Suggested Solutions to HW 1

* To deserve full credits, one has to find all NEs, and clearly explain how to obtain the desired resutls. Please consult the class notes posted. Problem 1 was not graded, and each prolem is worth 10 points. Any question/complaint is welcome; it can be directed to hwang@math.umass.edu or the instructor.

1. Suppose that you prefer life to death and more money to less and you are just willing to pay \$$X$ to get one bullet removed from a gun containing one bullet and \$$Y$ to get one bullet removed from a gun containg four bullets. Consider the prizes D = Dead, A = Alive, L_X = alive after paying \$$X$, L_Y = alive after paying Y. WOLG, we suppose that $u(D) = 0, u(A) = 1$. Then since you are indifferent between L_X and the lottery in which you get A with $\frac{1}{6}$ and D with $\frac{5}{6}$,

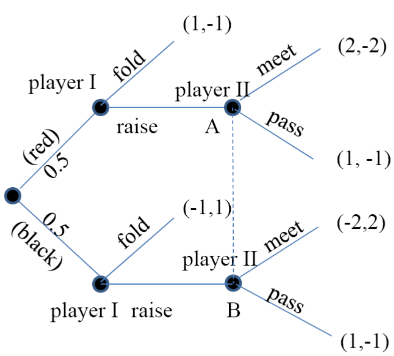
$$u(L_X) = \frac{5}{6}u(A) + \frac{1}{6}u(D) = \frac{5}{6}$$

   Similarly we have

$$\frac{1}{2}u(L_Y) + \frac{1}{2}u(D) = \frac{1}{3}u(A) + \frac{2}{3}u(D)$$

   and we find $u(L_Y) = \frac{2}{3}$. So we obtain $u(L_X) > u(L_Y)$ and this means $L_X \succ L_Y$. Since you prefer more money to less, conclude that $X < Y$.

2. The representation of the game in the extensive form is



   To emphasize that player II cannot distinguish whether she is at A or B, these points are connected by a dotted line. We have players = $\{I, II\}$ and strategy sets S_{II} = {meet, pass} and S_I = {RR, RF, FR, FF}, where RF means "Raise if red and Fold if black" and other pairs are interpreted similarly. Note that the strategy is a complete plan of the player about how to play the game. The normal form representation of this game is given by

	meet	pass
RR	$(0,0)$	$(1,-1)$
RF	$(\frac{1}{2}, -\frac{1}{2})$	$(0,0)$
FR	$(-\frac{1}{2}, \frac{1}{2})$	$(1,-1)$
FF	$(0,0)$	$(0,0)$

There is a unique mixed strategy NE, $\left(\left(\frac{1}{3}, \frac{2}{3}, 0, 0\right), \left(\frac{2}{3}, \frac{1}{3}\right)\right)$.

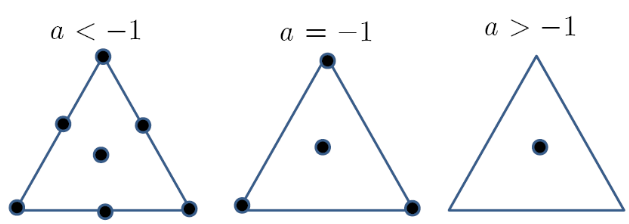
3. (a) Unique Mixed Strategy: $\left((\frac{4}{5}, \frac{1}{5}), \left(\frac{1}{2}, \frac{1}{2}\right)\right)$

   (b) Two pure strategy NEs : (s_2, t_1), (s_1, t_3); one mixed strategy NE: $((\frac{4}{5}, \frac{1}{5}), (0, \frac{3}{4}, \frac{1}{4}))$. Note that from the candidate $((\frac{10}{11}, \frac{1}{11}), \left(\frac{3}{5}, 0, \frac{2}{5}\right))$, player β can deviate to $(0, 1, 0)$, so obtain higher payoff; i.e. $((\frac{10}{11}, \frac{1}{11}), \left(\frac{3}{5}, 0, \frac{2}{5}\right))$ is not NE.

4. Suppose that p' is a NE for (Γ, S', π') and $\bar{p}'$ is the extension to (Γ, S, π) ,as is described in the problem. By a way of contradiction, assume that $\bar{p}'$ is not a NE of (Γ, S, π). Then from the definition, there exists $\gamma \in \Gamma$ (called a deviant) and $q_\gamma \in S_\gamma$ (called a deviation strategy) such that
$$\pi_\gamma(\bar{p}'_\gamma, \bar{p}'_{-\gamma}) < \pi_\gamma(q_\gamma, \bar{p}'_{-\gamma}) \tag{1}$$
If $q_\gamma \in S'_\gamma$, this contradicts to the fact that p' is a NE for (Γ, S', π'). So we suppose $q_\gamma \in S_\gamma \backslash S'_\gamma$. By the definition of weakly dominance, there exists a $r_\gamma \in S_\gamma$ such that q_γ is weakly dominated by
$$\pi_\gamma(q_\gamma, s_{-\gamma}) \leq \pi_\gamma(r_\gamma, s_{-\gamma}) \text{ for all } s_{-\gamma} \in S_{-\gamma} \tag{2}$$
In particular, we have $\pi_\gamma(q_\gamma, \bar{p}'_{-\gamma}) \leq \pi_\gamma(r_\gamma, \bar{p}'_{-\gamma})$. Thus, if $r_\gamma \in S'_\gamma$, inequality (1) leads to contradiction to the fact that p' is a NE for (Γ, S', π'). If $r_\gamma \notin S'_\gamma$, we find another weakly dominating strategy over it and do the same argument. Finally, by doing this process, if we have one last strategy, that strategy should be in S'_γ , and again we reach contradiction.

5. NE is given by $((0, 0, 1), (p, 1-p))$ for $p \in [0, 1]$.

6. Divide Cases:
(i) $a > -1$ (Generalized Rock-Paper-Scissor) A unique Mixed strategy NE, $(1/3, 1/3, 1/3)$
(ii) $a = -1$ Three pure strategy NEs, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ and one mixed NE $(1/3, 1/3, 1/3)$
(iii) $a < -1$ (Coordination game) Three pure strategy NEs (the same as (ii)) and 4 mixed strategy NEs. $(1/3, 1/3, 1/3)$, $(1 + \frac{1}{a}, -\frac{1}{a}, 0)$, $(0, 1 + \frac{1}{a}, -\frac{1}{a})$, $(-\frac{1}{a}, 0, 1 + \frac{1}{a})$. See the figure below.



7. There are 8 (even) NEs. (s_1, t_1, u_2), (s_2, t_1, u_1), (s_1, t_2, u_1), (s_2, t_2, u_2), $\left((\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{1}{3})\right)$, $\left((\frac{4}{9}, \frac{5}{9}), (\frac{6}{11}, \frac{5}{11}), (1, 0)\right)$, $\left((1, 0), (\frac{4}{9}, \frac{5}{9}), (\frac{6}{11}, \frac{5}{11})\right)$, $\left((\frac{6}{11}, \frac{5}{11}), (1, 0), (\frac{4}{9}, \frac{5}{9})\right)$