

Design of Broadband Vibration Energy Harvesting Devices

Professor Yuji Suzuki

Department of Mechanical Engineering

The University of Tokyo, Japan

e-mail: ysuzuki@mesl.t.u-tokyo.ac.jp



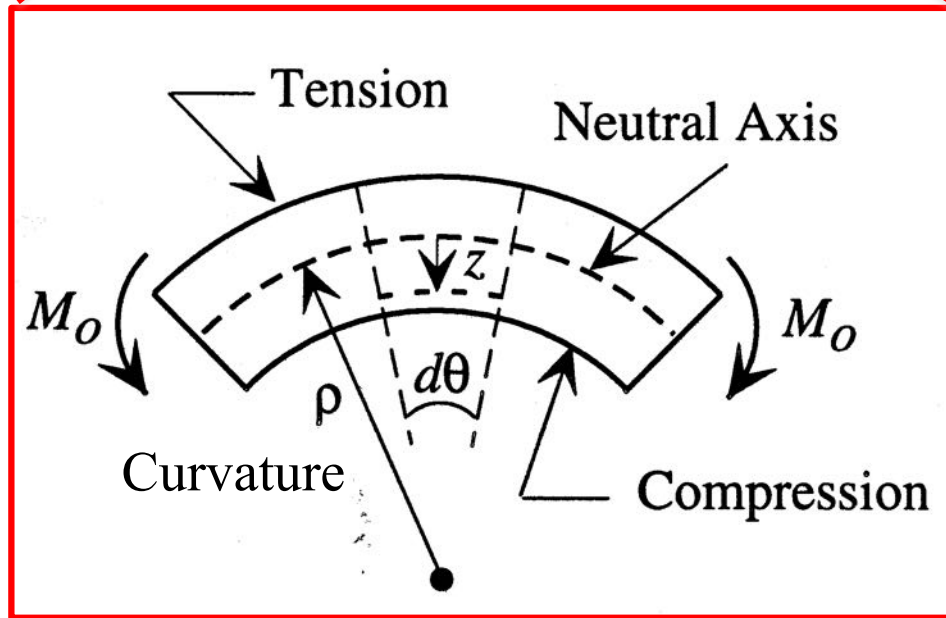
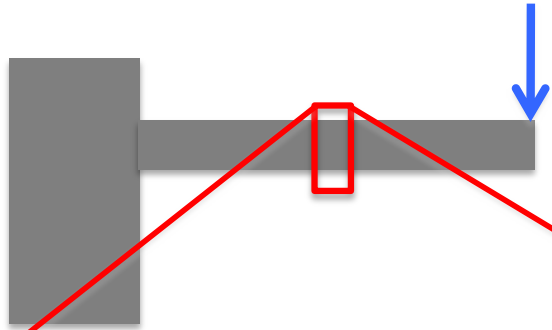
東京大学
THE UNIVERSITY OF TOKYO

Objective

Experience vibration energy harvesting using piezoelectric cantilever

- ✓ **Calculate the length of cantilever and the tip mass to satisfy the given resonance frequency.**
- ✓ **Carry out simple power generation experiment using the piezoelectric cantilever.**

