# The automated testing of randomness with multiple statistical batteries

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# Presentation structure

- RTT implementation – program and service, deployed on MetaCentrum
- Brief intro into statistical testing – battery, test, hypothesis, partial results, false positives/negatives
- Baseline experiment – default behaviour of the batteries, what is the rate of false negative tests, find the thresholds, use it in the subsequent testing
- Security margins experiment – compare to EACirc (?), compare strength of individual batteries in RTT, detect discrepancies
- Dieharder experiment – distribution of the partial results, assumed uniformity, however different results obtained, show how much.

# Statistical testing of randomness

**Statistical battery**
Software with purpose of detecting biases in data stream; collection of statistical tests.

**Statistical test**
Single unit in statistical battery, checks some property of the data; e.g. longest uninterrupted stream of ones.

**Null hypothesis – $H_0$**
Null hypothesis stating that the tested data were produced by a true random number generator.

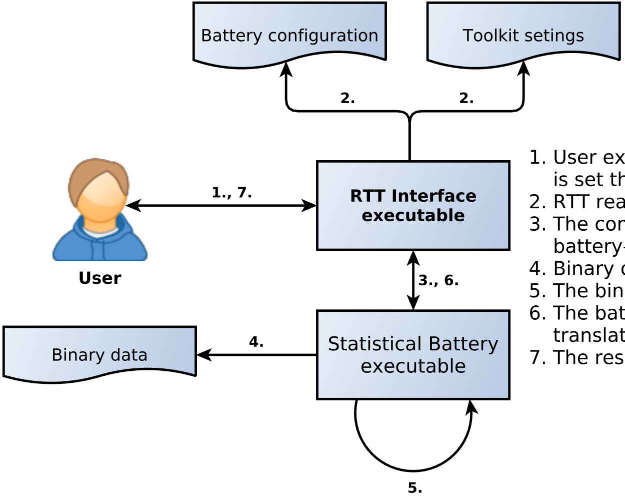
**False positive (Type I error)**
Happens when $H_0$ holds true but it is rejected – truly random stream is evaluated as non random. We assume that for random data the p-values have an uniform distribution – probability of Type I error is $\alpha$.

**False negative (Type II error)**
Happens when $H_0$ is false but it is not rejected – biased stream is evaluated as random.

# Randomness Testing Toolkit – motivation

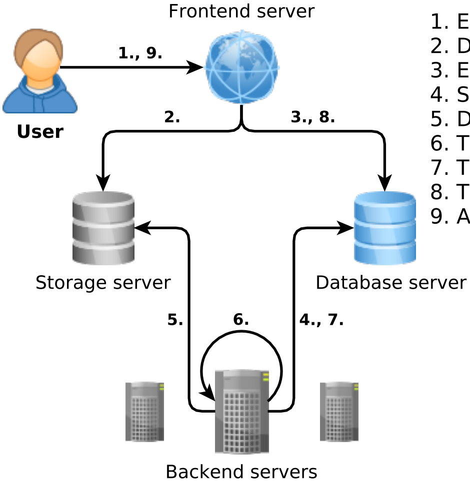
TODO

# Randomness Testing Toolkit – local interface




1. User executes the binary, basic configuration is set through command-line arguments.
2. RTT reads advanced settings in common format
3. The common settings are translated into battery-specific settings.
4. Binary data is read from file supplied to battery.
5. The binary stream is analysed.
6. The battery-specific results are gathered and translated into unified format by RTT.
7. The results are stored for user to inspect.

# Randomness Testing Toolkit – web service




1. Experiment and data are submitted.
2. Data is uploaded.
3. Experiment and related jobs are created.
4. Single pending job is selected.
5. Data is downloaded.
6. The data is analyzed.
7. The results of the analysis are stored.
8. The results in the database are presented.
9. Analysis of the data is inspected.

# Talk overview

- **Randomness Testing Toolkit project**
  - Developed framework for randomness testing
- **Introduction to statistical testing**
  - Theory behind the statistical tests
- **Experiments done with RTT**
  - Baseline experiment
  - Analysis of popular crypto functions
  - Dieharder outputs inspection

### Motivation

- Randomness testing used for benchmarking of the research tools developed in CRoCS. TODO: Add another reason, make it easy.
- Most of the existing statistical batteries are neither intuitive nor trivial to use.
- Inconsistent results among researchers.

### Randomness Testing Toolkit

- Project aiming to provide easy, fast and consistent randomness testing.
- Inclusion of multiple statistical batteries – NIST STS, Dieharder, TestU01.
- Available as a standalone executable or as a service.

