# Curriculum Vitae: Luc Rey-Bellet

#### **Contact Information**

arXiv:2407.11901

## **Educational History:**

- Ph.D. in Mathematics (1998): Department of Mathematics, University of Geneva, Geneva, Switzerland. Advisor: *Prof. Jean-Pierre Eckmann*
- Dipl. Phys. ETHZ (1994): , Department of Physics, Swiss Federal Institute of Technology (ETHZ), Zurich, Switzerland. Advisor: *Prof. Jürg Fröhlich*

## **Professional appointments:**

- Research Fellow: Rutgers University 1998-1999
- Whyburn Instructor: University of Virginia 1999-2002
- Assistant Professor: University of Massachusetts 2002-2008
- Associate Professor: University of Massachusetts 2008-2013
- Visiting Professor: University of Crete 2011 (Spring semester)
- Professor: University of Massachusetts since 2013

#### Selected recent service

- Editorial board of Journal of Statistical Physics 2013–2018
- Organizer of the Oberwolfach seminar on Markov processes, 2011
- Organizer of the Conference in honor of J.P. Eckmann 75th birthday, 2022
- Organizer of ICERM Workshop on Optimal Transport and Machine Learning, 2023

# **Grant funding**

- FNRS Bourse de jeune chercheur 1999 (SFr 45k)
- NSF Grant 2003 (3 year, PI, \$101K)
- NSF Grant 2006 (3 year, PI, \$106K)
- NSF Grant 2011 (3 Year, PI, \$99K)
- NSF Grant 2015 (3 year, PI (with 1 Co-PI) \$280K)
- DOE Grant 2016 (3 year, co-PI (with 4 PI) \$900K)
- AFOSR Grant 2019 (3 year, co-PI (with 2 PIs) \$900 K)
- NSF Grant 2020 (3 year, PI (with 1 Co-PI) \$ 370K)
- AFOSR Grant 2022 (4 year (with 2 PIs) \$1.950M)
- NSF Grant 2023 (3 year, PI (with 1 Co-PI) \$300K)

## Graduate students and postdoctoral advising:

- Dimitrios Tsagkarogiannis (Graduation 2006, University of L'Aquila)
- Michael Diehl (Graduation 2008, Endicott College)
- Sasanka Are (Graduation 2009, Cerner Corporation)
- Sung-Ha Hwang (Graduation 2011, KAIST, Korea)
- Yannis Pantazis (Postdoctoral fellow, 2012-2013, and 2014-2015, Forth Crete)
- Kostis Gourgoulias (Graduation 2017, JP Morgan)
- Georgios Arampatzis (Postdoctoral fellow, 2014-2015, University of Crete)
- Jinchao Feng (Graduation 2019, John Hopkins University )
- Jie Wang (Graduation 2019, Discover)
- Jeremiah Birrell (Postdoctoral fellow, 2017-2022, Texas State University)
- Pangiota Birmpa (Postdoctoral fellow, 2018-2022, Heriott-Watt University)
- Hyemin Gu (Graduate Student since 2021)
- Benjamin Zhang (Postdoctoral fellow, 2022-2024, Brown University)
- Ziyu Chen (Postdoctoral fellow, 2022-2025)

#### **Publications**

Hyemin Gu, Markos A. Katsoulakis, Luc Rey-Bellet, and Benjamin J. Zhang. Combining Wasserstein-1 and Wasserstein-2 proximals: robust manifold learning via well-posed generative flows. Submitted.
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Jeremiah Birrell, Markos A. Katsoulakis, Luc Rey-Bellet, Benjamin Zhang, and Wei Zhu. Nonlinear denoising score matching for enhanced learning of structured distributions.

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Ziyu Chen, Hyemin Gu, Markos A. Katsoulakis, Luc Rey-Bellet, and Wei Zhu.

Learning heavy-tailed distributions with Wasserstein-proximal-regularized - divergences. Submitted.

