Design of Broadband Vibration Energy Harvesting Devices

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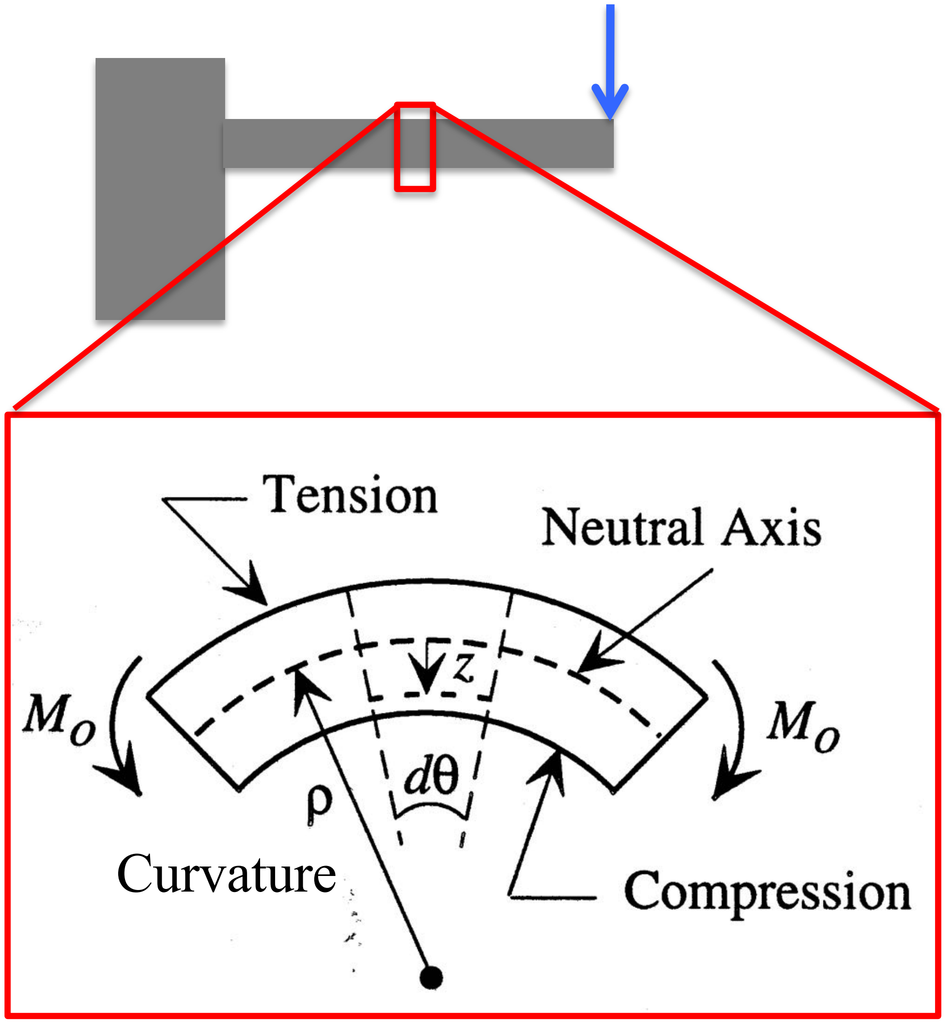
Objective

Experience vibration energy harvesting using piezoelectric cantilever

- ✓ **Calculate the length of cantilever and the tip mass to satisfy the given resonance frequency.**
- ✓ **Carry out simple power generation experiment using the piezoelectric cantilever.**

