Transformada de Laplace

1)

a) $\frac{dy(t)}{dt} = 3-2t$, $y(0)=0$

$$\mathcal{L}\left(\frac{dy(t)}{dt}\right) = \mathcal{L}(3-2t)$$

$$sY(s) - \cancel{y(0)}^{\to 0} = \frac{3}{s} - \frac{2}{s^2}$$

$$\boxed{Y(s) = \frac{3s-2}{s^3}}$$

b) $\frac{dy(t)}{dt} = e^{-3t}$, $y(0)=4$

$$sY(s) - y(0) = \frac{1}{s+3}$$

$$Y(s) = \frac{4s+12+1}{(s+3)s}$$

$$\boxed{Y(s) = \frac{4s+13}{(s+3)s}}$$

c) $\frac{dy(t)}{dt} + y(t) = f(t)$, $y(0)=a$

$$sY(s) - y(0) + Y(s) = \mathcal{L}(f(t))$$

$$Y(s)(1+s) = \mathcal{L}(f(t)) + a$$

$$\boxed{Y(s) = \frac{\mathcal{L}(f(t)) + a}{1+s}}$$

d) $\frac{dy(t)}{dt} + y(t) = e^{t}$, $y(0)=1$

$$sY(s) - y(0) + Y(s) = \frac{1}{s-1}$$

$$Y(s)(1+s) = \frac{s}{(s-1)}$$

$$\boxed{Y(s) = \frac{s}{s^2-1}}$$

e) $\frac{dy(t)}{dt} - 5y(t) = 0$, $y(0)=2$

$$sY(s) - 2 - 5Y(s) = 0$$

$$\boxed{Y(s) = \frac{2}{s-5}}$$

f) $\frac{dy(t)}{dt} - 5y(t) = e^{5t}$, $y(0)=0$

$$sY(s) - 0 - 5Y(s) = \frac{1}{s-5}$$

$$\boxed{Y(s) = \frac{1}{(s-5)^2}}$$

2

g) $\frac{d^2y(t)}{dt^2} = 1-t$, $y(0)=0$, $\dot{y}(0)=0$

$$s^2Y(s) - \cancel{sy(0)} - \cancel{\dot{y}(0)} = \frac{1}{s} - \frac{1}{s^2}$$

$$\boxed{Y(s) = \frac{s-1}{s^4}}$$

h) $\frac{d^2y(t)}{dt^2} + 16y(t) = 5\delta(t-1)$, $y(0)=0$, $\dot{y}(0)=0$

impulso

$$s^2Y(s) + 16Y(s) = 5e^{-s} \times 1$$

$$Y(s)(s^2+16) = 5e^{-s}$$

$$\boxed{Y(s) = \frac{5e^{-s}}{s^2+16}}$$

i) $\frac{d^2y(t)}{dt^2} + 16y(t) = 16u(t-3) - 16$, $y(0)=0$, $\dot{y}(0)=0$

degrau

$$s^2Y(s) + 16Y(s) = 16\frac{e^{-3s}}{s} - \frac{16}{s}$$

$$\boxed{Y(s) = \frac{16(e^{-3s}-1)}{s(s^2+16)}}$$

j) $\frac{d^2y(t)}{dt^2} + 4y(t) = \cos(t)$, $y(0)=a$, $\dot{y}(0)=b$

$$s^2Y(s) - as - b + 4Y(s) = \frac{s}{s^2+1}$$

$$Y(s)(s^2+4) = \frac{s+(s^2+1)(as+b)}{s^2+1}$$

$$\boxed{Y(s) = \frac{as^3+bs^2+(a+1)s+b}{(s^2+1)(s^2+4)}}$$

k) $\frac{d^2y(t)}{dt^2} + y(t) = te^{-t}$, $y(0)=a$, $\dot{y}(0)=b$

$$s^2Y(s) - as - b + Y(s) = \frac{1}{(s+1)^2}$$

$$Y(s)(s^2+1) = 1 + (as+b)\underbrace{(s+1)^2}_{s^2+2s+1}$$

$$\boxed{Y(s) = \frac{as^3+bs^2+2as^2+2bs+as+b+1}{s^2+1}}$$

l) $\frac{d^2y(t)}{dt^2} - y(t) = e^t$, $y(0)=1$, $\dot{y}(0)=0$

$$s^2Y(s) - s - Y(s) = \frac{1}{s-1}$$

$$Y(s)(s^2-1) = \frac{1+s^2-s}{s-1}$$

$$\boxed{Y(s) = \frac{s^2-s+1}{(s^2-1)(s-1)}}$$

m) $\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = 4t^2, \quad \begin{matrix} y(0)=1 \\ \dot{y}(0)=4 \end{matrix}$

$$s^2Y(s) - s - 4 - sY(s) + 1 - 2Y(s) = \frac{4\times 2}{s^3}$$

$$Y(s)\left(s^2 - s - 2\right) = \frac{8}{s^3} + (s+3)$$

$$\boxed{Y(s) = \frac{s^4 + 3s^3 + 8}{s^3(s^2-s-2)}}$$

n) $y(t) = t + \int_0^t -y(\tau)\,\mathrm{sen}(-t+\tau)\,d\tau$,

$$f_1 * f_2 = \int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau$$

$\mathrm{sen}(-t+\tau) = -\mathrm{sen}(t-\tau)$

→ convolução no domínio do tempo é multiplicação no domínio de s.

$$\mathcal{L}\left[f_1 * f_2\right] = F_1(s)\,F_2(s)$$
$$= -Y(s)\,\frac{-1}{s^2+1}$$

$$Y(s) = \frac{1}{s^2} + \frac{Y(s)}{s^2+1} \Rightarrow Y(s)\left(\frac{s^2+1-1}{s^2+1}\right) = \frac{1}{s^2}$$

$$\boxed{Y(s) = \frac{s^2+1}{s^4}}$$

o) $y(t) = t^2\cos^2 t$,

Sabe-se que:

$$\mathcal{L}\left[t^2 y(t)\right] = \frac{d^2Y(s)}{ds^2} \quad \text{e} \quad \cos^2 t = \frac{1}{2}\left[\cos(2t) + 1\right]$$

$$Y(s) = \mathcal{L}\left[t^2 \frac{1}{2}(\cos 2t + 1)\right]$$

$$= \frac{1}{2}\mathcal{L}\left[t^2 \cos 2t\right] + \frac{1}{2}\mathcal{L}\left[t^2\right]$$

$$= \frac{1}{2}\frac{d^2}{ds^2}\left(\frac{s}{s^2+4}\right) + \frac{1}{\cancel{2}}\frac{\cancel{2}}{s^3} =$$

$$\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = \frac{s^2+4-s(2s)}{(s^2+4)^2} = \frac{s^2+4-2s^2}{(s^2+4)^2} = \frac{-s^2+4}{(s^2+4)^2}$$

$$\frac{d^2}{ds^2}\left(\frac{s}{s^2+4}\right) = \frac{-2s(s^2+4)^2 - (-s^2+4)\,2(s^2+4)\,2s}{(s^2+4)^4}$$

$$= \frac{-2s(s^2+4) + (s^2-4)4s}{(s^2+4)^3} = \frac{-2s^3-8s+4s^3-16s}{(s^2+4)^3}$$

$$= \frac{2s^3-24s}{(s^2+4)^3}$$

$$\therefore \boxed{Y(s) = \frac{s(s^2-12)}{(s^2+4)^3} + \frac{1}{s^3}}$$

Transformada Inversa de Laplace

LISTA I

2) (nível 0)

a) $\frac{1}{s^2+b^2}$

$\mathcal{L}(\operatorname{sen}\omega t) = \frac{\omega}{s^2+\omega^2}$

$\therefore \mathcal{L}^{-1}\left(\frac{1}{s^2+b^2}\right) = \frac{1}{b}\operatorname{sen} bt$

b) $\frac{1}{\underbrace{(s+a)^2}+b^2}$

função translada de a em S = multiplicação por e^{-at} em t

$\therefore \mathcal{L}^{-1}\left(\frac{1}{(s+a)^2+b^2}\right) = \frac{e^{-at}}{b}\operatorname{sen} bt$

c) $\frac{1}{s^n}$

$\mathcal{L}^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$

d) $\frac{1}{(s-a)^n}$

$\therefore \mathcal{L}^{-1}\left(\frac{1}{(s-a)^n}\right) = \frac{e^{at}t^{n-1}}{(n-1)!}$

