & & k & 0 & 0 \end{array}$$

$$d = \left|\begin{array}{ccc|cc} 1 & 0 & k & 1 & 0 \\ k & 1 & 0 & k & 1 \\ 1 & k & 0 & 1 & k \end{array}\right| = k^3 - k$$

$$\begin{array}{ccccc} & & 0 & 0 & k^3 \end{array}$$

$k^3 - k \neq 0$

$k(k^2 - 1) \neq 0$

$k^2 - 1 \neq 0$

$k \neq \pm\sqrt{1}$

$k \neq -1, 1$

$S = \{k \in \mathbb{R} \,/\, k \neq 0, k \neq 1, k \neq -1\}$

A

⑤ $\begin{cases} -x + 2y - 3 = 0 \\ 3x - y + 3 = 0 \\ 2x - 4y + 6 = 0 \end{cases} \rightarrow \begin{cases} -x + 2y = 3 \\ 3x - y = -3 \\ 2x - 4y = -6 \end{cases} \Big]+ \rightarrow \begin{cases} 2x + y = 0 \\ 2x - 4y = -6 \end{cases}$

$d = \begin{vmatrix} 2 & 1 \\ 2 & -4 \end{vmatrix} = -8 - 2 = -10$ $\quad dx = \begin{vmatrix} 0 & 1 \\ -6 & -4 \end{vmatrix} = 0 + 6 = 6$

$dy = \begin{vmatrix} 2 & 0 \\ 2 & -6 \end{vmatrix} = -12 - 0 = -12$

Desta forma, concluímos que as soluções são determinadas.

B