Beam Theory



ρ: radius of curvature

$$dx = \rho d\theta$$

$$dL = (\rho - z)\, d\theta$$

$$= dx - \frac{z}{\rho} dx$$

$$\varepsilon_x = -\frac{z}{\rho}$$ Strain

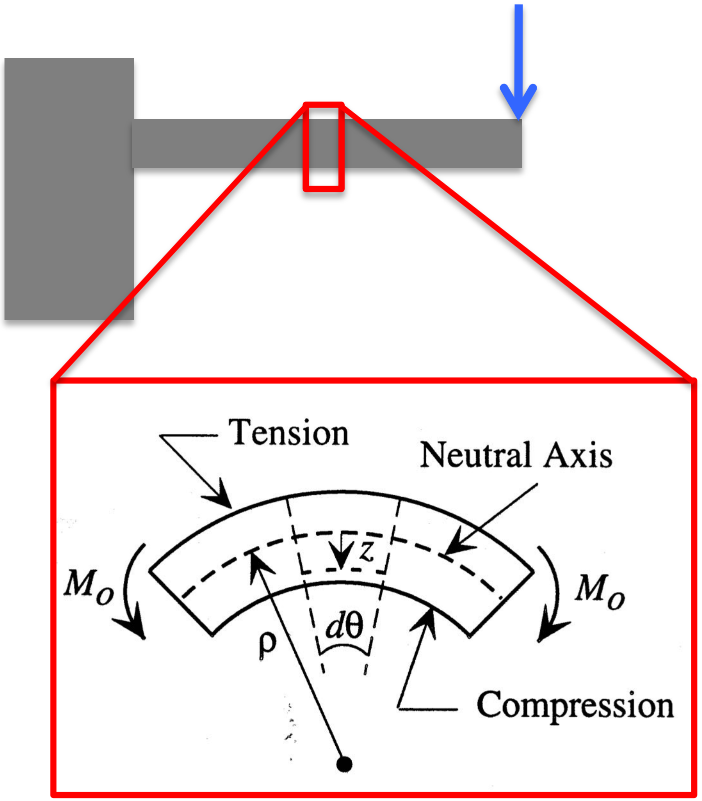
$$\sigma_x = -\frac{z}{\rho} E$$ Stress

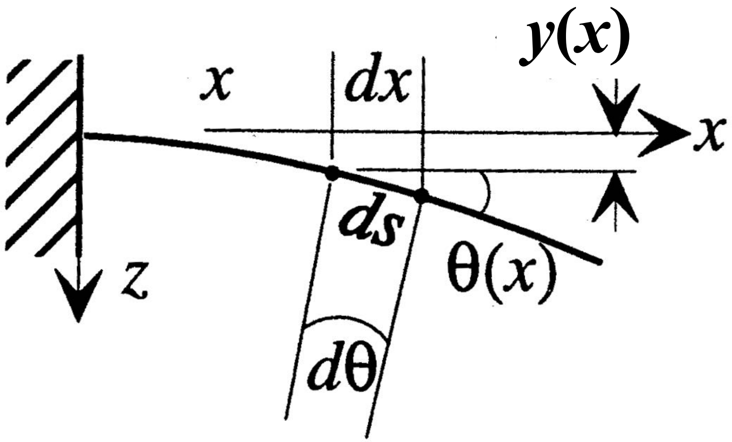
$$M = \int_{-H/2}^{H/2} W z \sigma_x dz$$ Moment

$$= -\frac{EI}{\rho}$$

$$I = \frac{1}{12} W H^3$$ Moment of inertia

Linear Beam Theory





$$ds = \rho d\theta$$

$$ds = \frac{dx}{\cos\theta}$$

$$\frac{dy}{dx} = \tan\theta$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2\theta}\frac{d\theta}{dx}$$

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}} \sim \frac{d^2y}{dx^2}$$

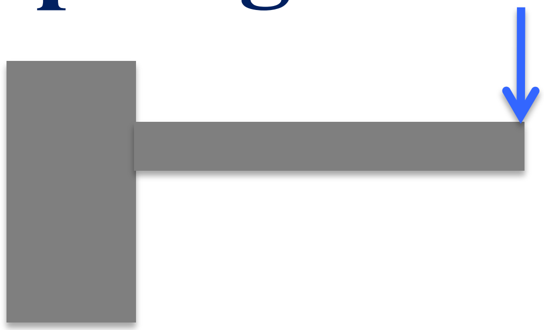
$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

$$= \frac{1}{EI}\cdot F(L-x)$$

B.C.

