# Math 523H–Homework 4

1. Determine which of the following series converge, and which ones converges absolutely or not. Justify your answer by stating the appropriate criterion (root test, ratio test, alternating series tests, comparison test.....)

   (a) $\sum \frac{n^2 3^n}{100^n - n}$.

   (b) $\sum \frac{2^{3n+2}}{n!}$.

   (c) $\sum \frac{\cos(\frac{n\pi}{2})}{\sqrt{n}}$.

   (d) $\sum (-1)^n \frac{n+1}{n^3+1}$

2. In class we prove the convergence of $\sum_n \frac{1}{n^2}$ by comparing to the alternating series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ .... Proceeding in the same way show that the series $\sum_n \frac{1}{n^{3/2}}$ converges by comparing it to the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ ...

   *Hint: Work on the expression* $\frac{1}{\sqrt{2n-1}} - \frac{1}{\sqrt{2n}}$.

3. In general it is hard to compute the actual values of an infinite series. Here are some examples where you can compute it.

   (a) Prove that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$.

   (b) Prove that $\sum_{n=1}^{\infty} \frac{n-1}{2^{n+1}} = \frac{1}{2}$. *Hint : Show that* $\frac{k-1}{2^{k+1}} = \frac{k}{2^k} - \frac{k+1}{2^{k+1}}$.

   (c) Use part (b) to compute $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

4. On a different piece of paper than problems 1–3, write a $2-3$-pages summary about all what you have learned about sequences and infinite series. Make a list of the important results and concepts? Make a list of all the convergence criteria we have obtained to study convergence of series and for sequences?