

MCTR 702

# Master Advanced Mechatronics

Lecturers : Luc Marechal, Christine Barthod, Georges Habchi

2026

# Contents

## Lecture 3

### RELIABILITY

- 1. Reliability Functions
- 2. Exponential Law
- 3. Weibull Law
- 4. Estimators
- 5. Reliability Modelling

# Definitions

**Quality** = Conformance to specifications or requirements defined by customer at time  $t = 0$

Improvement of the quality can be improved by different methods and techniques:

- ISO 9004:2008
- Total Quality Management (TQM)
- Statistical
- Process control
- Six Sigma
- Quality Function Deployment (QFD)
- Quality Circle,
- Taguchi method
- ....

# Definitions

**Reliability** = ability of a system or component to perform its required functions under stated conditions for a specified period of time

“ Reliability is quality based on time ”

# Definitions

**Maintainability** = ability of an item, under stated conditions of use, to be retained in, or restored to, a state in which it can perform its required functions, when maintenance is performed under stated conditions and using prescribed procedures and resources

- To reduce the chance of failures, **maintenance can be preventive or predictive**

# The 6 Types of Maintenance

## Preventive Maintenance

Includes regular and time-based schedules

## Corrective Maintenance

Occurs when an issue is noticed

### Predetermined Maintenance

Follows a factory schedule

### Condition-based Maintenance

Occurs when a situation or condition indicates maintenance is needed

### Predictive Maintenance

Is data-driven and impacted by present parameters

### Reactive Maintenance

Occurs when a total breakdown or failure appears

# Definitions

**Availability** = probability that a product or system is in operation at a specified time

The simplest representation of availability (A) is:

$$A = \frac{\text{Uptime of system}}{\text{Uptime of system} + \text{Downtime of system}}$$

Uptime depends on reliability of the system whereas downtime depends on maintainability of the system.

Thus availability is function of both reliability and maintainability.

# Definitions

**Failure** = is the opposite ability of reliability, it corresponds to the "cessation of the ability of an entity to perform a required function"

The faulty (failed) state corresponds to an unacceptable state (loss of function)

# Failures

**Failure** is inevitable for engineering systems.

## Impact of failures :

- minor inconvenience and costs
- personal injury
- significant economic loss
- environmental impact
- deaths

## Cause of failure :

- bad engineering design
- faulty manufacturing
- inadequate testing
- human error
- poor maintenance
- lack of protection against excessive stress



Fukushima  
Space X  
Chernobyl accident  
Bhopal gas tragedy  
space shuttle Columbia disaster



# Classification of failures

- Amplitude: Partial, Complete or Total
- Speed of manifestation: Progressive or Sudden
- Amplitude and speed of manifestation: By degradation, Catalectic
- Date of appearance: Early or Youth, Random or Mature, Aging
- Causes: First, Second, By control
- Effects: Minor, Significant, Critical, Catastrophic

# Why is Reliability Important?

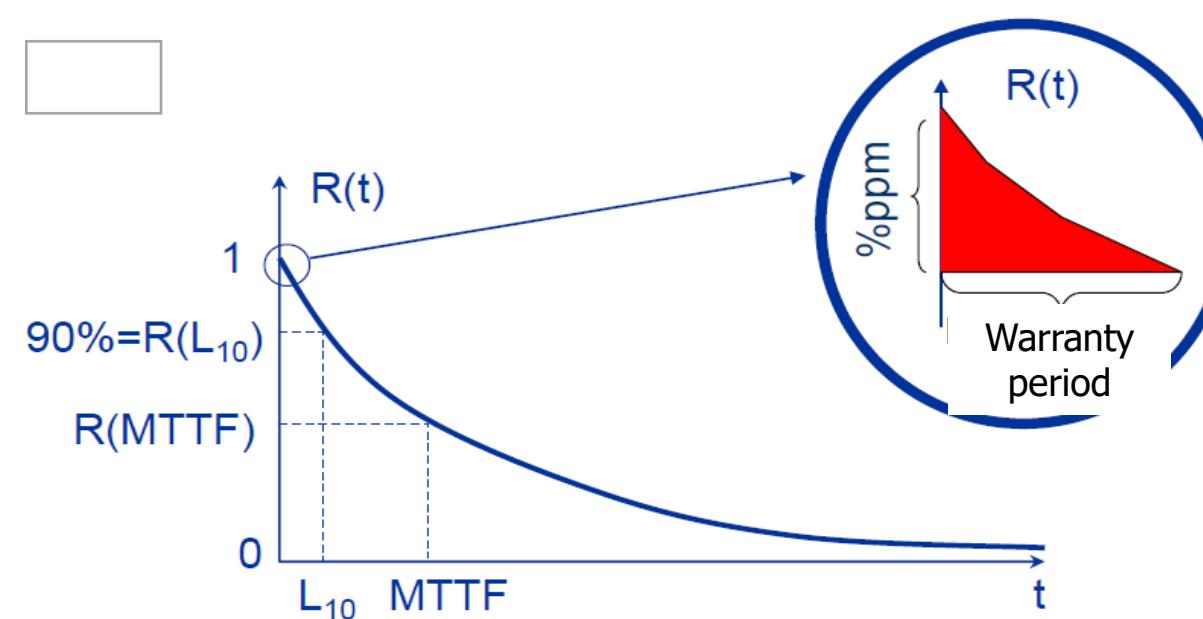
- Reputation. A company's reputation is very closely related to the reliability of its products. The more reliable a product is, the more likely the company is to have a favorable reputation.
- Customer Satisfaction. While a reliable product may not dramatically affect customer satisfaction in a positive manner, an unreliable product will negatively affect customer satisfaction severely. Thus high reliability is a mandatory requirement for customer satisfaction.
- Warranty Costs. If a product fails to perform its function within the warranty period, the replacement and repair costs will negatively affect profits, as well as gain unwanted negative attention. Introducing reliability analysis is an important step in taking corrective action, ultimately leading to a product that is more reliable.
- Repeat Business. A concentrated effort towards improved reliability shows existing customers that a manufacturer is serious about its product, and committed to customer satisfaction. This type of attitude has a positive impact on future business.

# Why is Reliability Important?

- Cost Analysis. Manufacturers may take reliability data and combine it with other cost information to illustrate the cost-effectiveness of their products. This life cycle cost analysis can prove that although the initial cost of a product might be higher, the overall lifetime cost is lower than that of a competitor's because their product requires fewer repairs or less maintenance.
- Customer Requirements. Many customers in today's market demand that their suppliers have an effective reliability program. These customers have learned the benefits of reliability analysis from experience.
- Competitive Advantage. Many companies will publish their predicted reliability numbers to help gain an advantage over their competitors who either do not publish their numbers or have lower numbers

# Reliability requirements used in contracts

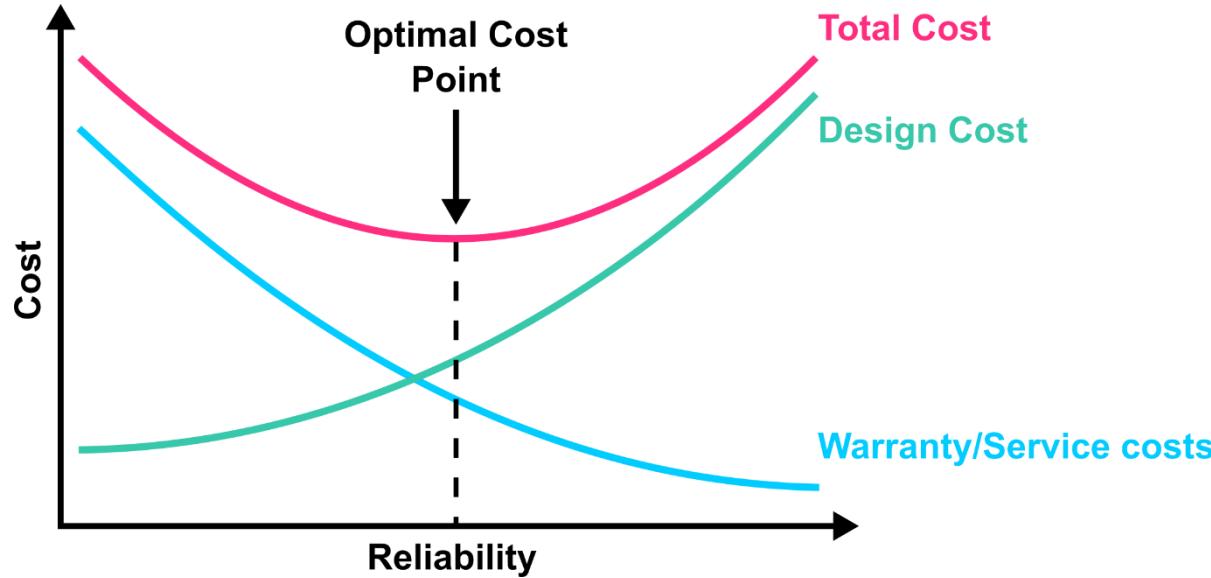
1. Maximum ppm rate observed over the warranty period
2. MTTF
3. L10



# What is the Difference Between Quality and Reliability?

- Even though a product has a reliable design, when the product is manufactured and used in the field, its reliability may be unsatisfactory. The reason for this low reliability may be that the product was poorly manufactured. So, even though the product has a reliable design, it is effectively unreliable when fielded, which is actually the result of a substandard manufacturing process. As an example, cold solder joints could pass initial testing at the manufacturer, but fail in the field as the result of thermal cycling or vibration. This type of failure did not occur because of an improper design, but rather it is the result of an inferior manufacturing process. So while this product may have a reliable design, its quality is unacceptable because of the manufacturing process.
- Just like a chain is only as strong as its weakest link, a highly reliable product is only as good as the inherent reliability of the product and the quality of the manufacturing process.

# Costs vs Reliability



- Effects of Over-reliability in Development
  - Product is too expensive for target market
  - Product is later getting to market
  - Company is behind technology leaders due to slow program development cycles
  
- Effects of Under-reliability in Development
  - High field Return Rate
  - High Warranty Cost
  - Loss of product sales once low reliability is known in market
  - Loss of market share in all product lines due to poor brand perception.