Linear Beam Theory



$$dx = \rho d\theta$$

$$dL = (\rho - z) d\theta$$

$$= dx - \frac{z}{\rho} dx$$

$$\epsilon_x = -\frac{z}{\rho}$$

Strain

$$\sigma_x = -\frac{z}{\rho} E$$

Stress

$$M = \int_{-H/2}^{H/2} W z \sigma_x dz$$

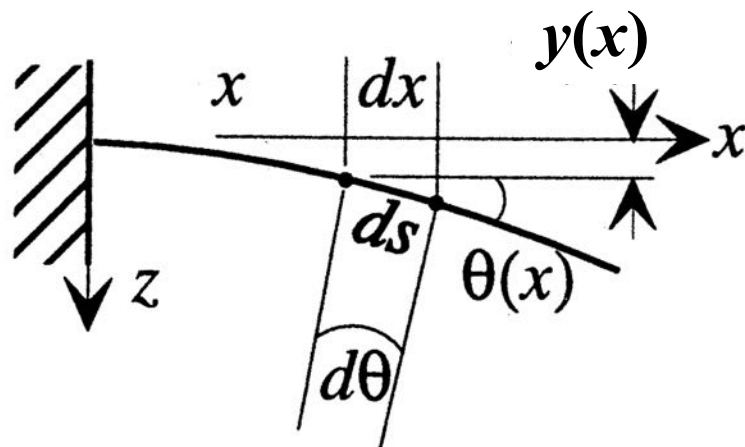
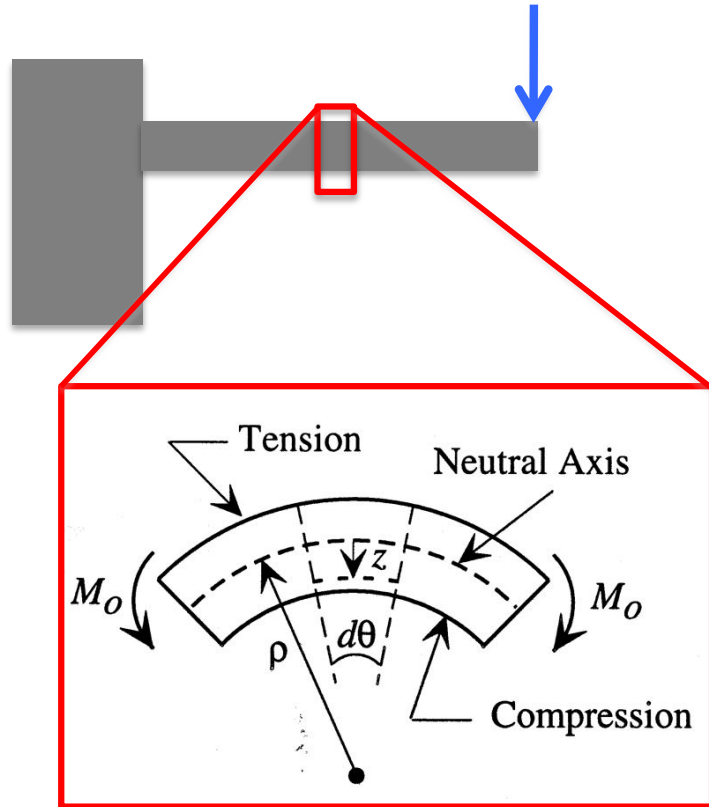
Moment

$$= -\frac{EI}{\rho}$$

$$I = \frac{1}{12} WH^3$$

Moment of
inertia

Linear Beam Theory



$$ds = \rho d\theta$$

$$ds = \frac{dx}{\cos \theta}$$

$$\frac{dy}{dx} = \tan \theta$$

$$\frac{d^2 y}{dx^2} = \frac{1}{\cos^2 \theta} \frac{d\theta}{dx}$$

$$\frac{1}{\rho} = \frac{d^2 y / dx^2}{\left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{3/2}} \sim \frac{d^2 y}{dx^2}$$

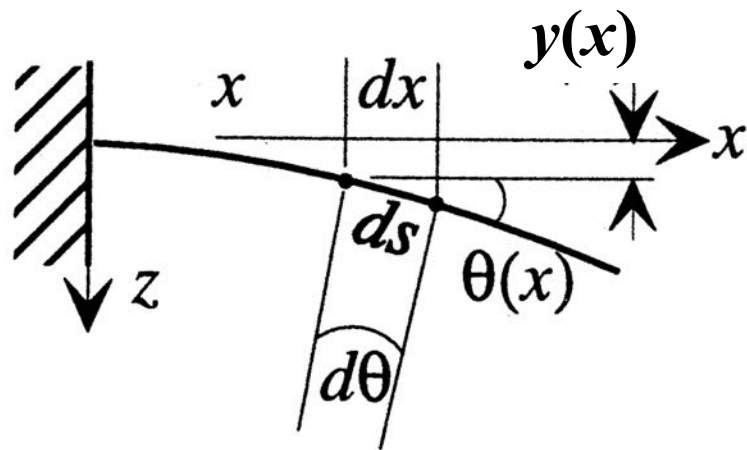
$$\frac{d^2 y}{dx^2} = -\frac{M}{EI}$$

$$= \frac{1}{EI} \cdot F(L - x)$$

B.C.

