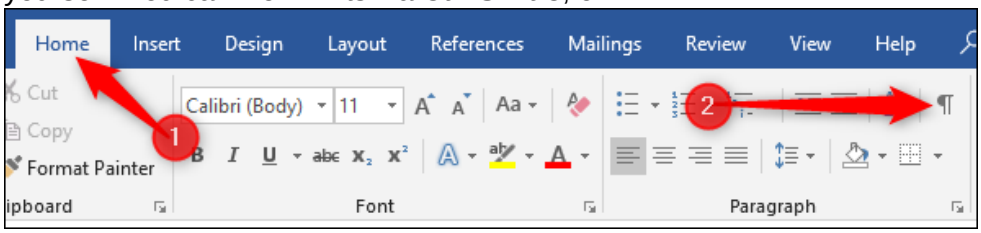
Module D1. Functional Programming

Summary

Functional Programming is a programming style where mathematical functions (that is, expressions with input parameters and an output result) are fundamental and running a program basically means evaluating a complex function. When compared to traditional imperative and OO programming styles, FP is much faster to develop and easier to test/debug, and is also very suitable for parallel applications. Lisp, Haskell, Scala, Elm are examples of purely functional programming languages very much used today.

This module aims at exposing you to the functional way of thinking and introducing FP concepts and techniques that have been proven useful in practice, like list comprehensions, generators, map, filter, reduce. We will work with Elm as default language, but will look at functional aspects of other languages as well.

Some exercises only have a brief description, such that you can design the sub functions yourself. You can view hints via Ctrl-Shift-8, or:



Learning outcomes

Learning outcome 1. Analysis

You are able to investigate, analyze and translate real world problems into abstract/mathematical models and solve them methodically and creatively.
You learn to use the functional programming style confidently, understand the pros and cons.

Learning outcome 2. Design

You are able to design abstract solutions (like algorithms and data structures) and able to reason about their validity/scope.
You design programs using functional programming paradigms (modular, reusable, without side-effects).

Learning outcome 3. Realization

You are able to translate abstract/mathematical solutions into concrete implementations and reason about the correctness of the implementation.
You program with basic functional concepts like filter, map, reduce, recursion.

Learning outcome 4. Testing

You are able to design and implement tests on the basis of abstract/mathematical specifications and reason about concepts like the coverage.
You reason about the correctness and time/space complexity of your programs, and measure their practical performance.

Learning outcome 5. Research

You are able to find and compare ideas and solutions from academic sources, such as research papers and are able to translate those abstract models/concepts into sustainable solutions.
Throughout the course, you adopt a curious attitude and take charge of your learning process.

Learning outcome 6. Interaction

You are able to effectively interact and communicate with academically trained people on abstract problems and solutions.
There will be enough opportunity for interaction with your teachers and colleagues.

Learning outcome 7. Communication and Leadership

You are able to communicate your ideas to other people on an academic level both in presentations and in writing.
There will be enough opportunity to describe and present your work, for instance in the research assignment.

Assessment

It will be based on the following elements:

- Presence and active participation in the lessons, especially for practical discussions on classroom exercises and on the weekly assignments listed in this document.
- Research report and presentation.
- Final exam.

The rest of this document contains weekly assignments. These are meant to be developed at home and then brought to the lessons to be discussed with the group. Make sure that you develop proper test cases for your weekly assignments.

The research assignment is also described below, under week 4. This assignment will be worked on in groups of two students. Discuss with your lecturer the language you wish to choose.

Assignments

Week 1

1. Getting started (not to be submitted in Canvas)

- (30 min.) Read the explanations and follow the tutorial “Learning the basics” section at https://elmbridge.github.io/curriculum/. (i.e. the paragraphs "Introduction", "Intermission: Why Elm?", "Getting Started" and The Basic Building Blocks of Elm")
- (10 min.) Do "Lab 1. First Steps Introduction to Functional Programming edX" (available on SharePoint)
- (10 min.) Do the beginning of https://elmprogramming.com/list.html (until "Sorting a List")

2. Caesar (part 1)

When we talk about cryptography these days, we usually refer to the encryption of digital messages, but encryption actually predates the computer by quite a long period. One of the best examples of early cryptography is the Caesar cipher, named after Julius Caesar because he is believed to have used it, even if he didn’t actually invent it. The idea is simple: take the message you want to encrypt and shift all letters by a certain amount between 0 and 26 (called the offset). For example: encrypting the sentence “THIS IS A BIG SECRET” with shifts of 5, would result in “YMNX NX F GNL XJHWJY”. See also https://cryptii.com/pipes/caesar-cipher

In this exercise you will be implementing a variant of the Caesar cipher. You can use the list of library functions linked from the course webpage, as well as those in the Appendix of this tutorial sheet.