# 1. RELIABILITY FUNCTIONS

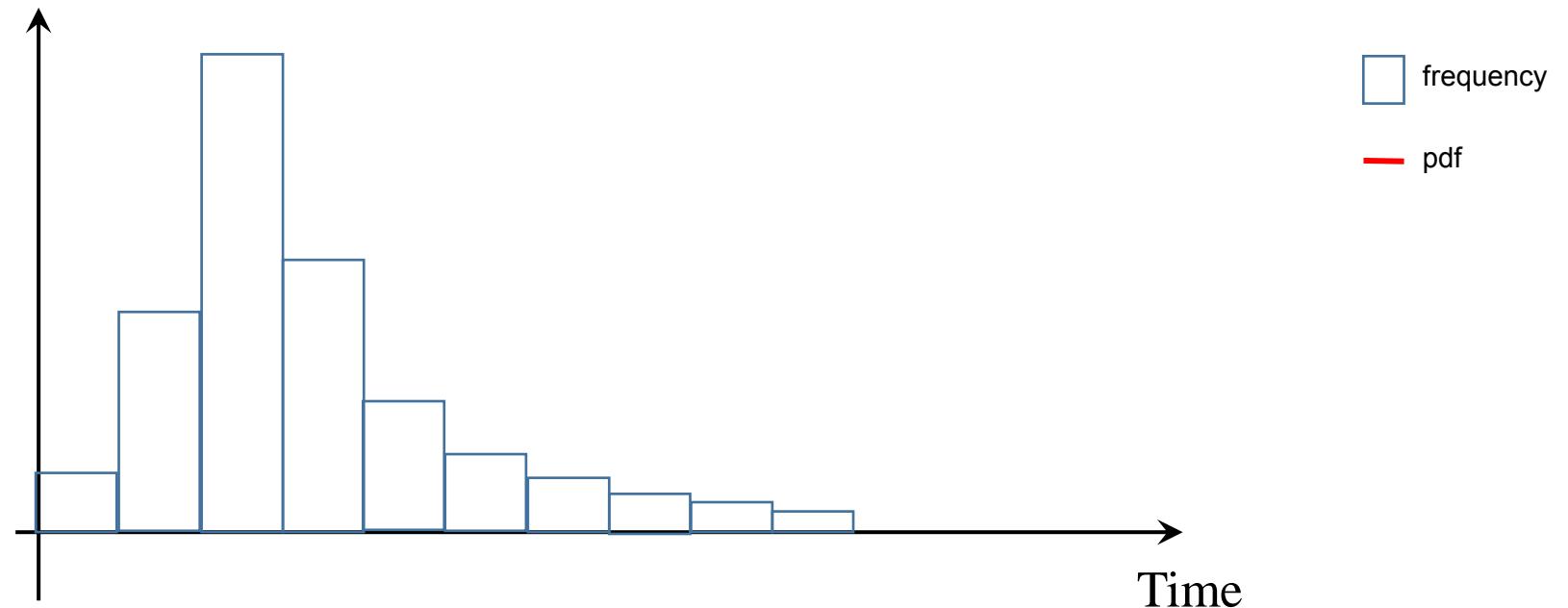
- The failure function  $F(t)$
- The reliability function  $R(t)$
- Complementarity between  $F(t)$  and  $R(t)$
- The failure rate  $I(t)$
- The density function  $f(t)$

# Reliability Functions & Estimators

- Reliability functions
  - The failure function  $F(t)$
  - The reliability function  $R(t)$
  - Complementarity between  $F(t)$  and  $R(t)$
  - The failure rate  $\lambda(t)$
  - The density function  $f(t)$
- Reliability function estimators
  - Estimation of  $F(t)$
  - Estimation of  $R(t)$
  - Estimation of  $\lambda(t)$
  - Estimation of  $f(t)$
  - Estimation of the mean  $E(t)$

# The density function: $f(t)$

It represents the **histogram of the relative failures** as a function of time or the probability frequency of the relative failures relative to the unit of time.



# The density function: $f(t)$

Example to better understand PDF

Take the example of heights of students in a class.

Let's say that the height of all the students is between 160 cm and 170 cm.

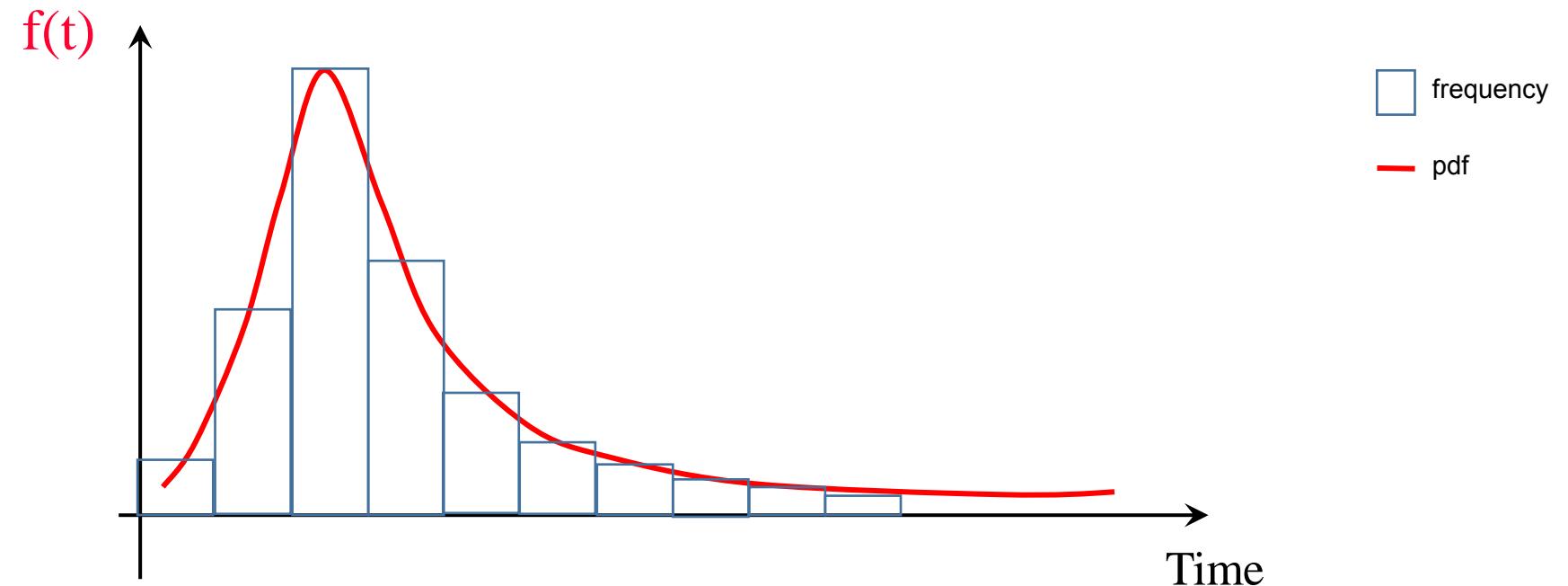
It makes no sense to ask the probability that the height of the student is EXACTLY 165.84 cm; that probability is zero.

We can, however, define the probability that the height of the student lies in the infinitesimal interval [163;166] cm for example.

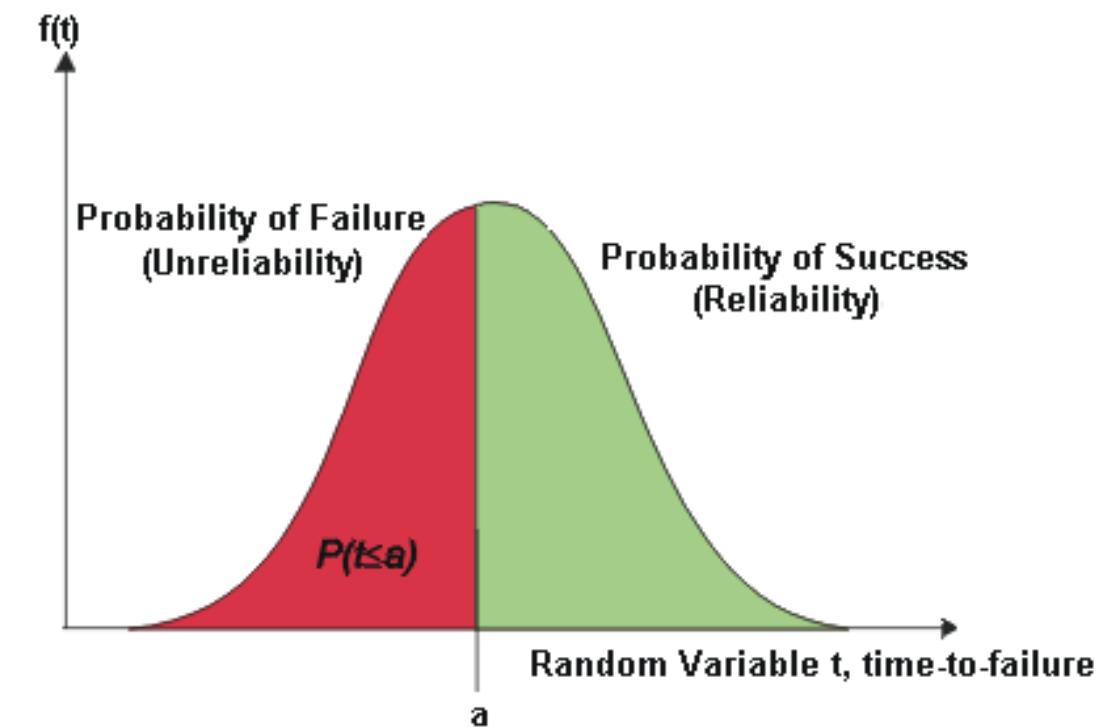
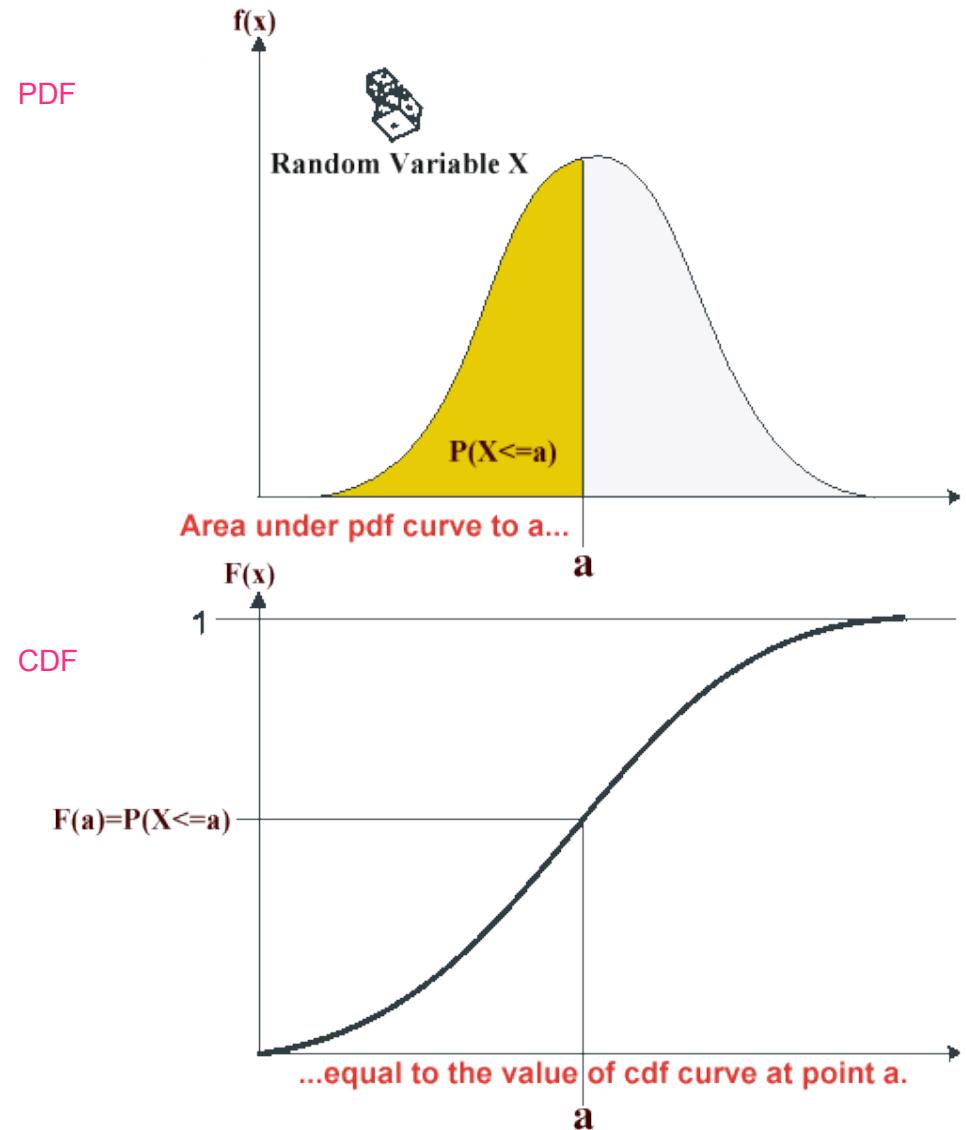
The function that gives this probability density is referred to as the probability density function or pdf.

