

# **Computer Vision**

**Dr. Syed Faisal Bukhari**

**Associate Professor**

**Department of Data Science**

**Faculty of Computing and Information Technology**

**University of the Punjab**

# Textbook

**Multiple View Geometry in Computer Vision,**  
Hartley, R., and Zisserman

Richard Szeliski, **Computer Vision: Algorithms and Applications,** 1<sup>st</sup> edition, 2010

# Reference books

Readings for these lecture notes:

- ❑ Hartley, R., and Zisserman, A. **Multiple View Geometry in Computer Vision**, Cambridge University Press, 2004, Chapters 1-3.
- ❑ Forsyth, D., and Ponce, J. **Computer Vision: A Modern Approach**, Prentice-Hall, 2003, Chapter 2.

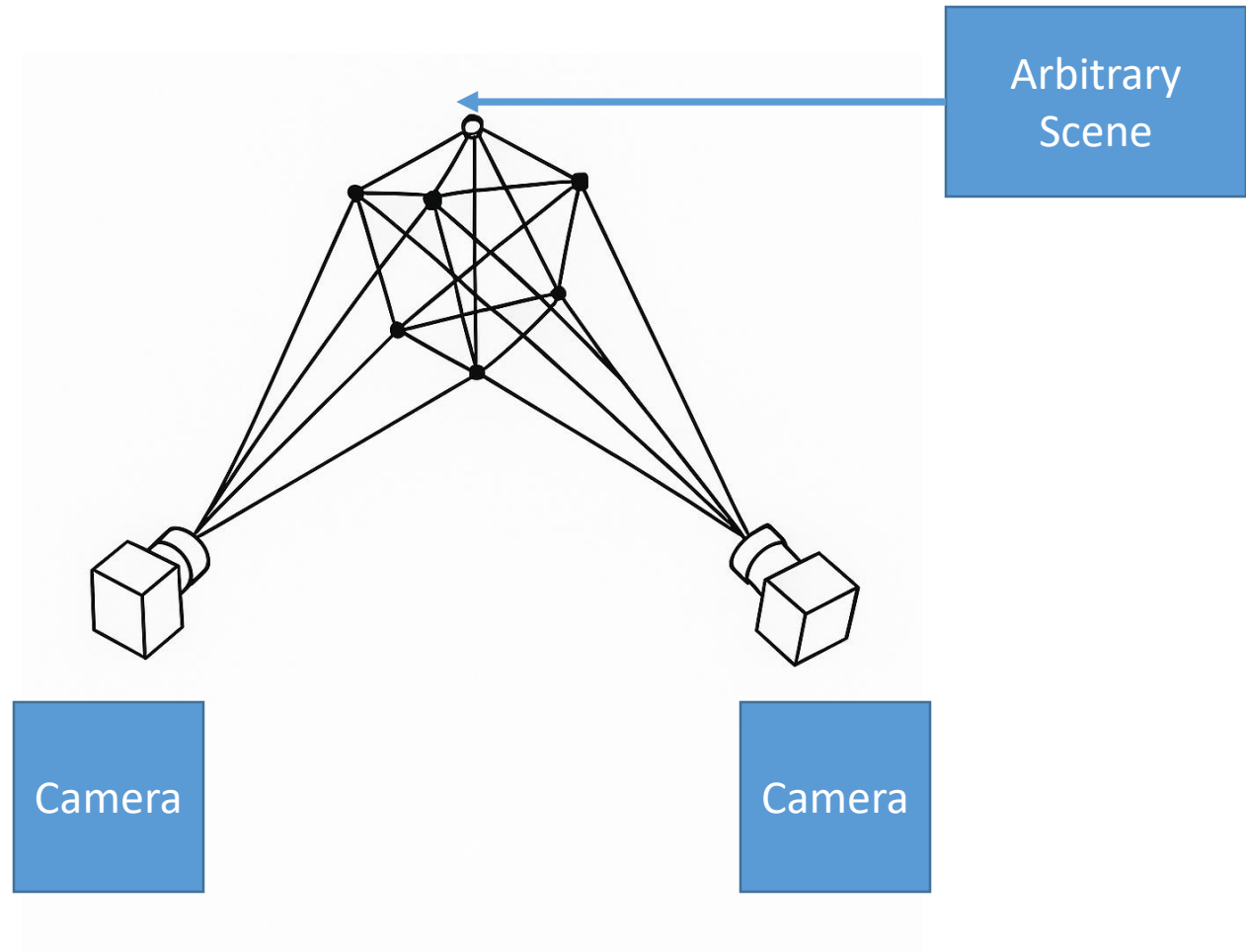
These notes contain material c Hartley and Zisserman (2004) and Forsyth and Ponce (2003).

