

$$\beta \frac{df_{free}}{d\phi_A} = \frac{1}{2} \frac{dy}{d\phi_A} \left( \frac{1}{y(y+1)} - \frac{\ln(y+1)}{y^2} \right) \sum_s \frac{\phi_s y_s}{v_s} + \frac{1}{2} \sum_s \frac{d\phi_s}{d\phi_A} \frac{1}{v_s} \left[ \frac{y_s}{y} \ln(1+y) - \ln(1+y_s) \right] + \sum_s \frac{d\phi_s}{d\phi_A} \frac{1+\ln\phi_s}{v_s}$$

$$\beta \frac{d^2 f_{free}}{d\phi_A^2} = \frac{dy}{d\phi_A} \left( \frac{1}{y(y+1)} - \frac{\ln(y+1)}{y^2} \right) \sum_s \frac{y_s}{v_s} * \frac{d\phi_s}{d\phi_A} + \frac{1}{2} \left( \frac{dy}{d\phi_A} \right)^2 \left( \frac{2\ln(y+1)}{y^3} - \frac{2+3y}{(y+y^2)^2} \right) \sum_s \frac{\phi_s y_s}{v_s} + \sum_s \frac{1}{v_s \phi_s}$$

For 2 substances (A and B):

$$\frac{dy}{d\phi_A} = (y_A - y_B)$$

$$\frac{d\phi_s}{d\phi_A} = \pm 1$$