# The density function: $f(t)$

It represents the **histogram of the relative failures** as a function of time or the probability frequency of the relative failures relative to the unit of time.



# The density function $f(t)$ and Failure function $F(t)$



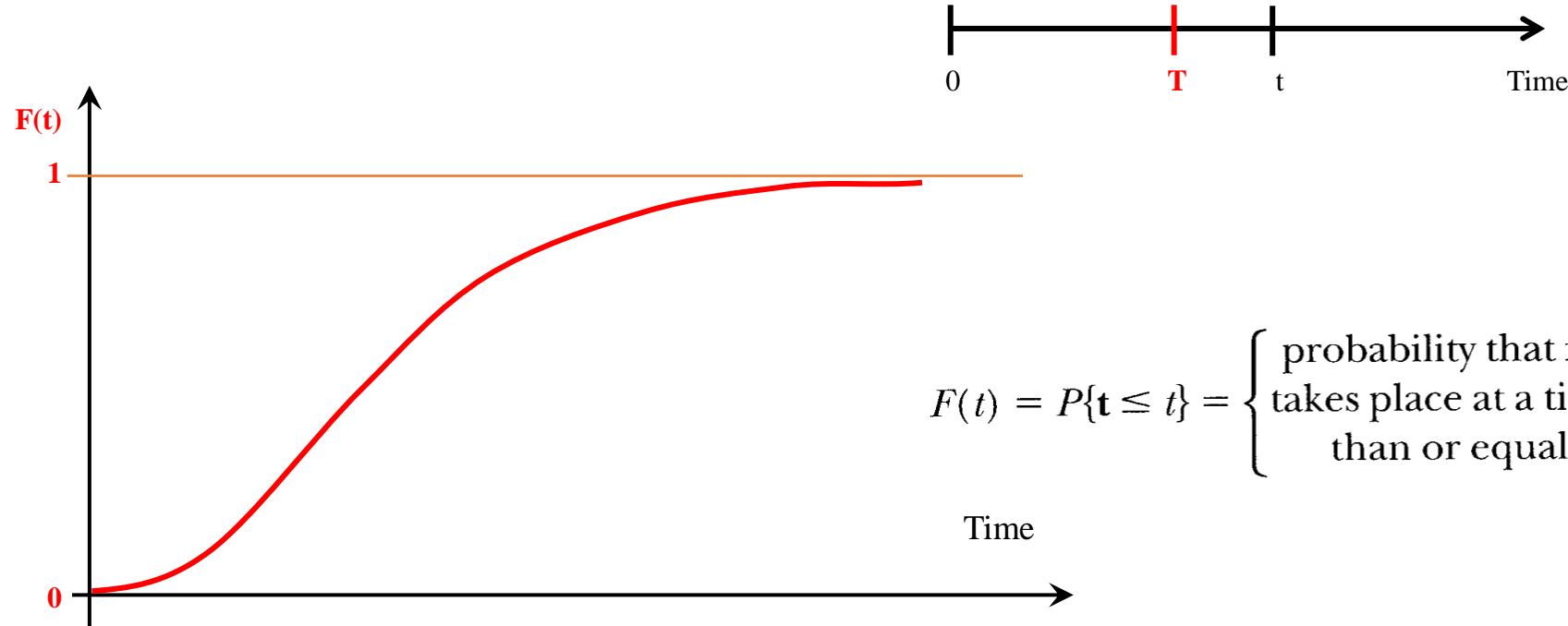
# The failure function: $F(t)$

It is measured by the probability that an entity E fails over the time interval  $[0, t]$ :

$$F(t) = \Pr(E \text{ failing on } [0, t])$$

That is, if we assume that an entity is failing at a date  $T$ :

$$F(t) = \Pr(T \leq t)$$

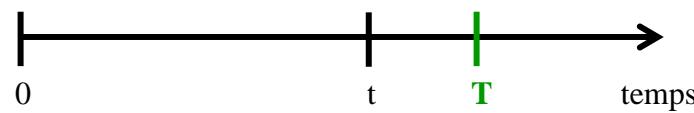
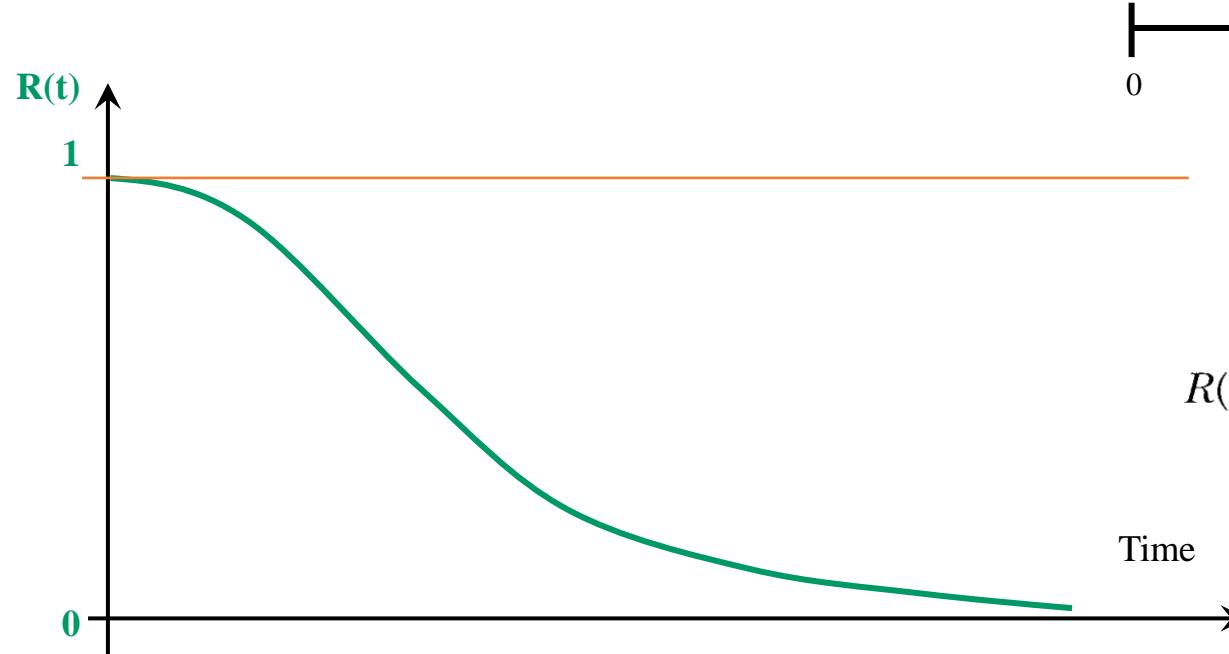


# The reliability function: $R(t)$

It is measured by the probability that an entity E is non-faulty over the time interval  $[0, t]$ :  
 $R(t) = \Pr(E \text{ not failing on } [0, t])$