# References


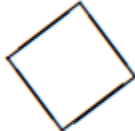
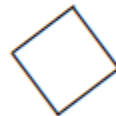


These notes are based

- ❑ Dr. Matthew N. Dailey's course: AT70.20: Machine Vision for Robotics and HCI
- ❑ Dr. Sohaib Ahmad Khan CS436 / CS5310 Computer Vision Fundamentals at LUMS

# Homography: Two views of an arbitrary 3D scene



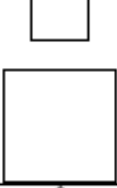
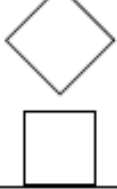


# Hierarchy of 2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \mathbf{I} &   & \mathbf{t} \end{bmatrix}_{2 \times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} &   & \mathbf{t} \end{bmatrix}_{2 \times 3}$	3	lengths	
similarity	$\begin{bmatrix} s\mathbf{R} &   & \mathbf{t} \end{bmatrix}_{2 \times 3}$	4	angles Length Ratios	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$	6	parallelism Length Ratios along a line	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$	8	straight lines Length Cross-Ratios along a line	

# 2D projective geometry

## A hierarchy of transforms

Group	Matrix	Distortion	Invariant properties
Projective 8 dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, order of contact, tangent discontinuities and cusps, cross ratios
Affine 6 dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallelism, ratio of areas, ratio of lengths on collinear or parallel lines, linear combinations of vectors, the line at infinity $l_\infty$
Similarity 4 dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratio of lengths, angle
Euclidean 3 dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Length, area

# Translation, Rigid Body Transformation, Similarity, Affine

$$1. \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$2. \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$3. \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$4. \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$



# Translation Group

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3 \times 1}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}_{3 \times 1}$$

# Rigid Body Transformation (Euclidean Transformation)

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3 \times 1}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} x\cos\theta - y\sin\theta + t_x \\ x\sin\theta + y\cos\theta + t_y \\ 1 \end{bmatrix}_{3 \times 1}$$

# Similarity Group

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} s \cos \theta & -s \sin \theta & t_x \\ s \sin \theta & s \cos \theta & t_y \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3 \times 1}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} s x \cos \theta - s y \sin \theta + t_x \\ s x \sin \theta + s y \cos \theta + t_y \\ 1 \end{bmatrix}_{3 \times 1}$$

# Affine Group

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3 \times 1}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} a_{11}x + a_{12}y + a_{13} \\ a_{21}x + a_{22}y + a_{23} \\ 1 \end{bmatrix}_{3 \times 1}$$

- Contains **rotation, scaling, shear, translation** and any combination thereof
- **Preserves Parallel lines**

# Projective Group (Homography)

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}_{3 \times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3 \times 1}$$

- Simulates out of plane rotations
- **Preserves straight lines**
- Physical Interpretation: Plane + Camera

# GL(3) - General Linear Group of Degree 3

- Set of all invertible  $3 \times 3$  matrices ( $\det \neq 0$ )
- Notation:  $GL(3) = \{ A \in \mathbb{R}^{3 \times 3} \mid \det(A) \neq 0 \}$
- Includes **all linear transformations** of 3D space
- Preserves vector space structure, not scale. GL(3) keeps the **“vector math”** consistent but allows geometric distortions (like stretching, skewing, and flipping) — it does **not** keep distances or proportions constant.
- Used in classical linear transformations

