Integers

October 7, 2023

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1 Expression

We can construct an integer as a pair with a boolean representing the sign and a natural number representing the magnitude.

$$-5 = \upsilon_2 \mathrm{F} 5_{\mathbb{N}}$$

$$+4 = \upsilon_2 \mathrm{T} 4_{\mathbb{N}}$$

We however need to make sure that -0 and +0 are equal, so any operation dealing with integers needs to reduce -0 to +0.

$$\mathrm{reduce0} = \lambda z.0?_{\mathbb{N}}(z\pi_2)(\upsilon_2 \mathrm{T}(z\pi_2))z$$

We can now create an expression to generate strict integers.

$$\mathbb{Z} = \lambda s \lambda m.\mathrm{reduce0}(\upsilon_2 sm)$$

2 Operations

For our standard operations we simply have to add a system for handling sign to our natural numbers.

$$
\begin{aligned}
\mathrm{add}_{\mathbb{Z}} = \quad & \lambda n \lambda m.\mathrm{eq}_b(n\pi_1)(m\pi_1)(\\
& \quad \mathbb{Z}(n\pi_1)(\mathrm{add}_{\mathbb{N}}(n\pi_2)(m\pi_2)) \\
&)(\\
& \quad \mathbb{Z}(\mathrm{gt}_{\mathbb{N}}(n\pi_2)(m\pi_2)(\\
& \quad \quad n\pi_1 \\
& \quad)(\\
& \quad \quad m\pi_1 \\
& \quad))(\mathrm{sub}_{\mathbb{N}}(n\pi_2)(m\pi_2)) \\
&)
\end{aligned}
$$

This single expression actually works for all combinations of signs, so we can do both addition and subtraction using this, so long as you are performing the equivalent of a + -(b). It may however be useful for us to create this shorthand.

$$\mathrm{neg}_{\mathbb{Z}} = \lambda a.\mathbb{Z}(\mathrm{not}_b(a\pi_1))(a\pi_2)$$

$$\mathrm{sub}_{\mathbb{Z}} = \lambda a \lambda b.\mathrm{add}_{\mathbb{Z}} a(\mathrm{neg}_{\mathbb{Z}} b)$$

We can do the same thing for multiplication.

$$\mathrm{mul}_{\mathbb{Z}} = \lambda n \lambda m.\mathbb{Z}(\mathrm{eq}_b(n\pi_1)(m\pi_2)\mathrm{TF})\mathrm{mul}_{\mathbb{N}} nm$$

We actually dont need to even define integral relational operators. We have all we need to define a real number system, after which all of this becomes irrelevant again.