That is, if we assume that an entity is failing at a date T:

$$R(t) = \Pr(T > t)$$

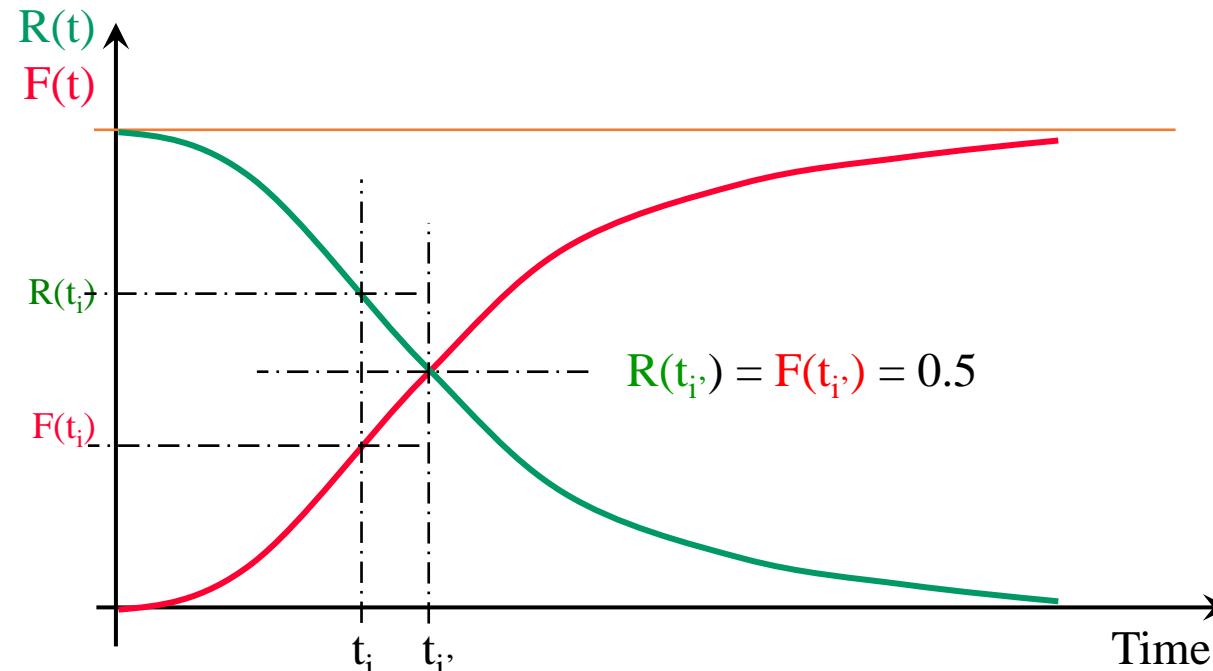


$$R(t) = P\{t > t\} = \begin{cases} \text{probability that a system} \\ \text{operates without failure} \\ \text{for a length of time } t \end{cases}$$

# Complementarity between $F(t)$ and $R(t)$

For a given system,  $\forall t = t_i$ : we have

$$R(t_i) + F(t_i) = 1$$



# Conditional Reliability Function

Conditional reliability is defined as :

The probability that a component or system will operate without failure for a mission time,  $t$ , given that it has already survived to a given time,  $T$

$$R(t|T) = \frac{R(t+T)}{R(T)}$$

# Reliability Function

Several distributions defined by parameters are commonly used for reliability models, including:

- Exponential
- Weibull
- Gamma
- normal (Gaussian)
- log-normal
- log-logistic.

The choice of parametric distribution for a particular application can be made using graphical methods or using formal tests of fit.

The **exponential law** is well adapted when **the failure rate ( $\lambda$ ) is constant**, (i.e. independent of time).

The **normal law** represents behaviours where the lifetime of the population is homogeneous, the probability of failure is **centred and symmetrical**. It can be used to model maturity behaviour (without breakdowns) and then rapid wear and tear.

The **Weibull's law** models each of the three phases of a material's life. It generalises the two previous laws but is more difficult to use and interpret.

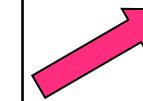
# Reliability distribution and parameters

Technology	Comments	Reliability distribution and parameters
Electronics	Known distribution and parameters available in Database	<b>EXPONENTIAL LAW :</b> MTTF* or $\lambda$ (failure rate)
Mechanics	<ul style="list-style-type: none"><li>Distribution are known for few standard elements</li><li>To find for most of specific components</li></ul>	<b>WEIBULL LAW :</b> <ul style="list-style-type: none"><li><math>\beta</math> (shape parameter)</li><li><math>\eta</math> (scale parameter)</li></ul>

\*MTTF : Mean Time To Failure



FIDES 2009  
(reliability database for electronic components)



Feedback or estimation of reliability parameters for mechanical components

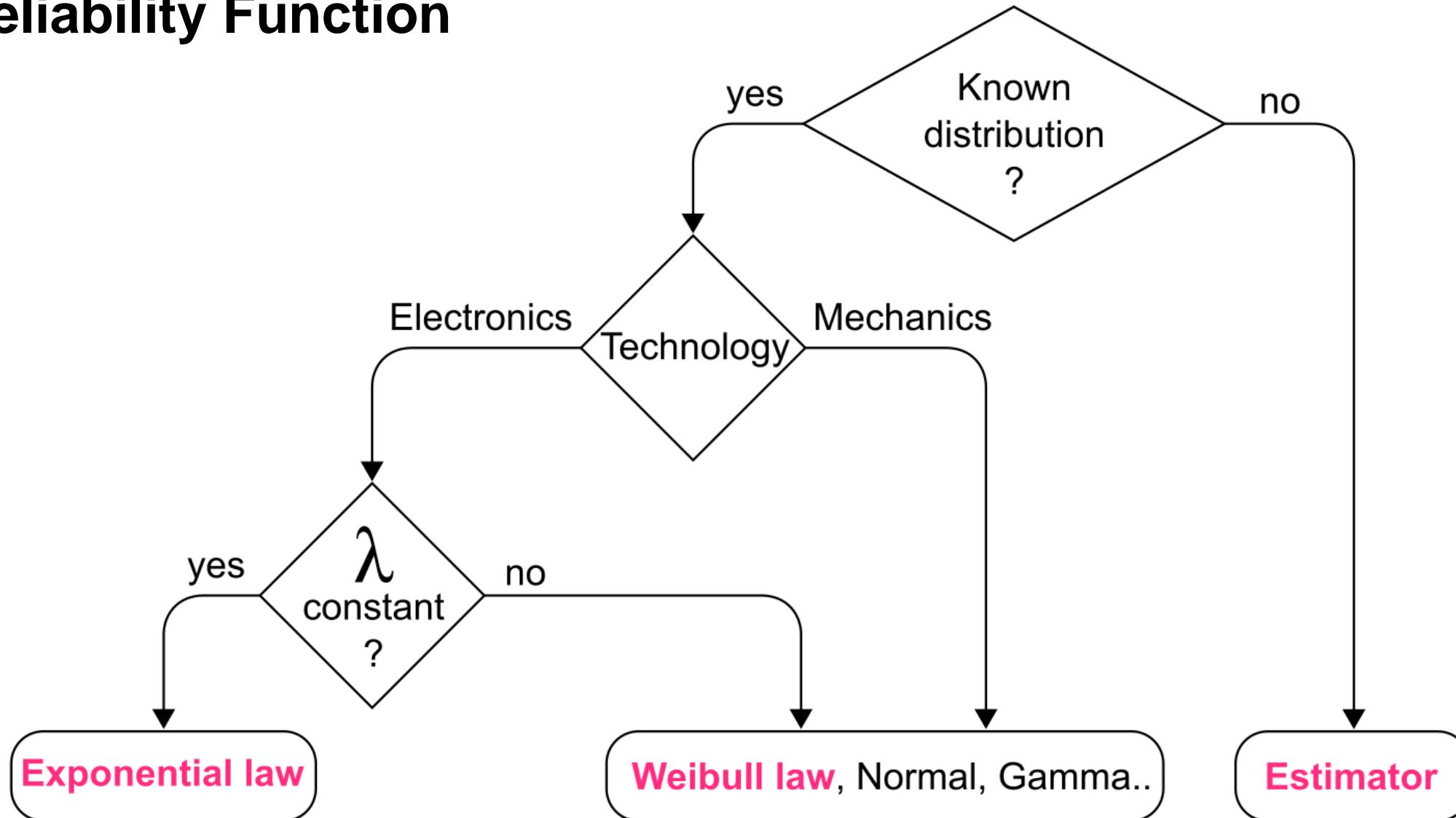
# FIDES database

The expression of the failure rate or the MTTF of electronic components depends on several factors that themselves depend on 3 steps:

- design technology
- manufacturing process
- environmental operation



# Reliability Function



## 2. Exponential Law

# Exponential distribution

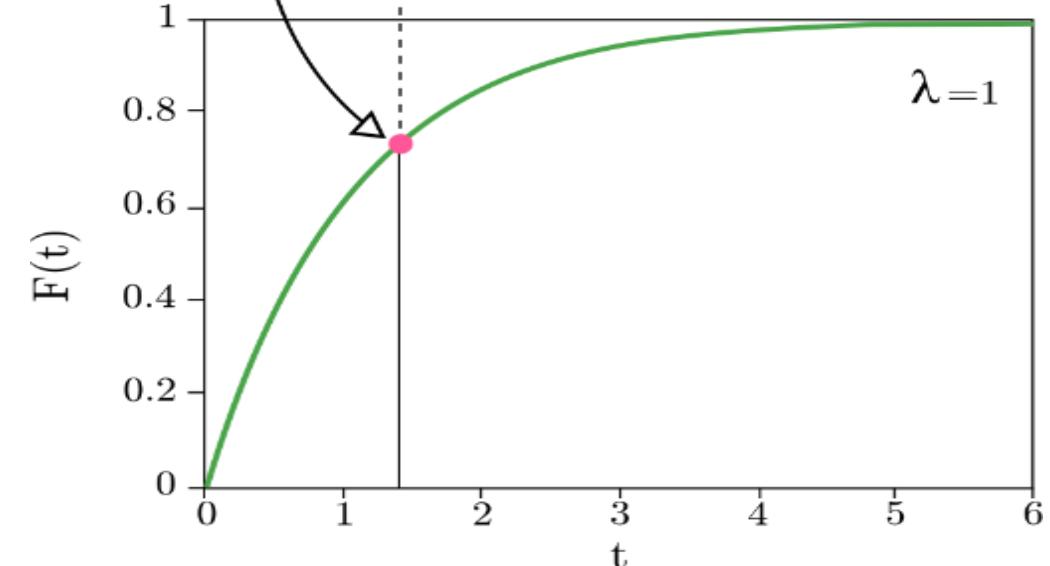
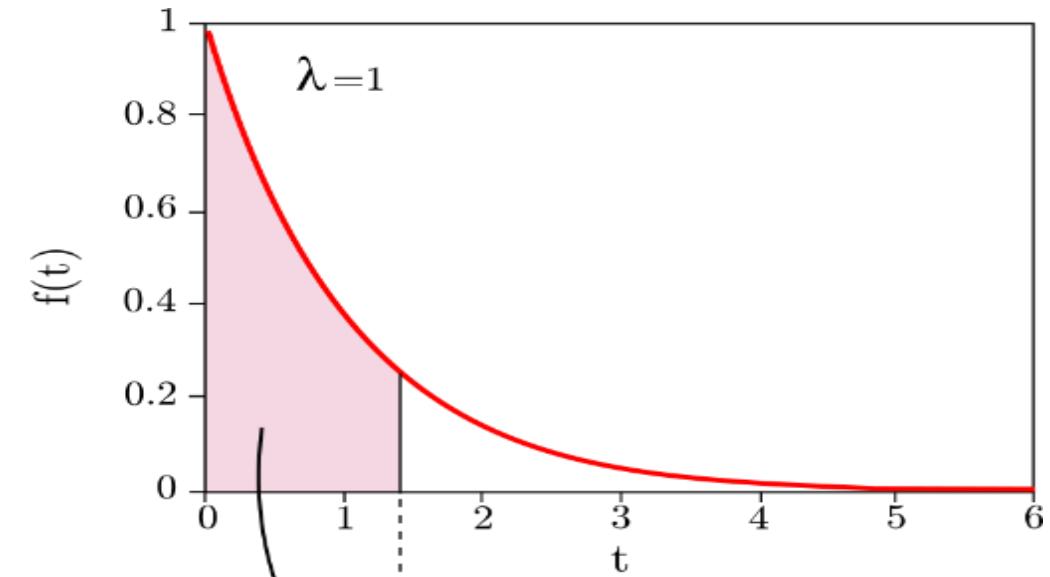
$F(t)$  in this particular case with the exponential distribution, is the Failure function of a product when the **failure rate  $\lambda$  is constant**. The failure density is linked to the failure rate by the following relation:

