分類問題

郭耀仁

分類問題有哪些？

- 貸款顧客是否會倒帳
- 圖片辨識
- 垃圾郵件辨識
- ...etc.

In [1]:

```
import pandas as pd

labeled_url = "https://storage.googleapis.com/kaggle_datasets/Titanic-Machine-Learning-from-Disaster/train.csv"
test_url = "https://storage.googleapis.com/kaggle_datasets/Titanic-Machine-Learning-from-Disaster/test.csv"
labeled_df = pd.read_csv(labeled_url)
test_df = pd.read_csv(test_url)
print(labeled_df.shape)
print(test_df.shape)
```

```
(891, 12)
(418, 11)
```

In [2]:

```
labeled_df.head()
```

Out[2]:

	PassengerId	Survived	Pclass	Name	Sex	Age	SibSp	Parch	
0	1	0	3	Braund, Mr. Owen Harris	male	22.0	1	0	A 2
1	2	1	1	Cumings, Mrs. John Bradley (Florence Briggs Th...	female	38.0	1	0	P
2	3	1	3	Heikkinen, Miss. Laina	female	26.0	0	0	S 3
3	4	1	1	Futrelle, Mrs. Jacques Heath (Lily May Peel)	female	35.0	1	0	1
4	5	0	3	Allen, Mr. William	male	35.0	0	0	3

In [3]:

```
labeled_df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 891 entries, 0 to 890
Data columns (total 12 columns):
PassengerId    891 non-null int64
Survived       891 non-null int64
Pclass         891 non-null int64
Name           891 non-null object
Sex            891 non-null object
Age            714 non-null float64
SibSp          891 non-null int64
Parch          891 non-null int64
Ticket         891 non-null object
Fare           891 non-null float64
Cabin          204 non-null object
Embarked       889 non-null object
dtypes: float64(2), int64(5), object(5)
memory usage: 83.6+ KB
```

*Although there was some element of luck involved in surviving the sinking, some groups of people were more likely to survive than others, such as **women**, **children**, and the **upper-class.***

In [4]:

```
labeled_df = labeled_df[labeled_df["Age"].notna()]
labeled_df.describe()
```

Out[4]:

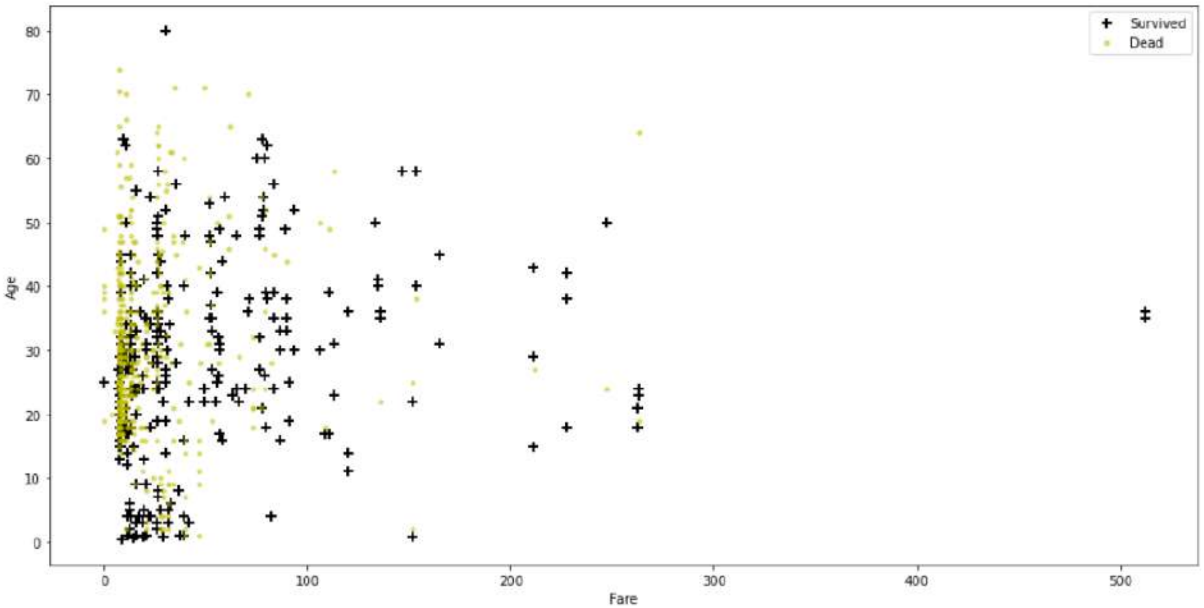
	PassengerId	Survived	Pclass	Age	SibSp	
count	714.000000	714.000000	714.000000	714.000000	714.000000	714
mean	448.582633	0.406162	2.236695	29.699118	0.512605	0.4
std	259.119524	0.491460	0.838250	14.526497	0.929783	0.8
min	1.000000	0.000000	1.000000	0.420000	0.000000	0.0
25%	222.250000	0.000000	1.000000	20.125000	0.000000	0.0
50%	445.000000	0.000000	2.000000	28.000000	0.000000	0.0
75%	677.750000	1.000000	3.000000	38.000000	1.000000	1.0
max	891.000000	1.000000	3.000000	80.000000	5.000000	6.0

