Exercícios: Propriedades dos determinantes

01- $\begin{vmatrix} p & 2 & 2 \\ p & 4 & 4 \\ p & 4 & 1 \end{vmatrix} = -18$ $\begin{vmatrix} p & -1 & 2 \\ p & -2 & 4 \\ p & -2 & 1 \end{vmatrix}$

$8p \quad 16p \quad 2p = 26p$

$\begin{vmatrix} p & 2 & 2 \\ p & 4 & 4 \\ p & 4 & 1 \end{vmatrix} \begin{matrix} p & 2 \\ p & 4 \\ p & 4 \end{matrix}$

$4p \quad 8p \quad 8p = 20p$

$20p - 26p = -18$

$-6p = -18$

$p = \frac{-18}{-6} = 3$

$-12 \quad -24 \quad -3 = -39$

$\begin{vmatrix} 3 & -1 & 2 \\ 3 & -2 & 4 \\ 3 & -2 & 1 \end{vmatrix} \begin{matrix} 3 & -1 \\ 3 & -2 \\ 3 & -2 \end{matrix}$

$-6 \quad -12 \quad -12 = -30$

$det = -30 - (-39)$

$det = -30 + 39$

$det = 9$//

E

02-

$$A = \begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix}_{4\times4} = -6 \rightarrow 2 \cdot A = \det B$$

$2 \cdot A = x - 97$

$\det B = k^n \cdot \det A$

$x - 97 = 2^4 \cdot (-6)$

$x - 97 = 16 \cdot (-6)$

$x - 97 = -96$

$x = -96 + 97$

$x = 1$

C

03 — Demonstração:

$$B = y \cdot \begin{bmatrix} \begin{vmatrix} a & b & c \\ d & e & f \\ \underbrace{g \quad h \quad i}_{\div x} \end{vmatrix} \end{bmatrix} \qquad A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$\det B = k \cdot \det A$

$\det B = \frac{1}{x} \cdot y \cdot \det A$

$\det B = \frac{y}{x} \cdot \det A = \det B = \frac{\det A}{\frac{x}{y}}$

C

Exemplo:

$$A = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{vmatrix} = 4 \qquad B = 5 \begin{bmatrix} \begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ \underbrace{0 \quad 0 \quad 2}_{2} \end{vmatrix} \end{bmatrix} =$$

$\det B = k \cdot \det A$

$\det B = \frac{4}{\frac{2}{5}}$

$\det B = 10$

O mesmo ocorre assim:

$\det B = \frac{5}{2} \cdot 4 = 10$

04-

$$0 \quad 4K \quad -2K = 2K$$

$$\left|\begin{array}{ccc|cc} 2 & 1 & 0 & 2 & 1 \\ K & K & K & K & K \\ 1 & 2 & -2 & 1 & 2 \end{array}\right|$$

$$-4K \quad K \quad 0 = -3K$$

$-3K - 2K = 10$

$-5K = 10$

$K = \frac{10}{-5} = -2$

$$\left|\begin{array}{ccc} 2 & 1 & 0 \\ k+4 & k+3 & k-1 \\ 1 & 2 & -2 \end{array}\right| \rightarrow \left|\begin{array}{ccc} 2 & 1 & 0 \\ -2+4 & -2+3 & -2-1 \\ 1 & 2 & -2 \end{array}\right|$$

$$0 \quad -12 \quad -4 = -16$$

$$\left|\begin{array}{ccc|cc} 2 & 1 & 0 & 2 & 1 \\ 2 & 1 & -3 & 2 & 1 \\ 1 & 2 & -2 & 1 & 2 \end{array}\right|$$

$$-4 \quad -3 \quad 0 = -7$$

$det = -7 - (-16)$

$det = -7 + 16$

$det = 9$ //

(C)

05-

$$\left|\begin{array}{ccc} 1 & -11 & 6 \\ -2 & 4 & -3 \\ -3 & -7 & 2 \end{array}\right| = 0$$

Analisando a matriz, percebemos que uma coluna (a_{13}, a_{23}, a_{33}) pode ser usada em conjunto com outra coluna (a_{12}, a_{22}, a_{32}) para originar a terceira coluna da combinação linear (a_{11}, a_{21}, a_{31}). Desta forma aplicamos a seguinte fórmula para verificarmos:

$$\left|\begin{array}{ccc} 1 & -11 & 6 \\ -2 & 4 & -3 \\ -3 & -7 & 2 \end{array}\right|$$

$a_{n1} = 2 \cdot a_{n3} + a_{n2}$

$a_{11} = 2 \cdot 6 + (-11) = 1$

$a_{21} = 2 \cdot (-3) + 4 = -2$

$a_{31} = 2 \cdot 2 + (-7) = -3$

(D)

Por serem filas com a combinação linear descrita acima, segundo as propriedades dos determinantes, ele anulará.

06-

$2x^2 \quad -12 \quad 9x$

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & -3 & 9 \end{vmatrix} \begin{matrix} 1 & x \\ 1 & 2 \\ 1 & -3 \end{matrix} = 0$$

$18 \quad 4x \quad -3x^2$

$18+4x-3x^2-(2x^2-12+9x)=0$

$-3x^2-2x^2+4x-9x+18+12=0$

$-5x^2-5x+30=0 \div 5$

$-x^2-x+6=0 \quad .(-1)$

x^2+x-6

$\underline{-3} + \underline{2} = -1$

$\underline{-3} \cdot \underline{2} = -6$

$X1 = -3$

$X2 = 2$

07-

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 4 & 2 & 3 & -2 & 0 \\ 5 & 1 & 2 & 3 & 3 \end{vmatrix}$$

Quando a matriz é triangular, podemos calcular o determinante multiplicando os elementos da diagonal principal:

$det = 1.2.1.(-2).3$

$det = -12$

D