Exercise 1.14   
Define P = {1, 2, 3}, Q = {a, b, c}.   
Which of the following are functions?   
a. { (1, a), (2, b), (3, a) }   
b. { (1, a), (2, b), (2, a) }   
c. { (3, a), (2, b), (2, c) }

Only A is a function, as B and C are not a set of unique input/output pairs, which is the very definition of a function.

Exercise 1.18   
Let R be the relation on ℕ defined by x + 3y = 12, so R = { (x, y) ∈ ℕ x ℕ | x + 3y = 12 }   
a. Write R in roster notation.   
b. Find the domain and range of R.   
c. Is R a function on its domain? If yes, determine the inverse of this function, if it exists.

A: { (12, 0), (9, 1), (6, 2), (3, 3), (0, 4) }

I just went trough values for y from 0 trough 4 inclusive, as y = 5 would make 3y 15 which does not satisfy the condition of 12. Then I subtracted the outcome of 3y from 12 to find the corresponding value for x given y.

B:

Dom(R) = { { 0, 3, 6, 9, 12 }, { 0, 1, 2, 3, 4 } }

I would say that the Range(R) = { 12 } given the restriction of ‘= 12’

C: I don’t know how to answer this

Exercise 2.4   
Check that the following logical equivalences are valid by making truth tables.   
( ≡ (unicode symbol u2261) is short for “is logically equivalent to” )

**Commutativity:** an OR gate is defined as a gate which opens if either one of its inputs is on. In other words, let A and B be the inputs and O be the output, when either A or B is 1, then O will be 1, regardless of the order, which should be evident by looking at the second and third row of the truth table. The same applies to the AND gate, which has its output defined as being on only when both A and B are on. Again, order does not matter here. Thus P ∨ Q ≡ Q ∨ P, as well as P ∧ Q ≡ Q ∧ P

|  |  |  |  |
| --- | --- | --- | --- |
| **OR GATE** | **A** | **B** | **O** |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

|  |  |  |  |
| --- | --- | --- | --- |
| **AND GATE** | **A** | **B** | **O** |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Exercise 4.1

For each of the following statements:  
- Write down what this statement amounts to, for n = 0, 1, 2, and 3.  
- Prove the statement using mathematical induction.

1. (for n ≥ 0): 1 + 2 + 3 + ... + n = n(n+1) / 2

**Proof for a**:

For base case (K) where n = 2, then   
1 + 2 = 3, and  
2(2+1) / 2 = 2 \* 3 / 2 = 6

For next case (K+1) where n = n + 1

1 + 2 + 3 = 6, and

TODO: figure out how to add the + 1 algebraically using brilliant

∎

1. (for n ≥ 1): 1 + 4 + 7 + ... + (3n - 2) = n(3n-1) / 2
2. (for n ≥ 0): 1 + 2 + 22 + 23 + ... + 2n = 2n+1 – 1
3. (for n ≥ 1): 1.2 + 2.3 + 3.4 + ... + n.(n+1) = 1/3 (n(n+1)(n+2) )
4. (for n ≥ 1): 1.2.3 + 2.3.4 + 3.4.5 + ... + n(n+1)(n+2) = 1/4 (n(n+1)(n+2)(n+3))