# Syllabus Math 605, Fall 2024

**Probability theory**

Luc Rey-Bellet

### Instructor

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### Office hours:

- Tuesday 3:00 PM-4:00 PM in LGRT 1423 K or on ZOOM
- Friday 1:00 PM-2:00 in LGRT 1423 K or on ZOOM
- By appointment is always possible, and/or ask your questions be email.

### Class Meeting

Tu-Th, 11:30AM–12:45AM in LGRT 204

### Class homepage

On Canvas at https://umamherst.instructure.com/courses/22961

### Class slides

I will post class slides here.

In the meantime you can look at my slides from last year

Math 605 Fall 23 Probability slides

The slides from this year will be an updated and reorganized version from those. I will also break them into smaller bits so they run better.

## Syllabus

This is the first part of a (newly renamed) 2-semester graduate sequence **Math605-Math606** which leads to the Stochastics qualifying exam. Prerequisites are

1. a solid working knowledge of undergaduate probablilty (at least STAT 515 or STAT 607 or equivalent).
2. a good working knowedge of analysis and proofs (at least Math 523 or equivalent)
3. some mathematical maturity

### Learning Objectives

One can do many interesting applications with undergaduate probability (i.e. probability without measure theory) but at some point one needs to up your game. The goal of the class is to bring your probability knowledge to the next level so that you can tackle more sophisticated problems and are able to take classes (or read by yourself) on, for exmaple, stochastic processes, stochastic differential equations, statistical learning theory (an important component of machine learning), large deviation theory, optimal transport, information theory, ergodic theory and so on....

In Math 605 we will cover some of the foundations of probability.

1. What is a probability space and what are random variables? How do compute their distribution and how do we simulate them?
2. Conditional probability and independence.
3. Some background on measure and integration, and the $L^p$ spaces.
4. The mode of convergences of random variable, almost sure, in probabilty, in distribution.
5. Characteristic and Moment generating functions.
6. The limit theorems: Law of Large numbers and Centra Limit Theorem and Concentration inequalities.
7. Conditional expectation, Martingale and Martingale convergence theorems.

In Math 606 we will cover various aspects of stochastic processes. One of the main goal in the class is to develop a "probabilist intuition and way of thinking". We will present some proofs and we will skip some others in order to provide a reasonably broad range of topics, concepts and techniques. We emphasize examples both in discrete and continuous time from a wide range of disciplines and with an eye on numerical implementation. Among the topics treated in the class are

1. Discrete time Markov chains on discrete spaces. Definition and basic properties, classification of states (positive recurrence, recurrence and transience), stationary distribution and limit theorems, analysis of transient behavior, optimal stopping, Monte-Carlo Markov chains.

2. Continuous-Time Markov chains. Poisson Process, Birth and Death Process, and Queueing models. Renewal processes and semi-Markov processes.
3. Martingales and applications
4. Brownian motion and applications.

## Grade/assignment

Weekly homework, one midterm and one final exam, each valued 1/3 of your grade.

The homework can be found at the end of each section in the slides. It will be submitted via gradescope https://www.gradescope.com/courses/829012

## Textbooks

The following text book

*Probability Theory and Stochastic Processes* by Pierre Bremaud. Universitext. Springer 2020. ISBN: 978-3030401825

covers all the material in 605-606 (and more). My class is designed in a similar spirit and is at the same "intermediate" level. I think it is a really good book to learn the basic ides and concepts, and you can move on to more comprehensive textbooks next (let's say the books by Durrett or Cinlar).

There are many other excellent textbooks on probability theory, which are often more comprehensive than our textbook and cover more material and in more depth. **I strongly encourage you to pick a book and read it in parallel to the class. This is a really important skill to learn for your future career.** Ask me if you want some advice which one to pick.

1. *Probability: Theory and Examples* by Rick Durrett, 4th edition. Cambridge University Press ISBN-13: 978-0521765398

2. *Probability and Stochastics* by Erhan Cinlar. Graduate Texts in Mathematics 261. Springer 2011

3. *Probability and Measure* by Patrick Billingsley. Wiley.

4. *Probability Essentials* by Jean Jacod and Philip Protter, 2n edition. Universitext. Springer 2004. ISBN: 978-3-540-43871-7

5. *A first look at rigorous probability* by Jeffrey Rosenthal, 2n edition. World Scientific 2006 ISBN-10: 9812703713 ISBN-13: 978-9812703712

6. *A Probability Path* by Sidney Resnick. Birkhauser 2014

7. *Probability: Theory and Examples* by Rick Durrett, 4th edition. Cambridge University Press ISBN-10: 0521765390 ISBN-13: 978-0521765398

8. *Real Analysis and Probability* by R.M Dudley, 2nd edition. Cambridge University Press 2004

9. *Probability* by A. Shiryaev, 2nd edition. Springer 1995


Also for references here are the books recommended for Math 606


1. *Markov Chains Gibbs Fields, Monte Carlo Simulation, and Queues*, 2nd edition (2007) by Pierre Bremaud, Springer.

2. *Introduction to Stochastic Processes*, 2nd edition (2007) by Gregory F. Lawler, Chapman&Hall.

3. *Adventures in Stochastic processes*, by Sidney I. Resnick, Birkhauser.

4. *Essentials of Stochastic Processes*, by Rick Durrett, Springer.

5. *Introduction to Probability Models*, by Sheldon M. Ross, Academic Press

6. *Lectures on Monte-Carlo Methods*, by Neal N. Madras, American Mathematical Society

7. *Simulation and the Monte Carlo Method*, by Reuven Y. Rubinstein and Dirk P. Kroese , Wiley

8. *Markov chain and Mixing Times*, by David A. Levin, Yuval Peres, and Elizabeth L. Wilmer, American Mathematical Society

9. *Stochastic Processes*, by Sheldon M. Ross, Wiley.

10. *A first course in Stochastic Processes*, by Samuel Karlin and Howard M. Taylor, Academic Press.