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Ziyu Chen, Markos A. Katsoulakis, Luc Rey-Bellet, and Wei Zhu.

Statistical Guarantees of Group-Invariant GANs. Submitted.

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Ziyu Chen, Markos A. Katsoulakis, Luc Rey-Bellet, and Wei Zhu.

Sample Complexity of Probability Divergences under Group Symmetry.

Proceedings of 40th International Conference on Machine Learning, PMLR 202 (2023) pp. 4713-4734.

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Hyemin Gu, Panagiota Birmpa, Yannis Pantazis, Luc Rey-Bellet, and Markos A. Katsoulakis. Lipschitz-regularized gradient flows and generative particle algorithms for high-dimensional scarce data.

To appear in Siam Journal on Mathematics of Data Science (2024). arXiv:2210.17230

Jeremiah Birrell, Paul Dupuis, Markos Katsoulakis, Yannis Pantazis, and Luc Rey-Bellet. Function-space regularized Rényi divergences.

Proceedings of The Eleventh International Conference on Learning Representations (ICLR 2023).

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Structure-preserving GANs.

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 ${\bf Concentration\ inequalities\ and\ performance\ guarantees\ for\ hypocoercive\ MCMC\ samplers.}$ 

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Paul Dupuis, Markos Katoulakis, Yannis Pantazis, and Luc Rey-Bellet.

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inequalities.

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Luc Rey-Bellet.

Open Classical Systems.

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Luc Rey-Bellet.

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Luc Rey-Bellet.

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Marco Lenci and Luc Rev-Bellet.

Large deviations in quantum lattice systems: one phase region.

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Luc Rey-Bellet.

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Lawrence E. Thomas and Luc Rey-Bellet.

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Jürg Fröhlich, Luc Rey-Bellet, and Daniel Ueltschi.

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Communications in Mathematical Physics 224 (2001), no. 1, pp. 33–63.

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Paul Dupuis, Markos Katoulakis, Yannis Pantazis, and Luc Rey-Bellet, Sensitivity analysis for rare events based on Rényi divergence, The Annals of Applied Probability, 30 (2020), no.4, pp. 1507-1533.

Jeremiah Birrell and Luc Rey-Bellet Concentration inequalities and performance guarantees for hypocoercive MCMC samplers, Preprint.

Jeremiah Birrell, Markos Katsoulakis, and Luc Rey-Bellet, Quantification of model uncertainty on path-space via goal-oriented relative entropy, ESAIM: Mathematical Modeling and Numerical Analysis 55 (2021), no.1, pp. 131-169.

Sung-Ha Hwang and Luc Rey-Bellet, Positive feedback in coordination games: stochastic evolutionary dynamics and the logit choice rule, Games and Economic Behavior 126 (2021), pp. 355-373.

Jeremiah Birrell, Paul Dupuis, Markos A. Katsoulakis, Luc Rey-Bellet, and Jie Wang, Distributional robustness and uncertainty quantification for rare events, Preprint.

Jeremiah Birrell, Paul Dupuis, Markos A. Katsoulakis, Luc Rey-Bellet, and Jie Wang, A variational formula for Rényi divergences, SIAM Journal on Mathematics of Data Science 3 (2021), no. 4, pp. 1093-1116

Jeremiah Birrell, Paul Dupuis, Markos Katsoulakis, Yannis Pantazis, and Luc Rey-Bellet,  $(f,\Gamma)$ -Divergences: Interpolating between f-Divergences and Integral Probability Metrics, Journal of Machine Learning Research 23 (2022), paper 39, pp.1—70.

Panagiota Birmpa, Jinchao Feng, Markos Katsoulakis and Luc Rey-Bellet, Model Uncertainty and Correctability for Directed Graphical Models, SIAM/ASA Journal on Uncertainty Quantification 10 (2022), no.4, pp 1461–1512.

Jeremiah Birrell, Markos Katsoulakis, Luc Rey-Bellet, Wei Zu Structure-preserving GANs, Proceedings of the 39th International Conference on Machine Learning, PMLR 162 (2022) pp.1982-2020.