(nível 1)

e) $\frac{4}{s-2} - \frac{3}{s+5}$ $\quad \mathcal{L}(e^{-at}) = \frac{1}{s+a}$

$y(t) = 4e^{2t} - 3e^{-5t}$

f) $\frac{s+5}{s^2+9} = \frac{s}{s^2+9} + \frac{5}{s^2+9}$

$y(t) = \cos 3t + \frac{5}{3}\operatorname{sen} 3t$

g) $\frac{8(s+2)-4}{(s+2)^2+25} = \frac{8(s+2)}{(s+2)^2+25} - \frac{4}{(s+2)^2+25}$

$y(t) = 8e^{-2t}\cos 5t - \frac{4}{5}e^{-2t}\operatorname{sen} 5t$

k) $\frac{4}{s} - \frac{1}{s^2} + \frac{5}{s^3} + \frac{2}{s^4}$

$y(t) = 4 - t + \frac{5}{2!}t^2 + \frac{2}{3!}t^3$

2

i) $\frac{10}{(s-5)^2} + \frac{2}{(s-5)^3}$

$y(t) = 10\, e^{5t}\, t + \frac{\cancel{2}}{\cancel{2!}} e^{5t} t^2$

$\boxed{y(t) = 10t\, e^{5t} + t^2 e^{5t}}$

j) $\frac{1}{s^2+6s+13}$

$\Delta = 6^2 - 4\times 13 < 0$

$\therefore$ senóide amortecida

$s^2+6s+13 = (s+3)^2+4$

$\therefore = \frac{1}{(s+3)^2+4}$

$\boxed{\therefore y(t) = \frac{1}{2} e^{-3t} \operatorname{sen} 2t}$

k) $\frac{6}{s} - \frac{1}{s-8} + \frac{4}{s-3}$

$\boxed{y(t) = 6 - e^{8t} + 4e^{3t}}$

l) $\frac{19}{s+2} - \frac{1}{3s-5} + \frac{7}{s^5}$

$= \frac{19}{s-(-2)} - \frac{1}{3(s-\frac{5}{3})} + \frac{4!}{4!}\frac{7}{s^{4+1}}$

$y(t) = 19\, e^{-2t} - \frac{e^{5t/3}}{3} + \frac{7}{4!} t^4$

$\boxed{y(t) = 19 e^{-2t} - \frac{1}{3} e^{5t/3} + \frac{7}{24} t^4}$

m) $\frac{6s}{s^2+25} + \frac{3}{s^2+25}$

$\boxed{y(t) = 6 \cos 5t + \frac{3}{5} \operatorname{sen} 5t}$

n) $\frac{8}{3s^2+12} + \frac{3}{s^2-49}$

$= \frac{8}{3} \frac{1}{(s^2+4)} + 3 \frac{1}{s^2-49}$

$\boxed{y(t) = \frac{8}{6} \operatorname{sen} 2t + \frac{3}{7} \operatorname{senh}(7t)}$

o) $\frac{6s-5}{s^2+7}$

$= \frac{6s}{s^2+7} - \frac{5}{s^2+7}$

$\boxed{y(t) = 6 \cos\sqrt{7}t - \frac{5}{\sqrt{7}} \operatorname{sen}\sqrt{7}t}$

(nível 2)

p) $\frac{1-3s}{s^2+8s+21}$

$s^2+8s+21 = s^2+8s+16+5$
$= (s+4)^2+5$

$\therefore \frac{1-3(s+4)+12}{(s+4)^2+5} = \frac{13-3(s+4)}{(s+4)^2+5} = \boxed{\frac{13}{\sqrt{5}} e^{-4t} \operatorname{sen}\sqrt{5}t - 3 e^{-4t} \cos\sqrt{5}t}$

q) $\dfrac{3s-2}{2s^2-6s-2} = \dfrac{3s-2}{2(s^2-3s-1)}$ 3

$s^2-3s-1 = s^2-3s+\frac{9}{4}-\frac{9}{4}-1 = \left(s-\frac{3}{2}\right)^2-\frac{13}{4}$

$$= \frac{3s-2}{2\left[\left(s-\frac{3}{2}\right)^2-\frac{13}{4}\right]} = \frac{3(s-3/2)+9/2-2}{2\left[\left(s-\frac{3}{2}\right)^2-\frac{13}{4}\right]} = \frac{3(s-3/2)+5/2}{2\left[\left(s-\frac{3}{2}\right)^2-\frac{13}{4}\right]}$$

$$= \frac{3}{2}\frac{(s-3/2)}{(s-3/2)^2-\frac{13}{4}} + \frac{5}{4}\frac{1}{\left[(s-3/2)^2-\frac{13}{4}\right]}$$

$$\therefore\ y(t) = \frac{3}{2}e^{3t/2}\cosh\frac{\sqrt{13}}{2}t + \frac{5}{2\cancel{4}}\frac{\cancel{\sqrt{4}}}{\sqrt{13}}e^{3t/2}\operatorname{senh}\frac{\sqrt{13}}{2}t$$

$$y(t) = \frac{e^{3t/2}}{2}\left[3\cosh\frac{\sqrt{13}}{2}t + \frac{5}{\sqrt{13}}\operatorname{senh}\frac{\sqrt{13}}{2}t\right]$$

(nível 3, expansão em frações parciais)

r) $\dfrac{s+7}{s^2-3s-10}$

$\Delta = 9+40=49$

$s_1 = \frac{10}{2}=5;\ s_2=-2$

$$\therefore = \frac{s+7}{(s-5)(s+2)} = \frac{A}{(s+2)} + \frac{B}{(s-5)} = \frac{A(s-5)+B(s+2)}{(s+2)(s-5)}$$

$s+7 = A(s-5)+B(s+2)$

$s=-2 \Rightarrow -2+7 = -7A \quad \therefore A = -\frac{5}{7}$

$s=5 \Rightarrow 12 = 7B \quad \therefore B = \frac{12}{7}$

$$\therefore Y(s) = -\frac{5/7}{(s+2)} + \frac{12/7}{(s-5)} = -\frac{5}{7}e^{-2t} + \frac{12}{7}e^{5t}$$

5) $\dfrac{1}{S(S^3+6S^2+11S+6)} =$

$S^3+6S^2+11S+6 = S(S^2+6S+11)+6$

$= S(S^2+6S+9)+6+2S$ 4

$= S(S+3)^2+2(S+3)$

$= (S+3)\left[S(S+3)+2\right]$

$S^2+3S+2 \Rightarrow S^2+2S+1+S+1$

$= (S+3)\left[(S+1)^2+(S+1)\right]$

$= (S+3)(S+1)(S+1+1)$

$= (S+1)(S+2)(S+3)$

$= \dfrac{1}{S(S+1)(S+2)(S+3)}$

$= \dfrac{A}{(S)} + \dfrac{B}{(S+1)} + \dfrac{C}{(S+2)} + \dfrac{D}{(S+3)}$

$A(S+1)(S+2)(S+3) + Bs(S+2)(S+3) + Cs(s+1)(S+3) + Ds(S+1)(S+2) = 1$

$-2(-1)(1)$ $\quad -3\ -2\ -1$

$S=-2 \rightarrow 2C=1 \quad C=1/2$

$S=0 \rightarrow 6A=1 \quad A=1/6$

$S=-3 \rightarrow -6D=1 \quad D=-1/6$

$S=-1 \rightarrow -2B=1 \quad B=-1/2$

$\therefore Y(S) = \dfrac{1}{6s} - \dfrac{1}{2(S+1)} + \dfrac{1}{2(S+2)} - \dfrac{1}{6(S+3)}$