$$\begin{cases} y(0) = 0 \\ \frac{dy}{dx}(0) = 0 \end{cases}$$

Spring Constant of a Cantilever Beam



$$\frac{d^2 y}{dx^2} = \frac{1}{EI} \cdot F(L - x)$$

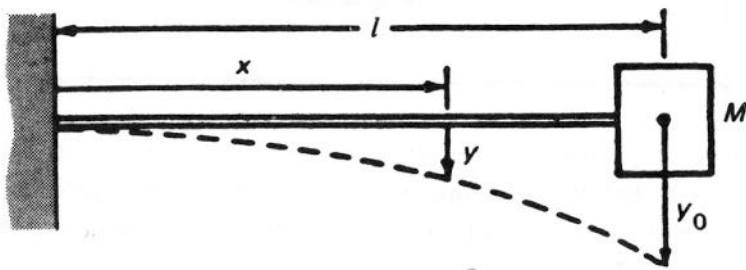
$$y(x) = \frac{FL}{2EI} x^2 \left(1 - \frac{x}{3L} \right)$$

$$y_{\max} = y(L) = \frac{L^3}{3EI} \cdot F$$

$$k = \frac{y_{\max}}{F} = \frac{3EI}{L^3} = \frac{WH^3}{4L^3} E$$

When the mass of cantilever is neglected, the resonant frequency becomes

$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{3EI}{ML^3}} = \sqrt{\frac{WH^3}{4ML^3}} E$$



How about ω with finite mass of the beam?

Derivation with the Energy Method

V : Kinetic energy

T : Strain energy

$$V + T = \text{const.}$$

$$\frac{d}{dt}(V + T) = 0$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right) = 0$$

$$\therefore m\ddot{x}\dot{x} + kx\dot{x} = 0$$

$$\therefore m\ddot{x} + kx = 0$$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

V_{\max} : Kinetic energy

T_{\max} : Strain energy

$$V_{\max} = T_{\max}$$

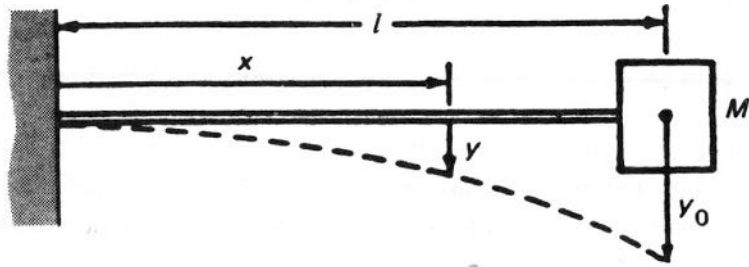
$$\frac{1}{2}m\dot{x}_{\max}^2 = \frac{1}{2}kx_{\max}^2$$

$$x_{\max} = x_0, \dot{x}_{\max} = \omega x_0$$

$$\therefore \frac{1}{2}m(\omega x_0)^2 = \frac{1}{2}kx_0^2$$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

Derivation with Energy Method



$$k = \frac{y_{\max}}{F} = \frac{3EI}{L^3}$$

$$\therefore 2T = k \cdot y(L)^2 = \frac{3EI}{L^3} y(L)^2$$

Beam shape with applied force F at the tip

$$\begin{aligned} y(x) &= \frac{FL}{2EI} x^2 \left(1 - \frac{x}{3L} \right) \\ &= \frac{y(L)}{2L^3} (3Lx^2 - x^3) \end{aligned}$$