Jeremiah Birrell, Paul Dupuis, Markos Katsoulakis, Yannis Pantazis, Luc Rey-Bellet Function-space regularized Rényi divergences. Proceedings of The Eleventh International Conference on Learning Representations (ICLR 2023).

Hyemin Gu, Panagiota Birmpa, Yannis Pantazis, Luc Rey-Bellet, and Markos A. Katsoulakis, Lipschitz-regularized gradient flows and generative particle algorithms for high-dimensional scarce data, Submitted.

Ziyu Chen, Markos A. Katsoulakis, Luc Rey-Bellet, and Wei Zhu, Sample Complexity of Probability Divergences under Group Symmetry,
Submitted

Ziyu Chen, Markos A. Katsoulakis, Luc Rey-Bellet, and Wei Zhu, *Statistical Guarantees of Group-Invariant GANs*, Submitted.

#### Invited talks

- Journées semi-classiques 1998, Grenoble, February 1998
- Statphys 20, Paris, July 1998
- 80th Statistical mechanics conference, Rutgers University, December 1998
- Non equilibrium statistical mechanics, Schrodinger Institute, February 1999
- The 2002 UAB International conference mathematical physics, March 2002
- Workshop on Fokker-Planck equations, Rennes, France, February 2003
- ODYN-II, Probability and statistical mechanics, CIRM, Marseilles, March 2003
- 89th Statistical mechanics conference, Rutgers university, May 2003
- Summer school on open quantum systems, Grenoble, July 2003
- International congress of mathematical physics, ICMP 2003, Lisbon, August 2003
- Young researcher symposium, Lisbon, July 2003
- Workshop on dynamics and statistical mechanics, CRM, Montreal, August 2004
- SIAM conference on applications of dynamical systems, Snowbird, May 2005
- International congress on the applications of mathematics, Santiago, March 2006
- Workshop on statistical mechanics, CPT-CNRS, Marseilles, July 2006
- 96th Statistical mechanics Conference, Rutgers University, December 2006
- Workshop on nonequilibrium statistical mechanics, Vienna, June 2008
- Workshop on stochastic dynamical systems, Bielefeld, November 2008
- Arizona school of analysis with applications I, University of Arizona, March 2009
- NORDITA Stockholm, Multiscale modeling and simulation in science, November 2009
- SAMSI, Theory and qualitative behavior of stochastic dynamics, February 2010
- Indam meeting, Corinaldo, Hyperbolic dynamical systems in the sciences, June 2010
- SIAM conference on nonlinear waves and coherent stuctures, August 2010
- Oberwolfach seminar on ergodic theory of Markov processes, October 2010
- 105th Statistical mechanics Conference, Rutgers University, May 2011
- FRG workshop on quantum spin systems, Harvard University, May 2011
- Workshop on Coarse-graining of many-body systems. Heraklion, June 2011
- ENUMATH conference. September 2011
- Three hours course on statistical mechanics), Ecole des Ponts, September 2011
- SIAM conference on material sciences, Philadelphia, June 2013
- SPA conference (Stochastic processes and applications), Boulder, August 2013
- AMS sectional meeting, Session on dynamical systems. Saint Louis, October 2013
- Randomness and Long-Time Dynamics, Radcliffe Institute, November 2014
- CIRM Workshop on Averaging and Homogenization, Marseille, May 2015
- CRiSM Workshop on Non-Reversible Markov Chains, Warwick, September 2015
- Meeting of the Canadian Mathematical Society, Montreal, December 2015
- 114th Statistical Mechanics Conference, Rutgers, December 2015
- Computational Statistics and Molecular Simulation (COSMOS), Paris, February 2016
- Numerical Aspects of Nonequilibrium Dynamics, Paris, April 2017
- BIRS-CMO Workshop, Oaxaca, November 2018