In [5]:

```
import matplotlib.pyplot as plt

def plot_data(data, label_x, label_y, label_pos, label_neg, label_target):
    fig, ax = plt.subplots(1, 1, figsize=(16, 8))
    neg = data[label_target] == 0
    pos = data[label_target] == 1
    ax.scatter(data[pos][label_x], data[pos][label_y], marker='+', c='k', s=60, linewidth=2, label=
label_pos)
    ax.scatter(data[neg][label_x], data[neg][label_y], c='y', s=10, label=label_neg, alpha = 0.5)
    ax.set_xlabel(label_x)
    ax.set_ylabel(label_y)
    ax.legend(frameon= True, fancybox = True)
```

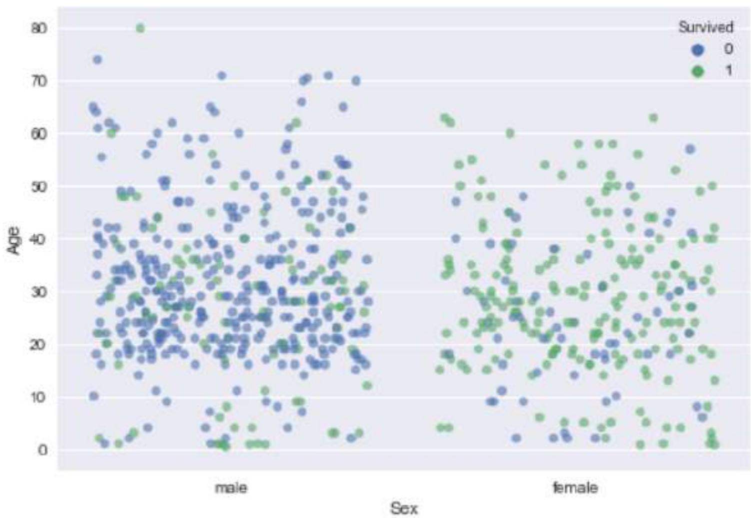
In [6]:
```
plot_data(labeled_df, "Fare", "Age", "Survived", "Dead", "Survived")
plt.show()
```



觀察性別的影響

In [7]:
```
import seaborn as sns

sns.stripplot(x="Sex", y="Age", data=labeled_df, jitter=0.4, hue="Survived", alpha=0.6, size = 6)
plt.show()
```



分類問題其實是迴歸問題的延伸

將 $\boldsymbol{h(X) = X\theta}$ 的結果利用特定函數轉換為簡單分類器

- 羅吉斯迴歸利用 **sigmoid** 函數

羅吉斯迴歸利用 sigmoid 函數

$$\hat{y} = H(g(X\theta))$$

其中，

$$g(X\theta) = \frac{1}{1 + e^{-X\theta}}$$

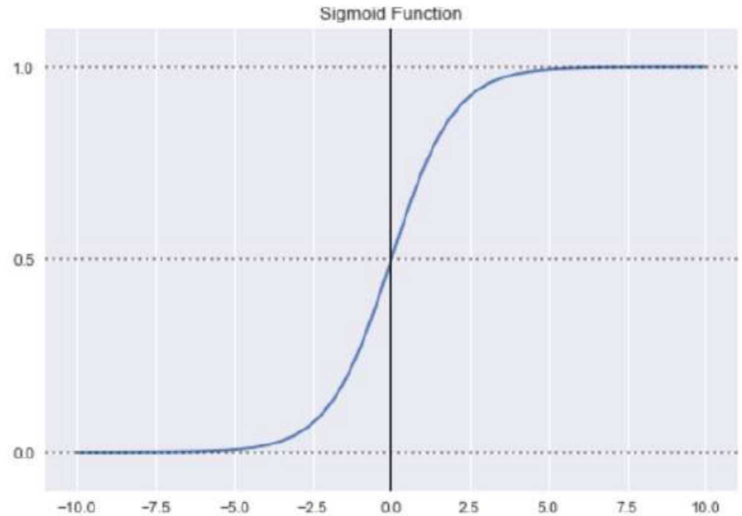
$$g(z) = \frac{1}{1 + e^{-z}}$$

In [8]:

```
def sigmoid(z):
    return(1 / (1 + np.exp(-z)))
```

隨堂練習：描繪出 Sigmoid 函數的外觀

In [10]:
```
plt.show()
```



接著決定

- $g(z)$ 輸出的機率該如何轉換至 $\hat{y} \in \{0, 1\}$？

$$\hat{y} = H(z) = \begin{cases} 1 & \text{if } z \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

拆解 Logistic 分類器的建立流程：

- 建立迴歸模型 $\hat{y} = X\theta$
- 將 h 的輸出作為 Sigmoid 函數的輸入，得到 $g(X\theta)$
- 將 $g(X\theta)$ 作為 H 的輸入，得到 $\hat{y}$