$$f(t) = \lambda \cdot e^{(-\lambda \cdot t)}$$

$$F(t) = 1 - e^{(-\lambda \cdot t)}$$

$$MTTF : E(t) = 1/\lambda$$

$$R(t) = e^{(-\lambda \cdot t)}$$

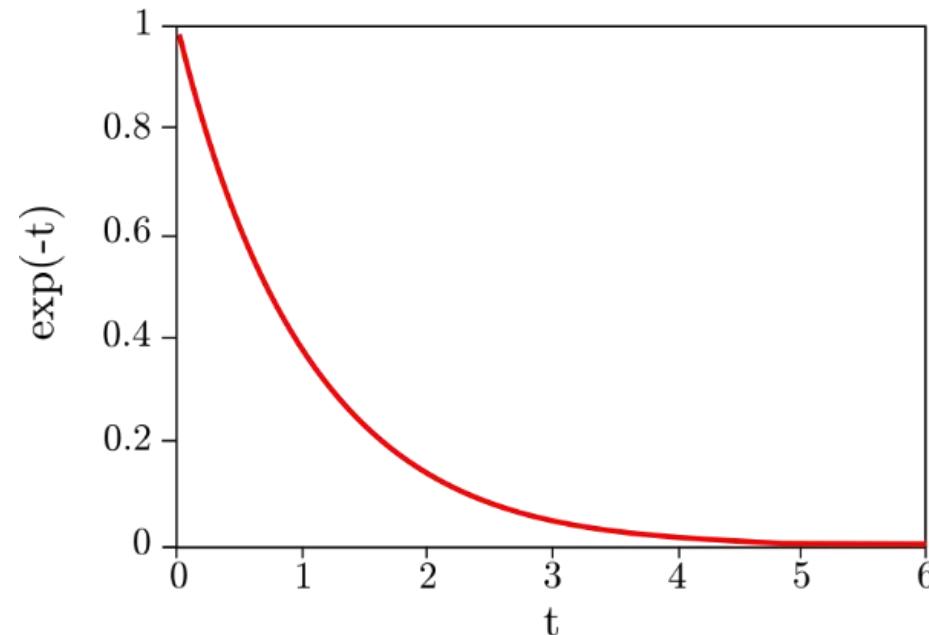


# Exponential distribution

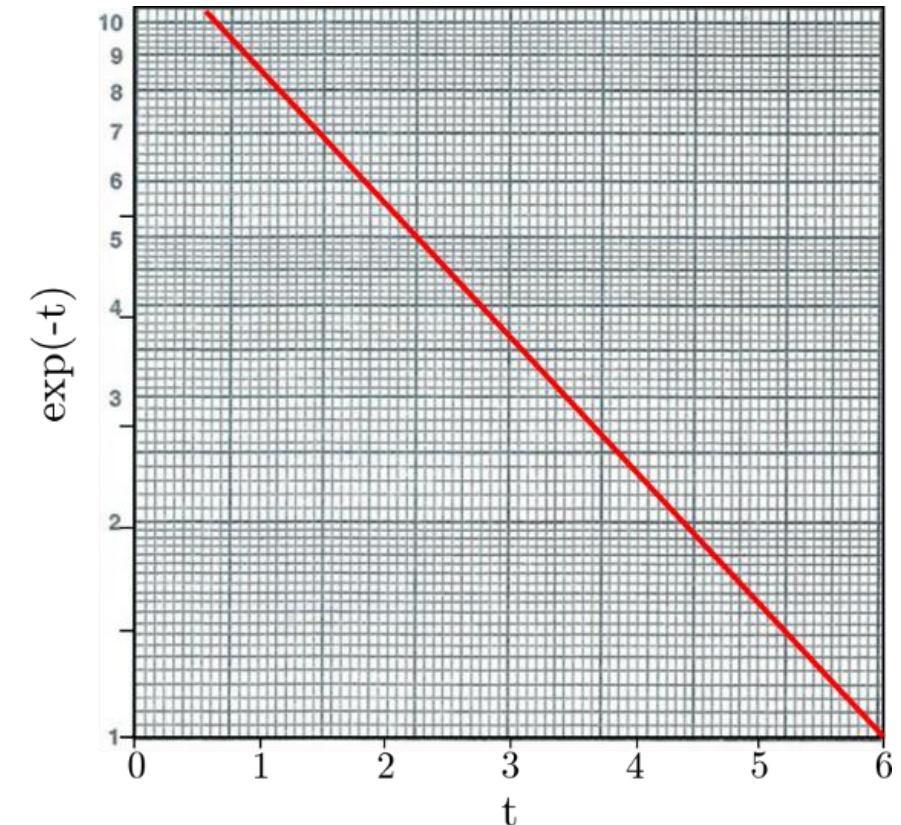
Recall

$$R(t) = e^{(-\lambda \cdot t)} \Leftrightarrow \ln(R(t)) = \ln(e^{(-\lambda \cdot t)})$$

$$\Leftrightarrow \ln(R(t)) = -\lambda \cdot t$$



$\log_{10}$       Semi-log plot



# Exponential distribution

## Example

Assume that the mean time to failure of an engineering system is 1500 hours.

Calculate the probability of failure of the engineering system during a 500-hour mission.

# Example

The failure time ( $T$ ) of an electronic circuit board follows exponentially distribution with failure rate  $\lambda = 10^{-4} / \text{h}$ .

- a) What is the probability that it will fail before 1000 h ?
- b) What is the probability that it will survive at least 10,000 h
- c) What is the probability that it will fail between 1000 and 10,000 h
- d) Suppose that the device has been successfully operated for 8000 hr. What is the probability that it will fail during the next 1500 hr of operation?

# Exponential distribution

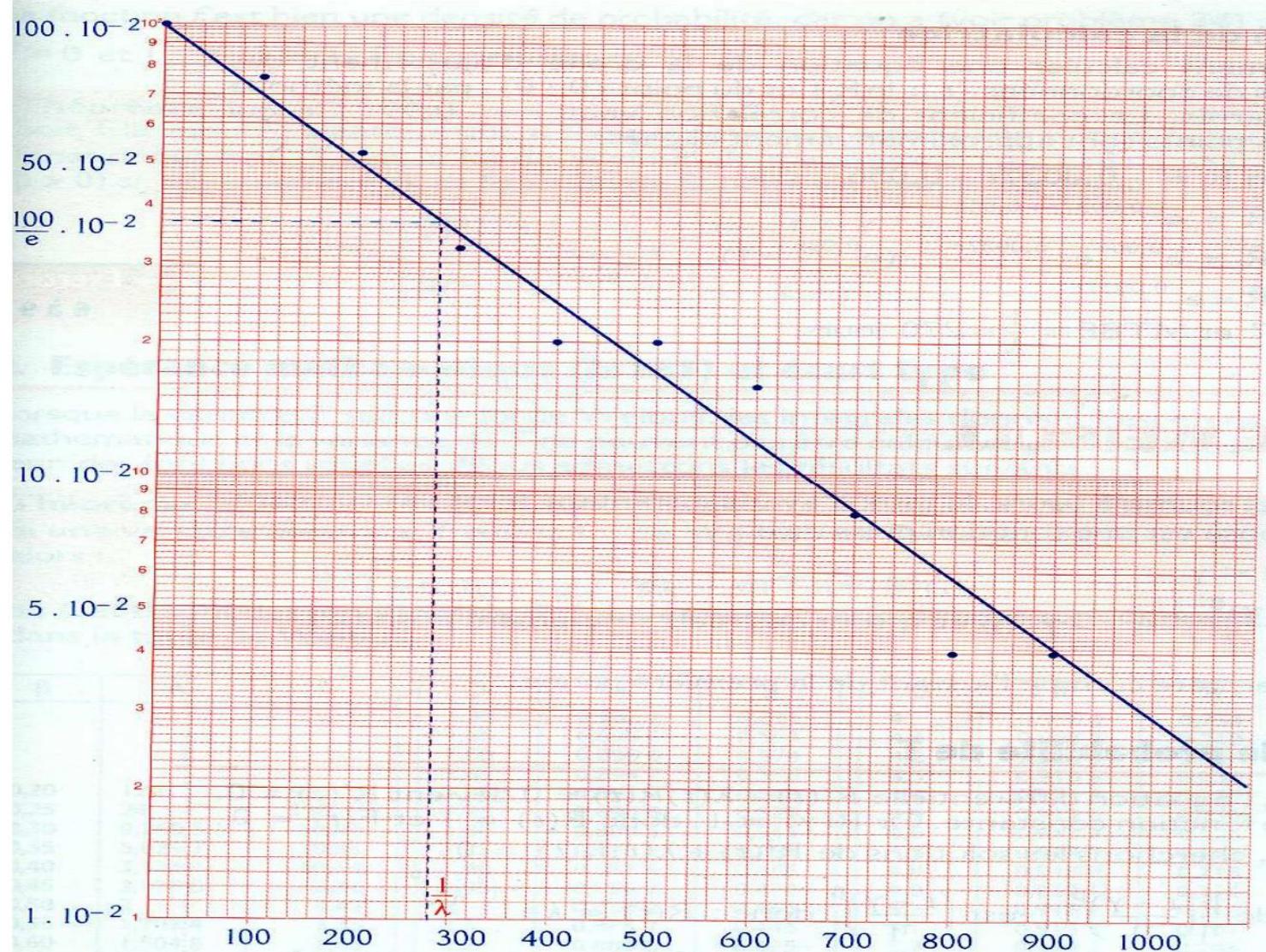
## Example

The reliability data of a machine is given :

Determine if this lifetime law follows an exponential distribution?

