Logic

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Logic operators

The 4 connectives ordered by rank.

1. $\leftrightarrow$ (bi-implication)
2. $\rightarrow$ (implication)
3. $\wedge$ and $\vee$ (Conjunction and disjunction)
4. $\neg$ (negation)

Quantors

- $\exists, \forall$ (existential, universal)

The quantors have the same rank as the negation.

Examples

$$P \wedge Q \rightarrow R == (P \wedge Q) \rightarrow R$$
$$P \vee Q \leftrightarrow R == (P \vee Q) \leftrightarrow R$$
$$\neg Q \rightarrow \neg P == (\neg Q) \rightarrow (\neg P)$$
$$\neg P \rightarrow Q \leftrightarrow R == (((\neg P) \rightarrow Q) \leftrightarrow R)$$

Truth table

Table: Truth table

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

When is a reasoning correct?

Standard structure:

$$\frac{\overline{P_1, \ldots, P_n \leftarrow \text{Premises}}}{Q \leftarrow \text{Conclusion}}$$

Correct

$$\frac{\overline{3 > 2 \rightarrow 2 > 1} \qquad \overline{3 > 2}}{2 > 1}$$

Standard stucture

$$\frac{\overline{P \rightarrow Q} \qquad \overline{P}}{Q}$$

Mudus ponens, very important rule.

Wrong structure

Wrong

$$\frac{\overline{3 > 2 \to 2 > 1} \qquad \overline{2 > 1}}{3 > 2}$$

Wrong standard stucture

$$\frac{\overline{P \to Q} \qquad \overline{Q}}{P}$$

If in every situation where $P_1, \ldots P_n$ are all true, then Q is also true.

Some more rules

Contraposition

$$\frac{\neg Q \to \neg P}{P \to Q}$$

Double negation

$$\frac{\dfrac{\dfrac{P}{\neg\neg P}}{P \vee Q} \qquad \dfrac{}{\neg P}}{Q}$$

Logic consequence