- Worskshop on Entropic Fluctuation Relations, Montreal, October 2018
- Ki-Net: Dimension reduction in physical and data sciences. Duke. April 2019
- Bernoulli Center, EPFL, May 2019
- Quantissima III, Venice, August 2019.
- Non Reversible Markovian Monte-Carlo Methods, Lorentz Center, August 2021
- Meetings on Foundations of Data Science, Tufts University, March 2022
- AFOSR PI Meeting, Arlington, VA, August 2022
- Conference in Honor of J.P. Eckmann 75th birthday, Geneva, Switzerland, June 2022
- ICML 2022, Baltimore, Poster presentation, July 2022
- Neurips 2022, New orleans, Poster presentation, December 2022
- ICLR 2023, Kigali, Rwanda, Poster presentation, April 2023
- AFOSR PI Meeting, Arlington, VA, August 2023
- AFOSR PI Meeting, Arlington, VA, August 2024
- ICERM Meeting. Providence, RI, September 2024

## Seminar talks

- Mathematical physics seminar, ETH, Zürich January 1998
- Mathematical physics seminar, CPT-CNRS, Marseille, January 1998
- Mathematical physics seminar, Institut Fourier Grenoble, February 1998
- Mathematical physics seminar, University Paris-XI Orsay, April 1998
- Mathematical physics seminar, University of Virginia, November 1998
- Colloquium, University of Ottawa, January 1999
- Statistical mechanics seminar, Princeton University, February 1999
- Mathematical physics seminar, University of Geneva, July 1999
- Dynamical systems seminar, SUNY at Stony Brook, November 1999
- Mathematical physics seminar, University of Geneva, July 2000
- Mathematical physics seminar, ETH, Zürich, August 2000
- Colloquium, University of Arizona, October 2000
- Statistical mechanics seminar, Princeton University, February 2001
- Applied mathematics seminar, University of Toronto, March 2001
- Theoretical physics seminar, University of Geneva, May 2001
- Mathematical physics seminar, ETH Zürich, June 2001
- Analysis seminar, University of Helsinki, August 2001
- Mathematical physics seminar, University of Texas at Austin, January 2002
- Colloquium, University of Massachusetts, Amherst, January 2002
- Colloquium, University of Maryland, College Park, January 2002
- Colloquium, University of Rochester, February 2002
- Colloquium, University of Notre Dame, February 2002
- Statistical mechanics seminar, Institute of Advanced Study, February 2002
- Nonlinear analysis seminar, Stevens Institute of Technology, May 2002
- Mathematical physics seminar, Rutgers University, October 2002

- Stochastic processes seminar, Brown University, November 2002
- Mathematical physics seminar, University of Virginia, January 2003
- Condensed matter seminar, University of Massachusetts, April 2003
- Probability seminar, EPFL, Lausanne, June 2003
- Mathematical physics seminar, University of Geneva, June 2003
- Applied mathematics seminar, Duke University, March 2004
- Mathematical physics seminar, University of Geneva, January 2005
- Analysis and mumerics seminar, Munich University, January 2005
- Mathematical physics seminar, Munich University, January 2005
- Dynamical systems seminar, Boston University, April 2005
- Nonlinear systems seminar, Stevens Institute of Technology, April 2005
- Mathematical physics seminar, Rutgers University, September 2005
- Mathematical physics seminar, University of Virginia, November 2006
- Mathematical physics seminar, Rutgers University, November 2006
- Applied mathematics seminar, Dartmouth College, April 2007
- Mathematical physics seminar, University of Arizona, October 2007
- Analysis seminar, University of Toronto, October 2007
- Applied mathematics seminar, Brown University, November 2007
- Colloquium, University of Connecticut, February 2009
- Probability seminar, University of Wisconsin, March 2009
- Theoretical physics Seminar, University of Geneva, April 2009
- Mathematical physics Seminar, University of Rome, April 2009
- Mathematical physics Seminar, University of Bologna, April 2009
- Mathematical physics Seminar, University of Grenoble, May 2009
- Applied mathematics Seminar, University of Crete, May 2009
- Mathematical physics seminar, Rutgers University, November 2009
- Operator algebra, University of Tokyo, January 2010
- Joint PDE seminar Boston University Brown University, November 2010
- Applied Mathematics Seminar, University of Crete, March 2011
- Probability Seminar, Warwick University, March 2011
- Probability Seminar, Boston University, September 2012
- Probability Seminar, University of Delaware, October 2013
- Probability Seminar, Concordia University, May 2015
- Probability Seminar, Brown University, November 2016
- Mathematical physics seminar, Rutgers University, December 2017
- Dynamical systems seminar, Courant Institute, NYU January 2018
- Applied PDE seminar, Imperial College, November 2020
- Probability seminar KTH, November 2020
- Probability and Statistics Seminar, Boston University, November 2024

