Homework 7

Exercise 1: The snowdrift game: Two drivers are caught in a snowstorm and a big snowdrift blocks the road. To go home they have to clear the path. The fairest solution is for them to clear the path together. If one simply refuses to do it, the other driver may just do it because he wants to go home. But if both drivers have the same idea then nobody goes home. A variant of this game with numerical payoff is the following: I will give to Robert and Chelsea each a gift worth $40 if I receive $30 in cash. Their options are to either to pay the fee or not pay the fee knowing that if both of them decide to pay then then they will share the fee and pay $15 each. Write down a payoff table for the game and find the Nash equilibria.

Exercise 2: The ultimatum game: Consider the following experiment where $100 is handed to Robert and he is given the task to split the amount of money between Robert and Chelsea any way he wants. Then Chelsea has the option to accept the deal and take the money offered, or to refuse in which case both go empty-handed. In most experiments Robert will propose a more or less fair deal, say 55-45 and Chelsea will accept. If Robert proposes a bad deal, like say 85-15 then often this deal will be rejected even though Chelsea would be better off accepting it and taking any money rather than having nothing.

   Let us construct a simple game which captures the essence of this relation. Robert has two option, offer a fair split, say 60-40, or offer a unequal split, 85-15. Chelsea has also two options, accepting any offer or accepting only the fair offer. Write down the payoff matrix for the game and compute the Nash equilbria.

Exercise 3:

1. Solve the following game by iteratively eliminating dominated strategies

		Player B		
		R	S	P
Player A	R	25 / 73	42 / 57	32 / 66
	S	26 / 80	12 / 35	54 / 32
	P	27 / 28	31 / 63	29 / 54

2. Find all the Nash equilibria of the game with payoff

<center>Player B</center>

		R	S	P
Player A	R	1 5	2 3	2 5
	S	7 4	11 0	12 2
	P	3 1	4 6	4 5

Exercise 4: Global warming or the "tragedy of the commons". Ten countries are considering fighting global warming. Country i must decide to spend an amount x_1 with $0 \le x_i \le 1$ to reduce its carbon emissions. The total benefits produced by theses expenditures is $6(x_1 + \cdots x_{10})$ and each country receives $1/10$ of the benefits. This is a game with ten players $1, 2, 3, \cdots 10$.

1. Write down the payoff for country i

$$P_i(x_1, \cdots, x_{10}) = \text{Benefits} - \text{Expenditures} = ?$$

2. Solve the game by showing that for each country $x_i = 0$ (i.e. spend nothing) is a dominating strategy.

Exercise 5: Three companies use water from a lake. When a company returns the water to the lake, it can either purify it or fail to purify it (and thereby pollute the lake). The cost of purifying the used water before returning it to the lake is 1. If two or more firms fail to purify the water before returning it to the lake, all three firms incur a cost of 3 to treat the water before they can use it. The payoffs are therefore as follows:

- If all three firms purify: -1 to each firm.

- If two firms purify and one pollutes: -1 to each firm that purifies, 0 to the polluter.

- If one firm purifies and two pollute: -4 to the firm that purifies, -3 to each polluter.

- If all three firms pollute: -3 to each firm.

By inspecting the four different cases above find the Nash equilibria. (Decide if a firm can improve his payoff by switching strategy).

Exercise 6: The facility location game. Our example is a game in which two firms compete through their choice of locations. Suppose that two firms A and B are each

Figure 1: The facility location game

planning to open a store in one of six towns located along six consecutive exits on a highway. We can represent the arrangement of these towns using a six-node graph as in Figure 1. Firm 1 has the option of opening its store in any of towns 1, 3, or 5, while Firm 2 has the option of opening its store in any of towns 2, 4, or 6. These decisions will be executed simultaneously. Once the two stores are opened, customers from the towns will go to the store that is closer to them. So for example, if Firm A open its store in town 3 and Firm B opens its store in town 2, then the store in town 2 will attract customers from 1 and 2, while the store in town 3 will attract customers from 3, 4, 5, and 6. If we assume that the towns contain an equal number of customers, and that payoffs are directly proportional to the number of customers, this would result in a payoff of 4 for Firm A and 2 for Firm B, since Firm A claims customers from 4 towns while Firm 2 claims customers from the remaining 2 towns.

1. Write down the payoff table for this game.

2. Solve the game by eliminating dominated strategies.

3. Find the Nash equilibria for the game.