Here goes the figure...

# Statistical testing – 1/3

**Statistical battery**

Software with purpose of detecting biases in data stream; collection of statistical tests.

**Statistical test**

Single unit in statistical battery, checks some property of the data; e.g. longest uninterrupted stream of ones.

**Statistical test results – REWORK**

- Single test is repeated multiple times, resulting in multiple first-level p-values.
- First-level p-values are processed by arbitrary statistical algorithm – the result is second-level p-value or statistic; e.g. Kolmogorov-Smirnov test for uniformity.
- Single test can produce multiple statistics (subtests, variations in test parameters, multiple uniformity tests, etc...).

## Statistical testing – 2/3

**Hypothesis $H_0$**

Null hypothesis stating that the tested data were produced by a true random number generator.

**p-value**

P-value represents the probability that the $H_0$ holds true.

**Significance level $\alpha$**

If p-value is lesser than $\alpha$ then the result is significant and $H_0$ is rejected. If the p-value of a test if lesser than $\alpha$ we consider the test failed.

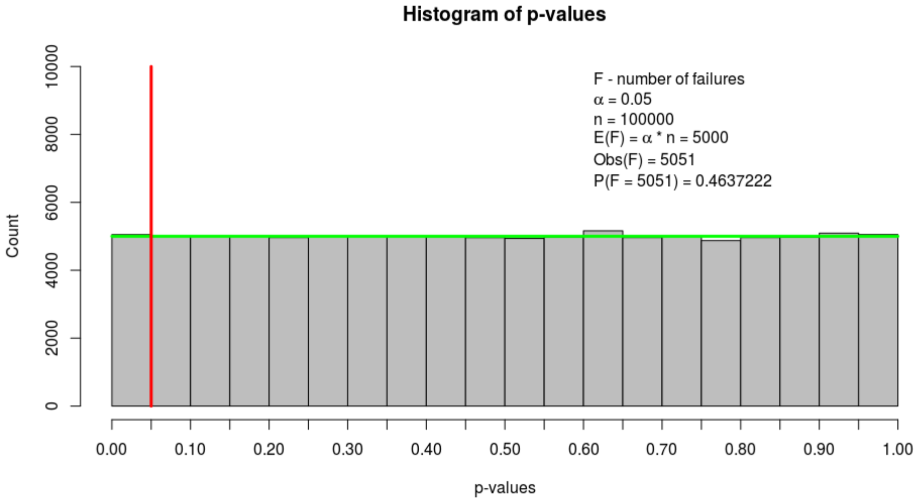
**False positive (Type I error)**

Happens when $H_0$ holds true but it is rejected – truly random stream is evaluated as non random. We assume that for random data the p-values have an uniform distribution – probability of Type I error is $\alpha$.

**False negative (Type II error)**

Happens when $H_0$ is false but it is not rejected – biased stream is evaluated as random.

# Statistical testing – 3/3



# Overview of the experiments

**Baseline experiment**

- Discovering failure rates of the batteries
- Finding the bound of test failure count in a battery

**Usable testbed analysis**

- Analyse outputs of well-known cryptographic algorithms (AES, DES, RC4, etc.)
- Observe the security margins of the algorithms
- Compare the results to other approach in CRoCS

**Analysis of Dieharder**

- Examine behavior of Dieharder battery during truly random data analysis
- Analyse the distribution of the results

# Baseline experiment – 1/4

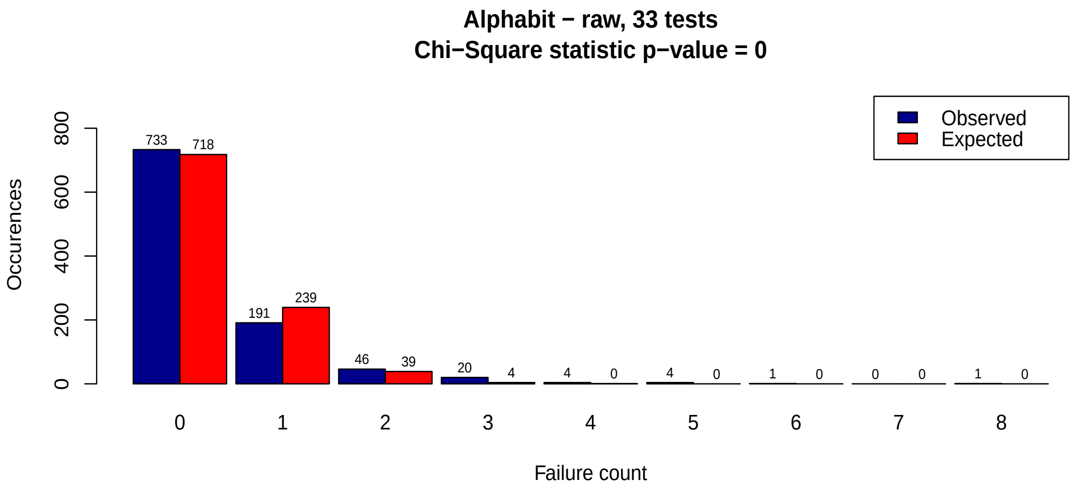
**Analysis of 8TB of quantum random data**