$$\begin{cases} y(0) = 0 \\ \frac{dy}{dx}(0) = 0 \end{cases}$$

Spring Constant of a Cantilever Beam

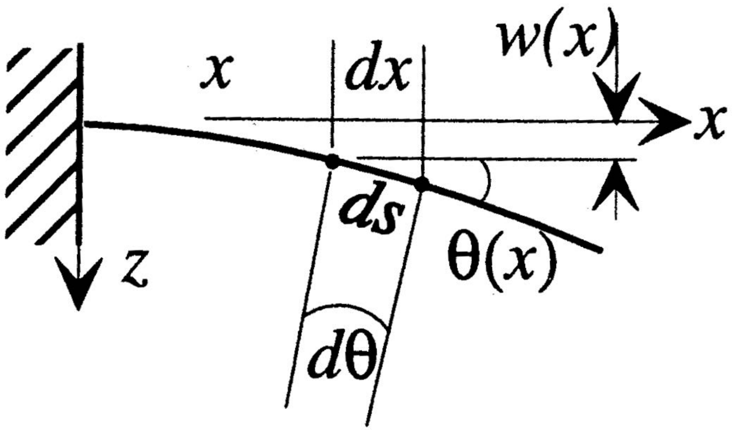


$$\frac{d^2w}{dx^2} = \frac{1}{EI} \cdot F(L-x)$$

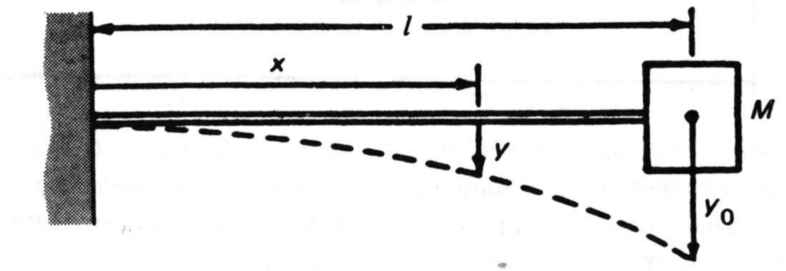
$$w = \frac{FL}{2EI} x^2 \left(1 - \frac{x}{3L}\right)$$

$$w_{\max} = w(L) = \frac{L^3}{3EI} \cdot F$$

$$k = \frac{w_{\max}}{F} = \frac{3EI}{L^3} = \frac{WH^3}{4L^3} E$$



When the mass of cantilever is neglected, the resonant frequency becomes



$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{3EI}{ML^3}} = \sqrt{\frac{WH^3}{4ML^3} E}$$

How about ω with a finite mass of the beam?

Derivation with the Energy Method

V : *Kinetic energy*

T : *Strain energy*

$$V+T=const.$$

$$\frac{d}{dt}(V+T)=0$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2\right)=0$$

$$\therefore m\dot{x}\ddot{x}+k\dot{x}x=0$$

$$\therefore m\ddot{x}+kx=0$$

$$\therefore \omega=\sqrt{\frac{k}{m}}$$

$V_{\max}$: *Kinetic energy*

$T_{\max}$: *Strain energy*

$$V_{\max}=T_{\max}$$

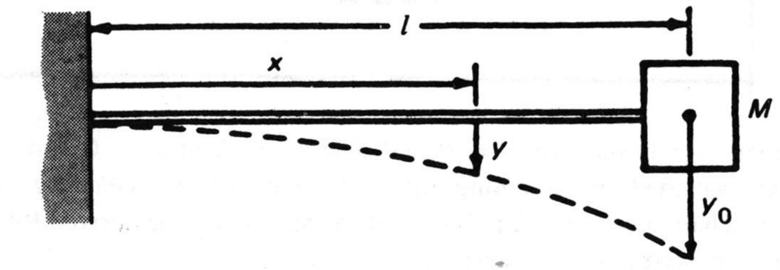
$$\frac{1}{2}m\dot{x}_{\max}^2=\frac{1}{2}kx_{\max}^2$$

