e-Yantra Robotics Competition

eYRC-BB#2403

Team Leader Name	Heethesh Vhavle
College	B.M.S. College of Engineering
E-mail	heetheshvn@yahoo.com
Date	09-12-2016

Models and Simulation

****

Instructions:

- There are no negative marks
- **Unnecessary explanation** will lead to less marks even if answer is correct
- Use the same **font and font size** for writing your answer

****

Q1:

The human eye has a biological control system that varies the pupil diameter to maintain constant light intensity to the retina. As the light intensity increases, the optical nerve sends a signal to the brain, which commands internal eye muscles to decrease the pupil's eye diameter. When the light intensity decreases, the pupil diameter increases. We define this closed loop system as *light-pupil system*.

A. Draw a function block diagram of the light-pupil system indicating the input, output, intermediate signals, the sensor, the controller and the actuator.

B. It has been found that it takes the pupil about 300 milliseconds to react to a change in the incident light. If light shines on the eye, describe the response of the pupil: (i) without delay and (ii) with delay.

Answer:

A.

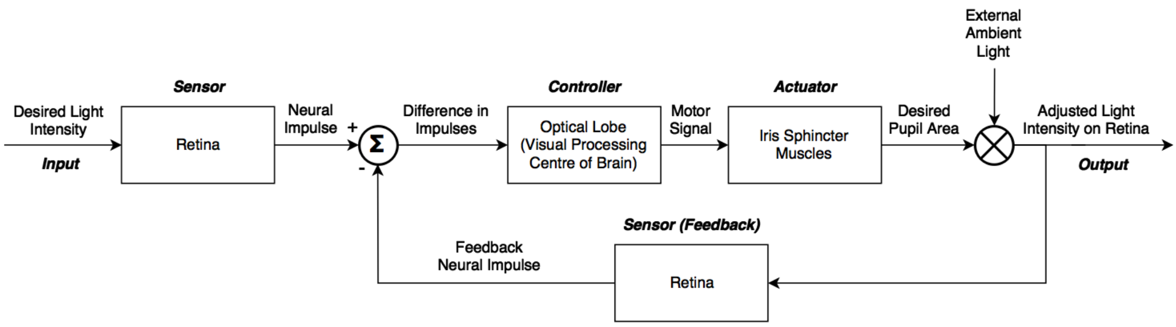


Figure 1.1: Functional block diagram representing the light-pupil system.

B.

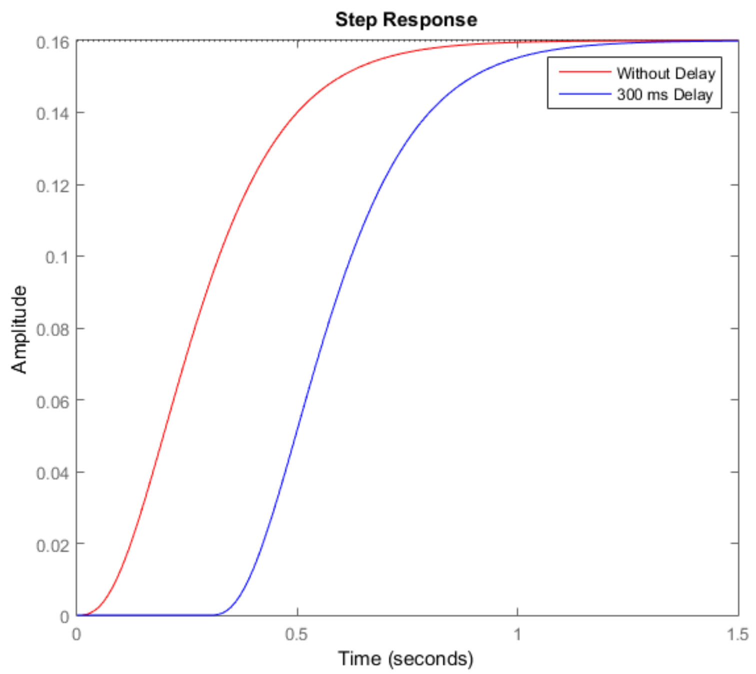
As per a model developed by Stark and Sherman (1957), on the *Servo – Analytic Study of Pupil Reflex Dynamics*, the suggested open loop transfer function for relative change in light intensity (Input) and relative compensation in pupil area (Output) is as follows:

$$G(s) = \frac{0.16}{(1+0.1s)^3} \quad (Without\ Delay)$$

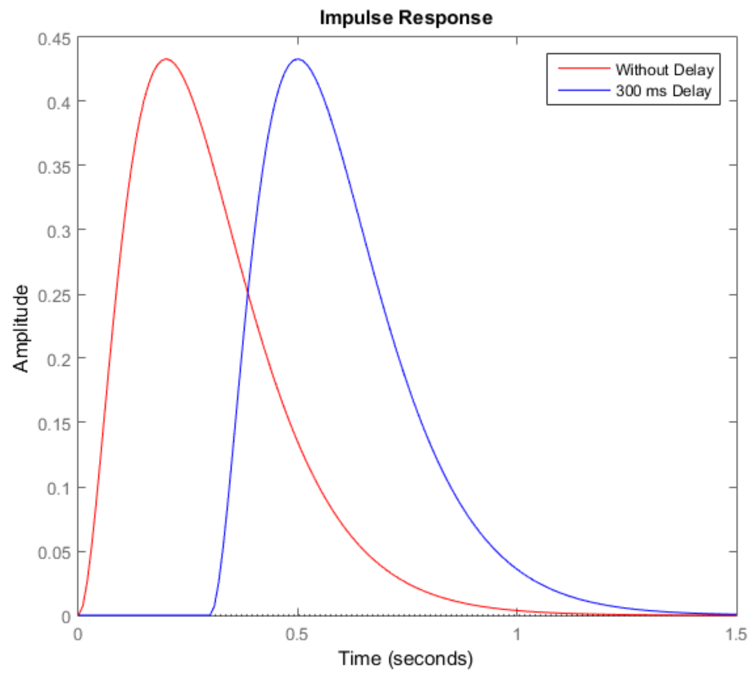
The response of the same light-pupil system with a delay of 300ms is expressed mathematically, where the exponential term represents the delay and is added to the forward path of the system.

$$G(s) = \frac{0.16e^{-0.300s}}{(1+0.1s)^3} \quad (With\ Delay)$$

For a step input (step change in light intensity), the step response for the above system (considering both with and without delay) is plotted below.



It is observed that with a delay, the step response remains unaltered in its slope except for a 300ms delay in the Settling Time (t_s) of the system, i.e., the pupil does not respond to the changed light intensity for the first 300ms. For an impulse input (burst of light), the response of the system is shown below.



Q2:

List any two applications where principle of Balance Bot can be used. Draw a functional block diagram of any one of the applications, indicating input, output, intermediate signals, sensors, controller and actuators.

Answer:

1. The principle of the Balance Bot, which is the Inverted Pendulum, is used in self-balancing two-wheeled human transporter (**Segway PT**). It uses accelerometer and gyroscopic sensors to detect the pitch angle and balances using feedback. The block diagram is shown in *Figure 2.3*.

Figure 2.1: Segway PT™

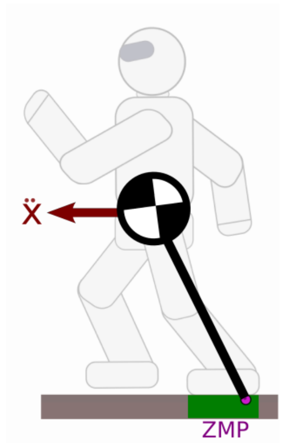


Figure 2.2: Linear Inverted Pendulum Model for Humanoid Robots.