TTF (day)	0	100	200	300	400	500	600	700	900
R(t)	1	0,76	0,52	0,32	0,20	0,20	0,13	0,08	0,04

# Exponential distribution

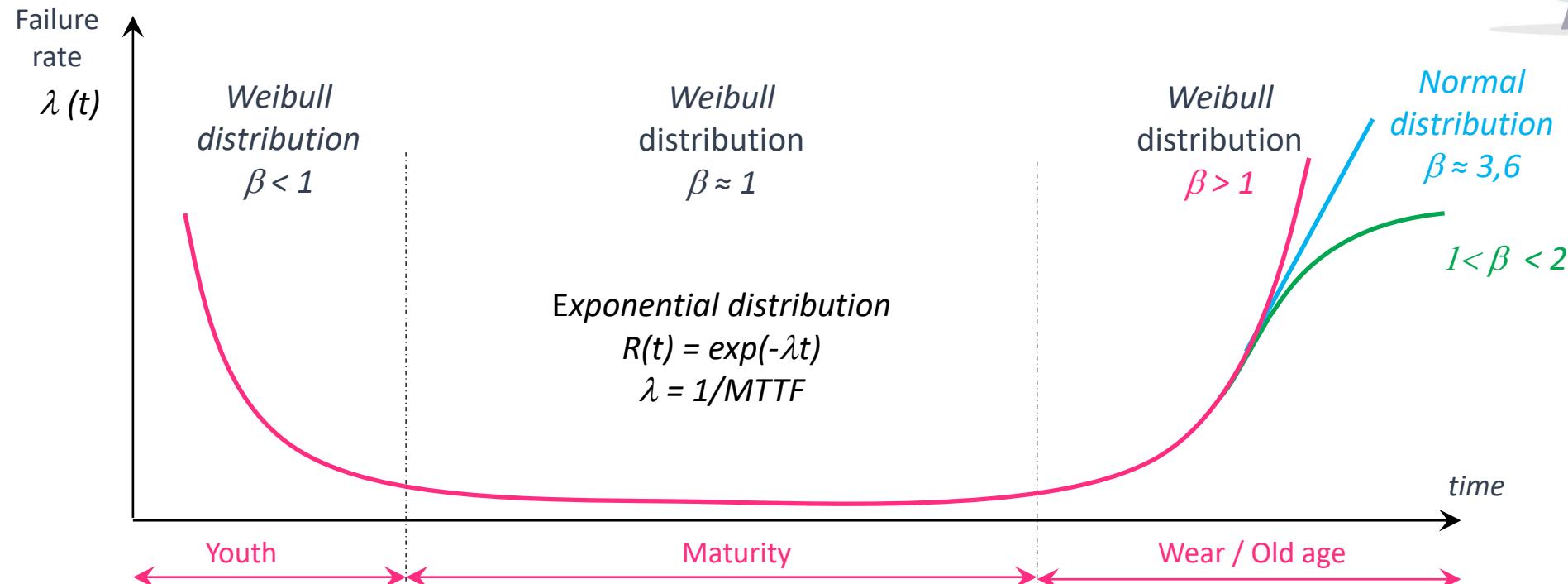


$$\frac{1}{\lambda} \approx 280, \text{ d'où } \lambda \approx 0,0036.$$

### 3. Weibull's Law

# Reliability distribution and parameters (Weibull's Law)

## Bathtub curve



$$\text{Weibull Distribution : } R(t) = \exp \left[ - \left( \frac{t-\gamma}{\eta} \right)^\beta \right]$$

$\beta$ : shape parameter ;  $\eta$  : scale parameter ;  $\gamma$  : position parameter

# The Weibull model Functions

- Reliability

$$R(t) = \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^\beta\right]$$

- Failure

$$F(t) = 1 - R(t) = 1 - \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^\beta\right]$$

- Failure probability density

$$f(t) = \frac{dF(t)}{dt} = \frac{\beta}{\eta} \left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^\beta\right]$$

# The Weibull model Functions

- Failure rate

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\beta}{\eta} \left( \frac{t - \gamma}{\eta} \right)^{\beta-1}$$

- Average

$$E(t) = A\eta + \gamma$$

- Standard deviation

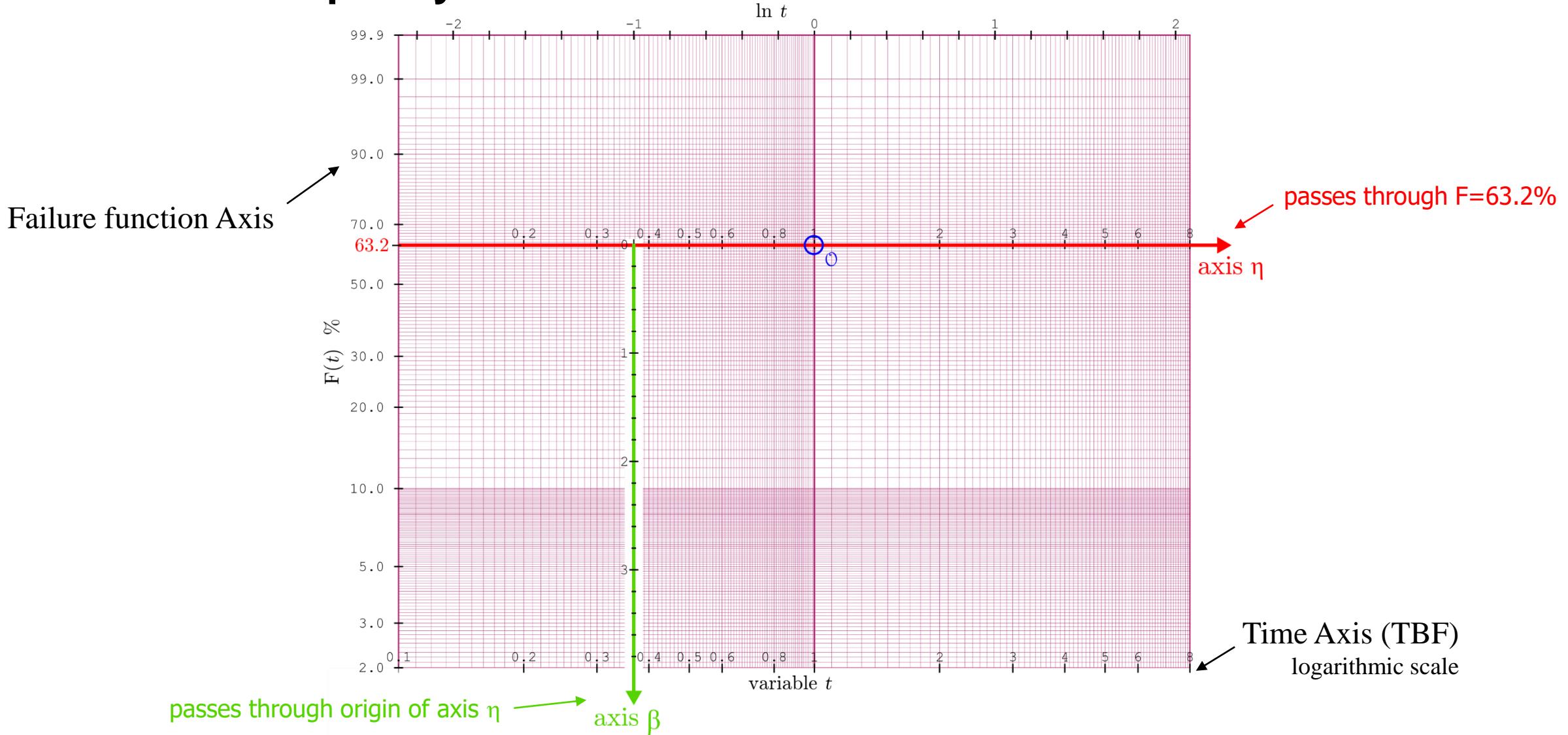
$$\sigma = B\eta$$

- Time associated with a reliability threshold

$$t = \eta \left( \ln \frac{1}{R} \right)^{\left(\frac{1}{\beta}\right)} + \gamma$$