$y(t) = \dfrac{1}{6} - \dfrac{1}{2}e^{-t} + \dfrac{1}{2}e^{-2t} - \dfrac{1}{6}e^{-3t}$

t) $\dfrac{S+1}{S(S^2+4S+4)} = \dfrac{S+1}{S(S+2)^2} = \dfrac{A}{S} + \dfrac{B}{(S+2)} + \dfrac{C}{(S+2)^2}$

$S+1 = A(S+2)^2 + BS(S+2) + CS$

$S+1 = AS^2 + 4AS + 4A + BS^2 + 2BS + CS$

$S+1 = (A+B)S^2 + (4A+2B+C)S + 4A$

$A+B=0$

$4A+2B+C=1$

$4A=1 \therefore A=1/4,\ B=-1/4,$

$\cancel{1} - 1/2 + C = \cancel{1} \rightarrow C = \frac{1}{2}$

$Y(S) = \dfrac{1}{4S} - \dfrac{1}{4(S+2)} + \dfrac{1}{2(S+2)^2} \quad \therefore y(t) = \dfrac{1}{4} - \dfrac{1}{4}e^{-2t} + \dfrac{1}{2}te^{-2t}$

u) $\dfrac{s^2+2s+3}{(s+1)^3} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{C}{(s+1)^3}$

$$s^2+2s+3 = A(s+1)^2 + B(s+1) + C$$
$$= As^2 + 2As + A + Bs + B + C$$
$$= As^2 + (2A+B)s + A + B + C$$

$A = 1$

$2A+B = 2 \quad \therefore B = \emptyset$

$A+B+C = 3 \quad \therefore C = 2$

maneira alternativa:

$s = -1 \to C = 2$

$A = 1$

$\therefore B = 2 - 2A = 0$

$$Y(s) = \frac{1}{(s+1)} + \frac{2}{(s+1)^3}$$

$$\therefore y(t) = e^{-t} + \frac{2}{2} e^{-t} t^2 = e^{-t}(1+t^2)$$

v) $\dfrac{86s-78}{(s+3)(s-4)(5s-1)} = \dfrac{A}{(s+3)} + \dfrac{B}{(s-4)} + \dfrac{C}{(5s-1)}$

$$86s-78 = A(s-4)(5s-1) + B(s+3)(5s-1) + C(s+3)(s-4)$$

$s = -3 \to -336 = A(-7)(-16) \Rightarrow A = -\dfrac{336}{112} = -3$ $\quad (2\times 8)$

$s = 4 \to 266 = B(7)(19) \Rightarrow B = \dfrac{266}{133} = 2$

$s = \dfrac{1}{5} \to \dfrac{-304}{\cancel{5}} = C\left(\dfrac{16}{\cancel{5}}\right)\left(-\dfrac{19}{5}\right) \Rightarrow C = \dfrac{1520}{304} = 5$

$$\therefore Y(s) = \frac{3}{s+3} + \frac{2}{s-4} + \frac{\cancel{5}}{\cancel{5}(s-1/5)}$$

$$\boxed{y(t) = -3e^{-3t} + 2e^{4t} + e^{t/5}}$$

x) $\dfrac{2-5s}{(s-6)(s^2+11)} = \dfrac{A}{(s-6)} + \dfrac{Bs+C}{(s^2+11)}$ 6

$2-5s = A(s^2+11) + Bs(s-6) + C(s-6)$

$2-5s = As^2+11A+Bs^2-6Bs+Cs-6C$

$2-5s = (A+B)s^2 + (-6B+C)s + (11A-6C)$

$A+B \quad = 0$

$-6B+C = -5$

$11A \quad -6C = 2$

$-36B+6C = -30$

$-11B-6C = 2$

$-47B = -28$

$\therefore B = \dfrac{28}{47}$

$\therefore A = -\dfrac{28}{47}$

$C = -5+6B$

$= -5+6\times\dfrac{28}{47} = -\dfrac{67}{47}$

$\therefore Y(s) = -\dfrac{28}{47}\dfrac{1}{(s-6)} + \dfrac{1}{47}\dfrac{(28s-67)}{(s^2+11)}$

$= \dfrac{1}{47}\left[-\dfrac{28}{(s-6)} + \dfrac{28s}{(s^2+11)} - \dfrac{67}{(s^2+11)}\right]$

$\therefore y(t) = \dfrac{1}{47}\left(-28e^{6t} + 28\cos\sqrt{11}t - \dfrac{67}{\sqrt{11}}\operatorname{sen}\sqrt{11}t\right)$

y) $\dfrac{25}{s^3(s^2+4s+5)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s^3} + \dfrac{Ds+E}{(s^2+4s+5)}$

$\Delta = 16-20 < 0$

$\therefore s = \dfrac{-4\pm\sqrt{4}i}{2}$

$\begin{cases} s_1 = -2+i \\ s_2 = -2-i \end{cases}$

$(s+2)^2+1$

$= \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s^3} + \dfrac{Ds+E}{(s+2)^2+1}$

$$25 = As^2(s^2+4s+5) + Bs(s^2+4s+5) + C(s^2+4s+5) + Ds^4 + Es^3$$

$$25 = As^4 + 4As^3 + 5As^2 + Bs^3 + 4Bs^2 + 5Bs + Cs^2 + 4Cs + 5C + Ds^4 + Es^3$$

$$25 = (A+D)s^4 + (4A+B+E)s^3 + (5A+4B+C)s^2 + (5B+4C)s + 5C$$

$A+D=0$ $\quad D=-11/5$

$4A+B+E=0$ $\quad 44/5-4+E=0 \quad E=-24/5$

$5A+4B+C=0$ $\quad 5A-16+5=0 \rightarrow \boxed{A=11/5}$

$5B+4C=0$ $\quad 5B=-20 \therefore \boxed{B=-4}$

$5C=25 \quad \therefore \boxed{C=5}$

$$\therefore Y(s) = \frac{11}{5}\frac{1}{s} - \frac{4}{s^2} + \frac{5}{s^3} - \frac{1}{5}\frac{(11s+24)}{(s+2)^2+1}$$

$$= \frac{1}{5}\left[\frac{11}{s} - \frac{20}{s^2} + \frac{25}{s^3} - \frac{11(s+2)-22+24}{(s+2)^2+1}\right]$$

$$= \frac{1}{5}\left[\frac{11}{s} - \frac{20}{s^2} + \frac{25}{s^3} - \frac{11(s+2)}{(s+2)^2+1} + \frac{2}{(s+2)^2+1}\right]$$

$$\therefore y(t) = \frac{1}{5}\left(11 - 20t + \frac{25}{2}t^2 - 11e^{-2t}\cos t + 2e^{-2t}\sin t\right)$$

8

4)

nível 4,

a) $2s + 1s^3 + s^2$

$\mathcal{L}\{\delta(t)\} = 1$, onde $\delta(t)$ é a função impulso

$s^3 F(s) + s^2 F(s) + 2s F(s)$

$\frac{d^3}{dt^3}\delta(t) + \frac{d^2}{dt^2}\delta(t) + 2\frac{d}{dt}\delta(t)$, p/ $t > 0^-$

b) $6s + 3s^2$

$6s F(s) + 3 s^2 F(s)$

$6\frac{d}{dt}\delta(t) + 3\frac{d^2}{dt^2}\delta(t)$,

p/ $t > 0^-$

c) $5s + 2s^3 + 5s^2 + 8s + 4$

$13\frac{d}{dt}\delta(t) + 2\frac{d^3}{dt^3}\delta(t) + 5\frac{d^2}{dt^2}\delta(t) + 4\delta(t)$

p/ $t > 0^-$

d) $\frac{1}{s^2(s^2+\omega^2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{(s^2+\omega^2)}$

$1 = As(s^2+\omega^2) + B(s^2+\omega^2) + Cs + D(s^2)$

$1 = As^3 + As\omega^2 + Bs^2 + B\omega^2 + Cs + Ds^2$

$1 = As^3 + (B+D)s^2 + (A\omega^2 + C)s + B\omega^2$

$A = 0$

$B + D = 0$

$A\omega^2 + C = 0 \quad \therefore C = 0$

$B\omega^2 = 1 \quad \therefore B = 1/\omega^2 \quad \therefore D = -\frac{1}{\omega^2}$

