Vibration Energy Harvesting

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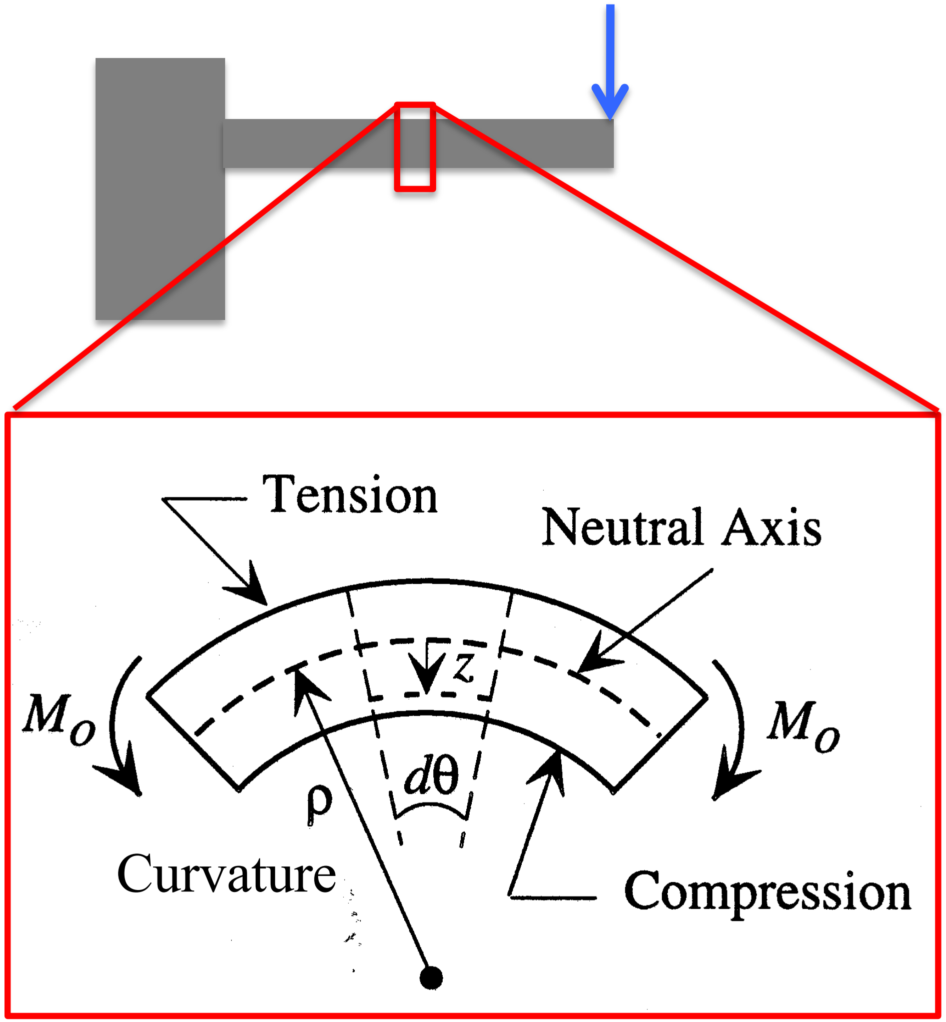
Objective

Experience vibration energy harvesting using piezoelectric cantilever

- ✓ **Calculate the length of cantilever and the tip mass to satisfy the given resonance frequency.**
- ✓ **Carry out simple power generation experiment using the piezoelectric cantilever.**

