

# MATHEMATICAL MODELING OF VISCERAL LEISHMANIASIS IN SUDAN

AGBATAN FIACRE LUC KOUDERIN  
SALAM MUSA  
PETER ABUGRI  
JOSEPH MAINA  
VICTORY CHUKWUDI

AFRICAN INSTITUTE FOR MATHEMATICAL  
SCIENCES (AIMS)

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- 2 EPIDIOLOGICAL CHARACTERISTICS OF LEISHMANIASIS
- 3 METHODOLOGY
- 4 RESULTS AND ANALYSIS
- 5 CONCLUSION
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# INTRODUCTION

- Leishmaniasis is still one of the world's most neglected diseases, affecting largely the poorest of the poor, mainly in developing countries
- 350 million people are considered at risk of contracting leishmaniasis, and some 2 million new cases occur yearly
- Mathematical models can help describe and understand how the disease spreads, providing insights for better control and elimination strategies

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# Leishmaniasis : Definition and Transmission mode

- Leishmaniasis is a vector-borne infectious disease caused by protozoan parasites of the genus *Leishmania*.
- The disease is transmitted to humans through the bite of infected female phlebotomine sandflies.
- Sandflies acquire the parasite by feeding on infected hosts (e.g., humans or animals) and transmission occurs when the infected sandfly bites another host during a blood meal.
- Different forms of leishmaniasis include cutaneous, mucocutaneous, and visceral

# Types of Leishmaniasis

**Cutaneous Leishmaniasis (CL):** Characterized by skin sores or ulcers at the site of the sandfly bite, often leading to scarring.



Figure 1: CL Example

# Types of Leishmaniasis

**Mucocutaneous Leishmaniasis (MCL):** A severe form that destroys mucous membranes of the nose, mouth, and throat, causing disfigurement.



Figure 2: MCL Example



# Types of Leishmaniasis

**Visceral Leishmaniasis (VL):** The most severe form, affecting internal organs like the spleen and liver, and can be fatal if untreated.



Figure 3: VL case example

# Prevention and Treatment of Visceral Leishmaniasis

- Possibles prevention measures are related to vector control :
  - Use of Insecticide-Treated Nets (ITNs)
  - Indoor residual spraying (IRS)
  - Environmental management

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- Possibles prevention measures are related to vector control :
  - Use of Insecticide-Treated Nets (ITNs)
  - Indoor residual spraying (IRS)
  - Environmental management
- Current Situation in Sudan :
  - VL has no vaccine available
  - Treatment relies on two major drugs:
    - Pentamidine: Commonly used for treatment
    - Sodium Stibogluconate: Another effective treatment option

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# Research Problem and Question

## Background

Visceral leishmaniasis (VL), is a life-threatening disease that disproportionately affects vulnerable populations in Sudan. Despite ongoing control efforts, VL remains a significant public health challenge.

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A key issue is determining the optimal level of treatment coverage required to eliminate VL. In fact, insufficient treatment allows the disease to persist.

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## Question

What level of treatment coverage is required to achieve the elimination of Visceral leishmaniasis in Sudan ?

# Description of the model : Assumptions

- ☼ Vectors (sandflies) are not considered in the model, but are implicitly included in the transmission rate
- ☼ There is no vertical transmission of the disease
- ☼ Our population is homogeneous
- ☼ The treatment is entirely efficacious.
- ☼ There is no longlife immunity. Once someone gets treated, he/she becomes susceptible again after 2 years.



# Description of the model : Key Parameters

- $\lambda$ : Force of infection
- $\gamma$ : Progression rate (1/latent period)
- $\epsilon$ : Proportion of the population receiving treatment;
- $\eta$ : Proportion of the population developing chronic infection;
- $\mu$ : Natural mortality rate
- $\mu_c$ : Mortality rate due to chronic infection
- $b$ : Birth rate
- $\alpha$ : Immunity loss rate

