MATH. 421 – EXAM #1

10/18/99 **NAME:**__________________________

1) (35 points) Let $z = \dfrac{6}{1 - i\sqrt{3}}$. Compute (If you prefer, you may leave your answer in polar form):

(a) $|z^3|$

(b) $\mathrm{Arg}(z^4)$

(c) $\mathrm{Log} z$

(d) All values of z^i

(e) $z^{2/3}$

(f) Sketch the values obtained in Part **(e)**

2) (5 points) Find the image of the vertical line $x = 2$ under the function $f(z) = e^{-z}$.

3) (10 points) Determine whether the following statements are true or false. Justify your answers.

(a) $\sin(iz) = i \sinh(z)$ for **all** complex numbers $z \in \mathbb{C}$.

(b) $|\cos z| \leq 1$ for **all** complex numbers $z \in \mathbb{C}$.

4) (10 points) Compute $\cos\left(\frac{\pi}{4} - \frac{i}{2}\ln 2\right)$. (Show all your work and simplify your answer as much as possible.)

5) (10 points) Let $z = x + iy$, compute $\mathrm{Re}(e^{-z^2})$ as a function of x and y.

6) (a) (5 points) Write down the Cauchy-Riemann equations in polar form.

(b) (5 points) Prove that the function $f(z) = \sqrt[3]{z} = \sqrt[3]{|z|}\, e^{i\mathrm{Arg}(z)/3}$ is analytic in the domain $D = \mathbb{C} - \{z : \mathrm{Re}(z) \leq 0\,;\ \mathrm{Im}(z) = 0\}$.

(c) (5 points) Compute the derivative $f'(z)$.

7) (a) (5 points) Prove that the function

$$u(x, y) = 2y - 3x^2y + y^3$$

is harmonic on all of $\mathbb{R}^2$.

(b) (10 points) Find an entire function $f(z)$ such that $\mathrm{Re}(f) = u$. Your answer must be expressed as a function of $z = x + iy$, not x and y.