

Q1 – [Asymptotic Relations - 1 point] For each of the following pairs of functions $f(n)$ and $g(n)$, determine the most appropriate symbol in the set $\{O, o, \Theta, \Omega, \omega\}$. ($\lg n = \log$ to the base 2 of n)

$$1. f(n) = 1005n^2 + 10n + 11$$

$$g(n) = n^3/1000$$

$$2. f(n) = \lg^7(n^7)$$

$$g(n) = (n^{1/2})^{1/2}$$

$$3. f(n) = (n^2 - 1)(n^2 + 1)\lg n$$

$$g(n) = n^4 \lg n^{1001}$$

$$4. f(n) = 32^{\lg(n)} \text{ (square root of } n\text{)}$$

$$g(n) = n^3$$

Q2 – [Step count analysis - 1 point] Analyze the following pseudocode and give a tight bound on the running time as a function of n . You can assume that all individual instructions take $O(1)$ time. Show your work.

```
l := 0; i := 1;

while i ≤ n {
    for j = 1 to i {
        l := l + 2 * n + 3 * j;
    }
    i = 2 * i
}
```

Q3 – [Logarithms – 1 point] Prove that $a^{\log_b x} = x^{\log_b a}$. Do not assume the statement to be true. Deduce your answer by applying logarithm principles.

Q4 – [Recurrence relations – 2 points] Please solve the following recurrences.

$$1. T(n) = 4T(n/3) + n$$

$$2. T(n) = 3T(n/3) + n/2$$

$$3. T(n) = 4T(n/2) + n^{2.5}$$

$$4. T(n) = T(n/2) + T(n/4) + T(n/8) + 1$$