Exercícios Matriz inversa

① $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$ $A = B^{-1} = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$

(C)

- Mudar posição, diagonal principal.
- Mudar sinal, diagonal secundária.

$\begin{pmatrix} 3 & -(-1) \\ -(y) & 2 \end{pmatrix} \rightarrow \begin{pmatrix} x & 1 \\ -5 & 3 \end{pmatrix}$ $y = -5$ $x = 2$

$x + y = 2 - 5 = -3$

②

$1 \quad 3k \quad 0 = 3k + 1$

$\begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix} = k^2 + 3 - 3k - 1$

$3 \quad 0 \quad k^2 = k^2 + 3$

$k^2 - 3k + 2 \neq 0$

$\Delta = 9 - 4 \cdot 1 \cdot 2 = 1$

$k_1 = \frac{3+1}{2} \neq 2$ $k_2 = \frac{3-1}{2} \neq 1$

$\{k \neq 1 \text{ e } k \neq 2\}$

(C)

③ $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} x & y \\ a & b \end{bmatrix}$ $\det_A = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} = 12 - 10 = 2$ ✓

- Troca de posição na diagonal principal e troca de sinal na secundária.

(C)

$\begin{bmatrix} 3 & -(5) \\ -(2) & 4 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2 = \begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & -\frac{3}{2} \end{bmatrix}$

(4)

$$\begin{array}{ccc|cc} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{array} = 1$$

$20 \quad 2x \quad 3x = 5x + 20$

$x^2 \quad 20 \quad 6 = x^2 + 26$

$1 = x^2 + 26 - 5x - 20$

$$\begin{cases} x^2 - 5x + 6 \neq 0 \\ \Delta = 25 - 4 \cdot 1 \cdot 6 = 1 \\ x_1 = \frac{5+1}{2} \neq 3 \qquad x_2 = \frac{5-1}{2} \neq 2 \\ \{x \neq 2 \text{ e } x \neq 3\} \end{cases}$$

(A)

(5)

$$\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2 \quad 2 \quad 2 = 6$

$$\det = \begin{array}{ccc|cc} -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{array} = 7 - 6 = 1$$

$1 \quad 2 \quad 4 = 7$

$$\begin{cases} -a - d + 2g = 1 \\ 2a + d - 2g = 0 \\ a + d - g = 0 \end{cases} + \rightarrow \boxed{a = 1}$$

$$\begin{cases} -1 - d + 2g = 1 \\ -d + 2g = 2 \\ g = 1 + d/2 \end{cases} \quad \begin{cases} 1 + d - 1 - d/2 = 0 \\ d - d/2 = 0 \\ d/2 = 0 \end{cases} \quad \begin{cases} \boxed{d = 0} \\ 1 + 0 - g = 0 \\ \boxed{g = 1} \end{cases}$$

$$\begin{cases} -b - e + 2h = 0 \\ 2b + e - 2h = 1 \\ b + e - h = 0 \end{cases} + \rightarrow \boxed{b = 1}$$

$$\begin{cases} -1 - e + 2h = 0 \\ -e + 2h = 1 \\ h = (1 + e)/2 \end{cases} \quad \begin{cases} 1 + e - (1 - e)/2 = 0 \\ 1 - 1/2 = -1 + 1/2 \\ e/2 = -1/2 \end{cases} \quad \begin{cases} \boxed{e = -1} \\ 1 - 1 - h = 0 \\ \boxed{h = 0} \end{cases}$$

$$\begin{cases} -c - f + 2i = 0 \\ 2c + f - 2i = 0 \\ c + f - i = 1 \end{cases} + \rightarrow \boxed{c = 0}$$

$$\begin{cases} 0 - f + 2i = 0 \\ i = \frac{f}{2} \end{cases} \quad \begin{cases} f - f/2 = 1 \\ f/2 = 1 \\ \boxed{f = 2} \end{cases} \quad \begin{cases} 0 + 2 - i = 1 \\ \boxed{i = 1} \end{cases}$$

$$\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

(B)

⑥ $(X \cdot A)^t = B$

$((X \cdot A)^t)^t = (B)^t$

$X \cdot A = B^t$

$X A \cdot A^{-1} = B^t \cdot A^{-1}$

$X \cdot I = B^t \cdot A^{-1}$

$X = B^t \cdot A^{-1}$

B

⑦ $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ $A = \frac{C}{B}$ $\rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$ $\det A = \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = -1$

$B = \begin{bmatrix} x \\ y \end{bmatrix}$ Cofator de $A = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div \det A$

D

$A = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$

⑧ $\det A^{-1} = \frac{1}{\det A}$ $\det A = \begin{vmatrix} 2 & K \\ -2 & 1 \end{vmatrix} = 2 + 2K$

$\det A^{-1} \cdot \det A = 1$

$2 + 2K \cdot (2 + 2K) = 1$

$4 + 4K + 4K + 4K^2 = 1$

$4K^2 + 8K + 3 = 0$

$\Delta = 64 - 4 \cdot 4 \cdot 3$ $K_1 = \frac{-8 + 4}{8} = \frac{-4}{8}$ $K_2 = \frac{-8 - 4}{8} = \frac{-12}{8}$

$\Delta = 64 - 48$

$\Delta = 16$

B

$K1 + K2 = \frac{-4}{8} + \left(\frac{-12}{8}\right) = \frac{-16}{8} = \boxed{-2}$

(9)

a) $(A+B) \cdot (A-B)$ $\qquad AB \neq BA$

$A^2 - AB + BA - B^2$

b) $(A+B)^2 = A^2 + 2AB + B^2$

AB precisa ser igual a BA, pois são matrizes que, se diferentes, a ordem de multiplicação mudará o resultado.

c) $\dfrac{\det A}{\det(-A)} \rightarrow \det(-A) = \det A \cdot (-1)^2$

$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ $\qquad \dfrac{\det A}{\det(-A)} = \dfrac{(ad-bc)}{(ad-bc)}$

$\det A \cdot (-1) \begin{vmatrix} -a & -b \\ -c & -d \end{vmatrix} = ad - bc$ $\qquad \dfrac{\det A}{\det(-A)} = \boxed{1}$

d) $B = A^{-1}$ $\quad \det B = \det A^{-1} \rightarrow \det^{-1} = \dfrac{1}{\det A}$

Logo:

$\boxed{\det B = \dfrac{1}{\det A}}$