那麼 Logistic 分類器的成本函數該怎麼寫呢？

- 成本函數一般式

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}\,log\,(h_\theta\,(x^{(i)})) - (1-y^{(i)})\,log\,(1-h_\theta(x^{(i)}))\right]$$

- 成本函數矩陣式（vectorized）

$$J(\theta) = -\frac{1}{m}\Big(\big(\,log\,(g(X\theta)\big)^T y + \big(\,log\,(1 - g(X\theta)\big)^T (1 - y)\Big)$$

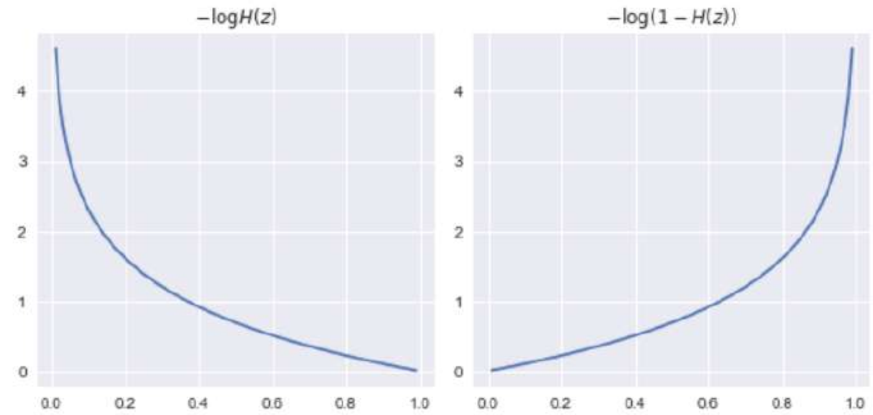
這個成本函數的含義為

- 讓錯誤分類的成本無限大

$$J(H(z),y) = \begin{cases} -\log(H(z)) & \text{if } y=1 \\ -\log(1-H(z)) & \text{if } y=0 \end{cases}$$

In [12]:

```
plt.show()
```



In [13]:
```
def cost_function(theta, X, y):
    m = y.shape[0]
    h = sigmoid(X.dot(theta))
    J = -1*(1/m)*(np.log(h).T.dot(y)+np.log(1-h).T.dot(1-y))
    if np.isnan(J[0]):
        return(np.inf)
    return(J[0])
```

利用 Gradient Descent 的方式求解 $\boldsymbol{\theta}$

- 偏微分

$$\frac{\delta J(\theta)}{\delta \theta_j} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$$

- 偏微分矩陣式

$$\frac{\delta J(\theta)}{\delta \theta_j} = \frac{1}{m}X^T(g(X\theta) - y)$$

In [14]:

```
def gradient_descent(theta, X, y):
    m = y.shape[0]
    h = sigmoid(X.dot(theta.reshape(-1, 1)))
    grad =(1/m)*X.T.dot(h-y)
    return(grad.ravel())
```

In [15]:

```
from sklearn.model_selection import train_test_split

train_df, validation_df = train_test_split(labeled_df, test_size=0.3, random_state=123)
X_train = train_df[["Fare", "Age"]].values
y_train = train_df["Survived"].values.reshape(-1, 1)
ones = np.ones(X_train.shape[0]).reshape(-1, 1)
X_train = np.concatenate([ones, X_train], axis=1)
initial_theta = np.zeros(X_train.shape[1])
cost = cost_function(initial_theta, X_train, y_train)
grad = gradient_descent(initial_theta, X_train, y_train)
print('Cost: \n', cost)
print('Grad: \n', grad)
```

```
Cost:
 0.69314718056
Grad:
 [ 0.08717435 -4.86218778  2.78531062]
```

利用 scipy 的 optimize() 函數

In [16]:
```
from scipy.optimize import minimize

res = minimize(cost_function, initial_theta, args=(X_train, y_train), method=None, jac=gradient_des
cent, options={'maxiter':400})
res
```

```
/Users/kuoyaojen/anaconda3/lib/python3.6/site-packages/ipykernel/__main__.py:4: RuntimeWarnin
g: divide by zero encountered in log
/Users/kuoyaojen/anaconda3/lib/python3.6/site-packages/ipykernel/__main__.py:4: RuntimeWarnin
g: divide by zero encountered in log
```

Out[16]:
```
      fun: 0.6249923100906284
 hess_inv: array([[  2.57707497e+01,  -9.67852173e-02,  -6.39438923e-01],
       [ -9.67852173e-02,   4.31442824e-03,  -1.31976410e-03],
       [ -6.39438923e-01,  -1.31976410e-03,   2.32935569e-02]])
      jac: array([  5.79538929e-08,   3.06075254e-06,   1.65741052e-06])
  message: 'Optimization terminated successfully.'
     nfev: 23
      nit: 14
     njev: 18
   status: 0
  success: True
        x: array([-0.62176132,  0.01692017, -0.01014703])
```