$$x_{\max}=x_0\,,\ \dot{x}_{\max}=\omega x_0$$

$$\therefore \frac{1}{2}m(\omega x_0)^2=\frac{1}{2}kx_0^2$$

$$\therefore \omega=\sqrt{\frac{k}{m}}$$

Derivation with Energy Method



$$k = \frac{y_{max}}{F} = \frac{3EI}{L^3}$$

$$\therefore 2T = k \cdot y(L)^2 = \frac{3EI}{L^3} y(L)^2$$

Beam shape with applied force F at the tip

$$y(x) = \frac{FL}{2EI} x^2 \left(1 - \frac{x}{3L}\right)$$

$$= \frac{y(L)}{2L^3}\left(3Lx^2 - x^3\right)$$

$$2V = \omega^2 \left\{ M \cdot y(L)^2 + \int_0^L \frac{m}{L} y^2 dx \right\}$$

$$\frac{2V}{\omega^2} \approx M \cdot y(L)^2 +$$

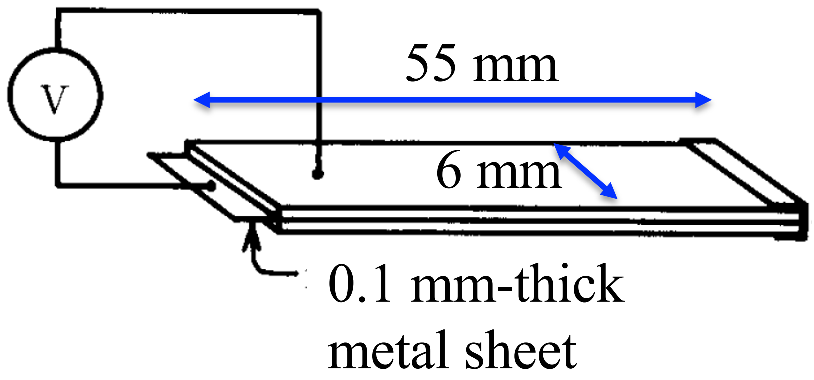
$$\int_0^L \frac{m}{L} \frac{y(L)^2}{4L^6} \left(3Lx^2 - x^3\right)^2 dx$$

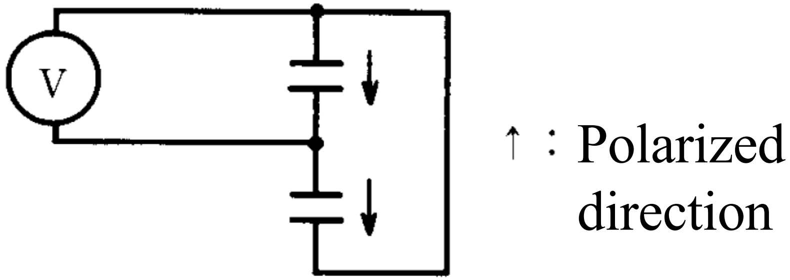
$$= y(L)^2 \left(M + \frac{33}{140} m \right)$$

With $2T = 2V$

$$\boxed{\therefore \omega^2 = \frac{3EI}{\left(M + \frac{33}{140} m\right) L^3}}$$

Specification of Piezoelectric Cantilever





Material: PZT - Pb(Zr·Ti)O_3
Thickness of PZT: 0.2 mm on each side
Young's modulus of PZT: 59 GPa
Density of PZT: 7.75 g/cm^3
Relative permittiity: 5500
Capacitance: 105 nF
d_{33}=640x10^{-12} C/N
d_{31}=-330x10^{-12} C/N