Linear Beam Theory



$$dx = \rho d\theta$$

$$dL = (\rho - z)d\theta$$

$$= dx - \frac{z}{\rho}dx$$

$$\varepsilon_x = -\frac{z}{\rho}$$ Strain

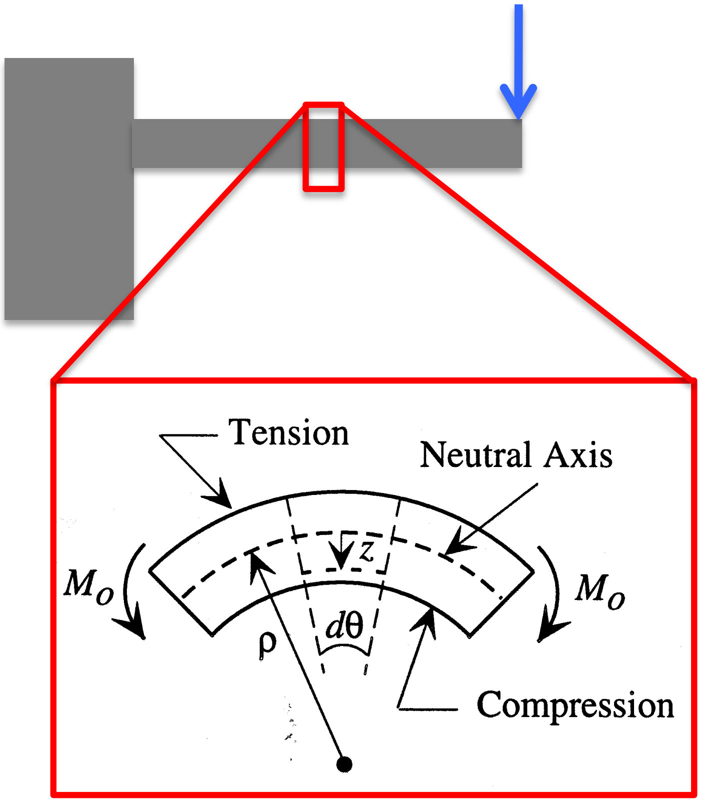
$$\sigma_x = -\frac{z}{\rho}E$$ Stress

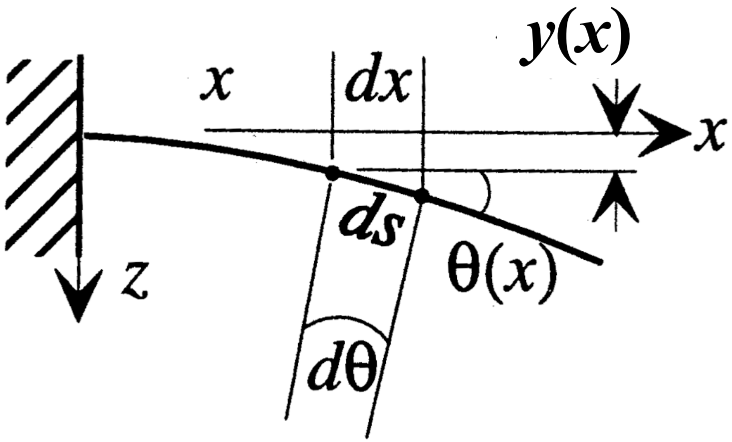
$$M = \int_{-H/2}^{H/2} Wz\sigma_x dz$$ Moment

$$= -\frac{EI}{\rho}$$

$$I = \frac{1}{12}WH^3$$ Moment of inertia

Linear Beam Theory





$$ds = \rho d\theta$$

$$ds = \frac{dx}{\cos\theta}$$

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2\theta}\frac{d\theta}{dx}$$

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}} \sim \frac{d^2y}{dx^2}$$

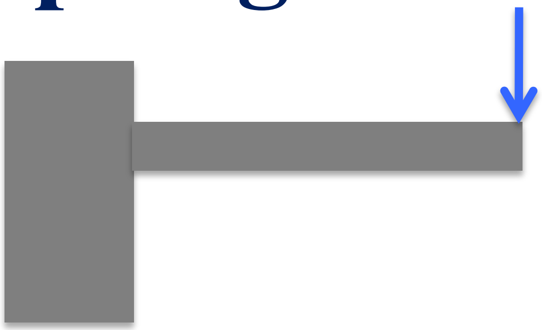
$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