In [17]:

```
thetas = res.x.reshape(-1, 1)
print(thetas)
```

```
[[-0.62176132]
 [ 0.01692017]
 [-0.01014703]]
```

In [18]:

```
# 將 validation_df 中的第一個觀測值拿出來試試看
X_validation = validation_df[["Fare", "Age"]].values
ones = np.ones(X_validation.shape[0]).reshape(-1, 1)
X_validation = np.concatenate([ones, X_validation], axis=1)
x_validation_0 = X_validation[0, :].reshape(-1, 1)
print(sigmoid(np.dot(thetas.T, x_validation_0))[0, 0])
```

```
0.397655381805
```

In [19]:
```
validation_df.head()
```

Out[19]:

	PassengerId	Survived	Pclass	Name	Sex	Age	SibSp	Parch
651	652	1	2	Doling, Miss. Elsie	female	18.0	0	1
694	695	0	1	Weir, Col. John	male	60.0	0	0
797	798	1	3	Osman, Mrs. Mara	female	31.0	0	0
875	876	1	3	Najib, Miss. Adele Kiamie "Jane"	female	15.0	0	0
505	506	0	1	Penasco y Castellana, Mr. Victor de Satode	male	18.0	1	0

In [20]:

```
def predict(thetas, X, threshold=0.5):
    p = sigmoid(np.dot(X, thetas)) >= threshold
    return(p.astype(int))
```

In [21]:

```
is_survived = predict(thetas, x_validation_0.ravel())[0]
print("羅吉斯迴歸模型的預測是: {}".format(is_survived))
print("真實的情況是 :{}".format(validation_df["Survived"].values[0]))
```

羅吉斯迴歸模型的預測是: 0
真實的情況是 :1

跟標準答案比對一下

In [22]:
```
y_hat = predict(thetas, X_validation)
y_validation = validation_df["Survived"].values
y_hat.ravel() == y_validation
```

Out[22]:
```
array([False,  True, False, False, False,  True,  True,  True,  True,
       False, False,  True,  True, False,  True,  True,  True,  True,
       False,  True, False, False, False, False,  True, False,  True,
       False,  True,  True,  True,  True,  True,  True,  True, False,
       False, False, False,  True,  True,  True,  True, False, False,
        True, False, False,  True,  True,  True,  True,  True,  True,
       False,  True, False,  True,  True,  True,  True, False,  True,
        True,  True,  True, False, False,  True, False,  True,  True,
        True,  True, False,  True,  True,  True,  True,  True,  True,
       False,  True, False, False,  True, False,  True, False,  True,
        True,  True, False,  True, False, False, False,  True,  True,
       False,  True, False,  True,  True,  True,  True,  True,  True,
        True,  True,  True,  True,  True, False, False,  True,  True,
        True, False,  True, False,  True,  True, False,  True,  True,
        True, False,  True, False, False, False, False,  True,  True,
        True,  True,  True,  True,  True, False,  True, False,  True,
        True, False,  True, False,  True,  True, False,  True,  True,
        True,  True,  True, False,  True,  True,  True,  True, False,
        True,  True,  True, False, False,  True,  True,  True, False,
       False,  True,  True,  True, False, False,  True,  True,  True,
       False,  True,  True,  True, False, False,  True,  True,  True,
       False,  True, False,  True,  True,  True,  True,  True,  True,
        True,  True,  True, False, False,  True, False,  True, False,
        True,  True,  True,  True, False,  True,  True,  True], dtype=bool)
```

In [23]:

```
accuracy = (y_hat.ravel() == y_validation).sum() / y_validation.size
print("預測準確率為 {:.2f}%".format(accuracy * 100))
```

預測準確率為 66.05%

什麼是準確率呢？

- 要談準確率，或者說明分類模型的評估，不得不提混淆矩陣

Phew...

- 我們終於又手動做出來了羅吉斯迴歸模型！

讓 Scikit-Learn 來找 θ 吧

In [24]:

```
from sklearn.linear_model import LogisticRegression

train_df, validation_df = train_test_split(labeled_df, test_size=0.3, random_state=123)
X_train = train_df[["Fare", "Age"]].values
y_train = train_df["Survived"].values
X_validation = validation_df[["Fare", "Age"]].values
y_validation = validation_df["Survived"].values
clf = LogisticRegression()
clf.fit(X_train, y_train)
y_hat = clf.predict(X_validation)
accuracy = (y_hat == y_validation).sum() / y_hat.size
print("預測準確率為 {:.2f}%".format(accuracy * 100))
```

```
預測準確率為 66.05%
```