- Data split into 1000 blocks – each battery executed on every block
- Sample size was 1000 trials.

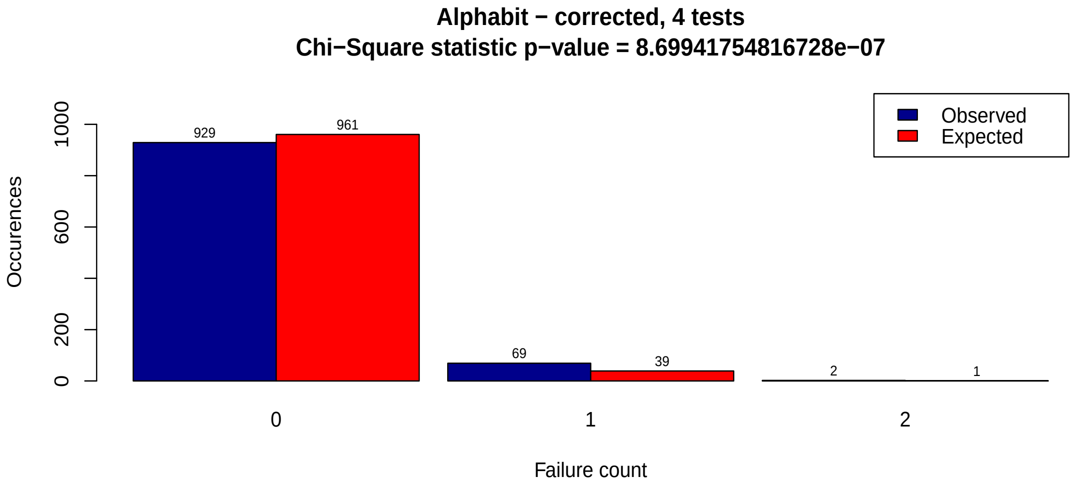
**Test failure observation**

- Raw second-level p-values – assuming that p-values are independent.
  - Single p-value will fail with probability $\alpha$
  - Out of $n$ p-values, $x$ will fail with probability $P(F = x) = \binom{n}{x} * \alpha^x * (1 - \alpha)^{n-x}$
- Corrected p-values – likely-dependent p-values grouped together
  - Products of a single test treated as a single result.
  - Each group of $n$ p-values has new corrected (partial) $\alpha_0 = 1 - (1 - \alpha)^{\frac{1}{n}}$
  - If any p-value in the group with $\alpha_0$ has lesser value than the $\alpha_0$, the entire group is treated as a failed test.

# Baseline experiment – 2/4



# Baseline experiment – 3/4



# Baseline experiment – 4/4

### Analysis result assessment

- Result of data analysis is assessed based on the number of failed corrected tests in the battery.
- $P(X = F) < 0.001$ – the analysed data is not considered random

### Summary

| Battery name | Raw $\chi^2$ | Corrected $\chi^2$ | Fail count bound |
|---|---:|---:|---:|
| Dieharder | 4.3e-17 | **0.50427** | 3/27 |
| NIST STS | **0.02975** | 5.5e-09 | 2/15 |
| TU01 Small Crush | **0.72575** | 0.15814 | 2/10 |
| TU01 Crush | 3.6e-15 | **0.00707** | 3/32 |
| TU01 Rabbit | **4.9e-22** | 1.4e-23 | 2/16 |
| TU01 Alphabit | 0.00000 | **8.6e-07** | 1/4 |
| TU01 Block Alphabit | 0.00000 | **1.4e-101** | 1/4 |

# Usable testbed analysis – 1/3

**Analysed algorithms**

- In total, 72 different data streams were analysed.
- The data streams were outputs from 16 distinct round-reduced cryptographic algorithms.
- The algorithms were chosed based on their popularity (AES, DES, RC4, ...) or their success in crypto competitions eSTREAM and SHA3 (Rabbit, Keccak, Grøstl, ...).