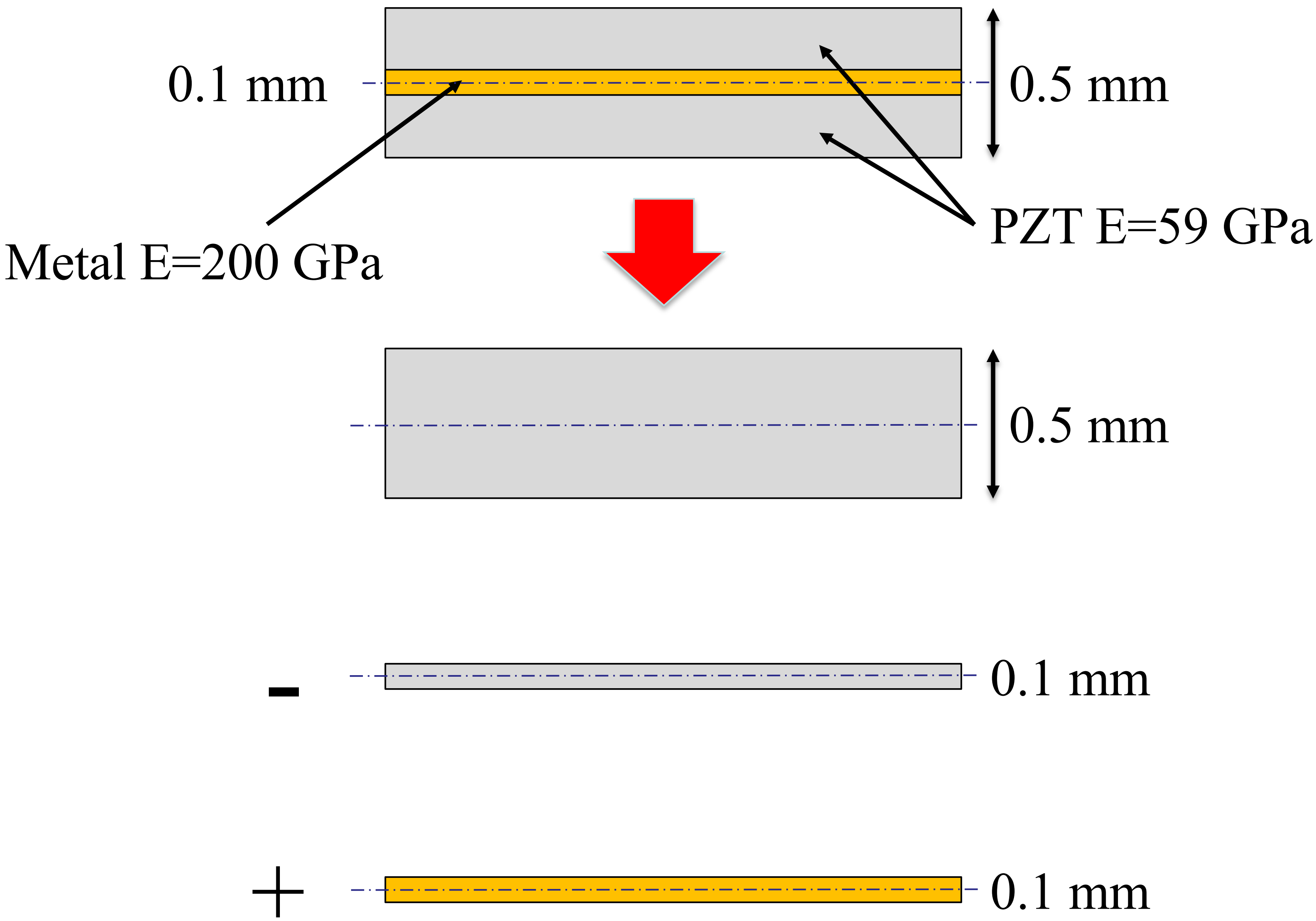
Total thickness: 0.5 mm
Width: 6 mm
Total length: 55 mm

