exemplo PARA $\alpha = 2$.

$$p(x,y) = \sum_{i+j \leq 2} a_{ij} x^i y^j$$

Aplicando o polinômio em cada par:

$$\begin{cases} p(x_1, y_1) \approx 0 \\ \vdots \\ p(x_m, y_m) \approx 0 \end{cases} \iff$$

$$\begin{cases} a_{00} + a_{10}x_1 + a_{01}y_1 + \cdots + a_{20}x_1^2 + a_{02}y_1^2 \approx 0 \\ \vdots \\ a_{00} + a_{10}x_m + a_{01}y_m + \cdots + a_{20}x_m^2 + a_{02}y_m^2 \approx 0 \end{cases} \iff$$

$$\underbrace{\begin{bmatrix} 1 & x_1 y_1^0 & x_1 y_1 & \cdots & x_1^2 y_1^0 & x_1^0 y_m^2 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & x_m y_m^0 & x_m y_m & & x_m^2 y_m^0 & x_m^0 y_m^2 \end{bmatrix}}_{A \in \mathbb{R}^{m \times n}} \underbrace{\begin{bmatrix} a_{00} \\ a_{10} \\ \vdots \\ a_{02} \end{bmatrix}}_{a \in \mathbb{R}^n} \approx 0$$

Termo fixo

A ideia é particionar 'a' e fazer com a00 = 1.

$$a = \begin{bmatrix} a_{00} \\ \tilde{a} \end{bmatrix}, \quad \tilde{a} \in \mathbb{R}^{n-1} \quad \text{e}$$

$$A = \begin{bmatrix} \uparrow & \\ \mathbb{1} & B \\ \downarrow & \end{bmatrix} \quad \text{onde } \mathbb{1} = (1, \cdots, 1) \in \mathbb{R}^m \text{ e } B \in \mathbb{R}^{m \times (n-1)}$$

Dessa forma:

$$Aa = (\mathbb{1} \; B)\begin{pmatrix} a_{00} \\ \tilde{a} \end{pmatrix} = a_{00} \cdot \mathbb{1} + B\tilde{a}$$

fazendo $a_{00} = 1$

$$Aa = B\tilde{a} + \mathbb{1} \approx 0 \Rightarrow B\tilde{a} \approx -\mathbb{1}$$

Dessa forma,

$$\min \|Aa\|_2^2 \iff \min \|B\tilde{a} + \mathbb{1}\|$$