$\therefore Y(s) = \frac{1}{\omega^2 s^2} - \frac{1}{\omega^2}\frac{1}{(s^2+\omega^2)}$

$\therefore y(t) = \frac{1}{\omega^2}t - \frac{1}{\omega^3}\operatorname{sen}\omega t$

(ver item 26, tabela 2.1, Ogata)

e) $\dfrac{5e^{-s}}{s+1}$

$$y(t) = 5\,u_1(t)\,e^{-(t-1)},$$

onde: $u_1(t) = u(t-1)$
(ver lista I, última página)

f) $\dfrac{10(s+2)(s+4)}{(s+1)(s+3)(s+5)^2} = \dfrac{A}{(s+1)} + \dfrac{B}{(s+3)} + \dfrac{C}{(s+5)} + \dfrac{D}{(s+5)^2}$

$$10(s^2+4s+2s+8) = A(s+3)(s+5)^2 + B(s+1)(s+5)^2 + C(s+1)(s+3)(s+5) + D(s+1)(s+3)$$

$s^2+3s+s+3$

$1 \; -4 \; -2 \; +8$

$9 \; -\cancel{12} \; -6 \; +\cancel{8}$

5

$s=-1 \rightarrow 30 = A(2)(16)$

$A = \dfrac{30}{32} = \dfrac{15}{16}$

$s=-3 \rightarrow -1 = B(-2)(2)^2$

$B = \dfrac{1}{8}$

$s^3+10s^2+25s+s^2+10s+25$

$s^3+10s^2+25s+3s^2+30s+75$

$$10s^2+60s+80 = A(s+3)(s^2+10s+25) + B(s+1)(s^2+10s+25) + C(s^2+4s+3)(s+5) + D(s^2+4s+3)$$

$s^3+4s^2+3s+5s^2+20s+15$

$$10s^2+60s+80 = A(s^3+13s^2+55s+75) + B(s^3+11s^2+35s+25) + C(s^3+9s^2+23s+15) + D(s^2+4s+3)$$

$A+B+C=0 \longrightarrow \dfrac{15}{16}+\dfrac{2}{16}+C=0 \;\therefore\; C = \dfrac{-17}{16}$

$13A+11B+9C+D=10$

$13\times\dfrac{15}{16} + 11\times\dfrac{2}{16} - 9\times\dfrac{17}{16} + \dfrac{16D}{16} = \dfrac{160}{16} \;\therefore\; D = \dfrac{96}{16}$

$55A+35B+23C+4D=60$

$75A+25B+15C+3D=80$

10

$$\therefore Y(s) = \frac{15}{16}\frac{1}{(s+1)} + \frac{2}{16}\frac{1}{(s+3)} - \frac{17}{16}\frac{1}{(s+5)} + \frac{96}{16}\frac{1}{(s+5)^2}$$

$$\therefore y(t) = \frac{1}{16}\left(15e^{-t} + 2e^{-3t} - 17e^{-5t} + 96e^{-5t}t\right)$$

g) $\dfrac{s^4+5s^3+6s^2+9s+30}{s^4+6s^3+21s^2+46s+30}$

MATLAB:

num = [1 5 6 9 30]

den = [1 6 21 46 30]

[r,p,k] = residue (num,den)

Denominador reescrito a partir do resultado do MatLab,

$$(s+3)(s+1)\left[(s+1)^2+3^2\right]$$

$$\frac{A}{(s+1)^2+3^2} + \frac{B}{(s+3)} + \frac{C}{(s+1)} + 1$$

$B = -0,1154$
$C = 1,2778$

$$A = (-1,0812 + 1,7051i)(s+1+3i) + (-1,0812 - 1,7051i)(s+1-3i)$$

$$A = -1,0812s + 1,0812 - 3\times 1,0812i + 1,7051is + 1,7051i - 3\times 1,705 + -1,0812s - 1,0812 + 3\times 1,0812i - 1,7051is - 1,7051i - 3\times 1,705 =$$

$$A = -2\times 1,0812s - 2\times 1,0812 - 6\times 1,705 = -2,1624s - 12,3924$$

$$\therefore Y(s) = \frac{-2,16s - 12,39}{(s+1)^2+3^2} - \frac{0,1154}{(s+3)} + \frac{1,2778}{(s+1)} + 1$$

$$Y(s) = \frac{-2,16(s+1)}{(s+1)^2+3^2} - \frac{10.23}{(s+1)^2+3^2} - \frac{0,12}{(s+3)} + \frac{1,28}{(s+1)} + 1$$

$$\therefore y(t) = -2,16e^{-t}\cos 3t - \frac{10.23}{3}e^{-t}\operatorname{sen} 3t - 0.12e^{-3t} + 1.28e^{-t} + \delta(t)$$

11

h) $\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$, $0<\zeta<1$

$\therefore$ $s^2+2\zeta\omega_n s+\omega_n^2$ possui raízes imaginárias conjugadas

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2-4\omega_n^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2}\, i$$
$$= -\zeta\omega_n \pm \omega_d\, i$$

onde $\omega_d = \omega_n\sqrt{1-\zeta^2}$

Pode-se escrever a equação como:

$(s+\zeta\omega_n)^2+\omega_d^2$

PROVA!!!

$$= s^2+\zeta^2\omega_n^2 + s\zeta\omega_n + \omega_n^2(1-\zeta^2)$$
$$= s^2+\cancel{\zeta^2\omega_n^2} + s\zeta\omega_n + \omega_n^2 - \cancel{\zeta^2\omega_n^2}$$
$$= s^2 + s\zeta\omega_n + \omega_n^2 \quad \text{cqd}$$

$\therefore$

$$Y(s) = \frac{\omega_n^2}{s\left[(s+\zeta\omega_n)^2+\omega_d^2\right]} = \frac{A}{s} + \frac{Bs+C}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$\omega_n^2 = A\left[s^2+2\zeta s\omega_n+\omega_n^2\right] + Bs^2 + Cs$$

$$\omega_n^2 = (A+B)s^2 + (2\zeta\omega_n A + C)s + \omega_n^2 A$$

$A = 1$

$A+B=0 \;\therefore\; B=-1$

$2\zeta\omega_n A + C = 0 \quad \therefore C = -2\zeta\omega_n$

$$Y(s) = \frac{1}{s} - \frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$= 1 - e^{-\zeta\omega_n t}\cos\omega_d t - \frac{2\zeta\omega_n e^{-\zeta\omega_n}}{\omega_d}\operatorname{sen}\omega_d t$$

a transf de Laplace de F(s), mas substituindo "t" por "t-c" 1

4) $\mathcal{L}^{-1}\{e^{-cs}F(s)\} = u_c(t)\,\mathcal{L}\{F(s)\}(t-c)$

retira-se e^{-cs} e escreve-se $u_c(t) = u(t-c)$

a) $\dfrac{e^{-2s}}{s} + \dfrac{6e^{-3s}}{s}$

$y(t) = u_2(t) + 6u_3(t)$,

onde

$u_2(t) = u(t-2)$

$u_3(t) = u(t-3)$

b) $e^{-3s}\left(\dfrac{1}{s^2} + \dfrac{5}{s^3}\right)$

$y(t) = u_3(t)\left[(t-3) + \dfrac{5}{2}(t-3)^2\right]$

c) $\dfrac{6}{s} + \dfrac{e^{-s}}{s^2+4}$

$y(t) = 6 + \dfrac{u_1(t)}{2}\,\text{sen}\,(2(t-1))$

d) $\dfrac{e^{-5s}(s+1)}{(s+1)^2+16}$

$y(t) = u_5(t)\, e^{-(t-5)} \cos 4(t-5)$

e) $\dfrac{4e^{-2s}}{s-3} + \dfrac{e^{-5s}}{s+9}$

$y(t) = 4u_2(t)\, e^{3(t-2)} + u_5(t)\, e^{-9(t-5)}$

f) $\dfrac{e^{-10s}}{(s-3)^2}$

$y(t) = u_{10}(t)\, e^{3(t-10)}(t-10)$

g) $\dfrac{e^{-7s}}{s} + \dfrac{e^{-11s}}{(s-2)^3}$

$y(t) = u_7(t) + \dfrac{u_{11}(t)}{2}\, e^{2(t-11)}(t-11)^2$

1

5) a) $2\ddot{y}+7\dot{y}+3y=0$ $\quad y(0)=3$, $\dot{y}(0)=0$

$$2\left[s^2Y(s)-s\,y(0)-\cancel{\overset{0}{\dot{y}(0)}}\right]+7\left[sY(s)-y(0)\right]+3Y(s)=\emptyset$$