Write a function

```
encode: Int -> Char -> Char
```

that encrypts the given single character using the given key (i.e. the offset). Note that lower case letters remain lower case, and upper case letters remain upper case. For example:

```
> encode 5 'x'
'c'
> encode 7 'T'
'A'
```

Write a function

```
decode: Int -> Char -> Char
```

that decrypts the given single character using the given offset. For example:

```
> decode 5 'c'
'x'
> decode 7 'A'
'T'
```

Tip:

- use `modBy`, `Char.toCode` and `Char.fromCode` (https://package.elm-lang.org/packages/elm/core/latest/Char)
- a look at the ASCII table (https://ascii-code.net) might be convenient

- when implementing `decode`: use `encode`
- when you see similar code lines appearing: make new functions

3. Pythagoras (part 1)

A Pythagorean triple is a set of three integers (a, b, c) which satisfy the equation $a^2 + b^2 = c^2$. For example, (3, 4, 5) is a Pythagorean triple, since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. See also https://en.wikipedia.org/wiki/Pythagorean_triple

Write a function

```
sqr: Int -> Int
```

which calculates a square, and write a function

```
isTriple: Int -> Int -> Int -> Bool
```

which tests for Pythagorean triples. Take care of negative or zero length. Make sure that it returns `True` for numbers that satisfy the equation (such as 3, 4 and 5) and `False` for numbers that don't (such as 3, 4 and 6).

Next we'll create some triples automatically. One formula for finding Pythagorean triples is as follows: $(x^2 - y^2, 2yx, x^2 + y^2)$ is a Pythagorean triple for all positive integers x and y with x > y.

Write functions `leg1`, `leg2` and `hyp` that generate the components of Pythagorean triples using the above formulas.

```
leg1 : Int -> Int -> Int
leg2 : Int -> Int -> Int
hyp  : Int -> Int -> Int
```

Write function `pythTriple` with input tuple `(x, y)`, generates a tuple `(a,b,c)` (according the formula above):

```
pythTriple : (Int, Int) -> (Int, Int, Int)
```

Examples:

```
> leg1 5 4
9
> leg2 5 4
40
> hyp 5 4
41
> isTriple 9 40 41
True
> pythTriple (5,4)
(9,40,41)
```

Write a function

```
isTripleTuple: (Int, Int, Int) -> Bool
```

which has a tuple as an input. Tip: use `isTriple` for its implementation. Examples:

```
> isTripleTuple (9,40,41)
True
> isTripleTuple (pythTriple (5,4))
True
```

Week 2

1. Caesar (part 2)

Write a function

```
normalize: String -> String
```

which removes spaces and all non-letters (like #, =, !, @). Example:

```
> normalize "Hello, Fontys!"
"HelloFontys"
```

Hints

Write the functions

```
encrypt: Int -> String -> String
decrypt: Int -> String -> String
```

to encrypt and decrypt a normalized string. Tip: same as `normalize`.
Do not use `String.map`. Example:

```
> encrypt 7 (normalize "Hello, Fontys!")
"OlssvMvuafz"
> decrypt 7 "OlssvMvuafz"
"HelloFontys"
```

2. Pythagoras (part 2)

Write functions

```
pythTriplesMap: List (Int, Int) -> List (Int, Int, Int)
pythTriplesRec: List (Int, Int) -> List (Int, Int, Int)
```

which calculates the a-b-c triples for the given x-y values. Make 2 implementations: with `List.map` and recursively. Example:

```
> pythTriplesMap [(5,4),(2,1),(35,7)]
[(9,40,41),(3,4,5),(1176,490,1274)]
> pythTriplesRec [(5,4),(2,1),(35,7)]
[(9,40,41),(3,4,5),(1176,490,1274)]
```

Write functions

```
arePythTriplesFilter: List (Int, Int, Int) -> List (Int, Int, Int)
arePythTriplesRec: List (Int, Int, Int) -> List (Int, Int, Int)
```

which removes all non-Pythagorean triples. Make 2 implementations: with `List.filter` and recursively. Example:

```
> arePythTriplesRec [(1,2,3), (9,40,41), (3,4,5), (100,2,500)]
[(9,40,41),(3,4,5)]
```

Week 3

1. Caesar (part 3)

One kind of brute-force attack on an encrypted string is to decrypt it using each possible key and then search for common English letter sequences in the resulting text. If such sequences are discovered then the key is a candidate for the actual key used to encrypt the plaintext. For example, the words "the" and "and" occur very frequently in English text: in the Adventures of Sherlock Holmes, "the" and "and" account for about one in every 12 words, and there is no sequence of more than 150 words without either "the" or "and".

The conclusion to draw is that if we try a key on a sufficiently long sequence of text and the result does not contain any occurrences of "the" or "and" then the key can be discarded as a candidate.