隨堂練習：讓 Scikit-Learn 幫我們算出來混淆矩陣

決策邊界 Decision Boundary

- 在散佈圖上將 $X\theta$ 畫出來
- 該如何解讀圖形？

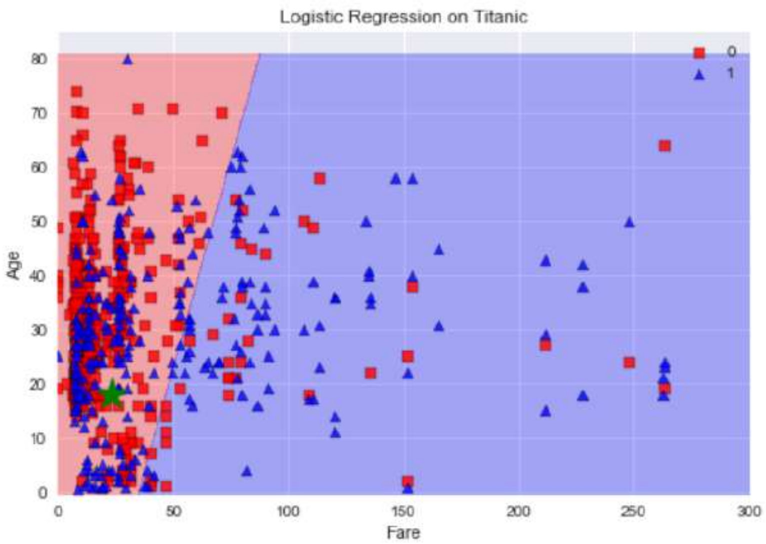
利用 mlxtend 模組的 plot_decision_regions()

In [25]:
```
from mlxtend.plotting import plot_decision_regions
import matplotlib.pyplot as plt

validation_x_0 = validation_df.iloc[0, [5, 9]]
plot_decision_regions(labeled_df[["Fare", "Age"]].values, labeled_df["Survived"].values, clf = clf,
res = 0.1)
plt.scatter(validation_x_0[1], validation_x_0[0], marker="*", s=500, color="g")
plt.xlabel('Fare')
plt.ylabel('Age')
plt.xlim(0, 300)
plt.title('Logistic Regression on Titanic')
```

Out[25]:
```
<matplotlib.text.Text at 0x1a20d236d8>
```

In [26]:
```
plt.show()
```



或是自己畫

In [27]:

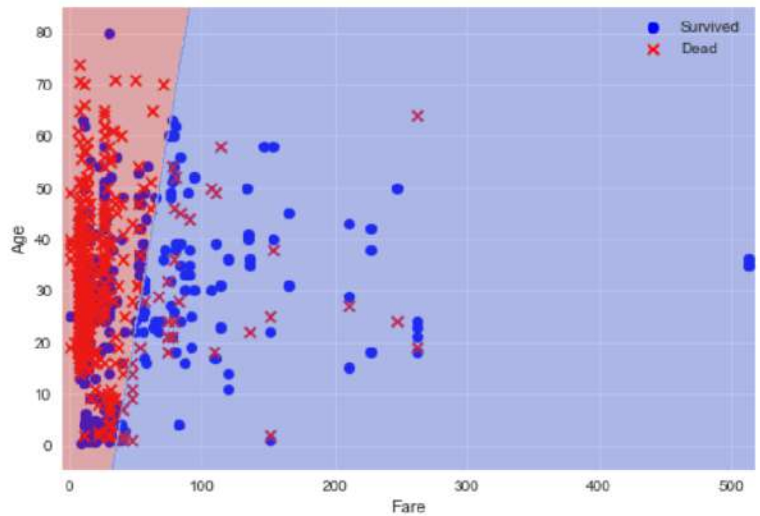
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression

def sigmoid(z):
    return 1/(1 + np.exp(-z))

def step(g_y_hat, threshold=0.5):
    return np.where(g_y_hat >= threshold, 1, 0).reshape(-1, 1)