$$2s^2Y(s)-6s+7sY(s)-21+3Y(s)=0$$

$$Y(s)\left[2s^2+7s+3\right]=6s+21$$

$$\boxed{Y(s)=\frac{6s+21}{2s^2+7s+3}=\frac{6s+21}{2(s+1/2)(s+3)}}$$

$\mathcal{L}^{-1}[Y(s)]=y(t)$

$-2s^2+7s+3=0$

$$s=\frac{-7\pm\sqrt{49-24}}{4}=\frac{-7\pm5}{4}\begin{cases}-1/2\\-3\end{cases}$$

$$Y(s)=\frac{(6s+21)/2}{(s+1/2)(s+3)}=\frac{A}{(s+1/2)}+\frac{B}{(s+3)}=\frac{A(s+3)+B(s+1/2)}{(s+1/2)(s+3)}$$

$-\frac{6}{2}+\frac{1}{2}=$

$$3s+\frac{21}{2}=A(s+3)+B(s+1/2)$$

$s=-3 \to -9+\frac{21}{2}=B\left(-\frac{5}{2}\right) \Rightarrow B=-3/5$

$s=-1/2 \to -\frac{3}{2}+\frac{21}{2}=A\left(\frac{5}{2}\right) \Rightarrow A=18/5$

$$Y(s)=\frac{1}{5}\left[\frac{18}{(s+1/2)}-\frac{3}{(s+3)}\right] \therefore \boxed{y(t)=\frac{1}{5}\left(18e^{-1/2t}-3e^{-3t}\right)}$$

b) $5\ddot{y}+20\dot{y}+15y=30u-4\dot{u}$ $\qquad y(0)=5$

$\dot{u} \to \delta(t)$ $\qquad \dot{y}(0)=1$

2

$$5s^2Y(s)-5sy(0)-5\dot{y}(0)+20sY(s)-20y(0)+15Y(s)=30X(s)-4Z(s)$$

$$Y(s)(5s^2+20s+15)-25s-5-100=30\frac{1}{s}-4$$

$$Y(s)(5s^2+20s+15)s=30-4s+25s+105$$

$$Y(s)=\frac{21s+135}{(5s^2+20s+15)}$$

```
num=[21 135];
den=[5 20 15];
[r,p,k]=residue(num,den);
```

$$\therefore Y(s)=\frac{-7.2}{(s+3)}+\frac{11.4}{(s+1)}=\frac{1}{5}\left(-\frac{36}{s+3}+\frac{57}{s+1}\right)$$

$$\therefore y(t)=\frac{1}{5}\left(-36e^{-3t}+57e^{-t}\right)$$

o) $\ddot{y}(t)+\dot{y}(t)+y(t)=g(t)$

$$g(t)=\begin{cases}0, & 0\leq t<1\\ 1, & t\geq 1\end{cases} \qquad y(0)=1 \quad \dot{y}(0)=0$$

$$g(t)=u(t-1)$$

$$\mathcal{L}[g(t)]=\frac{e^{-s}}{s}$$

$$\therefore s^2Y(s)-s\overset{1}{y(0)}-\cancel{\dot{y}(0)}_{0}+sY(s)-\cancel{y(0)}^{1}+Y(s)=\frac{e^{-s}}{s}$$

$$Y(s)(s^2+s+1)=\frac{e^{-s}}{s}+(s+1)\Rightarrow Y(s)=\frac{e^{-s}}{(s^2+s+1)s}+\frac{s+1}{(s^2+s+1)}$$

Agora temos que encontrar a inversa para $Y(s)$

$$\frac{1}{s(s^2+s+1)}=\frac{A}{s}+\frac{Bs+C}{s^2+s+1}$$

$$1=A(s^2+s+1)+Bs^2+Cs=s^2(A+B)+s(A+C)+A$$

$A=1$

$A+B=0 \therefore B=-1$

$A+C=0 \therefore C=-1$

$$\therefore Y(s)=e^{-s}\left[\frac{1}{s}-\frac{(s+1)}{s^2+s+1}\right]+\frac{s+1}{(s^2+s+1)}$$

$$\frac{s+1}{s^2+s+1}=\frac{s+1}{(s+1/2)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{(s+1/2)+1/2}{(s+1/2)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

4

$$\therefore \mathcal{L}^{-1}\left[\frac{(s+1/2)+1/2}{(s+1/2)^2+(\sqrt{3}/2)^2}\right] = \left(\cos\frac{\sqrt{3}}{2}t + \frac{1}{2}\frac{2}{\sqrt{3}}\,\text{sen}\,\frac{\sqrt{3}}{2}t\right)e^{-t/2}$$

$$= \left(\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\,\text{sen}\,\frac{\sqrt{3}}{2}t\right)e^{-t/2}$$

$$\therefore y(t) = u(t-1) - e^{-(t-1)/2}\left[\cos\frac{\sqrt{3}}{2}(t-1) + \frac{1}{\sqrt{3}}\,\text{sen}\,\frac{\sqrt{3}}{2}(t-1)\right]u(t-1) +$$

$$+ e^{-t/2}\left[\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\,\text{sen}\,\frac{\sqrt{3}}{2}t\right]$$

$$\boxed{y(t) = u(t-1)\left[1 - e^{-(t-1)/2}\left(\cos\frac{\sqrt{3}}{2}(t-1) + \frac{1}{\sqrt{3}}\,\text{sen}\,\frac{\sqrt{3}}{2}(t-1)\right)\right] + e^{-\frac{t}{2}}\left[\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\,\text{sen}\,\frac{\sqrt{3}}{2}t\right]}$$

d) $\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = u(t)$

$\ddot{y}(0) = 0$

$\dot{y}(0) = 0$

$y(0) = 0$

$s^3Y(s) - s^2\ddot{y}(0) - s\dot{y}(0)$

$$s^3Y(s) - s^2\cancel{y(0)} - s\cancel{\dot{y}(0)} - \cancel{\ddot{y}(0)} + 6\left[s^2Y(s) - s\cancel{y(0)} - \cancel{\dot{y}(0)}\right] + 11\left[sY(s) - y(0)\right] + 6Y(s) = U(s)$$

$$Y(s)(s^3+6s^2+11s+6) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s(s^3+6s^2+11s+6)}$$

$$Y(s) = \frac{1}{s(s+1)(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{s+3}$$

$A = 1/6,\ B = -1/2,\ C = 1/2,\ D = -1/6$

$$Y(s) = \frac{1}{6s} - \frac{1}{2(s+1)} + \frac{1}{2(s+2)} - \frac{1}{6(s+3)}$$

$$\therefore \boxed{y(t) = \frac{1}{6} - \frac{1}{2}e^{-t} + \frac{1}{2}e^{-2t} - \frac{1}{6}e^{-3t}}$$

6) $3\ddot{y}+39\dot{y}+120y=u(t)$ $\quad y(0)=0,\ \dot{y}(0)=0$ e $y(0)=0,\ \dot{y}(0)=10$