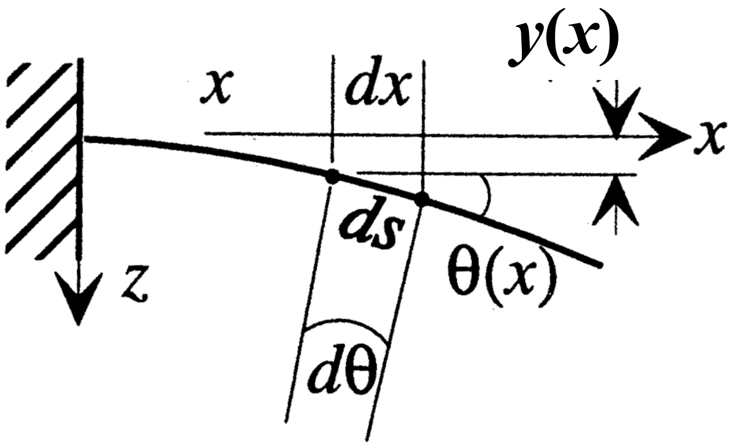
$$= \frac{1}{EI}\cdot F(L-x)$$

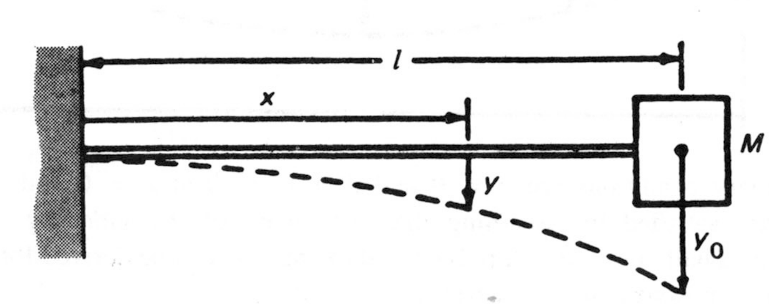
B.C.

$$\begin{cases} y(0) = 0 \\ \dfrac{dy}{dx}(0) = 0 \end{cases}$$

Spring Constant of a Cantilever Beam







$$\frac{d^2y}{dx^2} = \frac{1}{EI} \cdot F(L - x)$$

$$y(x) = \frac{FL}{2EI} x^2 \left(1 - \frac{x}{3L}\right)$$

$$y_{\max} = y(L) = \frac{L^3}{3EI} \cdot F$$

$$\boxed{k = \frac{y_{\max}}{F} = \frac{3EI}{L^3} = \frac{WH^3}{4L^3} E}$$

When the mass of cantilever is neglected, the resonant frequency becomes

$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{3EI}{ML^3}} = \sqrt{\frac{WH^3}{4ML^3} E}$$

How about ω with finite mass of the beam?

Derivation with the Energy Method

$V : Kinetic\ energy$

$T : Strain\ energy$

$V + T = const.$

$$\frac{d}{dt}(V+T)=0$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2\right)=0$$

$$\therefore m\ddot{x}\dot{x}+k\dot{x}x=0$$

$$\therefore m\ddot{x}+kx=0$$

$$\therefore \omega=\sqrt{\frac{k}{m}}$$

$V_{\max} : Kinetic\ energy$

$T_{\max} : Strain\ energy$

$V_{\max} = T_{\max}$

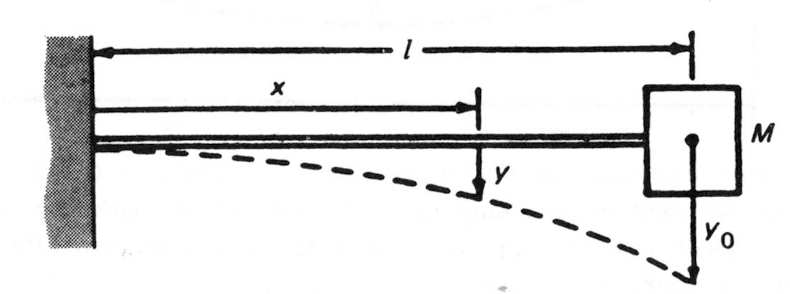
$$\frac{1}{2}m\dot{x}_{\max}^2=\frac{1}{2}kx_{\max}^2$$

