Lista Modelagem

Q1 $F(s) = \dfrac{2s+12}{s^2+2s+5}$

↳ Utilizando Poly2sym e ilaplace chegamos no seguinte Resultado:

$$f(t) = 2e^{-t}\cdot\left(\cos(2t) + \frac{5\,\text{sen}(2t)}{2}\right)$$

Q2

$$F(s) = \frac{s^2+2s+3}{(s+1)^3} = \frac{A}{(s+1)^3} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)}$$

$$A = s^2+2s+3\Big|_{s\to-1} = 1-2+3 = 2$$

$$B = \frac{d}{ds}\left[s^2+2s+3\right]\Big|_{s=-1} = 2s+2\Big|_{s=-1} = 0$$

$$C = \frac{1}{2!}\frac{d^2}{ds^2}\left[s^2+2s+3\right] = \frac{1}{2!}\cdot 2 = 1$$

$$F(s) = \frac{2}{(s+1)^3} + \frac{1}{(s+1)} \Rightarrow f(t)\, 2\cdot\frac{1}{(3-1)!}\cdot t^{3-1}e^{-(+1t)} + e^{-t} = t^2e^{-t} + e^{-t}$$

$$f(t) = e^{-t}(t^2+1)$$

Q3

$$F(s) = \frac{\overbrace{A(s)}^{G(s)}}{B(s)}\;\frac{s^4+2s^3+3s^2+4s+5}{s(s+1)} = G(s),\frac{(-1{,}288\pm0{,}8579j)(0{,}2878\pm1{,}1416j)}{s(s+1)}$$

$$f(t) = 2\delta(t) - 3e^{-t} + \delta(t-1) + \delta(t-2) + 5$$

Q4 $F(s) = \dfrac{3}{s^3+2s^2+5s} \rightsquigarrow f(t) = \dfrac{3}{5} - \dfrac{3}{5}e^{-t}\left(\cos(2t) + \dfrac{1\,\text{sen}(2t)}{2}\right)$

Q5 $2\ddot{x} + 7\dot{x} + 3x = 0 \qquad x(0) = 3 \quad \dot{x}(0) = 0$

↳ $2s^2X(s) - 2sX(0) - 2\cancel{\dot{X}(0)} + 7sX(s) - 7X(0) + 3X(s) = 0$

$$X(s)\left(2s^2+7s+3\right) - 6s - 21 = 0$$

$$X(s) = \frac{6s+21}{2s^2+7s+3} \to x(t) = \frac{18}{5}e^{-\frac{t}{2}} - \frac{3}{5}e^{-3t}$$

Q6 $\dot{x}+2x=\delta(t)$; $x(0_-)=0$

$$sX(s)-\cancel{x(0)}+2X(s)=1 \rightarrow X(s)=\frac{1}{s+2} \rightarrow x(t)=e^{-2t}$$

Q7 $\ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2=0$ $\quad x(0)=a$ $\quad \dot{x}(0)=b$

$$s^2X(s)-s\cancel{X(0)}-\dot{x}(0)+2\xi\omega_n sX(s)-2\xi\omega_n\cancel{X(0)}+\omega_n^2\cdot\frac{1}{s}=0$$

$$X(s)\left(s^2+2\xi\omega_n s\right)-x(0)\left(s+2\xi\omega_n\right)-\dot{x}(0)+\frac{\omega_n^2}{s}=0$$

$$X(s)(s^2+2\xi\omega_n s)-a(s+2\xi\omega_n)-b+\frac{\omega_n^2}{s}=0$$

$$X(s)(s^3+2\xi\omega_n s^2)=a(s^2+2\xi\omega_n s)+bs-\omega_n^2$$

$$X(s)=\frac{as^2+s(2\xi\omega_n a+b)-\omega_n^2}{s^3+2\xi\omega_n s^2}$$

$$\Rightarrow x(t)=\frac{4a\omega_n\xi^2-2b\xi+\omega_n}{4\omega_n\xi^2}-\frac{t\omega_n}{2\xi}-e^{-2\xi\omega_n t}\frac{(\omega_n-2b\xi)}{4\omega_n\xi^2}$$

$$x(t)=a+\frac{\omega_n-2b\xi}{4\omega_n\xi^2}-\frac{\omega_n}{2\xi}t-\frac{e^{-2\xi\omega_n t}}{4\xi^2}+\frac{e^{-2\xi\omega_n t}}{2\omega_n\xi}\cdot b$$

Q8 $\dot{x}+ax=A\,sen(\omega t)$, $x(0)=b$

$$sX(s)-x(0)+aX(s)=\frac{A\omega}{s^2+\omega^2}\rightarrow X(s)(s+a)-b=\frac{A\omega}{s^2+\omega^2}$$

$$X(s)=\frac{A\omega}{(s+a)(s^2+\omega^2)}+\frac{b}{(s+a)}$$

$$X(s)=\frac{A\omega}{s^3+\omega^2 s+as^2+a\omega^2}+\frac{b}{s+a}=\frac{A\omega}{s^3+as^2+\omega^2 s+a\omega^2}+\frac{b}{s+a}$$

$$x(t)=\frac{A\cdot a\cdot sen(\omega t)-A\cdot\omega\cdot\cos(\omega t)}{a^2+\omega^2}+\frac{A\omega e^{-at}}{a^2+\omega^2}+be^{-at}$$

Q9 $\ddot{x}+3\dot{x}+6x=0$, $x(0)=0$ $\quad \dot{x}(0)=3$

$$s^2X(s)-s\cancel{X(0)}-\dot{x}(0)+3sX(s)-3\cancel{X(0)}+6X(s)=0$$

$$X(s)(s^2+3s+6)-3=0$$

$$\Rightarrow X(s)=\frac{3}{s^2+3s+6}$$

$$x(t)=3t-2{,}7916\,sen(0{,}9306\cdot t)\cosh(0{,}9306\cdot t)-2{,}7918\cos(0{,}93t)\,senh(0{,}93$$

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