2. The second application is for developing the **Gait Pattern for controlling Humanoid Robots**. The control and stabilization of humanoid robots is developed using the Linear Inverted Pendulum Model. Humanoid robots are currently under R&D and can be used in Space Exploration.

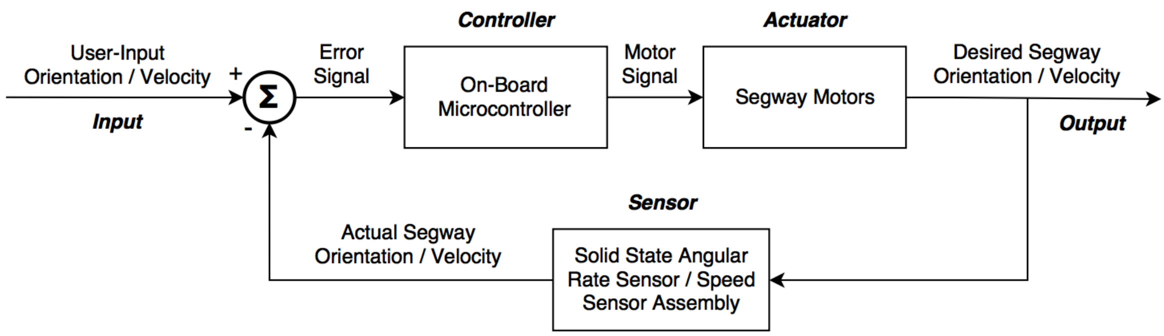


Figure 2.3: Block diagram representing the working of Segway PT.

Q3:

Consider the problem of controlling an inverted pendulum on a moving base. The transfer function of the system is:

$$G(s) = \frac{-1/(M_b L)}{s^2 - (M_b + M_s)g/(M_b L)}$$

Where,

M_b **= Mass of the base,**

M_s **= Mass at the center of gravity of inverted pendulum**

L **= height of the pendulum**

g **= acceleration due to gravity**

The design objective is to balance the pendulum in the presence of disturbance inputs.

Let M_s **= 2 Kg,** M_b **= 3 Kg,** L **= 0.3 m,** g **= 9.81 m/s^2.**

The design specifications (based on a unit step disturbance) are as follows:

a. Settling time (with a 2% criterion) < 10 seconds
b. Percent overshoot < 40%
c. Steady-state tracking error < 0.1° in the presence of the disturbance.

Analyze the system response to unit step disturbance. Design the controller to meet the specifications.

Answer:

<u>Without Controller</u>

Substituting the given values in the transfer function, we get,

$$G(s) = \frac{-1.111}{s^2 - 54.5}$$

Creating a simple block diagram in XCOS with the above transfer function, we get the following output for a unit step input. The system response increases exponentially and never settles. Thus, the system is unstable. *[XCOS file is included in the folder along with the other documents.]*

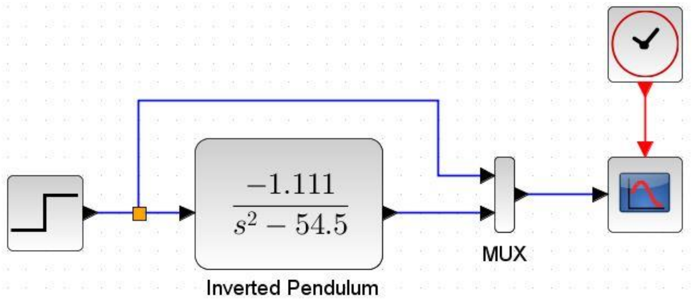


Figure 3.1: Open Loop Block Diagram.

