Tarefa básica - Determinantes, matrizes de ordem 1, 2 e 3

01 - a) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \rightarrow det = (2 \cdot 5) - (1 \cdot 3)$

$det = 10 - 3 = 7$

b) $\begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix} \rightarrow det = (3 \cdot (-4)) - ((-2) \cdot 6)$

$det = -12 + 12 = 0$

c) $\begin{array}{ccccc} & & 1 & -12 & 4 \\ \begin{bmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{bmatrix} & & \begin{matrix} 3 \\ 2 \\ 1 \end{matrix} & \begin{matrix} -1 \\ 1 \\ 4 \end{matrix} & \\ & -6 & 1 & 8 & \end{array}$

$det = (-6 + 1 + 8) - (1 - 12 + 4)$

$\rightarrow det = 3 - (-7)$

$det = 3 + 7 = 10$

data / fecha

1)
$$\begin{vmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{vmatrix}\begin{matrix} 3 & 2 \\ 2 & 3 \\ 1 & 1 \end{matrix}$$
$36 \quad 2 \quad -2$

$\det = (36+2-2)-(-3+3+16)$
$\rightarrow \det = 36-16$
$\det = 20$

02- $a_{ij} = \begin{cases} -3, \text{ se } i = j \\ 0, \text{ se } i \neq j \end{cases}$

$a11 = -3 \quad a12 = 0 \quad a13 = 0$
$a21 = 0 \quad a22 = -3 \quad a23 = 0$
$a31 = 0 \quad a32 = 0 \quad a33 = -3$

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}\begin{matrix} -3 & 0 \\ 0 & -3 \\ 0 & 0 \end{matrix}$$
$-27 \quad 0 \quad 0$

$\rightarrow \det A = -27 - 0 = -27$

(A)

03-
$$\begin{vmatrix} x & 1 & x \\ 3 & x & 4 \\ 1 & 3 & 3 \end{vmatrix}\begin{matrix} x & 1 \\ 3 & x \\ 1 & 3 \end{matrix}$$
$3x^2 \quad 4 \quad 9x$

$\rightarrow (3x^2+4+9x-(x^2+12x+9) = -3$
$3x^2 - x^2 - 12x + 9x - 9 + 4 + 3 = 0$
$2x^2 - 3x - 2 = 0$
$\Delta = (-3)^2 - 4 \cdot 2 \cdot (-2)$
$\Delta = 9 + 16$
$\Delta = 25$
$x_1 = \frac{-(-3)+\sqrt{25}}{2 \cdot 2} = \frac{3+5}{4} = \frac{8}{4} = 2$
$x_2 = \frac{-(-3)-\sqrt{25}}{2 \cdot 2} = \frac{3-5}{4} = \frac{-2}{4} = \frac{-1}{2}$

(E)

4)
$$\begin{vmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{vmatrix}$$
$x-1 \quad -1 \quad 0$
$0 \quad x+1 \quad -1$

$a = (x-1) \cdot (x+1) \cdot (x+1)$
$a = x^2 + x - x - 1 \cdot (x+1)$
$a = x^2 - x - 1$
$b = 0 \cdot (-1) \cdot 0 = 0$
$c = 2 \cdot (-1) \cdot (-1) = 2$
$d = 2(x+1) \cdot 0 = 2x$
$e = (-1) \cdot (-1) \cdot (-1) = -1$
$f = 0 \cdot (-1) \cdot x+1 = 1$

$x^2 - x - 1 + 0 + 2 + 2x - 1 + 1 = 2$
$x^2 - x + 2x + 0 + 2 - 1 + 1 - 2 = 0$
$x^2 + x = 0$
$x(x+1) = 0$
$x = 0 \quad x = -1$
$-1 + 0 = -1$

(C)

05 - $A \rightarrow a_{ij} = 2i - 3j$ $\quad B \rightarrow b_{jk} = k - j$

$$A = \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & -3 \end{bmatrix}_{3\times2}$$

$a_{11} = 2.1 - 3.1 = -1$
$a_{21} = 2.2 - 3.1 = 1$
$a_{31} = 2.3 - 3.1 = 3$

$a_{12} = 2.1 - 3.2 = -4$
$a_{22} = 2.2 - 3.2 = -2$
$a_{32} = 2.3 - 3.3 = -3$

$$B = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}_{2\times3}$$

$b_{11} = 1 - 1 = 0$
$b_{21} = 1 - 2 = -1$

$b_{12} = 2 - 1 = 1$
$b_{22} = 2 - 2 = 0$

$b_{13} = 3 - 1 = 2$
$b_{23} = 3 - 2 = 1$

$$A.B = \begin{bmatrix} 0+4 & -1-0 & -2-4 \\ 0+2 & 1+0 & 2-2 \\ 0+3 & 3-0 & 6-3 \end{bmatrix} = \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix} \begin{matrix} 4 & -1 \\ 2 & 1 \\ 3 & 3 \end{matrix}$$

18 0 -6

12 18 -36

$\det AB = (12 + 0 - 36) - (18 + 0 - 6)$
$\det AB = -24 + 24 = 0$

C

06 - $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}_{2\times3}$ $\quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}_{3\times2}$

D

$$A.B = \begin{bmatrix} 2+0-0 & -2+0-2 \\ -1-1+0 & 1+1+0 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix}$$

$\det AB = (2.2) - (-2.(-4))$
$\det AB = 4 - 8 = -4$