# PL(3) - Projective Linear Group of Degree 3

- $PL(3) = GL(3)$  modulo scalar matrices
- Two matrices are equivalent if **they differ by a non-zero scalar**
- Works with **homogeneous coordinates** (projective geometry)
- $PL(3)$  transformations preserve projective structure
- Common in computer vision and image **homographies**

# Summary: $GL(3)$ vs $PL(3)$

## $GL(3)$ :

- Invertible  $3 \times 3$  matrices
- Includes scale
- Linear transformations

## $PL(3)$ :

- Equivalence classes of  $GL(3)$
- Ignores scale
- Projective transformations



# Example: Matrix Equivalence in PL(3)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = 2A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- In **GL(3)**:  $A \neq B$
- In **PL(3)**:  $A \sim B$  (same **transformation up to scale**)

# 2D projective geometry

## A hierarchy of transformations

We can create a hierarchy of transformations based on the **restrictions** that we put on a **linear transformation  $H$** .

- ❑ The **real linear group  $GL(3)$**  consists of all invertible real  **$3 \times 3$  matrices**.<sup>1</sup>
- ❑ When we place all members of  **$GL(3)$**  related by **scale in an equivalence class**, we obtain the **projective linear group  $PL(3)$** .

<sup>1</sup> **Recall** that a group is a set paired with an operation that has an inverse, associativity, an identity, and closure.

# 2D projective geometry

## A hierarchy of transformations

There are three important **subgroups of  $PL(3)$** :

- The **affine group** in which the bottom row is **constrained to  $(0, 0, 1)$** ;
- The **similarity group** in which the rows and columns of the upper-left  **$2 \times 2$  submatrix are orthogonal**;
- The **Euclidean or isometry** group in which the **upper-left  $2 \times 2$  submatrix is orthonormal**.

# Projective Linear Group PL (3)

- **PL(3):** The elements of this set are  $3 \times 3$  homogenous invertible matrices, where we consider  $kH$  to be equal to  $H$ . We have a variety of **different subgroups**:
- The **affine group** within **PL(3)** is just the set of  $3 \times 3$  homogeneous invertible matrices such that the **bottom row** is **0 0 1**. Homographies having this property has some special characteristics.

# Projective Linear Group PL (3)

- The **similarity group** in which the rows and columns of the **upper left  $2 \times 2$  submatrix are orthogonal**. In this case, we place an additional restriction on homography, which creates the upper left  **$2 \times 2$  submatrix should be orthogonal**. The rows and columns are orthogonal to each other which creates a group called the similarity group.
- Finally**, we go even further and restrict this homography matrix even more then we obtain the **Euclidean or isometry group**. In this case, we have upper left  **$2 \times 2$  submatrix as orthonormal**.

# Projective Linear Group PL (3)

- In the **similarity group**, we say the  **$2 \times 2$  submatrix** is **orthogonal**. It means rows and columns have to be **orthogonal** to each other. The dot product between each row or each column has to be **orthogonal, i.e., zero**.
- In the **Euclidean group**, we say the **upper left  $2 \times 2$**  is **orthonormal**, which means the **norm of each row or each column is 1**.

# Projective Linear Group PL (3)

- First of all, the most **general type of homography** is called the **projective group**. This is called  $PL(3)$ . In it, we are allowed to **write whatever co-efficient we want** for the elements of this matrix as long as this **matrix is invertible**, then we have elements of  $PL(3)$ .
- What kind of transformation this group model?
- Projective homography can do a lot of different types of distortions.
- We can take a **trapezoid** and map it to a **different type of trapezoid**.

# Projective Linear Group PL (3)

## 1. Affine transformation:

Here we are saying that the bottom row of this matrix is **0 0 1**. In this case, we are effectively preventing a certain type of distortion. We have **6 degrees of freedom**. Now the transformations, we can model are called using the affine transformations: **Rotations, scale, translations, and shear**.