$$2V = \omega^2 \left\{ M \cdot y(L)^2 + \int_0^L \frac{m}{L} y^2 dx \right\}$$

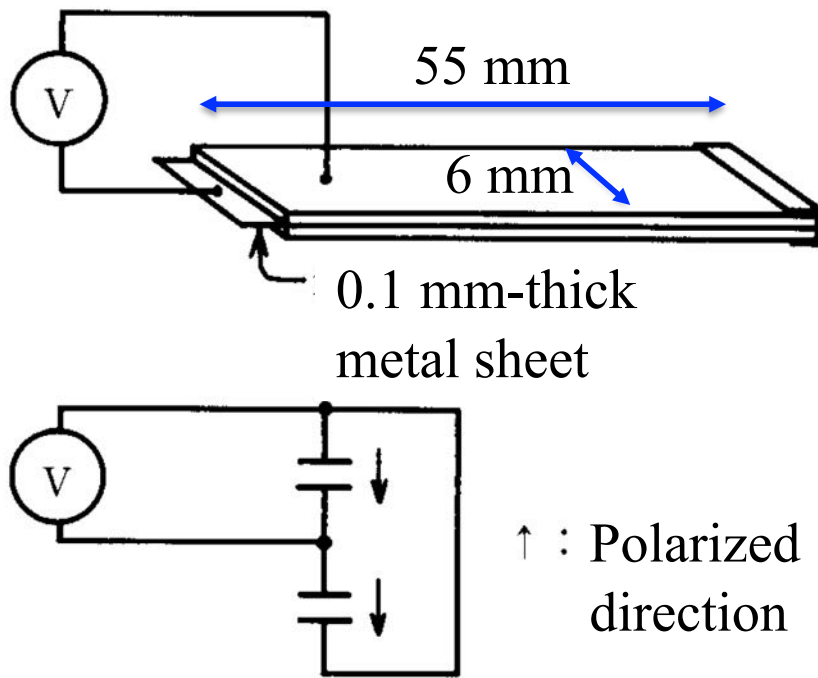
$$\frac{2V}{\omega^2} \approx M \cdot y(L)^2 +$$

$$\begin{aligned} &\int_0^L \frac{m}{L} \frac{y(L)^2}{4L^6} (3Lx^2 - x^3)^2 dx \\ &= y(L)^2 \left(M + \frac{33}{140} m \right) \end{aligned}$$