Thickness of metal: 0.1 mm
Density of metal: 7.93 g/cm^3
Young's modulus of metal: 200 GPa

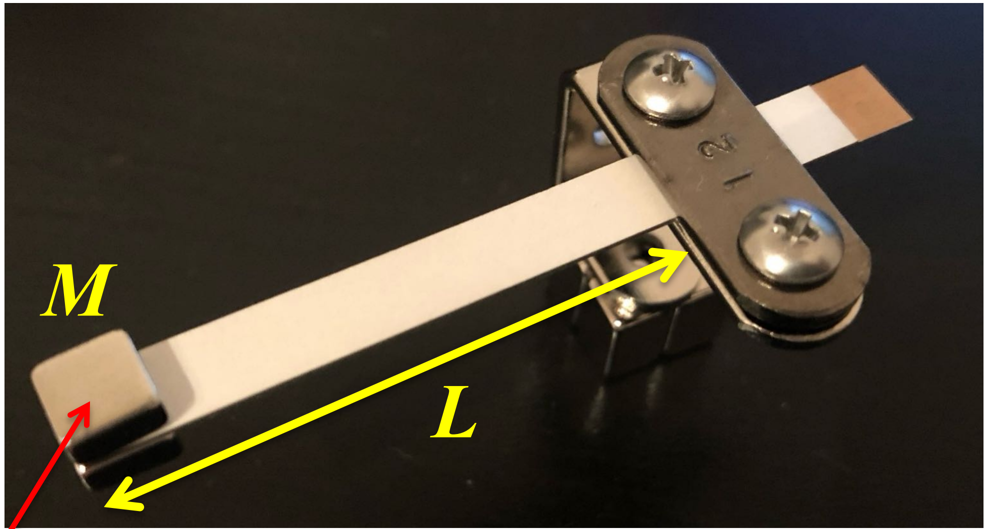
Design Target

Choose the added mass to get the resonant frequency of XX Hz.

Estimate of EI for Multi-layer Structure



Parameters of Piezoelectric Cantilever

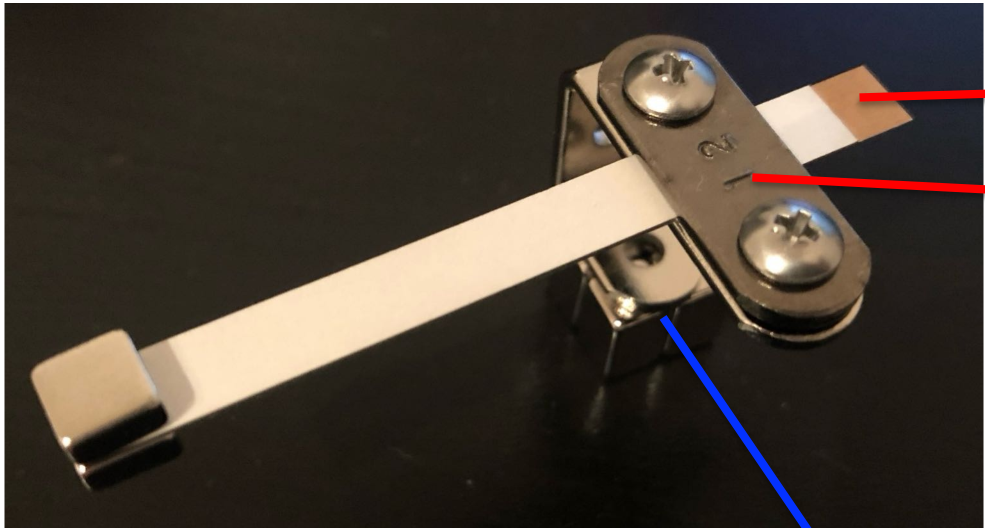


L: Length of the cantilever
M: Mass of the magnet (At least, two magnets are needed; one magnet on each side of the cantilever.)