1

$$3s^2\cancel{Y(s)}-s\cancel{y(0)}^{0}-\dot{y}(0)+39sF(s)-\cancel{y(0)}^{0}+120Y(s)=X(s)$$

$$Y(s)(3s^2+39s+120)=\frac{10}{s}+\dot{y}(0)$$

$$Y(s)\,s(3s^2+39s+120)=10+s\dot{y}(0)$$

$$Y(s)=\frac{10+s\dot{y}(0)}{s(3s^2+39s+120)}$$

a) Para $\dot{y}(0)=\emptyset$

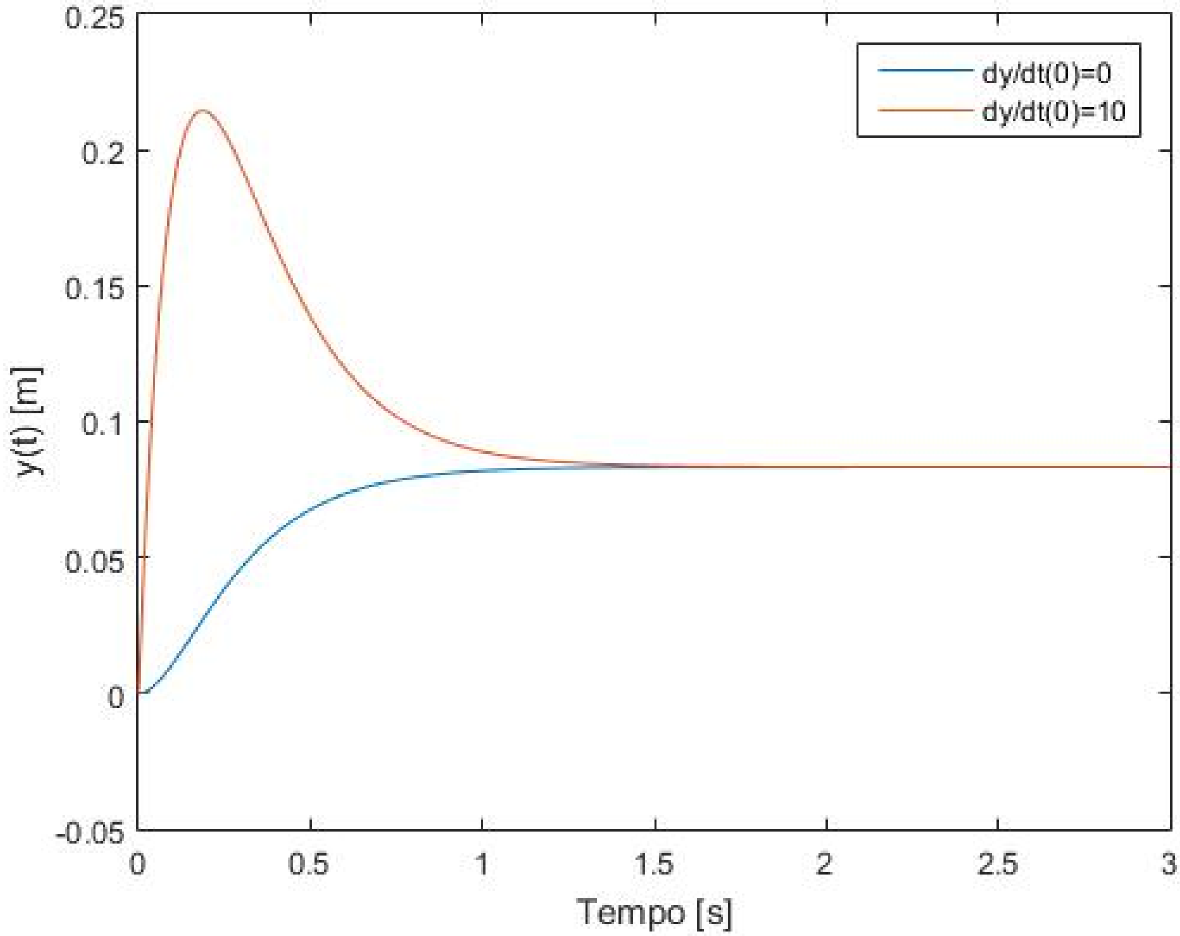
$$Y(s)=\frac{10}{3s^3+39s^2+120s+0}=\frac{5}{36(s+8)}-\frac{2}{9(s+5)}+\frac{1}{12s}$$

$$\boxed{y(t)=\frac{5}{36}e^{-8t}-\frac{2}{9}e^{-5t}+\frac{1}{12}u(t)}$$

b) Para $\dot{y}(0)=10$

$$Y(s)=\frac{-35}{36(s+8)}+\frac{8}{9(s+5)}+\frac{1}{12s}$$

$$\boxed{y(t)=-\frac{35}{36}e^{-8t}+\frac{8}{9}e^{-5t}+\frac{1}{12}u(t)}$$




```
clear all; close all; clc
t=0:0.01:3;
y1=5/36.*exp(-8.*t)-2/9.*exp(-5.*t)+1/12;
y2=-35/36.*exp(-8.*t)+8/9.*exp(-5.*t)+1/12;
plot(t,y1,t,y2);
xlabel('Tempo [s]');
ylabel('y(t) [m]');
legend('dy/dt(0)=0','dy/dt(0)=10')
```

7) Ver eq. 5, pág. 6 da apostila:

Entrada 1

$$\begin{cases} m_w \ddot{x}_w + b_s \dot{x}_w + (k_t + k_s) x_w = b_s \dot{x}_c + k_s x_c + k_t x_r \\ m_c \ddot{x}_c + b_s \dot{x}_c + k_s x_c = b_s \dot{x}_w + k_s x_w \end{cases}$$

$w = 2$
$c = 1$
$r =$ rodovia

$m_w = 40\ kg$, $k_t = 150\ KN/m$
$m_c = 250\ kg$, $k_s = 15\ KN/m$, $b_s = 1917\ Ns/m$

$$40s^2 X_2(s) - s\cancel{x_2(0)} - \cancel{\dot{x}_2(0)} + 1917 s X_2(s) - \cancel{x_2(0)} + 165\times10^3 X_2(s) = 1917 s X_1(s) - \cancel{x_1(0)} + 15\times10^3 X_1(s) + 150\times10^3 X_r(s)$$

$$250 s^2 X_1(s) - s\cancel{x_1(0)} - \cancel{\dot{x}_1(0)} + 1917 s X_1(s) - \cancel{x_1(0)} + 15\times10^3 X_1(s) = 1917 s X_2(s) - \cancel{x_2(0)} + 15\times10^3 X_2(s)$$

$$X_2(s)\left(40s^2 + 1917s + 165000\right) = X_1(s)\left(1917s + 15000\right) + X_r(s)\,150000$$

$$X_1(s)\left(250s^2 + 1917s + 15000\right) = X_2(s)\left(1917s + 15000\right)$$

$$X_1(s)\left[\frac{(250s^2+1917s+15000)}{(1917s+15000)}(40s^2+1917s+165000) - (1917s+15000)\right] = X_r(s)\,150000$$

$$\therefore \frac{X_1(s)}{X_r(s)} = \frac{15000\,(1917s+15000)}{(250s^2+1917s+15000)(40s^2+1917s+165000)-(1917s+15000)^2}$$

$$X_2(s)\left[(40s^2+1917s+165000) - \frac{(1917s+15000)^2}{250s^2+1917s+15000}\right] = X_r(s)\,150000$$

$$\frac{X_2(s)}{X_r(s)} = \frac{15000\,(250s^2+1917s+15000)}{(40s^2+1917s+165000)(250s^2+1917s+15000)-(1917s+15000)^2}$$

Para função rampa: $X_r(s) = \dfrac{1}{s}$

$$X_1(s) = \frac{0{,}1}{s} + \frac{0{,}005 + 0{,}0045i}{(s + 24{,}39 - 56{,}55i)} + \frac{0{,}005 - 0{,}0045i}{(s + 24{,}39 + 56{,}55i)} +$$