## 2. Similarity transformation:

We have imposed further restrictions. It has only **4 degrees of freedom**. We will write the upper left  $2 \times 2$  submatrix using the coefficient  $r_{11} \ r_{12} \ r_{13} \ r_{14}$ . The rows and columns of this matrix are **orthogonal to each other**. It allows us to model **translations, scales, and rotations**. But, **shear** is not part of similarity transformations.



# Projective Linear Group PL (3)

## 3. Euclidean transformation:

It allows us to model **rotations and translations but not scales**. These transformations are very interesting, as we see later when we get to **3D because they model the possible motion of the rigid body**.

# Invariant properties of PL (3)

- **Invariant property under Euclidean transformations:** In Euclidean transformations, if an object has length or area before applying transformations, then it will have the same **length** and **area** after applying the transformations.



- **Invariant property under affine:** The most important invariant property under affine is **parallelism**. If two lines are parallel in the first image and then you transform that image by affinities, then parallel lines still are parallel in the resulting image.

# Invariant properties of PL (3)

- **Invariant property under Projective transformation:**
- **Projective transformation** are real-life transformations between points that are taken when the point lies in the same plane, and you take pictures of them with different cameras.
- It turns out there are some properties that are preserved. For example, **collinearity**. If you have a line in image #1 and you transform it by any homography. That **line** will still be a **line in a resulting image**.

## 2D projective geometry

### Action of projectivities and affinities on ideal points

The key difference between a **projective** and **affine transformation** is that the vector  $\mathbf{v}$  is not null for a projectivity.

What happens when we apply a homography  $H$  to a point at infinity?

**Affinities** map **ideal points** to **ideal points**:

$$\begin{bmatrix} A & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ 0 \end{bmatrix}$$

but **general projectivities** can map ideal points to **finite points**:

$$\begin{bmatrix} A & \mathbf{t} \\ \mathbf{V}^T & v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ v_1 x_1 + v_2 x_2 \end{bmatrix}$$

## 2D projective geometry

### Action of projectivities and affinities on ideal points

- What happens when we apply a homography to a point at infinity?

Affinities map ideal points to ideal points.

**“Parallel lines remain parallel under affine transformations.”**

- How can we interpret it?

If you have transformations between two images. You would like to prove that **parallel lines** are still going to be **parallel** under **affinities**. It is very simple to prove because **affinities preserve parallelism**. When you have 2 parallel lines then their **point of intersection on a line at infinity**.

## 2D projective geometry

### Action of projectivities and affinities on ideal points

- **Point at infinity in homogenous representation of a 2D point:**

The third component of your vector representing the point is zero. If the **bottom row is 0 0 1** then **points remain ideal**.

- **Point at finite in homogeneous representation of a 2D point:**

If you have **arbitrary bottom row** of your homography then that means you have arbitrary homography. In this case **ideal points are not preserved**.

# 2D projective geometry

## Action of projectivities and affinities on ideal points



Similarity: circularity is invariant



Affinity: parallelism is invariant



Projectivity: the line at infinity becomes finite (parallel lines on the plane intersect at finite points on  $l_\infty$ )

Hartley and Zisserman (2004), Fig. 2.6

## 2D projective geometry

### Action of projectivities and affinities on ideal points



(a)



(b)



(c)