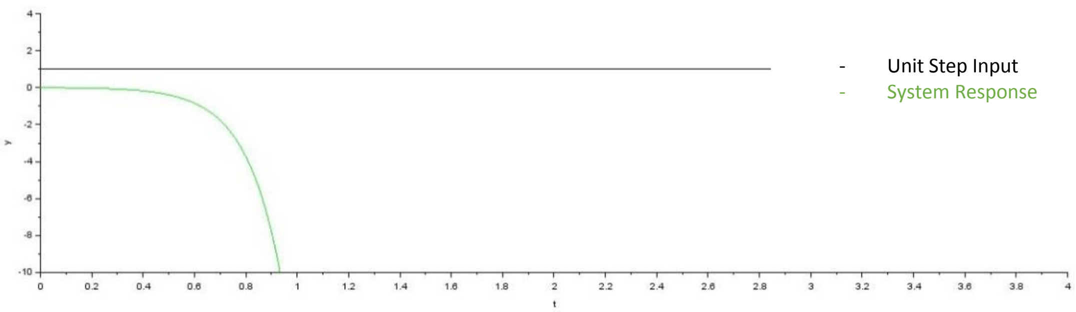


Figure 3.2: Unit Step System Response without a Controller or Feedback.

With Controller

In order to stabilize the system, some sort of feedback is required with the help of a controller to meet the desired time response specification. We will be using a **PID Controller** and a unit negative feedback to stabilize this system. The controller will be a reverse acting controller.

PID Controller

The PID Controller helps in minimizing the error between the output and the input. Proportional component helps in minimizing current error, Integral sums the error over time and Derivative component anticipates the future behavior of the error. It can be expressed mathematically as,

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$$

Where Kp, Ki and Kd are the PID gains and e(t) is the error value. To design our controller, we need to first find the PID gains or tune our system. The following table summarizes how each of these gains of PID, when increased, affect the closed loop step response of the system.

Gain (Increase)	Rise Time	Overshoot	Settling Time	Steady State Error
Kp	Decreases	Increases	Small Change	Decreases
Ki	Decreases	Increases	Increases	Eliminates
Kd	Small Change	Decreases	Decreases	No Change

Creating a new block diagram in XCOS with the PID controller and unity negative feedback, we get the following output for a unit step input. A unit negative gain block is added to forward path, since we are designing a reverse acting controller. The disturbance input is currently switched off.

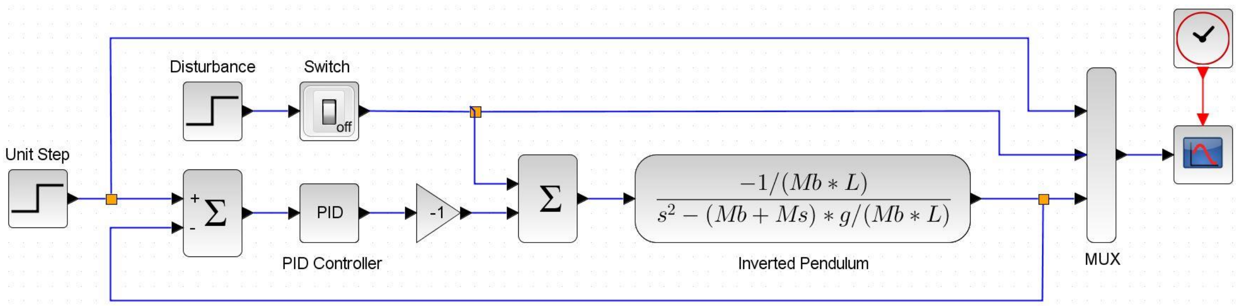


Figure 3.3: Block Diagram with PID Controller (Pendulum parameters are as specified in the question).

On increasing Kp to 500, the rise time decreased but the response was oscillatory. Ki was increased to 400 to reduce the steady state error. Furthermore, to reduce the overshoot and settling time, Kd was increased to 50. An even better response was achieved by varying the Kp and Ki values.

The gains for the PID controller that were finally chosen are **Kp = 700**, **Ki = 600** and **Kd = 50**. The response clearly met the specifications with **Overshoot < 12.25%, Settling Time (2%) < 2.2 seconds** and **Steady State Error < 0.1^{o}**. When a disturbance is applied as shown in *Figure 3.5*, the system recovers well with the help of PID controller.