With $2T = 2V$

$$\therefore \omega^2 = \frac{3EI}{\left(M + \frac{33}{140} m \right) L^3}$$

Specification of Piezoelectric Cantilever



Material: PZT - $\text{Pb}(\text{Zr} \cdot \text{Ti})\text{O}_3$

Thickness of PZT: 0.2 mm on each side

Young's modulus of PZT: 59 GPa

Density of PZT: 7.75 g/cm^3

Relative permittivity: 5500

Capacitance: 105 nF

$d_{33} = 640 \times 10^{-12} \text{ C/N}$

$d_{31} = -330 \times 10^{-12} \text{ C/N}$

↑ : Polarized
direction

Total thickness: 0.5 mm

Width: 6 mm

Total length: 55 mm

Thickness of metal: 0.1 mm

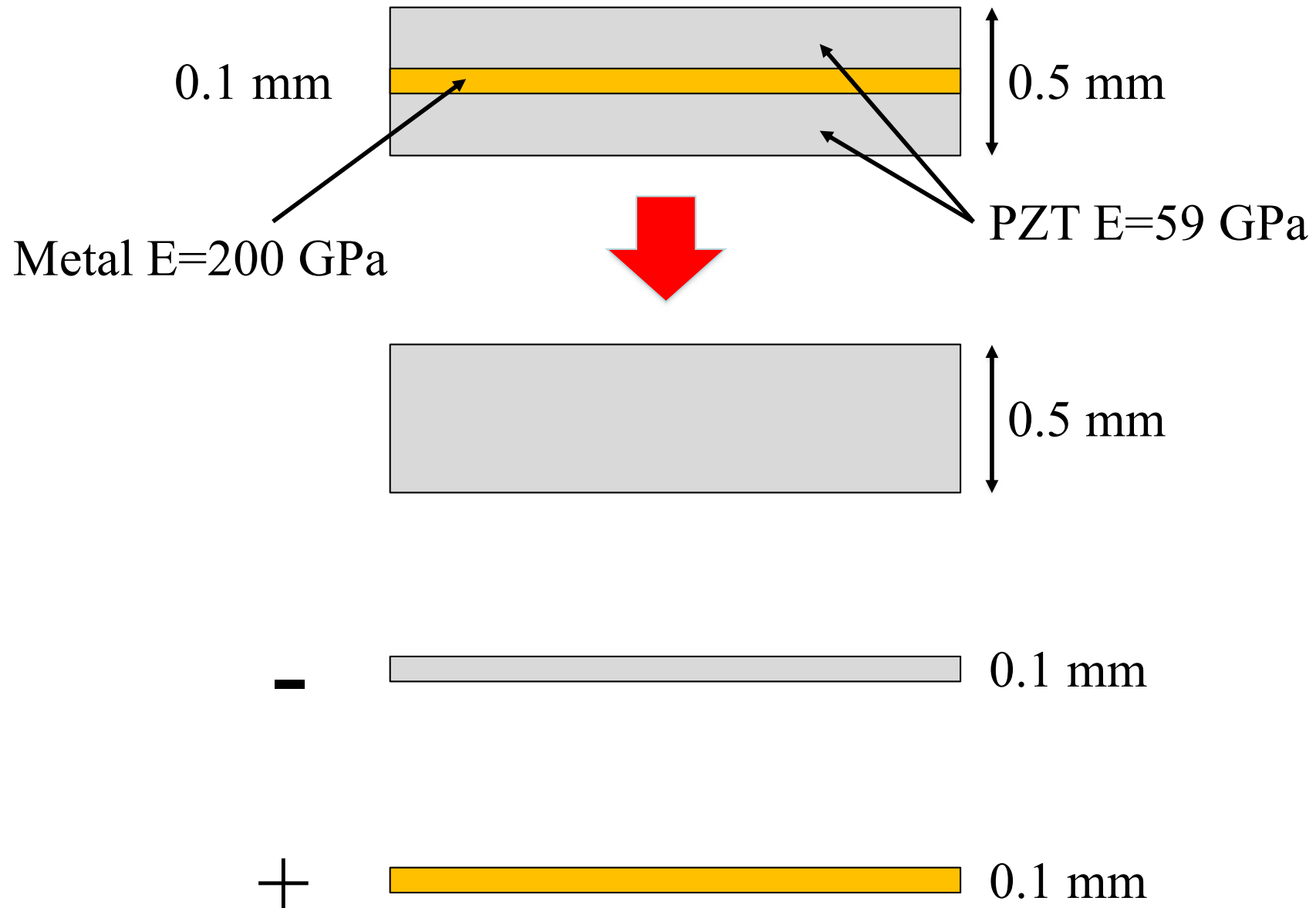
Density of metal: 7.93 g/cm^3

Young's modulus of metal: 200 GPa

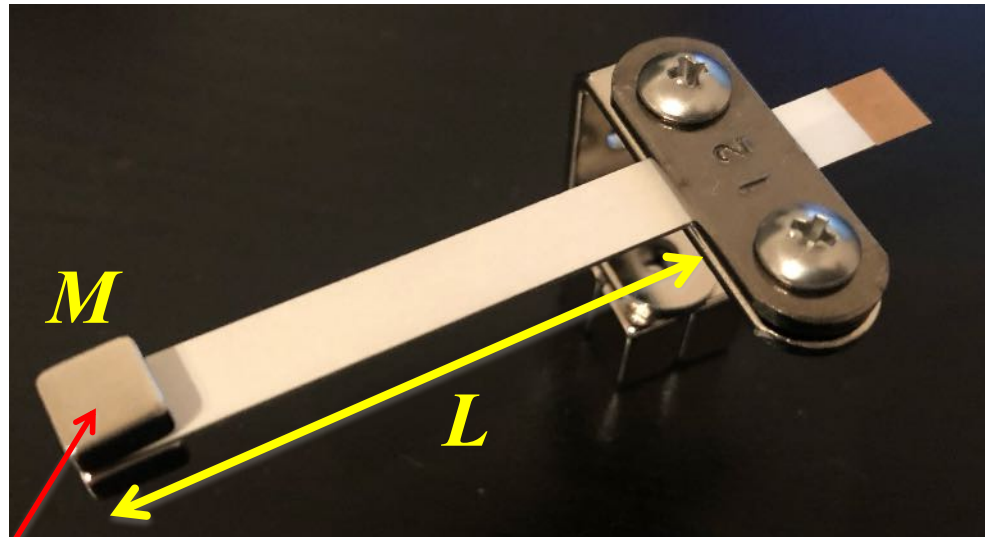
Design Target

Choose the added mass and the cantilever length to get the resonant frequency of XX Hz.