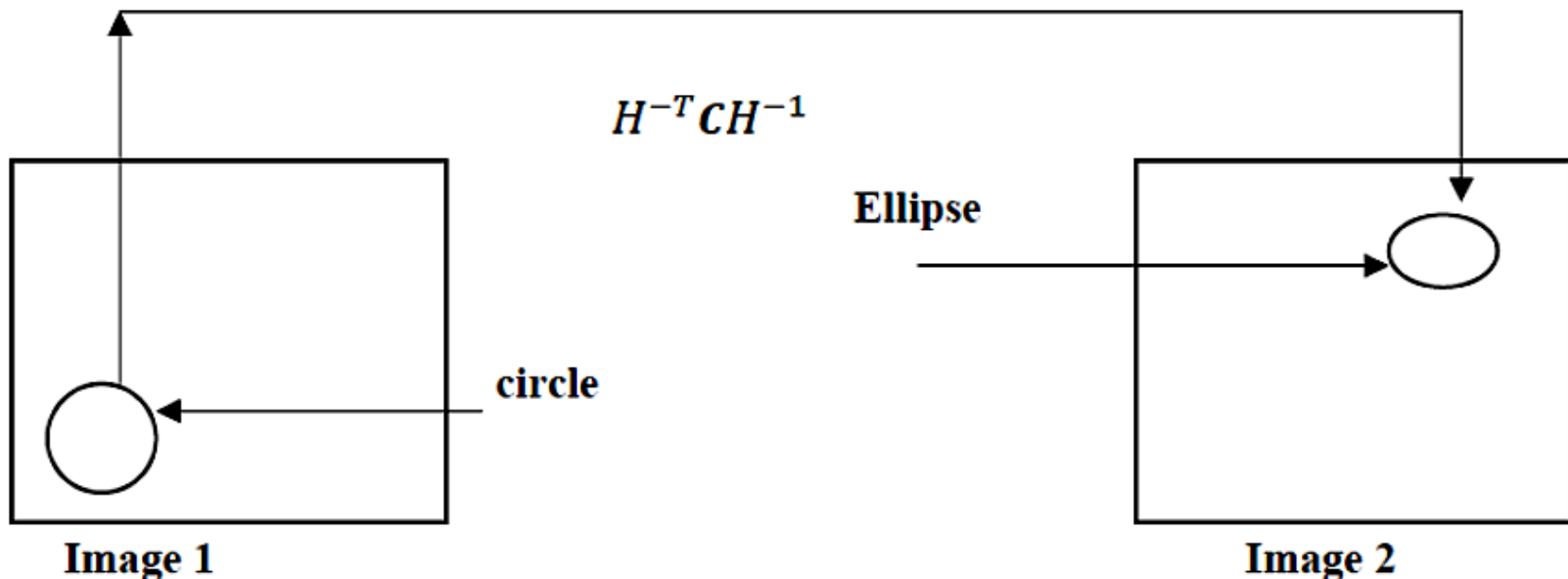
**Distortions arising under central projection.** Images of a tiled floor. **(a) Similarity:** the circular pattern is imaged as a circle. A square tile is imaged as a square. Lines which are parallel or perpendicular have the same relative orientation in the image. **(b) Affine:** The **circle** is imaged as an **ellipse**. Orthogonal world lines are not imaged as orthogonal lines. However, the sides of the square tiles, which are parallel in the world are parallel in the image. **(c) Projective:** Parallel world lines are imaged as converging lines. Tiles **closer to the camera** have a **larger image** than those further away.



# Conic Homography

The corresponding conic homography is

$$C' = H^{-T} C H^{-1}$$



Suppose we have circle in image 1 and we want to map it to ellipse in image 2. We can this transformation using the relationship  $H^{-T} C H^{-1}$

# Conic

The equation of a conic in **inhomogeneous coordinates** is

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

i.e. a polynomial of degree 2. “**Homogenizing**” this by the replacements:

$$x \mapsto \frac{x_1}{x_3}, y \mapsto \frac{x_2}{x_3} \text{ gives}$$

$$\Rightarrow ax_1^2 + bx_1x_2 + cx_2^2 + dx_1x_3 + ex_2x_3 + fx_3^2 = 0 \text{ -----(1)}$$

or in matrix form

$$\vec{x}^T C \vec{x} = 0$$

where the conic coefficient matrix C is given by

$$C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

# Conic

- C is homogeneous because scaling by non-zero constant does not change the conic
- It is a symmetric matrix. It has 9 elements but only 6 are unique. Also, it is a homogeneous matrix. We can scale C and we can still satisfy the equation  $\vec{x}^T C \vec{x} = 0$
- It means we lose one degree of freedom.
- **Note:** If we divide every element by f then we fix the scale factor by f. As a result, we have **5 degrees of freedom** of this matrix