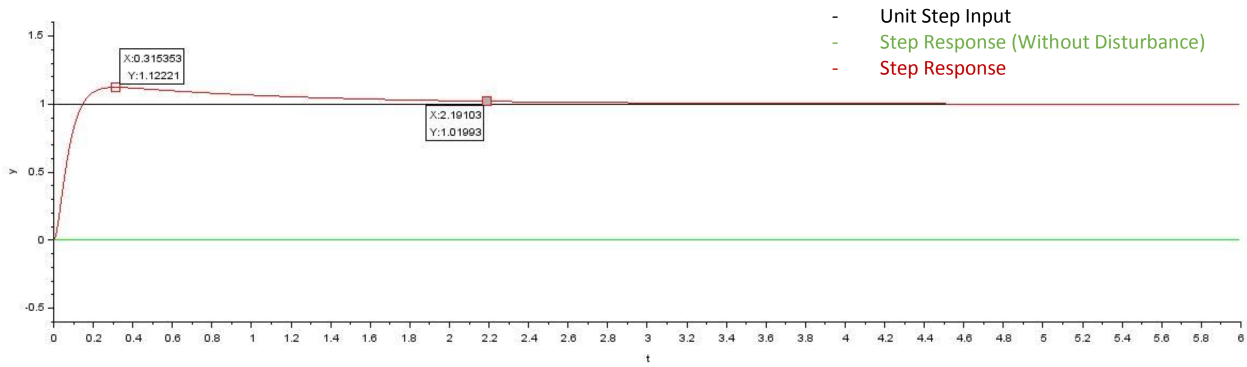


Figure 3.4: Step Response without Disturbance.

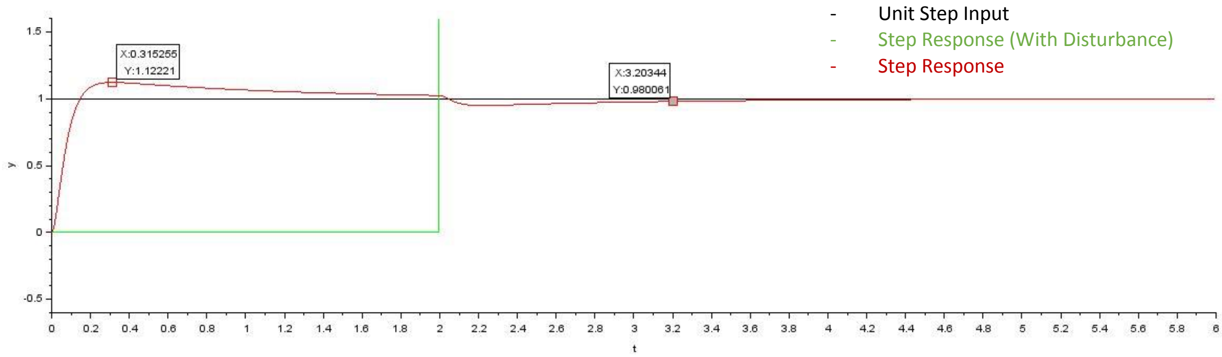


Figure 3.5: Step Response with Disturbance (Unit step disturbance $= 50u(t-2)$).

Parameter Variation

For the PID controller designed above, on increasing the mass of the base (Mb), the overshoot also increased. A similar observation was made when the mass at COG (Ms) was increased with an increase in the overshoot, but the settling time and rise time also increased slightly. However, on increasing the height of the pendulum, the system tends to oscillate slightly followed by an increase in the settling time, making it more unstable.