labeled = pd.read_csv("https://storage.googleapis.com/kaggle_datasets/Titanic-Machine-Learning-from
-Disaster/train.csv")
# Removed observations without Age
labeled = labeled[~labeled["Age"].isna()]
survived = labeled[labeled["Survived"] == 1]
dead = labeled[labeled["Survived"] == 0]
train, validation = train_test_split(labeled, test_size=0.3, random_state=123)
X_train = train.loc[:, ["Fare", "Age"]].values
y_train = train.loc[:, "Survived"].values
logistic_clf = LogisticRegression()
logistic_clf.fit(X_train, y_train)
fit_intercept = logistic_clf.intercept_.reshape(-1, 1)
fit_coef = logistic_clf.coef_.reshape(-1, 1)
thetas = np.concatenate([fit_intercept, fit_coef])
# Decision boundary plot
fare_min, fare_max = labeled["Fare"].min(), labeled["Fare"].max()
age_min, age_max = labeled["Age"].min(), labeled["Age"].max()
fare_arr = np.linspace(fare_min - 5, fare_max + 5, 1000)
age_arr = np.linspace(age_min - 5, age_max + 5, 1000)
xx, yy = np.meshgrid(fare_arr, age_arr)
ones = np.ones(xx.size).reshape(-1, 1)
X_grid = np.concatenate([ones, xx.reshape(-1, 1), yy.reshape(-1, 1)], axis=1)
```

In [28]:
```
plt.show()
```



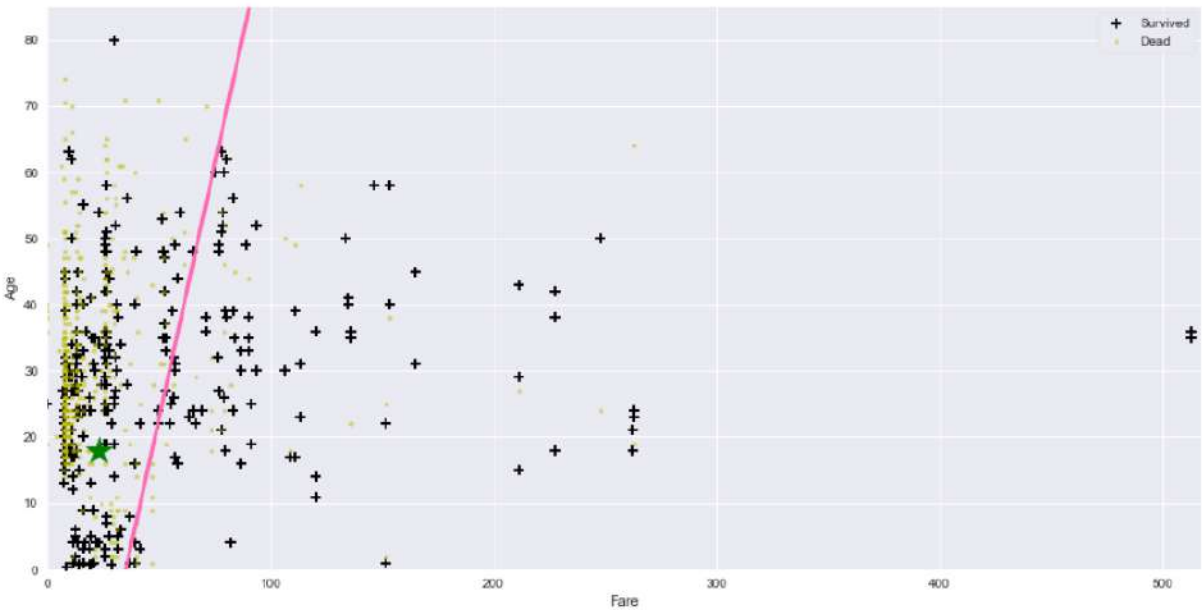
In [29]:

```
from matplotlib.colors import ListedColormap

def decision_boundary(X, labeled_df, clf):
    x1_min, x1_max = X[:,0].min()-5, X[:,0].max()+5,
    x2_min, x2_max = X[:,1].min()-5, X[:,1].max()+5,
    xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max))
    thetas = np.hstack((clf.intercept_, clf.coef_.ravel())).reshape(-1, 1)
    ones = np.ones(xx1.size).reshape(-1, 1)
    xx_concat = np.concatenate([ones, xx1.reshape(-1, 1), xx2.reshape(-1, 1)], axis=1)
    h = sigmoid(xx_concat.dot(thetas))
    h = h.reshape(xx1.shape)
    plot_data(labeled_df, "Fare", "Age", "Survived", "Dead", "Survived")
    plt.contour(xx1, xx2, h, [0.5], linewidths=3, colors='#FF69B4')
    #cmap=ListedColormap(["y", "k"])
    #plt.contourf(xx1, xx2, h, alpha=0.3, cmap=cmap, antialiased=True)
    plt.xlim(0, x1_max)
    plt.ylim(0, x2_max)
```

In [30]:

```
decision_boundary(labeled_df[["Fare", "Age"]].values, labeled_df, clf)
validation_x_0 = validation_df.iloc[0, [5, 9]]
plt.scatter(validation_x_0[1], validation_x_0[0], marker="*", s=500, color="g")
plt.show()
```



增強決策邊界

- 我們可以如同在迴歸問題中增加 X 的次方項

In [31]:

```
from sklearn.preprocessing import PolynomialFeatures

train_df, validation_df = train_test_split(labeled_df, test_size=0.3, random_state=123)
X_train = train_df[["Fare", "Age"]].values
y_train = train_df["Survived"].values
X_validation = validation_df[["Fare", "Age"]].values
y_validation = validation_df["Survived"].values