# Five points define a conic

- Suppose we wish to compute the conic which passes through a set of points,  $x_i$ .
- How many points are we free to specify before the conic is determined uniquely?
- The question can be answered constructively by providing an algorithm to determine the conic. From (1) each point  $x_i$  places one constraint on the conic coefficients, since if the conic passes through  $(x_i, y_i)$  then

$$ax_i^2 + bx_iy_i + cy_i^2 + dx_i + ey_i + f = 0$$

This constraint can be written as

$$[x_i^2 \quad x_iy_i \quad y_i^2 \quad x_i \quad y_i \quad 1]_{1 \times 6} \vec{c}_{6 \times 1} = 0$$

Where  $\vec{c}_{6 \times 1} = [a \quad b \quad c \quad d \quad e \quad f]^T$  is the conic C represented as a 6-vector

# Five points define a conic

○Stacking the constraints from five points we obtain

$$\begin{bmatrix} x_1^2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2 y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3 y_3 & y_3^2 & x_3 & y_3 & 1 \\ x_4^2 & x_4 y_4 & y_4^2 & x_4 & y_4 & 1 \\ x_5^2 & x_5 y_5 & y_5^2 & x_5 & y_5 & 1 \end{bmatrix}_{5 \times 6} \quad \vec{c}_{6 \times 1} = \vec{0}_{5 \times 1}$$

○The **conic** is the **null vector** of this  $5 \times 6$  matrix. This shows that a conic is determined uniquely (up to scale) by **five points** in general position.

# Transformation of conics

$$\mathbf{x}^T \mathbf{C} \mathbf{x} = 0 \text{ -----(2)}$$

$$\because \mathbf{x}' = \mathbf{H}\mathbf{x}$$

Pre multiply by  $\mathbf{H}^{-1}$

$$\mathbf{H}^{-1} \mathbf{x}' = \mathbf{H}^{-1} \mathbf{H}\mathbf{x}$$

$$\Rightarrow \mathbf{H}^{-1} \mathbf{x}' = \mathbf{x}$$

or

$$\mathbf{x} = \mathbf{H}^{-1} \mathbf{x}'$$

Substitute the value of  $\mathbf{x}$  in (2), we get

$$(\mathbf{H}^{-1} \mathbf{x}')^T \mathbf{C} \mathbf{H}^{-1} \mathbf{x}' = 0$$

$$\mathbf{x}'^T \mathbf{H}^{-1T} \mathbf{C} \mathbf{H}^{-1} \mathbf{x}' = 0$$

which is a quadratic form  $\mathbf{x}'^T \mathbf{C}' \mathbf{x}'$  with  $\mathbf{C}' = \mathbf{H}^{-1T} \mathbf{C} \mathbf{H}^{-1}$ . This gives the transformation rule for a conic:

Under a point transformation  $\mathbf{x}' = \mathbf{H}\mathbf{x}$ , a conic  $\mathbf{C}$  transforms to

$$\mathbf{C}' = \mathbf{H}^{-1T} \mathbf{C} \mathbf{H}^{-1}$$

# Exact homography using 4 points correspondences

$$P_1(x_1, y_1, 1) \mapsto P'_1(x'_1, y'_1, 1)$$

$$P_2(x_2, y_2, 1) \mapsto P'_2(x'_2, y'_2, 1)$$

$$P_3(x_3, y_3, 1) \mapsto P'_3(x'_3, y'_3, 1)$$

$$P_4(x_4, y_4, 1) \mapsto P'_4(x'_4, y'_4, 1)$$

$$\begin{bmatrix} -x_1 & -y_1 & -1 & 0 & 0 & 0 & x_1 x'_1 & y_1 x'_1 & x'_1 \\ 0 & 0 & 0 & -x_1 & -y_1 & -1 & x_1 y'_1 & y_1 y'_1 & y'_1 \\ -x_2 & -y_2 & -1 & 0 & 0 & 0 & x_2 x'_2 & y_2 x'_2 & x'_2 \\ 0 & 0 & 0 & -x_2 & -y_2 & -1 & x_2 y'_2 & y_2 y'_2 & y'_2 \\ -x_3 & -y_3 & -1 & 0 & 0 & 0 & x_3 x'_3 & y_3 x'_3 & x'_3 \\ 0 & 0 & 0 & -x_3 & -y_3 & -1 & x_3 y'_3 & y_3 y'_3 & y'_3 \\ -x_4 & -y_4 & -1 & 0 & 0 & 0 & x_4 x'_4 & y_4 x'_4 & x'_4 \\ 0 & 0 & 0 & -x_4 & -y_4 & -1 & x_4 y'_4 & y_4 y'_4 & y'_4 \end{bmatrix}_{8 \times 9} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}_{9 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{8 \times 1}$$