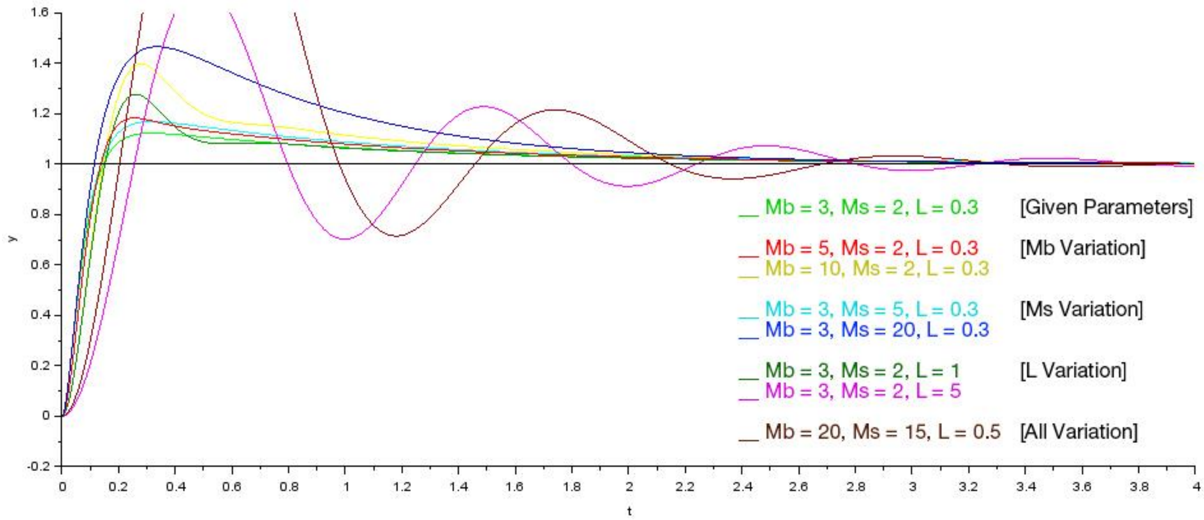


Figure 3.6: Step Response with Parameter Variation of Mb, Ms and L.

Q4:

Ball and Stick Experiment

Consider balancing a meter stick upright on the tip of your finger. After balancing meter stick, attach a ball at one end and again try to balance the stick. Is it easier to balance with the ball or without it?

Answer:

__Experimental Result:__ It was easier to balance the stick with the ball present.

__Theoretical Analysis:__

Consider a meter stick of length **L** and mass **M$_{stick}$**. The stick can be considered as a uniform rod with the moment of inertia about its centre, and the position of CG as,

$$I_1 \propto M_{stick} R_1^2 \qquad\qquad CG_{stick} = \frac{L}{2}\ (= R_1)$$

Now consider the same rod but with a ball on one end with a radius **r** and mass **M$_{ball}$**. The moment of inertia of this new object, at its centre, can be expressed using the parallel axis theorem as,

$$I_2 \propto (M_{stick} + M_{ball}) R_2^2 \qquad\qquad CG_{stick+\ ball} = \frac{M_{stick}L + M_{ball}(L+r)}{M_{stick} + M_{ball}}\ (= R_2)$$

When the body is tipping in either direction, it does with an angular acceleration **α** and the torque of the body can be expressed as, $\tau = I\alpha$. From the above equations, it is evident that $I_2 > I_1$. As the CG increases, the mass moment of inertia also increases and thus, the angular acceleration decreases. For the same external torque acting on the bodies,

$$\tau = I_1\alpha_1 = I_2\alpha_2$$

$$\alpha_2 = \frac{I_1}{I_2}\alpha_1 \quad \Rightarrow \quad \alpha_2 < \alpha_1 \ \ (\because I_2 > I_1\ )$$

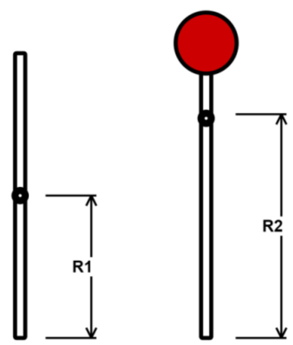


Figure 4.1: Diagram showing the position of centre of gravity with and without the ball.

Therefore, the stick with the ball present tips at a slower rate compared to the stick without the ball, giving us more time to balance it back to its upright position. Consequently, the stick with the ball is said to be more stable and is easier to balance.