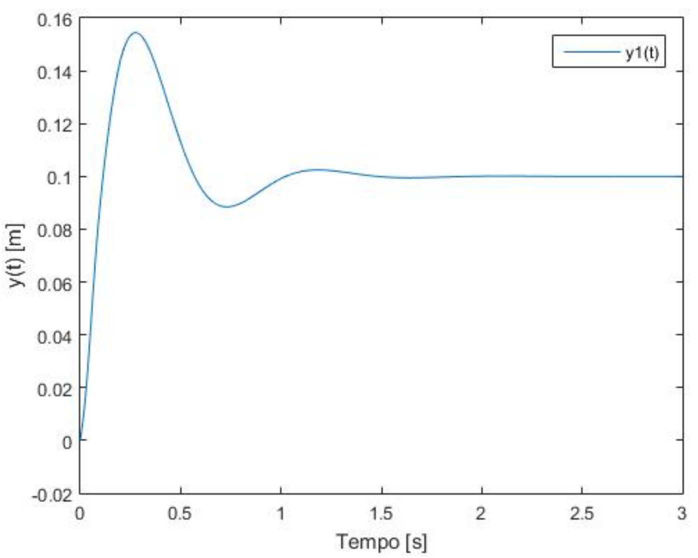
$$+ \frac{-0{,}055 - 0{,}027i}{(s + 3{,}409 - 6{,}91i)} + \frac{-0{,}055 + 0{,}027i}{(s + 3{,}409 + 6{,}91i)}$$

$$X_1(s) = \frac{0{,}1}{s} + \frac{0{,}010s + 0{,}26}{(s+24{,}39)^2 + 56{,}55^2} - \frac{0.11s + 0{,}004}{(s+3{,}409)^2 + 6{,}91^2}$$

$$X_1(s) = \frac{0{,}1}{s} + \frac{0.010\,(s+24{,}39) + 0.016}{(s+24{,}39)^2 + 56{,}55^2} - \frac{0.11(s+3.409) - 0{,}371}{(s+3.409)^2 + 6.91^2}$$

$$\therefore\; y_1(t) = 0{,}1 + e^{-24{,}39t}\left[0.01 \cos 56{,}55t + \frac{0{,}16}{56{,}55} \operatorname{sen} 56{,}55t\right] +$$

$$- e^{-3{,}409t}\left[0.11 \cos 6.91t - \frac{0{,}37}{6{,}91} \operatorname{sen} 6{,}91t\right]$$



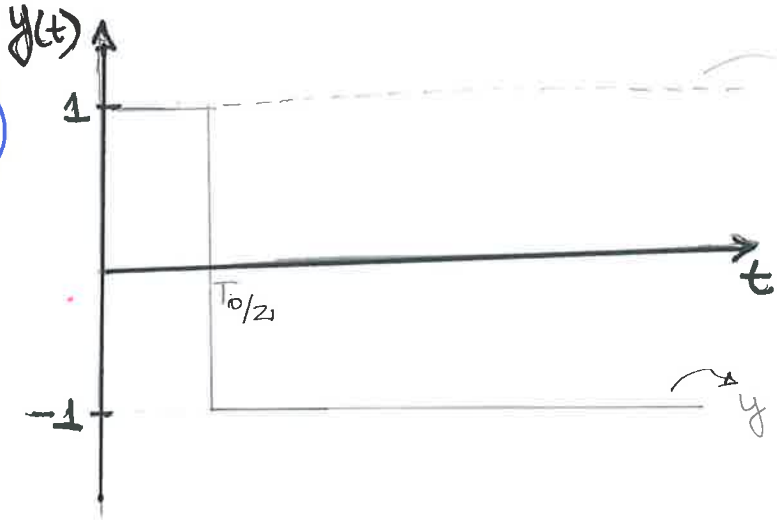

```
clear all; close all; clc
mw=40;      %kg
kt=150000; %kN/m
mc=250;     %kg
ks=15000;  %kN/m
bs=1917;   %Ns/m

syms s;
xa = mc*s^2+bs*s+ks;
xb = mw*s^2+bs*s+(kt+ks);
xc=bs*s+ks;
eq1=expand(s*(xa*xb-xc^2));
eq2=15000*expand(xc);
den = sym2poly(eq1);
num = sym2poly(eq2);
[r,p,k]=residue(num,den)
aux1=2*real(r(1));
aux2=r(1)*p(2)+r(2)*p(1);
aux3=2*real(r(3));
aux4=r(3)*p(4)+r(4)*p(3);
w1=abs(imag(p(1)));
w2=abs(imag(p(3)));
t=0:0.01:3;
y1=0.1+exp(real(p(1)).*t).*(aux1.*cos(w1.*t)+(aux2-aux1*w1)/w1.*...
    sin(w1.*t))+exp(real(p(3)).*t).*(aux3.*cos(w2.*t)+(aux4-aux3*w2)/w2.*sin(w2.*t));
% plot
 plot(t,y1);
 xlabel('Tempo [s]');
 ylabel('y(t) [m]');
 legend('y1(t)')
```

$y_2(t)$ é similar!!! Você faz... eu cansei!

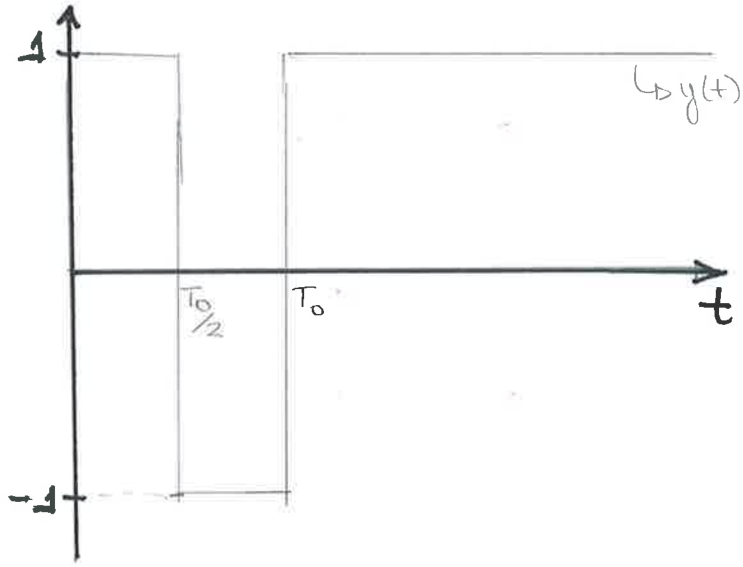
8)

a)



$\rightarrow y(t) = u(t)$

$\rightarrow y(t) = u(t) - 2u(t - T_0/2)$



$\rightarrow y(t) = u(t) - 2u(t - T_0/2) + 2u(t - T_0)$

$\vdots$

$$\therefore y(t) = u(t) - 2u(t - T_0/2) + 2u(t)(t - T_0) - 2u\left(t - \tfrac{3}{2}T_0\right) + 2u(t)(t - 2T_0)\ldots$$

$$\mathcal{L}[u(t)] = \frac{1}{s}$$

$$\mathcal{L}[u(t - \alpha T_0)] = \frac{e^{-\alpha s}}{s}$$

$$\therefore Y(s) = \frac{1}{s} - \frac{2}{s}e^{-T_0 s/2} + \frac{2}{s}e^{-T_0} - \frac{2}{s}e^{-3T_0 s/2} + \frac{2}{s}e^{-2T_0}$$