$$x_{\max}=x_0,\ \dot{x}_{\max}=\omega x_0$$

$$\therefore \frac{1}{2}m(\omega x_0)^2=\frac{1}{2}kx_0^2$$

$$\therefore \omega=\sqrt{\frac{k}{m}}$$

Derivation with Energy Method



$$k = \frac{y_{\max}}{F} = \frac{3EI}{L^3}$$

$$\therefore 2T = k \cdot y(L)^2 = \frac{3EI}{L^3} y(L)^2$$

Beam shape with applied force F at the tip

$$y(x) = \frac{FL}{2EI} x^2 \left(1 - \frac{x}{3L}\right)$$

$$= \frac{y(L)}{2L^3}\left(3Lx^2 - x^3\right)$$

$$2V = \omega^2 \left\{ M \cdot y(L)^2 + \int_0^L \frac{m}{L} y^2 dx \right\}$$

$$\frac{2V}{\omega^2} \approx M \cdot y(L)^2 +$$

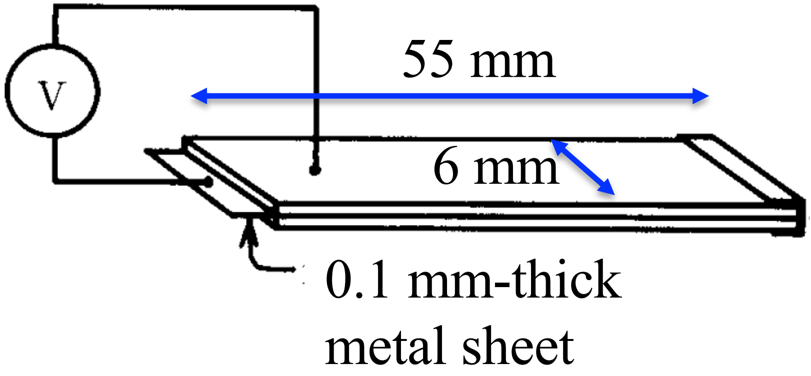
$$\int_0^L \frac{m}{L} \frac{y(L)^2}{4L^6}\left(3Lx^2 - x^3\right)^2 dx$$

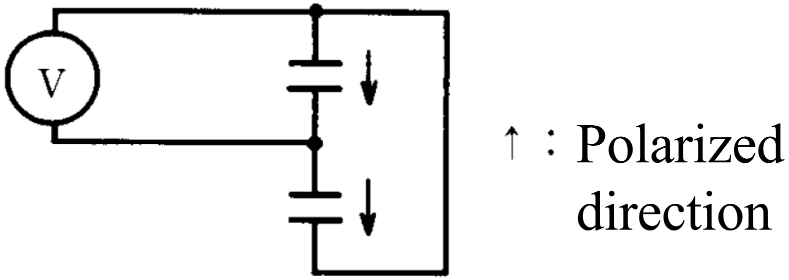
$$= y(L)^2 \left(M + \frac{33}{140} m \right)$$

With $2T = 2V$

$$\boxed{\therefore \omega^2 = \frac{3EI}{\left(M + \frac{33}{140} m \right) L^3}}$$