Permanent magnet:

Small: 6 mm x 6 mm x 3 mm, 0.81 g
Large: 6 mm x 6 mm x 6 mm, 1.62 g

Experimental Setup



Oscillo-scope

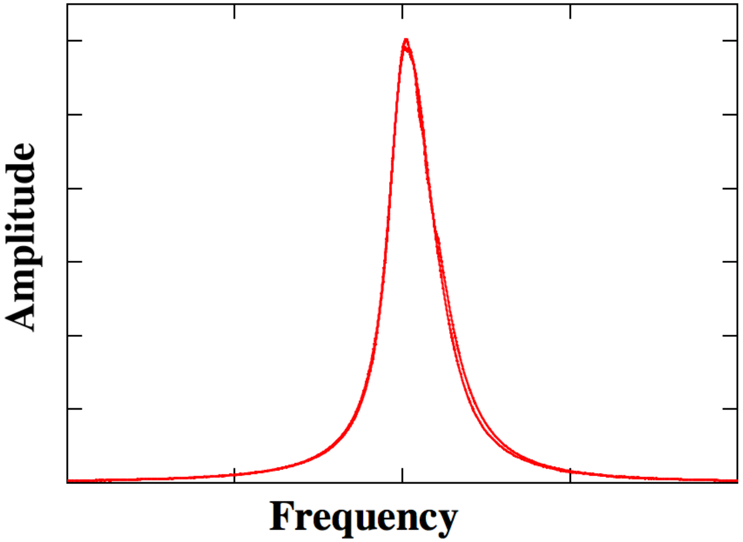
Monitor the voltage output

Accelerometer

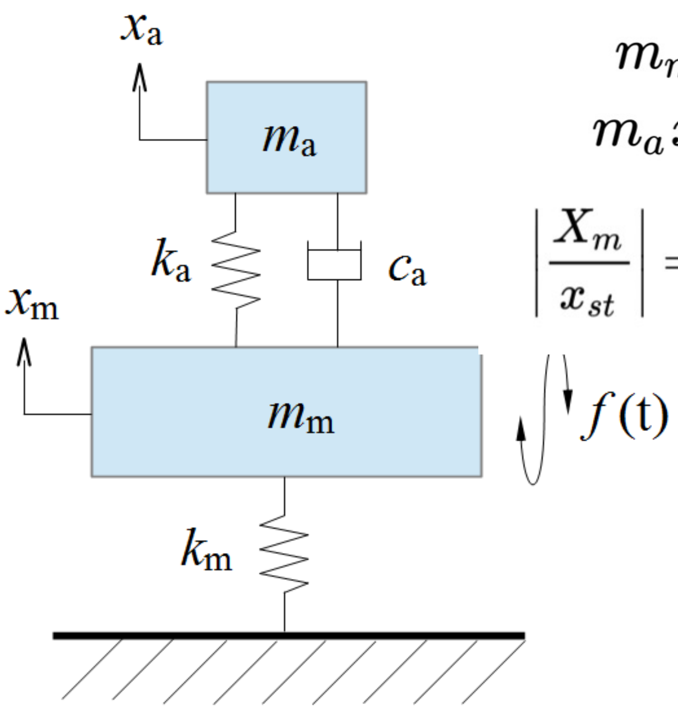
Monitor the acceleration

Amplifier

Change frequency and acceleration



2 DOF System with Damping

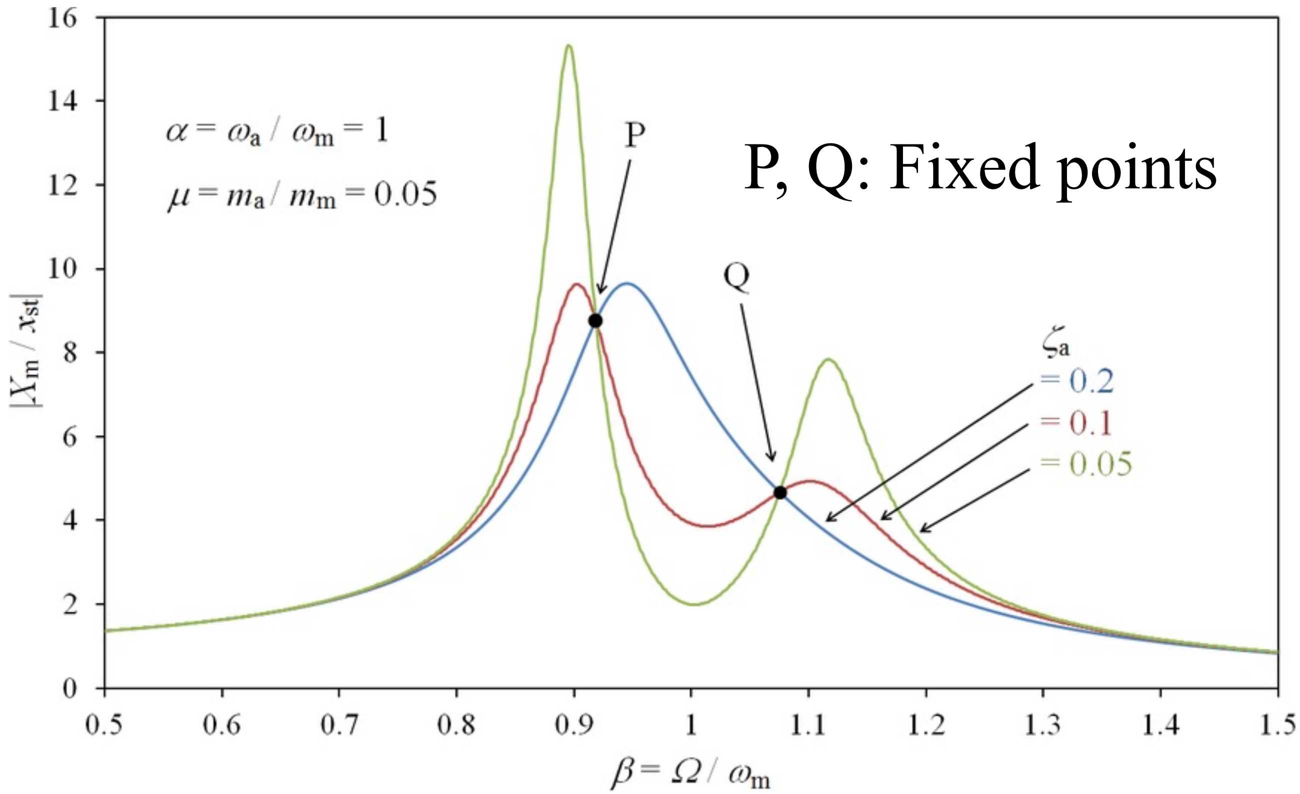


$$m_m\ddot{x}_m + k_m x_m + c_a(\dot{x}_m - \dot{x}_a) + k_a(x_m - x_a) = f_0 e^{i\Omega t}$$

$$m_a\ddot{x}_a + c_a(\dot{x}_a - \dot{x}_m) + k_a(x_a - x_m) = 0$$

$$\left|\frac{X_m}{x_{st}}\right| = \sqrt{\frac{(\alpha^2-\beta^2)^2 + (2\zeta_a\alpha\beta)^2}{\left[(\alpha^2-\beta^2)(1-\beta^2) - \mu\alpha^2\beta^2\right]^2 + (2\zeta_a\alpha\beta)^2(1-\beta^2-\mu\beta^2)^2}}$$

$$\alpha = \frac{\omega_a}{\omega_m},\ \beta = \frac{\Omega}{\omega_m},\ c_{ca} = 2\sqrt{m_a k_a},\ \zeta_a = \frac{c_a}{c_{ca}},\ x_{st} = \frac{f_0}{k_m}$$



Sueoka et al. (2002)
Wikipedia

Demonstration of Broadband EH with Two Mass Approach