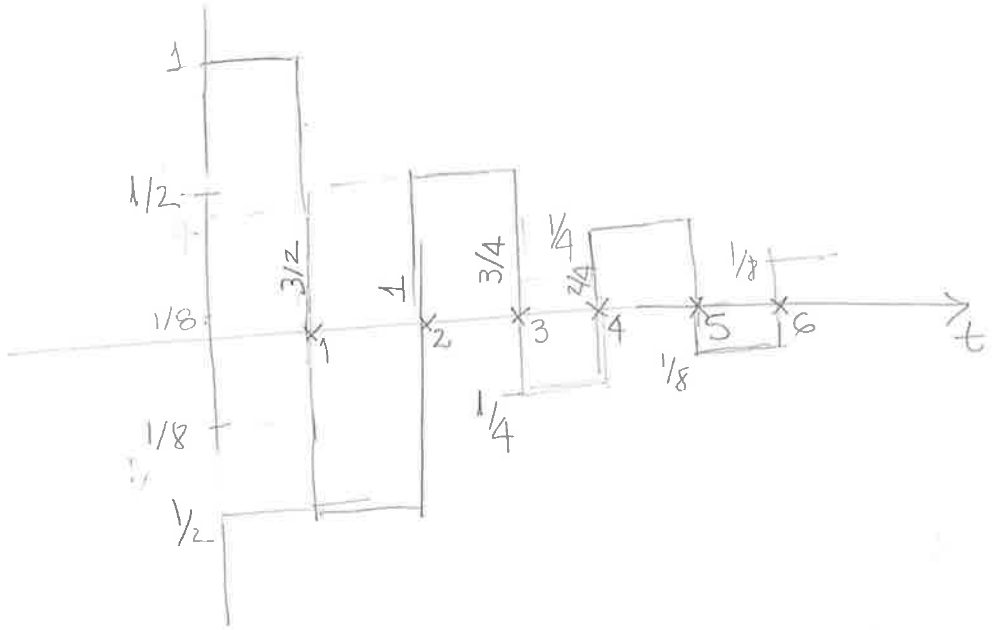
$\lambda = -e^{-T_0 s/2}$

$$\therefore Y(s) = \frac{1}{s} + \frac{2}{s}\lambda + \frac{2}{s}\lambda^2 + \frac{2}{s}\lambda^3 + \frac{2}{s}\lambda^4 + \ldots$$

$$= \frac{1}{s} + \frac{2}{s}\underbrace{(\lambda + \lambda^2 + \lambda^3 + \ldots)}_{\text{Soma PG infinita: } \Sigma = \frac{a_1}{1-q} = \frac{\lambda}{1-\lambda}} = \frac{1}{s} + \frac{2}{s}\left(\frac{\lambda}{1-\lambda}\right)$$

$$\therefore Y(s) = \frac{1 - \lambda + 2\lambda}{s(1-\lambda)} = \frac{1+\lambda}{1-\lambda} = \frac{1 - e^{-T_0 s/2}}{1 + e^{-T_0 s/2}}$$

Porém, o exercício pede uma "onda quadrada e decrescente"



$$y(t) = u(t) - \frac{3}{2}u(t-1) + \frac{2}{2}u(t-2) - \frac{3}{4}u(t-3) + \frac{2}{4}u(t-4) - \frac{3}{8}u(t-5) + \frac{2}{8}u(t-6) + \dots$$

$$Y(s) = \frac{1}{s} - \frac{3}{2}e^{-s} + \frac{2}{2}e^{-2s} - \frac{3}{4}e^{-3s} + \frac{2}{4}e^{-4s} - \frac{3}{8}e^{-5s} + \frac{2}{8}e^{-6s} + \dots$$

$$= \frac{1}{s} - \frac{3e^{-s}}{2} - \frac{3e^{-s}}{2}\left(\frac{1}{2}e^{-2s} + \frac{1}{4}e^{-4s} + \dots\right) + 2\left(\frac{1}{2}e^{-2s} + \frac{1}{4}e^{-4s} + \frac{1}{8}e^{-6s} + \dots\right)$$

$$\lambda = \frac{1}{2}e^{-2s}$$

$$\therefore Y(s) = \frac{1}{s} - \frac{3}{2}e^{-s}\underbrace{\left(1 + \lambda + \lambda^2 + \dots\right)}_{\frac{1}{1-\lambda}} + 2\underbrace{\left(\lambda + \lambda^2 + \lambda^3 + \dots\right)}_{\frac{\lambda}{1-\lambda}}$$

$$Y(s) = \frac{1}{s} - \frac{3}{2}e^{-s}\frac{1}{1-\lambda} + \frac{2\lambda}{1-\lambda} \qquad 1 - \lambda = \frac{2 - e^{-2s}}{2}$$

$$Y(s) = \frac{1}{s} - \frac{3e^{-s}}{\cancel{2}} \times \frac{\cancel{2}}{2 - e^{-2s}} + \frac{e^{-2s}}{2 - e^{-2s}} \times 2$$

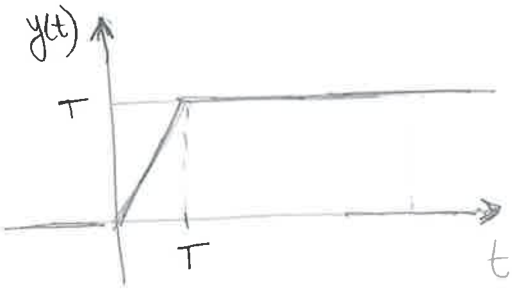
$$\boxed{Y(s) = \frac{1}{s} - \frac{1}{2 - e^{-2s}}\left(3e^{-s} - 2e^{-2s}\right)}$$

se não errei em conta

:)

3

b)



Sinal rampa unitária,

$$r(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

$$y(t) = r(t) - r(t-T)$$

$$\mathcal{L}[r(t)] = \frac{1}{s^2} \quad , \quad \mathcal{L}[u(t)] = \frac{1}{s}$$

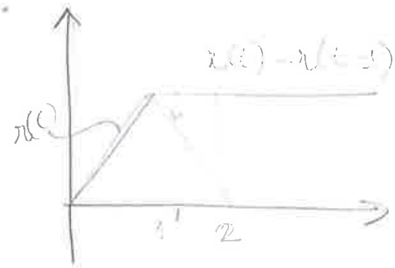
$$\therefore \; Y(s) = \frac{1}{s^2} - \frac{e^{-Ts}}{s^2} = \boxed{\frac{1}{s^2}\left(1 - e^{-Ts}\right)}$$

9) $\dot{y}(t)+y(t)=g(t)$

$g(t) = r(t) - 2r(t-1) + r(t-2)$

$u(t) = r(t) - 2r(t-1) + r(t-2)$

$\dot{y}(t)+y(t) = r(t) - 2r(t-1) + r(t-2)$



$$sY(s) + \cancel{y(0)} + Y(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

0

$$\boxed{Y(s) = \frac{1-2e^{-s}+e^{-2s}}{s^2(1+s)}}$$

Resolvendo a inversa

$$Y(s) = \frac{1-2e^{-s}+e^{-2s}}{s^2(1+s)} = \frac{1}{s^2(1+s)} - 2e^{-s}\left[\frac{1}{s^2(1+s)}\right] + 2e^{-2s}\left[\frac{1}{s^2(1+s)}\right]$$

$$\frac{1}{s^2(1+s)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{1+s}$$

$$1 = As(1+s) + B(1+s) + Cs^2$$

$$1 = s^2(A+C) + s(A+B) + B$$

$B = 1$

$A+B=0 \;\therefore\; A=-1$

$A+C=0 \;\therefore\; C=1$

$$\frac{1}{s^2(1+s)} = -\frac{1}{s} + \frac{1}{s^2} + \frac{1}{1+s}$$

$$Y(s) = -\frac{1}{s} + \frac{1}{s^2} + \frac{1}{1+s} - 2e^{-s}\left(-\frac{1}{s} + \frac{1}{s^2} + \frac{1}{1+s}\right) + 2e^{-2s}\left(-\frac{1}{s} + \frac{1}{s^2} + \frac{1}{1+s}\right)$$

$$y(t) = -u(t) + t + e^{-t} - 2\left[-u(t-1) + (t-1) + e^{-(t-1)}\right] + 2\left[-u(t-2) + (t-2) + e^{-(t-2)}\right]$$

$$\boxed{y(t) = -u(t) + 2u(t-1) - 2u(t-2) + t - 2 + e^{-t} - 2e^{-(t-1)} + 2e^{-(t-2)}}$$

1

10)

$$y(t) = 2u(t-10)e^{-2,5(t-10)}\left[\frac{e^{\frac{t}{2}-5} - e^{-2,5+5}}{2}\right]$$

$$= u(t-10)\left(e^{-2,5t+25+0,5t-5} - e^{-2,5t+25-0,5t+5}\right)$$

$$= u(t-10)\left[e^{-2t+20} - e^{-3t+30}\right]$$

$$\boxed{y(t) = u(t-10)\left[e^{-2(t-10)} - e^{-3(t-10)}\right]}$$