# Description of the model : Model Diagram

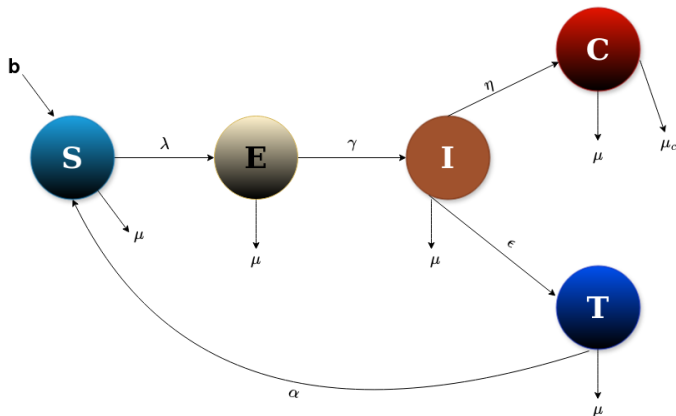


Figure 4: Model Diagram

# Description of the model : PDEs

- **Susceptible ( $S$ ):**

$$\frac{dS}{dt} = bN - \lambda S + \alpha T - \mu S \quad (1)$$

- **Exposed ( $E$ ):**

$$\frac{dE}{dt} = \lambda S - \gamma E - \mu E \quad (2)$$

- **Infectious ( $I$ ):**

$$\frac{dI}{dt} = \gamma E - \epsilon I - \eta I - \mu I \quad (3)$$

# Description of the model : PDEs

- **Treated ( $T$ ):**

$$\frac{dT}{dt} = \epsilon I - \alpha T - \mu T \quad (4)$$

- **Chronic ( $C$ ):**

$$\frac{dC}{dt} = \eta I - \mu_c C - \mu C \quad (5)$$

- **Constant Population:**

$$\frac{dN}{dt} = 0 \quad (6)$$

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# RESULTS AND ANALYSIS : Key Parameters Values

Table 1: Values and of Parameters and sources

Parameters	Values	Sources
Transmission rate	0.4	5
Initials Values :	$S = 1200, E = T = C = 0, I = 100$	Optional
Treatment rate (per day)	$[< 1\%, 2\%, 10\%]$	Optional
Immunity time	2 years	2
Infected Time	50 days	4

# RESULTS AND ANALYSIS : Dynamics of VL

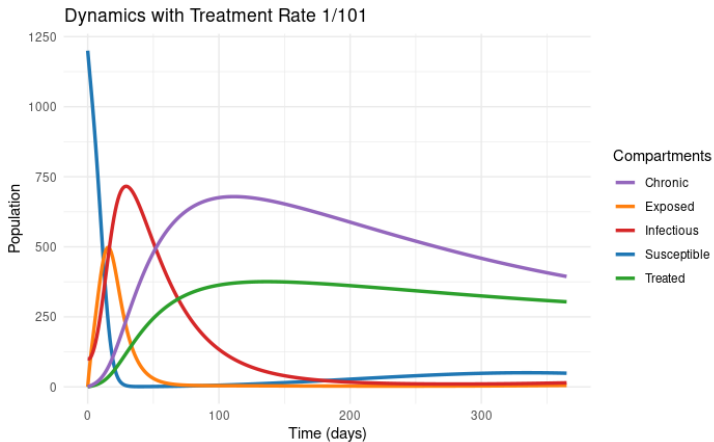


Figure 5: Dynamics of VL

# RESULTS AND ANALYSIS : Dynamics of VL

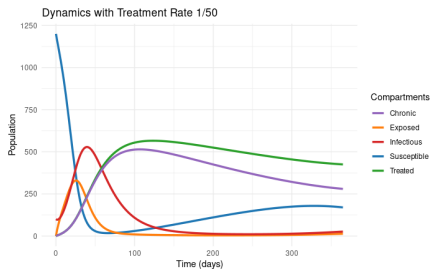


Figure 6: Treatment rate = 2%

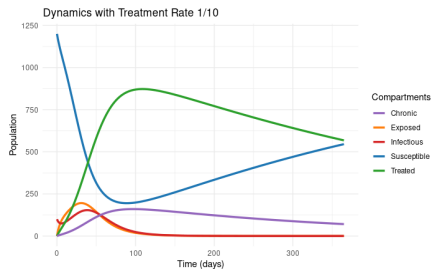


Figure 7: Treatment rate = 10%



# RESULTS AND ANALYSIS : General Interpretations

Based on the previous graphs, we can deduce that :

- As we increase the treatment rate, we can remark that the number of people treated is increasing and the number of infectious people is decreasing. This means that the treatment rate impacts significantly how the disease spread
- As we increase the treatment rate, the time needed before the infectious people decrease at almost zero is reducing; which means that by increasing the treatment rate, the disease will be eliminated more quickly.
- Knowing the limited resources in Sudan, we will advise the Public health responsables to administrate at least 10% of the treatment **per day** to be able to eliminate the disease almost three months.

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# CONCLUSION

- The dynamics reveal that increasing the treatment rate significantly enhances disease control by reducing the number of infectious and chronic cases and maintaining a larger susceptible population through effective management.
- These findings are essential for: public health planning ; allocating resources effectively to maximize treatment efficacy and minimizing the overall impact of the disease.

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# **Thank You!**

# **Merci!**

Questions are welcome.