Of course do not use `String.contains`, `String.startsWith`, `String.endsWith`, `String.indexes`, `String.indices`, `List.any` for any of the functions described below.

Hints

Write a function

```
candidates: List String -> String -> List (Int, String)
-- semantics:
-- candidates canaries encryptedText
```

that decrypts the input string with each of the 25 possible keys and, when the decrypted text contains one of the search strings (`canaries`), then include the decryption key and the decrypted text in the output list. Example:

```
> candidates ["THE", "AND"] "DGGADBCOOCZYMJHZYVMTOJOCZHVS"
[(5, "YBBVYWXJJXUTHECUTQHOJEJXUCQN"),
 (14,"PSSMPNOAAOLKYVTLKHYFAVAOLTHE"),
 (21,"ILLFIGHTTHEDROMEDARYTOTHEMAX")]
```

2. Validating Credit Card Numbers

Use the template CreditCard_template.elm. Make sure that *at least once* you have applied each of the following features:

- function recursion
- list recursion
- lambda expression (aka. anonymous function)
- `List.map`
- `List.filter`
- `List.foldr` or `List.foldl`

and do not use the trivial functions like:

- `List.reverse`
- `List.sum`

Have you ever wondered how websites validate your credit card number when you shop online? They don't check a massive database of numbers, and they don't use magic. In fact, most credit providers rely on a checksum formula for distinguishing valid numbers from random collection of digits (or typing mistakes).

In this exercise, you will implement a **validation algorithm for credit cards**. The algorithm follows these steps:

- Double the value of every second digit beginning with the rightmost.
- Add the digits of the doubled values and the undoubled digits from the original number.
- Calculate the modulus of the sum divided by 10.

If the result equals 0, then the number is valid. See also: https://www.creditcardvalidator.org/articles/luhn-algorithm. Here is an example of the results of each step on the number "*4012888888881881*".

- In order to start with the rightmost digit, we produce a reversed list of digits. Then, we double every second digit.

  Result: [1,16,8,2,8,16,8,16,8,16,8,16,2,2,0,8].

- We sum all of the digits of the resulting list above. Note that we must again split the elements of the list into their digits (e.g. 16 becomes [1, 6]).

  Result: 90.

- Finally, we calculate the modulus of 90 over 10.

  Result: 0.

Since the final value is 0, we know that the above number is a valid credit card number. If we make a mistake in typing the credit card number and instead provide *4012888888881891*, then the result of the last step is 2, proving that the number is invalid.

Hints

Write the function

```
numValid: List String -> Int
```

that tells how many valid credit card numbers there are in a list.

Calculate and show how many valid credit card numbers are in the given list `creditcards`.

Week 4

1. Sorting

Implement the merge sort (https://www.geeksforgeeks.org/merge-sort/) in a proper and efficient FP way; in this case: by *only* using list recursion.

Hints

Write the function

```
msort: List comparable -> List comparable
```

Function `msort` sorts a list via the mergesort algorithm.

2. Modelling Math functions (part 1)

Given the type Function, write function

```
print: Function -> String
eval: Float -> Function -> Float
graph: Function -> Int -> Int -> Int -> Int -> String
```

Function `print` gives a human readable infix representation of the Function.

Function `eval` calculates the function values of the given input values.

Function `graph` makes an graph of the function, within the ranges xmin-xmax and ymin-ymax. You may choose another return type if you prefer. Tip: it's easier to rotate 90 degrees (so the y-axis is from left to right, and the x-axis is from top to bottom). And perhaps you need to enlarge the variable `page_width` of the template.

You can use https://www.mathsisfun.com/calculus/derivative-plotter.html to create your own functions and to check your results.

Examples:

```
> f = Plus (Mult (Plus (Const 3) X) (Minus X (Poly X 5))) (Const 2)
> print f
"(((3 + x) * (x - (x ^ 5))) + 2)"

> eval 2 f
-148
```

```
-- g is the function g(x) = (x/5 - 1) ^ 4 - ((x/-2) + 2) ^ 2 + 6
-- print with x-range: -10 to 20, and y-range: -10 to 10
> graph g -10 20 -10 10
```