Estimate of EI for Multi-layer Structure



Parameters of Piezoelectric Cantilever



L : Length of the cantilever (from the tip to the clamp $30 \text{ mm} < L < 45 \text{ mm}$)

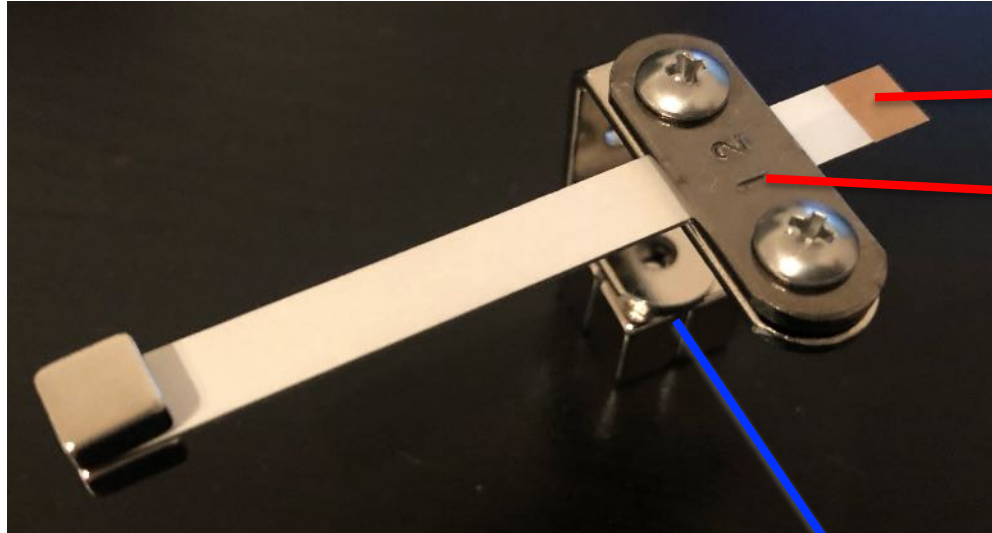
M : Mass of the magnet (At least a pair of magnet should be put on both sides of the cantilever.)

Permanent magnet:

Small: 6 mm x 6 mm x 3 mm, 0.81 g

Large: 6 mm x 6 mm x 6 mm, 1.62 g

Experimental Setup



Oscillo-
scope

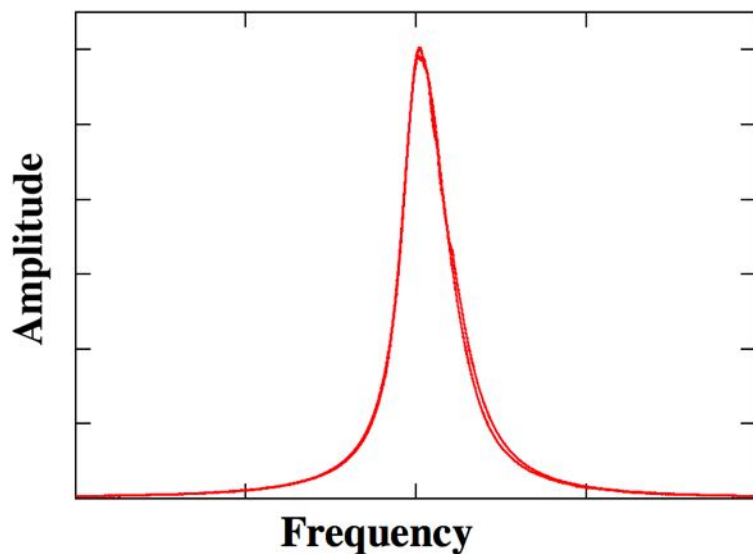
Monitor the voltage
output

Accelerometer

Monitor the
acceleration

Amplifier

Change frequency and
acceleration



Demonstration of Broadband EH with Two Mass Approach