# The Weibull model

## Functional Paper by Allan Plait



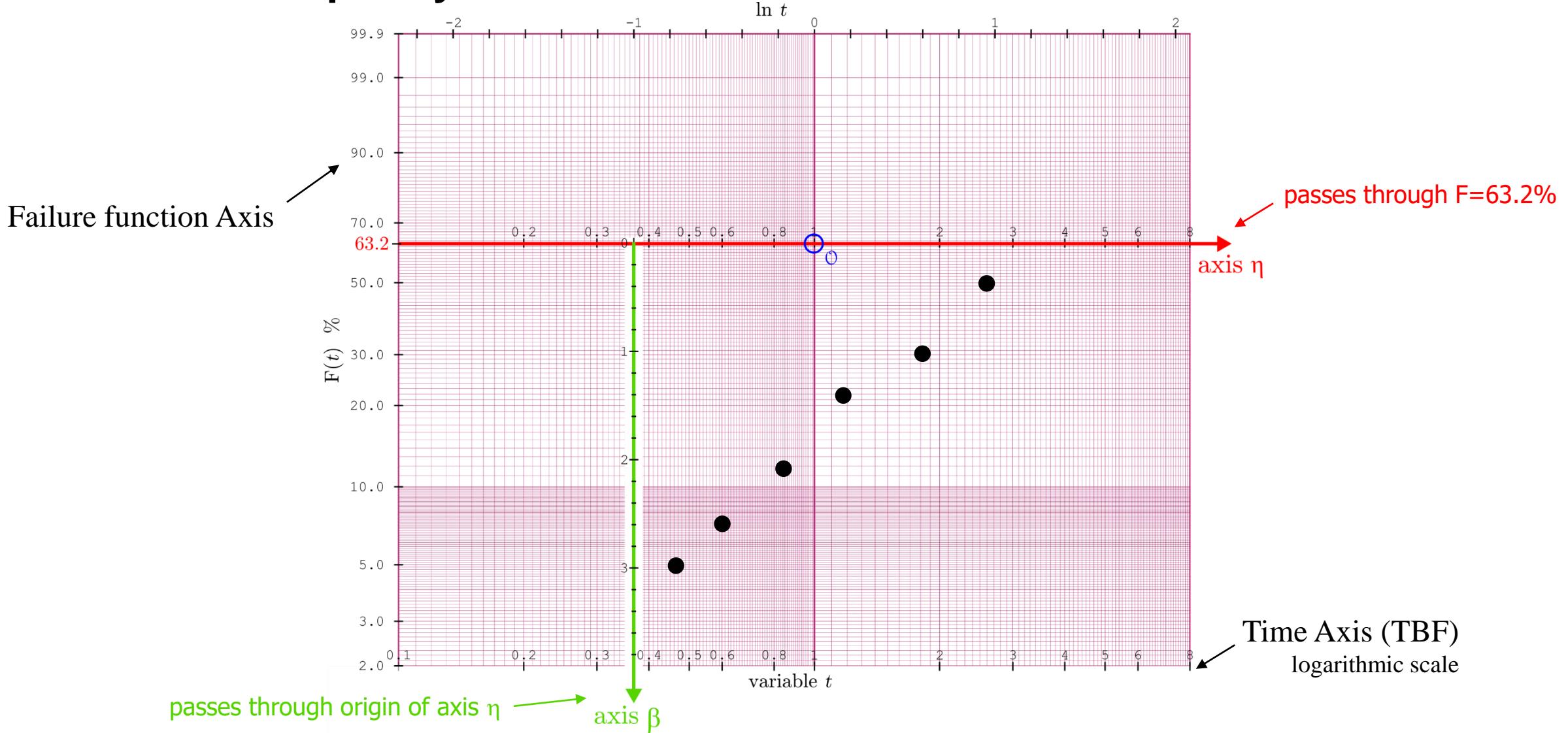
# The Weibull model

## Functional Paper by Allan Plait

1. Sort the data table from the smallest to the largest TTF or TBF.
2. Estimate the failure function  $F(t)$  for each unit.
3. Put the points of coordinates  $(t, F(t))$  on the Weibull paper

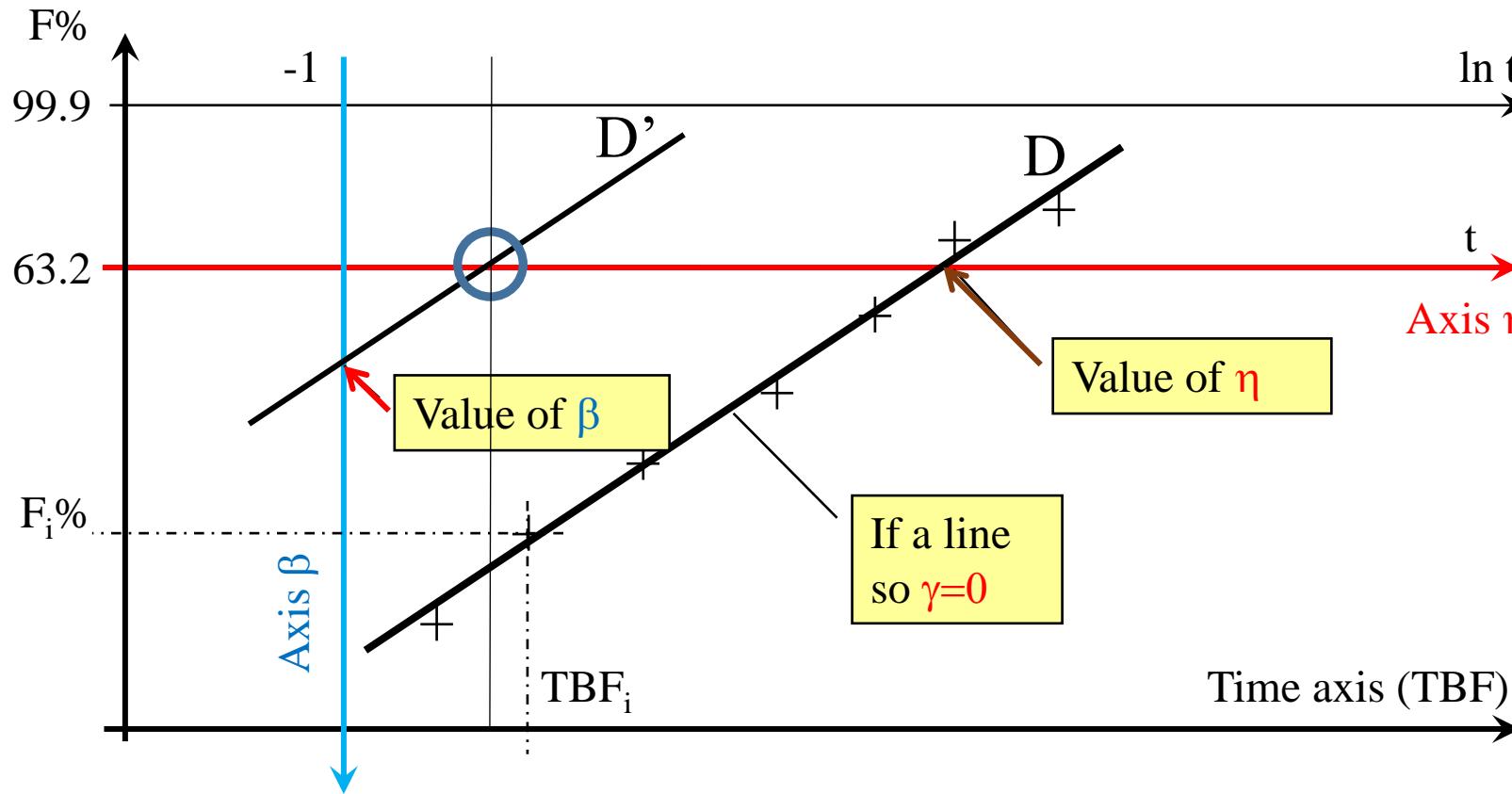
# The Weibull model

## Functional Paper by Allan Plait



# The Weibull model

## Determination of parameters (case of a line)



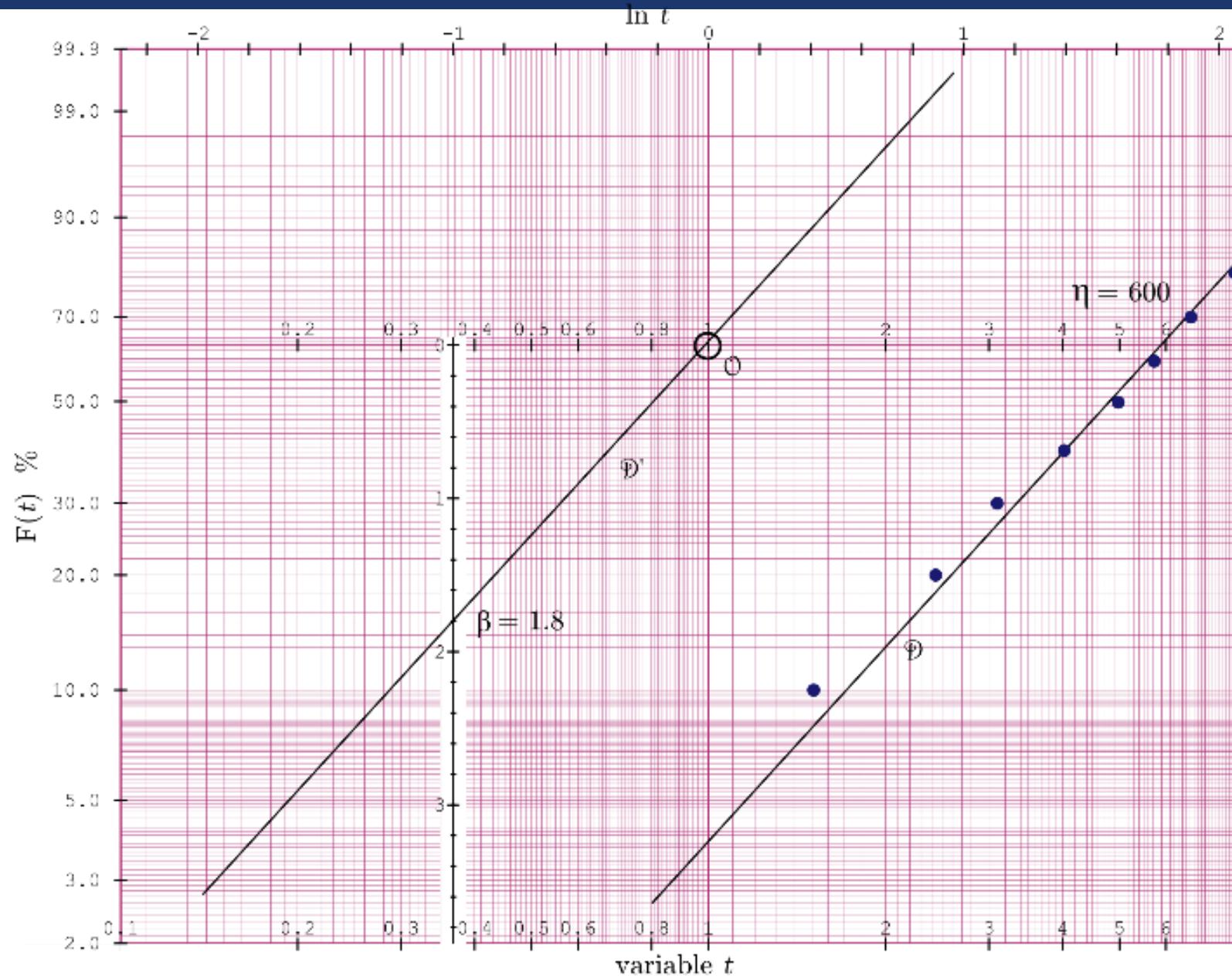
$$R(t) = \exp \left[ -\left( \frac{t - \gamma}{\eta} \right)^{\beta} \right]$$

If  $\gamma = 0$ , then  $D$  is a line.

$D'$  is parallel to line  $D$  and passing through the point noted on the graph with the O-ring

$\eta$  is found at the intersection of line  $D$  and  $F(t) = 63.2\%$ .

$\beta$  is found at the intersection of line  $D'$  and the vertical axis named  $\beta$ .