**Analysis conditions**

- Each datastream was 8GB long.
- The conditions of analysis were same as in the previous experiment.
- The interpretation of the result was based on the results of the baseline experiment.

# Usable testbed analysis – 2/3

| Algorithm | Biased round RTT | Biased round EACirc | Security Margin |
|---|---|---|---|
| AES | 3 | 3 | 7 – 70% |
| BLAKE | 1 | 1 | 15 – 93.7% |
| Grain | 6* | 2 | 7 – 53.8% |
| Grøstl | 2 | 2 | 12 – 85.7% |
| HC-128 | – | – | 0 – 100% |
| JH | 6 | 6 | 36 – 85.7% |
| Keccak | 3 | 2 | 21 – 87.5% |
| MD6 | 10* | 8 | 94 – 90.3% |
| Rabbit | 4* | 0 | 0 – 0% |
| RC4 | 0* | – | 0 – 0% |
| Salsa20 | 2 | 2 | 18 – 90% |
| SINGLE-DES | 5 | 4 | 11 – 68.7% |
| Skein | 4 | 3 | 68 – 94.4% |
| SOSEMANUK | 4 | 4 | 21 – 84% |
| TEA | 5 | 4 | 27 – 84.3% |
| TRIPLE-DES | 3 | 2 | 13 – 81.2% |

# Usable testbed analysis – 3/3

**Notable results**

- **Grain** – Tests smarsa_MatrixRank and scomp_LinearComp (Crush, Rabbit) will fail in 3, 4, 5 and 6-round configuration.
- **MD6** – Tests smarsa_MatrixRank and sspectral_Fourier3 (Crush, Rabbit) will fail in 9 and 10-round configuration.
- **Rabbit** – Tests sstring_HammingIndep and sstring_PeriodsInStrings (Crush, Rabbit, Alphabit, Block Alphabit) will fail in **full** configuration.
- **RC4** – Tests sknuth_SimpPoker and sknuth_Gap (Crush) will fail in **full** configuration.

# Analysis of Dieharder – 1/2

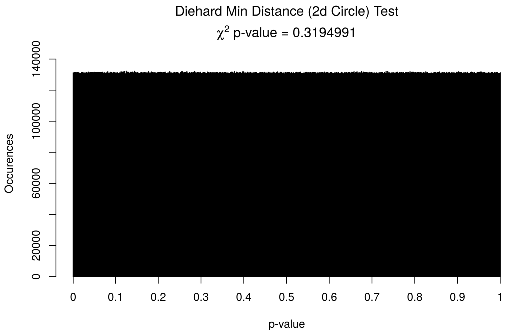
### Analysed data

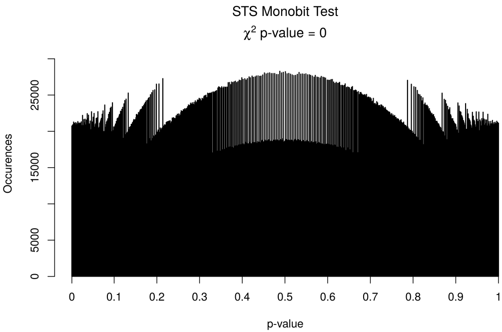
- 8TB of quantum random data processed continuously by the tests - single application of a test to a data stream will yield single first-level p-value
- Uniformity of the first level p-values was analysed.
- The p-values should be uniformly distributed on the interval $< 0, 1 >$.
- Total of 110 sets of p-values (single set per raw, uncorrected test) was inspected.
- Each set had a different size – usually between 1 to 2 millions of p-values per set.

### Experiment results

- Out of 110 p-value sets, 39 sets were not uniformly distributed
- Chi-Square ($\chi^2$) statistic used for uniformity testing. When the p-value of the statistic was less than 0.001, the inspected set was considered non-uniform.
- Flawed non-uniform distributions can have impact on Dieharder results.

# Analysis of Dieharder – 2/2



# References

- **Randomness Testing Toolkit**
  `https://github.com/crocs-muni/randomness-testing-toolkit`
- **EACirc**
  `https://github.com/crocs-muni/eacirc`
- **NIST Statistical testing suite**
  `http://csrc.nist.gov/groups/ST/toolkit/rng/documentation_software.html`
- **Dieharder**
  `http://www.phy.duke.edu/~rgb/General/dieharder.php`
- **TestU01**
  `http://simul.iro.umontreal.ca/testu01/tu01.html`