Math 331.2: Homework 10 (Section 6.1, 6.2, and (part of) 6.3)

1. Compute the Laplace transform of $f(t) = t$ and $f(t) = t^2$.

2. Find the real part and imaginary parts of

$$\frac{2}{7-2i}\,, \quad \frac{3-2i}{2-5i}\,, \quad \frac{e^{-i\pi/4}}{1-2i}$$

3.

(a) Find the real part and imaginary part of $\dfrac{1}{s-(a+ib)}$.

(b) Compute the Laplace transform of $e^{at}\cos(bt)$ and $e^{at}\sin(bt)$ by taking first the Laplace transform of the exponential function $e^{(a+ib)t}$ and then using Euler's formula and **(a)**.

4. For the following function find the inverse Laplace transform.

(a) $\dfrac{5}{s^2+9}$

(b) $\dfrac{2s-3}{s^2+9}$

(c) $\dfrac{2s+3}{s^2-9}$

(d) $\dfrac{3s}{s^2+4s+5}$

(e) $\dfrac{2}{s(s^2+4)}$

5. Use Laplace transform to solve the following initial value problems

(a) $y' + 3y = \sin(3t)\,,\ y(0) = -1$

(b) $y'' + 3y' + 2y = 0\,,\ y(0) = 2\,,\ y'(0) = 1$

(c) $y'' + 2y = 5\cos(3t)\,,\ y(0) = 0\,,\ y'(0) = -2$

(d) $y'' + 3y' + \dfrac{5}{2}y = 0\,,\ y(0) = 1\,,\ y'(0) = -3$

7. Graph the following functions

$$g_1(t) = u_2(t) - 3u_5(t) + 4u_6(t)$$
$$g_2(t) = (t-3)u_2(t) + (t-2)u_3(t)$$

8. Consider the function

$$f(t) = \begin{cases} 1 & 0 \le t < 2 \\ -2 & 2 \le t < 5 \\ 3 & 5 \le t < \infty \end{cases}$$

(a) Graph $f(t)$

(b) Express f in terms of the step function $u_c(t)$.

(c) Compute the Laplace transform of $f(t)$.