Specification of Piezoelectric Cantilever





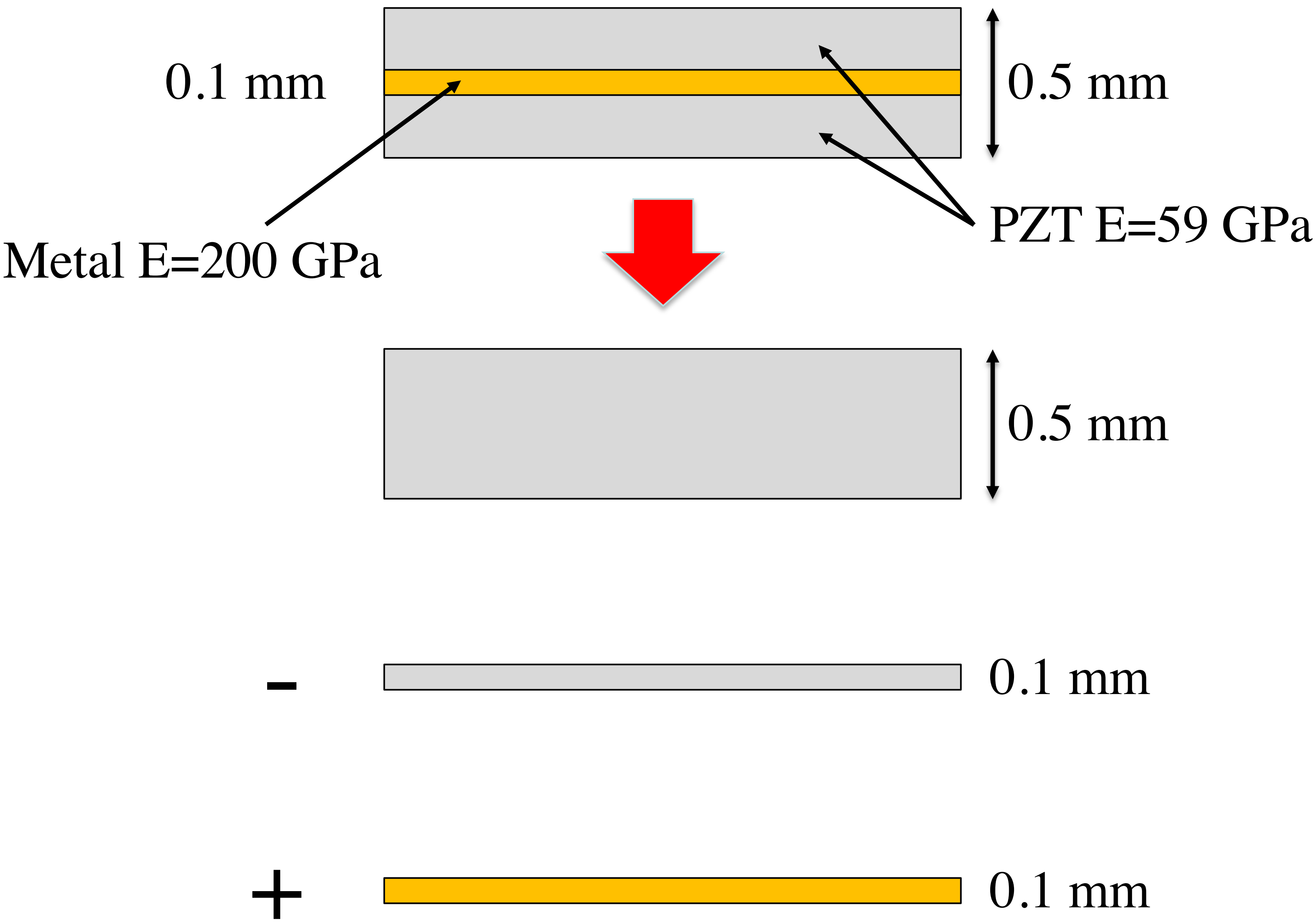
Material: PZT - Pb(Zr·Ti)O_3
Thickness of PZT: 0.2 mm on each side
Young's modulus: 59 GPa
Density: 7.75 g/cm^3
Relative permittiity: 5500
Capacitance: 105 nF
d_{33}=640x10^{-12} C/N
d_{31}=-330x10^{-12} C/N

