Tarefa básica – Determinante, matrizes de ordem 1, 2 e 3

01 – a) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ → $\det = (2.5) - (1.3)$

$\det = 10 - 3 = 7$

b) $\begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix}$ → $\det = (3.(-4)) - ((-2).6)$

$\det = -12 + 12 = 0$

c) $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{bmatrix} \begin{matrix} 3 & -1 \\ 2 & 1 \\ 1 & 4 \end{matrix}$

$\det = (-6+1+8) - (1-12+4)$

→ $\det = 3 - (-7)$

$\det = 3 + 7 = 10$

2)

$$\begin{vmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{vmatrix} \begin{array}{cc} 3 & 2 \\ 2 & 3 \\ 1 & 1 \end{array}$$

$det = (36+2-2)-(-3+3+16)$

$det = 36 - 16$

$det = 20$

02-

$a_{ij} = \begin{cases} -3, & \text{se } i=j \\ 0, & \text{se } i \neq j \end{cases}$

$a11 = -3 \quad a12 = 0 \quad a13 = 0$
$a21 = 0 \quad a22 = -3 \quad a23 = 0$
$a31 = 0 \quad a32 = 0 \quad a33 = -3$

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{array}{cc} -3 & 0 \\ 0 & -3 \\ 0 & 0 \end{array}$$

$det A = -27 - 0 = -27$

(A)

03-

$$\begin{vmatrix} x & 1 & x \\ 3 & x & 4 \\ 1 & 3 & 3 \end{vmatrix} \begin{array}{cc} x & 1 \\ 3 & x \\ 1 & 3 \end{array}$$

$3x^2 + 4 + 9x - (x^2 + 12x + 9) = -3$

$3x^2 - x^2 - 12x + 9x + 9 + 4 + 3 = 0$

$2x^2 + x - 2 = 0$

$\Delta = (-3)^2 - 4 \cdot 2 \cdot (-2)$

$\Delta = 9 + 16$

$\Delta = 25$

$x_1 = \dfrac{-(-3) + \sqrt{25}}{2 \cdot 2} = \dfrac{3+5}{4} = \dfrac{8}{4} = 2$

(E)

$x_2 = \dfrac{-(-3) - \sqrt{25}}{2 \cdot 2} = \dfrac{3-5}{4} = \dfrac{-2}{4} = -\dfrac{1}{2}$

(4)

$$\begin{vmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{vmatrix}$$

$a = (x-1)(x+1)(x+1)$

$a = (x^2 + x - x - 1)(x+1)$

$a = x^3 + x^2 - x - 1$

$b = 0 \cdot (-1) \cdot 0 = 0$

$c = 2 \cdot (-1) \cdot (-1) = 2$

$d = 2 \cdot (x+1) \cdot 0 = 0$

$x = (x+1)(-1)(-1) = -$

$l = -x+1 \cdot (-1) = x - 1$

$1 = 0 \cdot (-1) \cdot (x+1) = 0$

$a_{inc} = x^3 + x^2 - x - 1 + 0 + 2$

$a_{inc} = x^3 + x^2 - x + 1$

$S_{ac} = 0 + x + 1 + 0 = x - 1$

$x^3 + x^2 - x + 1 - x - 1 - 2 = 0$

$x^3 + x^2 - 2x - 2 = 0$

$\dfrac{-b}{a} = \dfrac{-1}{1} = -1$

$\dfrac{-b}{a} = x_1 + x_2$

(C)

05 – $A \to a_{ij} = 2i-3j$ $B \to b_{jk} = k-j$

A

$$\begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & -3 \end{bmatrix}_{3\times2}$$

$a_{11} = 2\cdot1-3\cdot1 = -1$
$a_{21} = 2\cdot2-3\cdot1 = 1$
$a_{31} = 2\cdot3-3\cdot1 = 3$

$a_{12} = 2\cdot1-3\cdot2 = -4$
$a_{22} = 2\cdot2-3\cdot2 = -2$
$a_{32} = 2\cdot3-3\cdot3 = -3$

B

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}_{2\times3}$$

$b_{11} = 1-1 = 0$
$b_{21} = 1-2 = -1$

$b_{12} = 2-1 = 1$
$b_{22} = 2-2 = 0$

$b_{13} = 3-1 = 2$
$b_{23} = 3-2 = 1$

$$A\cdot B = \begin{bmatrix} 0+4 & -1-0 & -2-4 \\ 0+2 & 1+0 & 2-2 \\ 0+3 & 3-0 & 6-3 \end{bmatrix} = \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix}$$

Ⓒ

$\det AB = (12+0-36)-(-18+0-6)$
$\det AB = -24+24 = 0$

06 – $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}_{2\times3}$ $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}_{3\times2}$

Ⓓ

$$A\cdot B = \begin{bmatrix} 2+0-0 & -2+0-2 \\ -1-1+0 & 1+1+0 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix}$$

$\ast$ $\det AB = (2\cdot2)-(-2\cdot(-4))$
$\det AB = 4-8 = -4$