Math 331.02: Spring 2017 midterm practice problems

1. Solve the initial value problem $\dfrac{dy}{dt} = y^2\left[t + \sin(t)\right]$ with $y(0) = 6$.

2. Solve the initial value problem $y' = \dfrac{3x^2 - 1}{3 + 2y}$ with $y(0) = 2$.

3. Mary initially deposits \$1000 in a savings account that pays interest at the rate of 5% per year (compounded continuously). She also arranges for \$25 per week to be deposited automatically into her account.

   (a) Assume that weekly deposits can be approximated by continuous deposits. Write down an initial value problem for her account balance $S(t)$ over time (t measured in years).

   (b) How long does she needs to save to buy a \$5000 car?

4. The half-life of a radioactive substance is 2 days. Find the time required for a given amount of the material to decay to 1/10 of its original mass.

5. A radioactive material loses 25% of its mass in 10 minutes. What is its half-life?

6. At what rate of interest, compounded continuously, will a bank deposit double in value in 8 years?

7. A person deposits \$25,000 in a bank that pays 5% per year interest, compounded continuously. The person continuously withdraws from the account at the rate of \$750 per year. Find $V(t)$, the value of the account at time t after the initial deposit.

8. To buy a new car, you take a 5 year loan of an amount of \$10,000 with a yearly interest rate of 2.50% compounded continuously. Assume that payment are made continuously. What is the total amount you have to pay back to the bank over the duration of the loan?

9. Newtons law of cooling states that if an object with temperature T(t) at time t is in a medium with temperature T_m the rate of change of T at time t is proportional to $T(t) - T_m$, thus T satisfies a differential equation of the form
$$T' = -k(T - T_m)$$

   Here $k > 0$, since the temperature of the object must decrease if $T > T_m$, or increase if $T < T_m$. Well call k the temperature decay constant of the medium.

   (a) A thermometer is moved from a room where the temperature is 70F to a freezer where the temperature is 12F . After 30 seconds the thermometer reads 40F. What does it read after 2 minutes?

   (b) An object is placed in a room where the temperature is 20C. The temperature of the object drops by 5C in 4 minutes and by 7C in 8 minutes. What was the temperature of the object when it was initially placed in the room?

10. Solve the initial value problem $\dfrac{dy}{dt} = -3y/t - 2 - t^{-4}$, $y(1) = 4$.

11. Solve the initial value problem $\dfrac{dy}{dt} = -e^t/y$, $y(0) = -2$.

12. Solve the initial value problem $\dfrac{dy}{dt} = 6y + e^{2t}$, $y(0) = 3$.

13. Solve the initial value problem $\dfrac{dy}{dt} = 6y + 2t - 4$, $y(0) = 3$.

14. Solve the initial value problem $\dfrac{dy}{dt} = \dfrac{1 + y^2}{y}$, $y(0) = -2$.

15. Solve the initial value problem $\dfrac{dy}{dt} = 1 + y^2$, $y(0) = -2$.

16. Find the general solution of $x^2 + y^2 + 2xyy' = 0$.

17. Solve the initial value problem $(\sin(x) - y\sin(x) - 2\cos x) + \cos(x)y' = 0$, $\quad y(0) = -1$.

18. Find the general solution of $xy^2 + 2xyy' = 0$.

19. Find the general solution of $y' = \dfrac{y}{x} + e^{-y/x}$ Hint: This one calls for a change of variable.

20. Find the general solution of $\dfrac{dy}{dx} - y = xy^2$. Hint: This one calls for a change of variable.

21. Find all functions $M(x,y)$ such that the equation $M(x,y)dx + (x^2 - y^2)dy = 0$ is exact.

22. A tank initially contains a solution of 10 pounds of salt in 60 gallons of water. Water with 1/2 pound of salt per gallon is added to the tank at 6 gal/min, and the resulting solution leaves at the same rate. Find the quantity $Q(t)$ of salt in the tank at time $t > 0$.

23. After a drought a water reservoir of capacity 200 liters is only half-full with pure water. The rate at which water is taken from the reservoir is 1 liters per hour. To replenish the reservoir it is decided to pump water into the half empty reservoir at a rate of 2 liters per hour until the reservoir is full. The water pumped into the reservoir is mistakenly polluted with salt with a concentration of 25g per liter. Write down the equation for the amount of salt in the pound at time t. How much salt will there be in the reservoir when the reservoir is full?

24. Solve the initial value problem $\dfrac{dy}{dt} = yt + 2t$, $\ y(3) = 2$. Hint: There is a hard way and there is a easy way.

25. Find the solution of the initial value problem $2y'' - 3y' + y = 0$, $y(0) = 2, y'(0) = \dfrac{1}{2}$. Sketch a graph of the solution. Determine the maximum value of the solution. Find the point where the solution is 0.

26. Solve the initial value problems and *sketch a graph of the solution*.

27. $y'' - y' - 2y = 0, y(0) = -1, y'(0) = 2$

28. $y'' + 25y = 0, y(0) = 1, y'(0) = -1$

29. $y'' + 2y' + 3y = 0, y(0) = 1, y'(0) = 0$

30. $y'' - 4y' + 13y = 0, y(0) = 4, y'(0) = 3$

31. $y'' - 4y' + 4y = 0, y(0) = 2, y'(0) = -1$

32. Consider the initial value problems $y'' + y' - 2y = 0$, $y(0) = 2, y'(0) = \beta$.

    (a) For which value of β the solution satisfies $\lim_{x \to \infty} y(x) = 0$?

    (b) For which values of β does the solution never hit 0?