Total thickness: 0.5 mm
Width: 6 mm
Length: 55 mm

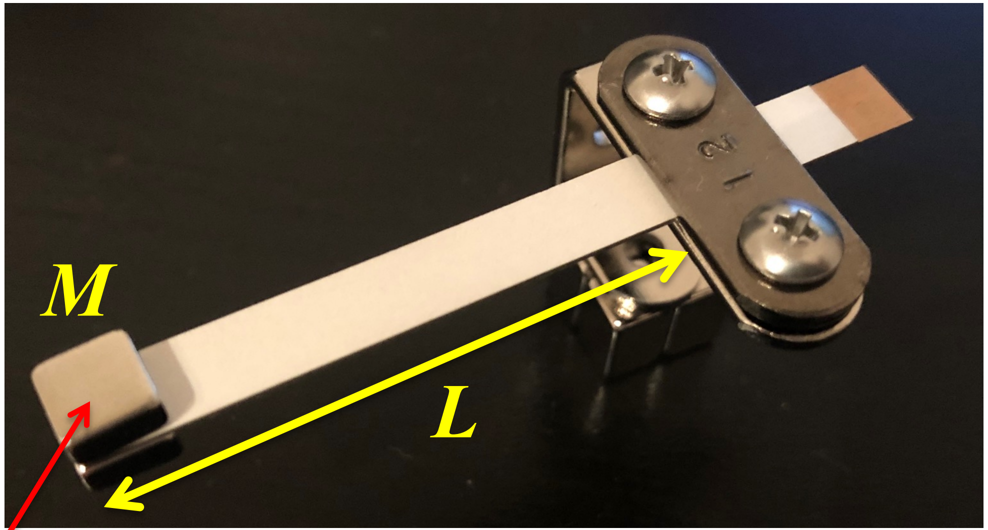
Design Target

Choose the length of the piezoelectric cantilever and the added mass to get the resonant frequency of 30Hz.

Estimate of EI for Multi-layer Structure



Parameters of Piezoelectric Cantilever

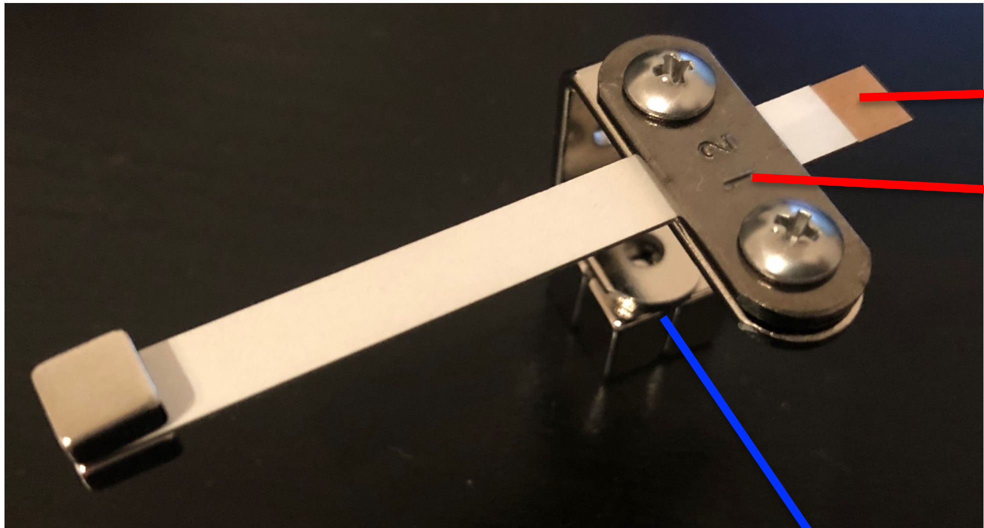


L: **Length of the cantilever (from the tip to the clamp)**
M: **Mass of the magnet (At least a pair of magnet should be put on both sides of the cantilever.)**

Permanent magnet:

Small: 6 mm x 6 mm x 3 mm, 0.81 g
Large: 6 mm x 6 mm x 6 mm, 1.62 g

Experimental Setup



Oscillo-
scope

Monitor the voltage output

Accelerometer

Monitor the acceleration

Amplifier

Change frequency and acceleration