## **Teaching Experience**

## University of Massachusetts Amherst (since 2002)

Fall 2002: "Math 131 Calculus I

Spring 2003: Math 132 Calculus II

Spring 2003: Math 421 Complex variables

Fall 2003: " Math 597/697 Introduction to stochastic processes

Spring 2004: Math 131 Calculus I

Fall 2004: " Math 645 Differential equations and dynamical systems

Spring 2005: Math 646 Ergodic theory

Spring 2005: Math 131 Calculus I

Fall 2005: "Math 645 Differential equations and dynamical systems

Fall 2005: "Stat 515 Introduction to Statistics I

Spring 2006: Math597/697 Introduction to stochastic processes

Fall 2006: "Math 623 Real analysis I

Fall 2006: "Math 131H Honor calculus I

Spring 2007: Math 624 Real analysis II

Fall 2007: "Math 697U Introduction to stochastic processes

Fall 2007: "Math 331 Differential equations

Spring 2008: Math 331 Differential equations

Fall 2008: "Math 331 Differential equations

Fall 2008: "Math 645 Differential equations and dynamical systems

Fall 2009: "Math 697EG Evolutionary game theory

Spring 2010: Math 697U Introduction to stochastic processes

Spring 2010: Math 331 Differential equations

Fall 2010: Math 623 Real analysis I

Fall 2010: Math 131H Honor calculus I

Fall 2011: Math 623 Real analysis I

Spring 2012: Math 624 Real analysis II

Spring 2012: Math 456 Mathematical Modeling

Spring 2013: Math 456 Mathematical Modeling

Spring 2013: Math797fn Functional Analysis

Fall 2013: Math 623 Real Analysis I

Spring 2014: Math 624 Real Analysis II

Spring 2014: Math 331 Differential equations

Fall 2014: "Math697U Introduction to stochastic processes

Spring 2015: Math 456 Mathematical Modeling

Spring 2015: Math 797MR Information Theory

Fall 2015: Math 797AB Agent-based modeling

Spring 2016: Math 456 Mathematical Modeling

Spring 2016: Math 523H Real analysis

Fall 2016: Math 797SM Statistical mechanics

Spring 2017: Math 331 Differential equations

Spring 2017: Math 523H Real analysis

Fall 2017: Stat 605 Probability Theory

Fall 2017: Math 331 Differential equations

Spring 2018: Math697U Stochastic Processes

Spring 2019: Math 456 Mathematical Modeling

% % % % %%\begin{tabular}{lll} %{ Courses:} & Fall 2002& Math 131 Calculus I \ % & Spring 2003 & Math 132 Calculus II \ % & Spring 2003& Math 421 Complex variables \ % & Fall 2003& Math 597/697 Introduction to stochastic processes\ % & Spring 2004& Math 131 Calculus I \ % & Fall 2004& Math 645 Differential equations and dynamical systems \ % & Spring 2005& Math 646 Ergodic theory \ % & Spring 2005& Math 131 Calculus I \ % & Fall 2005& Math 645 Differential equations and dynamical systems\ % & Fall 2005& Stat 515 Introduction to Statistics I \ % & Spring 2006& Math597/697 Introduction to stochastic processes\ % & Fall 2006& Math 623 Real analysis I \ % & Fall 2006& Math 131H Honor calculus I \ % & Spring 2007& Math 624 Real analysis II \ % & Fall 2007& Math 697U Introduction to stochastic processes \ % & Fall 2007& Math 331 Differential equations \ % & Spring 2008& Math 331 Differential equations \ % & Fall 2008& Math 645 Differential equations and dynamical systems\ % & Fall 2009 & Math 697EG Evolutionary game theory \ % & Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % %%\begin{tabular}{1} % & Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % Spring 2010 & Math 697U Introduction to stochastic processes % \end{tabular} % Spring 2010 & Math

