Tarefa básica - Determinantes, matrizes de ordem 1, 2 e 3

01 - a) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ → $det = (2 \cdot 5) - (1 \cdot 3)$

$det = 10 - 3 = 7$ //

b) $\begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix}$ → $det = (3 \cdot (-4)) - ((-2) \cdot 6)$

$det = -12 + 12 = 0$ //

c) $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{bmatrix}$ $\begin{matrix} 3 & -1 \\ 2 & 1 \\ 1 & 4 \end{matrix}$

$det = (-6 + 1 + 8) - (1 - 12 + 4)$

→ $det = 3 - (-7)$

$det = 3 + 7 = 10$ //

-6 1 8

1 -12 4

1) $\begin{vmatrix} 3 & 2 & -1 \\ 2 & 3 & 4 \\ 1 & 1 & 4 \end{vmatrix}$ $\begin{matrix} 3 & 2 \\ 2 & 3 \\ 1 & 1 \end{matrix}$ $\begin{matrix} 16 \\ 3 \\ -2 \end{matrix}$

$det = (36+2-2)-(-3+3+16)$

$det = 36-16$

$det = 20$

02- $a_{ij} = \begin{cases} -3, \text{ se } i=j \\ 0, \text{ se } i \neq j \end{cases}$

$a11=-3 \quad a12=0 \quad a13=0$

$a21=0 \quad a22=-3 \quad a23=0$

$a31=0 \quad a32=0 \quad a33=-3$

$A = \begin{vmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{vmatrix}$ $\begin{matrix} 0 & 0 \\ -3 & 0 \\ 0 & -3 \end{matrix}$ $\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$-27 \quad 0 \quad 0$

$detA = -27-0 = -27$

(A)

03- $\begin{vmatrix} x & 1 & x \\ 3 & x & 4 \\ 1 & 3 & 3 \end{vmatrix}$ $\begin{matrix} x^2 \\ 12x \\ 9 \end{matrix}$ $\begin{matrix} 1 \\ x \\ 3 \end{matrix}$

$3x^2 \quad 4 \quad 9x$

$3x^2+4+9x-(x^2+12x+9) = -3$

$3x^2-x^2-12x+9x+9+4+3=0$

$2x^2+3-2=0$

$\Delta = (-3)^2-4 \cdot 2 \cdot (-2)$

$\Delta = 9+16$

$\Delta = 25$

$x_1 = \frac{-(-3)+\sqrt{25}}{2 \cdot 2} = \frac{3+5}{4} = \frac{8}{4} = 2$

$x_2 = \frac{-(-3)-\sqrt{25}}{2 \cdot 2} = \frac{3-5}{4} = \frac{-2}{4} = \frac{-1}{2}$

(E)

(4) $\begin{vmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{vmatrix}$ $\begin{matrix} x-1 & -1 \\ 0 & x+1 \\ 2 & -1 \end{matrix}$

$a = (x-1)(x+1)(x+1)$

$a = x^2+x-x-1 \cdot (x+1)$

$a = x^3+x^2-x-1$

$b = 0 \cdot (-1) \cdot 0 = 0$

$c = 2 \cdot (-1) \cdot (-1) = 2$

$d = 2(x+1) \cdot 0 = 0$

$x = (x-1)(-1) \cdot (-1) =$

$1 = -x+1 \cdot (-1) = x-1$

$1 = 0 \cdot (-1) \cdot (x+1) = 0$

$G_{1,2} = x^3+x^2-x-1+0+2$

$G_{1,2} = x^3+x^2-x+1$

$S_{G_1} = 0+x+1+0 = x-1$

$x^3+x^2-x+1-x+1-2=0$

$x^3+x^2-2x=0$

$\frac{-b}{a} = \frac{-1}{1} = -1$

$\frac{-b}{a} = x_1+x_2$

(C)

$\frac{-b}{a}$

05 - $A = a_{ij} = 2i - 3j$ $\quad B = b_{jk} = k - j$

A (3×2)

$$\begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & -3 \end{bmatrix}$$

$a_{11} = 2.1 - 3.1 = -1$
$a_{21} = 2.2 - 3.1 = 1$
$a_{31} = 2.3 - 3.1 = 3$
$a_{12} = 2.1 - 3.2 = -4$
$a_{22} = 2.2 - 3.2 = -2$
$a_{32} = 2.3 - 3.3 = -3$

B (2×3)

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$b_{11} = 1 - 1 = 0$
$b_{21} = 1 - 2 = -1$
$b_{12} = 2 - 1 = 1$
$b_{22} = 2 - 2 = 0$
$b_{13} = 3 - 1 = 2$
$b_{23} = 3 - 2 = 1$

$$A \cdot B = \begin{bmatrix} 0+4 & -1-0 & -2-4 \\ 0+2 & 1-0 & 2-2 \\ 0+3 & 3-0 & 6-3 \end{bmatrix} = \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix}$$

$\det AB = (12 + 0 - 36) - (-18 + 0 - 6)$
$\det AB = -24 + 24 = 0$

(C)

06 - $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$ (2×3) $\quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$ (3×2)

(D)

$$A \cdot B = \begin{bmatrix} 2+0-0 & -2+0-2 \\ -1-1+0 & 1+1+0 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix}$$

$\det AB = (2.2) - (-2.(-4))$
$\det AB = 4 - 8 = -4$