$$A_{8 \times 9} \mathbf{h}_{9 \times 1} = \mathbf{0}_{8 \times 1}$$

$$\mathbf{h} = \text{null}(\mathbf{A})$$

**% Simple example of computing a homography from 4 exact point  
%correspondences**

**% The 4-points in the left image**

$X = \begin{bmatrix} 200 & 250 & 210 & 255 & 150 & 145 & 170 & 162 & 1 & 1 & 1 & 1 \end{bmatrix};$

**% The 4-points in the right image**

$X_{\text{prime}} = \begin{bmatrix} 200 & 250 & 200 & 250 & 150 & 150 & 170 & 170 & 1 & 1 & 1 & 1 \end{bmatrix};$

**% We have  $X_{\text{prime}} = H * X$**

**% Let's loop over the points and extract the linear equations in H**

$A = [];$



**% size(X,2) will give us column of X, which is 4**

**for** iPoint = 1:size(X,2)

x = X(:, iPoint); % for each point in left image

xprime = Xprime(:,iPoint); % for each point in right image

**% The equation is**

**% xprime(1) = ( h11 x(1) + h12 x(2) + h13 ) / ( h31 x(1) + h32 x(2) + h33 )**

**% Put this in the form ( coeffs \* (h11 h12 h13 h21 h22 h23 h31 h32 h33)' )**

coeffs = [ -x(1), -x(2), -1, 0, 0, 0, xprime(1) \* x(1), xprime(1) \* x(2), xprime(1)];

A = [ A ; coeffs ];

**% Do the same for the equation**

**% xprime(2) = ( h21 x(1) + h22 x(2) + h23 ) / ( h31 x(1) + h32 x(2) + h33 )**

coeffs = [0, 0, 0, -x(1), -x(2), -1, xprime(2) \* x(1), xprime(2) \* x(2), xprime(2)];

A = [ A ; coeffs ];

**end**

**% The solution is the right null space of A**

```
h = null(A);
```

**% Form the 9 parameters of H into a matrix**

```
H = reshape( h, 3, 3)';
```

**% Generate 100 random.**

**% diag([640,480]) creates a diagonal matrix 640 0; 0 480**

```
Xtest = [ ( rand(2,100)' * diag([640,480]) )' ; ones(1,100)];
```

**% Plot themfigure(1);**

```
figure(1);
```

```
plot(Xtest(1,:), Xtest(2,:), 'rx' );
```

```
hold on;
```

```
plot(X(1,:), X(2,:), 'bo');
```

```
axis([0,640,0,480],'equal','ij');
```

```
title('Left image points X');
```

```
hold off;
```

## **% Transfer the test points**

```
Xtestprime = H * Xtest;
```

```
Xtestprime = Xtestprime ./ repmat(Xtestprime(3,:),3,1);
```

```
Xprimeest = H * X;Xprimeest = Xprimeest ./ repmat(Xprimeest(3,:),3,1);
```

```
% Plot themfigure(2);plot( Xtestprime(1,:), Xtestprime(2,:), 'rx' );
```

```
figure(2);
```

```
plot( Xtestprime(1,:), Xtestprime(2,:), 'rx' );
```

```
hold on;
```

```
plot(Xprimeest(1,:),Xprimeest(2:), 'bo');
```

```
axis([0,640,0,480],'equal','ij');
```

```
title('Right image points X');
```

```
hold off;
```

# Resulting right image after applying homography on the left image