2010 & Math 331 Differential equations \ % & Fall 2010 & Math 623 Real analysis I \ % & Fall 2010 & Math 131H Honor calculus I \ % & Fall 2011 & Math 623 Real analysis I \ % & Spring 2012 & Math 624 Real analysis II \ % & Spring 2012 & Math 456 Mathematical Modeling \ % & Spring 2013 & Math 797fn Functional Analysis \ % & Fall 2013& Math 623 Real Analysis I \ % & Spring 2014& Math 624 Real Analysis II \ % & Spring 2014& Math 623 Real Analysis I \ % & Fall 2014& Math 697U Introduction to stochastic processes \ % & Spring 2015 & Math 456 Mathematical Modeling \ % & Spring 2015 & Math 797MR Information Theory \ % & Fall 2015 & Math 797AB Agent-based modeling \ % & Spring 2016 & Math 456 Mathematical Modeling \ % & Spring 2016 & Math 523H Real analysis \ % & Fall 2016 & Math 797SM Statistical mechanics \ % & Spring 2017 & Math 331 Differential equations \ % & Fall 2017 & Math 331 Differential equations \ % & Fall 2017 & Math 331 Differential equations \ % & Fall 2017 & Math 331 Differential equations \ % & Spring 2018 & Math697U Stochastic Processes \ % & Spring 2019 & Math 456 Mathematical Modeling % \end{tabular}

{ University of Virginia} 1999-2002

Fall 1999: "APMA 310 Probability

Spring 2000: APMA 206, Differential equations (2 sections)

Fall 2000: "APMA 310 Probability

Fall 2000: "Math 531 Introduction to real analysis I

Spring 2001: Math 532 Introduction to real analysis II

Fall 2001: "Math 131 Calculus I (2 sections)

Spring 2002: Math 845 Ergodic Theory

% %%\begin{tabular}{lll} %{ Courses:} & Fall 1999& APMA 310 Probability \ % & Spring 2000& APMA 206, Differential equations (2 sections) \ % & Fall 2000& APMA 310 Probability \ % & Fall 2000& Math 531 Introduction to real analysis I \ % & Spring 2001& Math 532 Introduction to real analysis II \ % & Fall 2001& Math 131 Calculus I (2 sections) \ % & Spring 2002& Math 845 Ergodic Theory %\end{tabular}

% %KK %% Research Fellow 1998–1999}, Center for Mathematical Sciences Research, %Department of Mathematics, Rutgers University, New Brunswick NJ 08854.

{ University of Geneva}, 1994-1998 (Teaching assistant)

1994-1995: Mathematics II for computer scientists

1994-1995: Function of several complex variables

1995-1996: Analysis I

1995-1996: Dynamical systems and ergodic theory

1996-1997: Mathematics for engineers

1996-1997: Numerical methods for PDE

1997-1998: Analysis II

%~%~%~%~% begin{tabular}{lll} %{ Courses:} & 1994-1995& Mathematics II for computer scientists \ % & 1994-1995& Function of several complex variables \ % & 1995-1996& Analysis I \ % & 1995-1996& Dynamical systems and ergodic theory \ % & 1996-1997& Mathematics for engineers \ % & 1996-1997& Numerical methods for PDE \ % & 1997-1998& Analysis II %\end{tabular}