d = 6
X_train_poly = PolynomialFeatures(d).fit_transform(X_train)
X_validation_poly = PolynomialFeatures(d).fit_transform(X_validation)
```

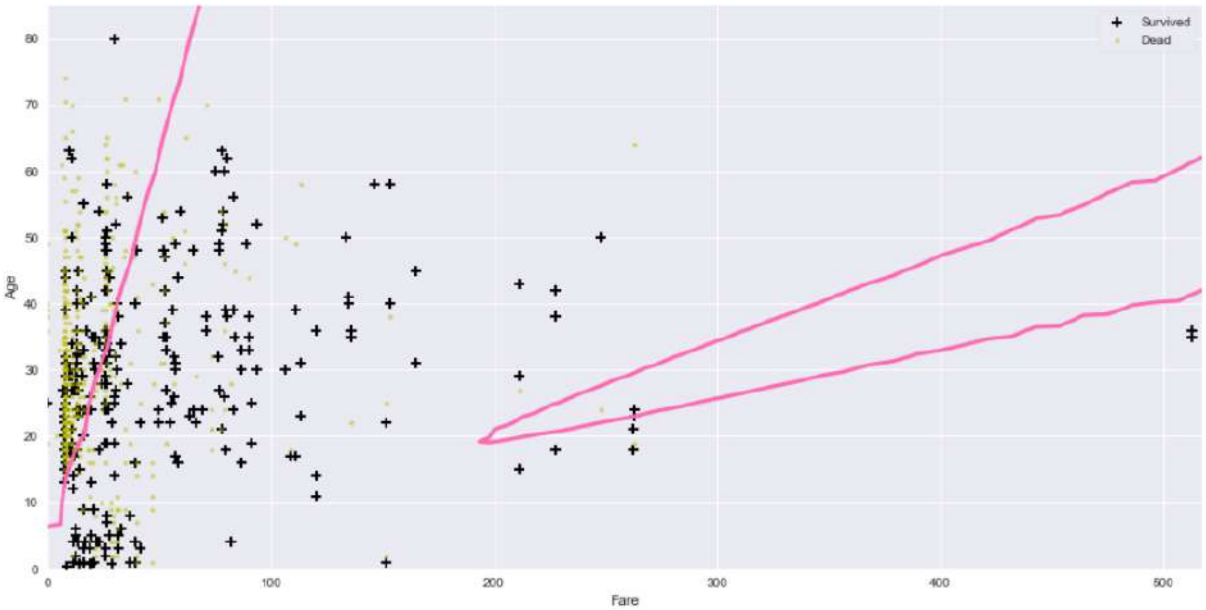
In [32]:
```
clf_poly = LogisticRegression()
clf_poly.fit(X_train_poly, y_train)
y_hat = clf_poly.predict(X_validation_poly)
accuracy = (y_hat == y_validation).sum() / y_hat.size
print("預測準確率為 {:.2f}%".format(accuracy * 100))
```

```
預測準確率為 69.30%
```

In [33]:
```
def decision_boundary(X, labeled_df, clf):
    x1_min, x1_max = X[:,0].min()-5, X[:,0].max()+5,
    x2_min, x2_max = X[:,1].min()-5, X[:,1].max()+5,
    xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max))
    thetas = np.hstack((clf.intercept_, clf.coef_.ravel())).reshape(-1, 1)
    xx_concat = np.concatenate([xx1.reshape(-1, 1), xx2.reshape(-1, 1)], axis=1)
    d = 6
    xx_concat_poly = PolynomialFeatures(d).fit_transform(xx_concat)
    ones = np.ones(xx1.size).reshape(-1, 1)
    xx_concat_poly = np.concatenate([ones, xx_concat_poly], axis=1)
    h = sigmoid(xx_concat_poly.dot(thetas))
    h = h.reshape(xx1.shape)
    plot_data(labeled_df, "Fare", "Age", "Survived", "Dead", "Survived")
    plt.contour(xx1, xx2, h, [0.5], linewidths=3, colors='#FF69B4')
    #cmap=ListedColormap(["y", "k"])
    #plt.contourf(xx1, xx2, h, alpha=0.2, cmap=cmap, antialiased=True)
    plt.xlim(0, x1_max)
    plt.ylim(0, x2_max)
```

In [34]:

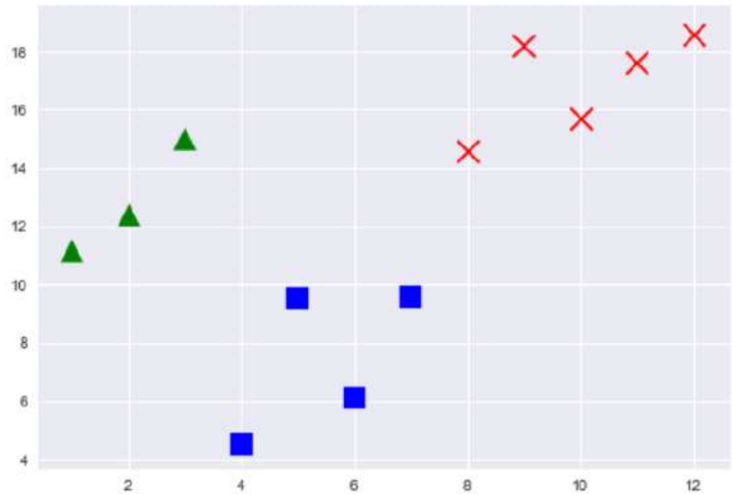
```
decision_boundary(labeled_df[["Fare", "Age"]].values, labeled_df, clf_poly)
plt.show()
```



延伸二元分類到多元分類問題：One-vs-all

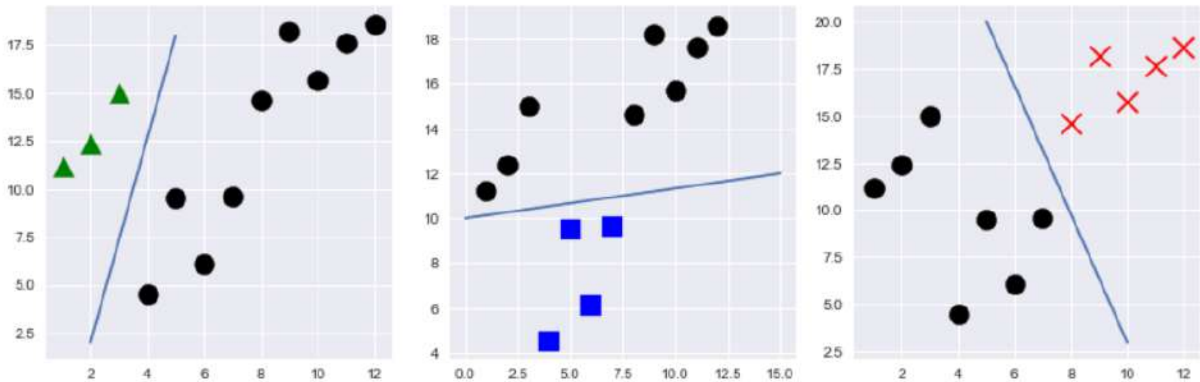
$$
\begin{gathered}
y \in \{0, 1, 2\} \\
h^0(x) = P(y = 0 \mid x; \theta) \\
h^1(x) = P(y = 1 \mid x; \theta) \\
h^2(x) = P(y = 2 \mid x; \theta) \\
\text{prediction: } max(h^0(x), h^1(x), h^2(x))
\end{gathered}
$$

In [36]:
```
plt.show()
```



In [38]:

```
plt.tight_layout()
plt.show()
```



手寫數字圖片辨識

In [39]:

```
import pandas as pd

labeled_df_url = "https://storage.googleapis.com/kaggle_datasets/Digit-Recognizer/train.csv"
test_df_url = "https://storage.googleapis.com/kaggle_datasets/Digit-Recognizer/test.csv"

labeled_df = pd.read_csv(labeled_df_url)
test_df = pd.read_csv(test_df_url)
```

In [40]:

```
print(labeled_df.shape)
print(test_df.shape)
```

```
(42000, 785)
(28000, 784)
```

In [41]:
```
labeled_df.head()
```

Out[41]:

	label	pixel0	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8
0	1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0
3	4	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0

5 rows × 785 columns

In [42]:
```
test_df.head()
```
Out[42]:

	pixel0	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixe
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0

5 rows × 784 columns

In [43]:

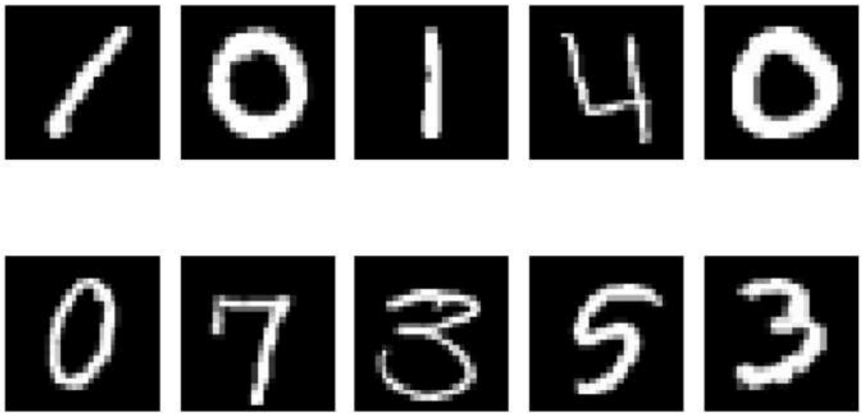
```
X_train_arr = labeled_df.iloc[:, 1:].values.astype(float)
y_train_arr = labeled_df.iloc[:, 0].values.astype(float)
X_test_arr = test_df.values.astype(float)
```

In [44]:
```
def plot_first_10(arr_set):
    for i, k in enumerate(range(1, 11)):
        plt.subplot(2, 5, k)
        plt.imshow(arr_set.reshape(arr_set.shape[0], 28, 28)[i], cmap = "gray")
        plt.xticks([]), plt.yticks([])

    plt.tight_layout()
    plt.show()
```

In [45]:

```
plot_first_10(X_train_arr)
```



In [46]:

```
plot_first_10(X_test_arr)
```

In [47]:

```
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import cross_val_score

clf = LogisticRegression(C=100000)
acc = np.mean(cross_val_score(clf, X_train_arr[:3000, :], y_train_arr[:3000], cv = 10, scoring = 'a
ccuracy')) # not to takes too long...
```

In [48]:

```
print("準確率：{:.2f}%".format(acc*100))
```

準確率：81.43%