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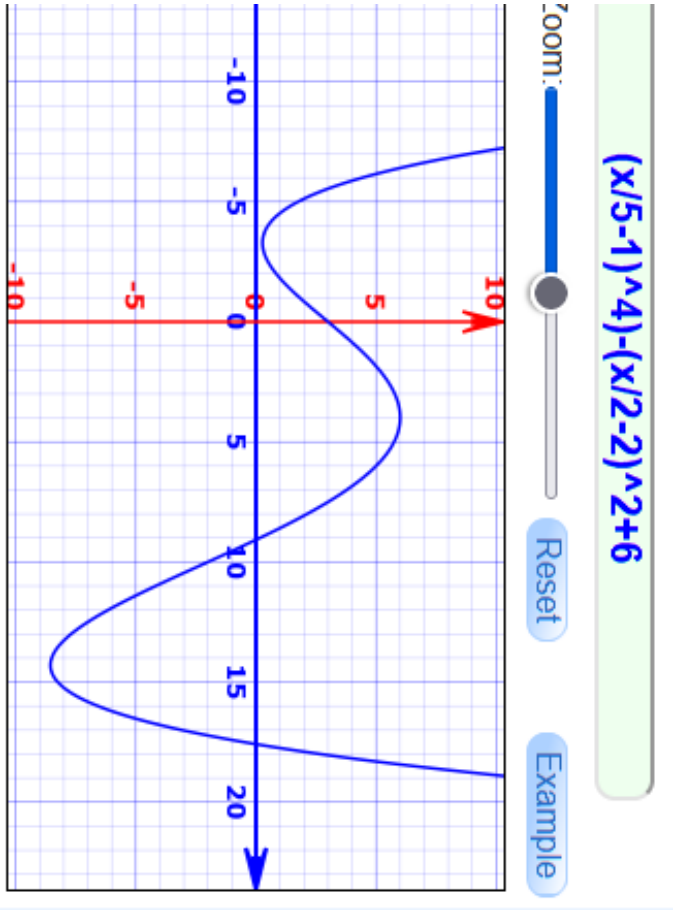

3. Research

Many modern programming languages support some form of functional programming, ranging from purely functional to external plugins. The goal of this research assignment is to get a good impression of how FP looks like in a popular language of your choice: Python, F#, JavaScript, Java etc.

To this aim, choose a language X that you are familiar with, interested in, or completely new to. Then research how functional X is, by applying any appropriate research methods. See http://ictresearchmethods.nl for inspiration.

As part of this research, a prototype needs to be built of the ***mergesort problem*** in the most functional way.

Week 5

1. Higher order functions

Write the function

```
repeatUntil: (a -> Bool) -> (a -> a) -> a -> a
```

which can be used in the following way:

```
above100: Int -> Bool
above100 x =
```

```
    x > 100

double: Int -> Int
double x =
    x * 2

repeatUntil above100 double 7
```

In this case, `repeatUntil` is repeatingly applying function `double` on initial value 7 (and its result etc.) until the value is greater than 100 (i.e. the predicate function `above100` holds).

`above100 7` is `False`, so apply: `double`(7) = 14
`double`(14) = 28
`double`(28) = 56
`double`(56) = 112, and now it stops because `above100 112` is True

So: `repeatUntil p f init` is continuously calculating f(f(f...f(f(f(init))))) until predicate `p` holds.

- Check that

  ```
  repeatUntil above100 ((+) 1) 42
  ```

  indeed returns 101.

- Change `above100` into a new function such that it can stop above any given value.
- Calculate $^{3}\log 100$ (for natural numbers only). Spoiler: answer=5 because $3^5=243$ (and $3^4 = 81$).
  Make sure that the calculations can be done for other base numbers and target numbers as well.
- Examine if the Collatz sequence of a given number indeed ends at 1 (see https://en.wikipedia.org/wiki/Collatz_conjecture).
  Now we don't choose the application function `f` being `Int -> Int` (because the final value is 'always' 1, too boring):

  ```
  repeatUntil ((==)1) (\x->if even x then x//2 else 3x+1) 19
  ```

  but instead: `List Int -> List Int`, collecting the whole sequence. Example:

  ```
  > repeatUntil myPredicate myCollatz [19]
  [1,2,4,8,16,5,10,20,40,13,26,52,17,34,11,22,44,88,29,58,19]
  ```

2. Modelling Math functions (part 2)

Given the type Function, write functions

```
derivative: Function -> Function
simplify: Function -> Function
```

Function `derivative` gives the derivative of a function (following all rules like chain-rule, etc.)

Function `simplify` prunes unnecessary branches like +0, *0, *1, ^1, ^0, /1. It also simplifies the multiplication, addition and subtraction of constants, like 3+6, 4^5, 9*5, -2--2.

Examples:

```
> print f
"(((3+x)*(x-(x^5)))+2)"
> print <| derivative f
"((((0+1)*(x-(x^5)))+((3+x)*(1-((5*(x^4))*1))))+0)"
> print <| simplify <| derivative f
"((x-(x^5))+((3+x)*(1-(5*(x^4)))))"
```

Make your own (nasty) test cases to check that it also works flawlessly for all kind of combinations of nested simplifications etc.

Also here: use https://www.mathsisfun.com/calculus/derivative-plotter.html to validate your answer: if you have calculated a (simplified) derivative, you can print it in the lower figure (and this screen shot shows that my calculation is not yet correct…)