Mechatronics common framework  
Lecture 3

# The Weibull model

## Functional Paper by Allan Plait

$$MTBF = A\eta + \gamma$$

$$\sigma = B\eta$$

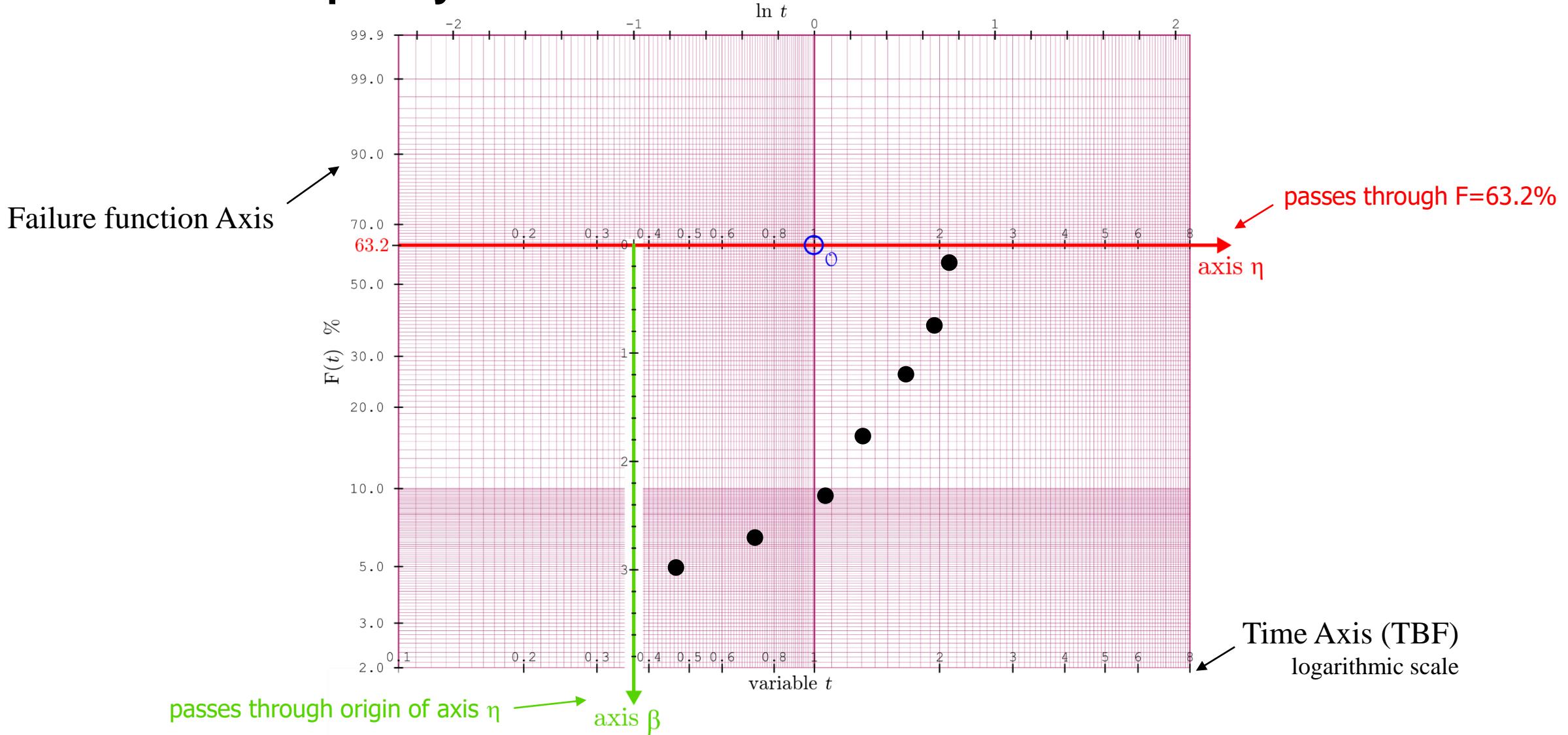
$\beta$	A	B	$\beta$	A	B	$\beta$	A	B
0,20	120	1901	1,50	0,9027	0,613	4	0,9064	0,254
0,25	24	199	1,55	0,8994	0,593	4,1	0,9077	0,249
0,30	9,2605	50,08	1,60	0,8966	0,574	4,2	0,9089	0,244
0,35	5,0291	19,98	1,65	0,8942	0,556	4,3	0,9102	0,239
0,40	3,3234	10,44	1,70	0,8922	0,540	4,4	0,9114	0,235
0,45	2,4786	6,46	1,75	0,8906	0,525	4,5	0,9126	0,230
0,50	2	4,47	1,80	0,8893	0,511	4,6	0,9137	0,226
0,55	1,7024	3,35	1,85	0,8882	0,498	4,7	0,9149	0,222
0,60	1,5046	2,65	1,90	0,8874	0,486	4,8	0,9160	0,218
0,65	1,3663	2,18	1,95	0,8867	0,474	4,9	0,9171	0,214
0,70	1,2638	1,85	2	0,8862	0,463	5	0,9182	0,210
0,75	1,1906	1,61	2,1	0,8857	0,443	5,1	0,9192	0,207
0,80	1,1330	1,43	2,2	0,8856	0,425	5,2	0,9202	0,203
0,85	1,0880	1,29	2,3	0,8859	0,409	5,3	0,9213	0,200
0,90	1,0522	1,77	2,4	0,8865	0,393	5,4	0,9222	0,197
0,95	1,0234	1,08	2,5	0,8873	0,380	5,5	0,9232	0,194
1	1	1	2,6	0,8882	0,367	5,6	0,9241	0,191
1,05	0,9803	0,934	2,7	0,8893	0,355	5,7	0,9251	0,186
1,10	0,9649	0,878	2,8	0,8905	0,344	5,8	0,9260	0,185
1,15	0,9517	0,830	2,9	0,8917	0,334	5,9	0,9269	0,183
1,20	0,9407	0,787	3	0,8930	0,325	6	0,9277	0,180
1,25	0,9314	0,750	3,1	0,8943	0,316	6,1	0,9286	0,177
1,30	0,9236	0,716	3,2	0,8957	0,307	6,2	0,9294	0,175
1,35	0,9170	0,687	3,3	0,8970	0,299	6,3	0,9302	0,172
1,40	0,9114	0,660	3,4	0,8984	0,292	6,4	0,9310	0,170
1,45	0,9067	0,635	3,5	0,8997	0,285	6,5	0,9318	0,168
			3,6	0,9011	0,278	6,6	0,9325	0,166
			3,7	0,9025	0,272	6,7	0,9333	0,163
			3,8	0,9038	0,266	6,8	0,9340	0,161
			3,9	0,9051	0,260	6,9	0,9347	0,160

*Numerical calculation of the mean and the standard deviation for the Weibull model*

$$R(t) = \exp \left[ - \left( \frac{t - \gamma}{\eta} \right)^\beta \right]$$

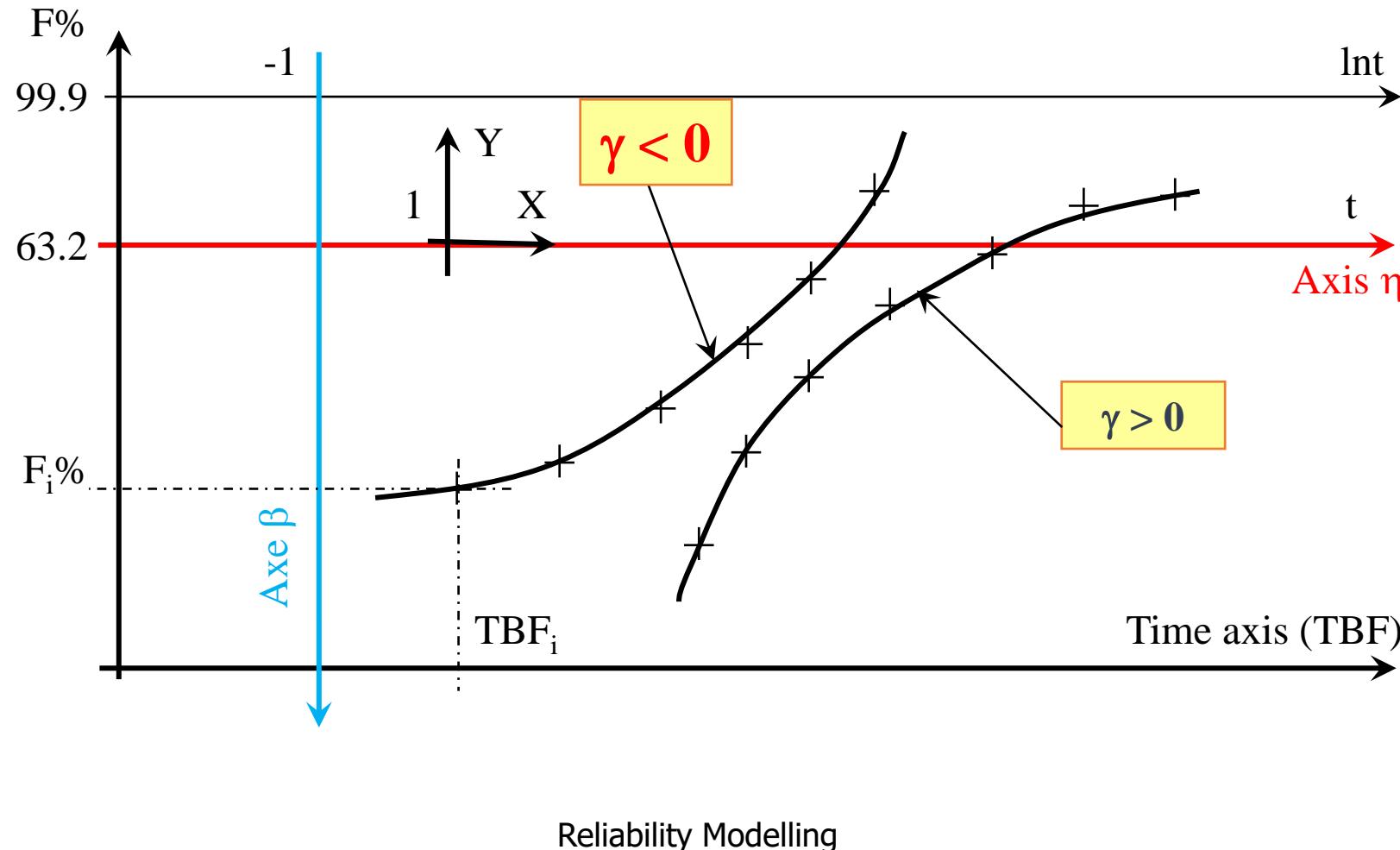
# The Weibull model

## Functional Paper by Allan Plait



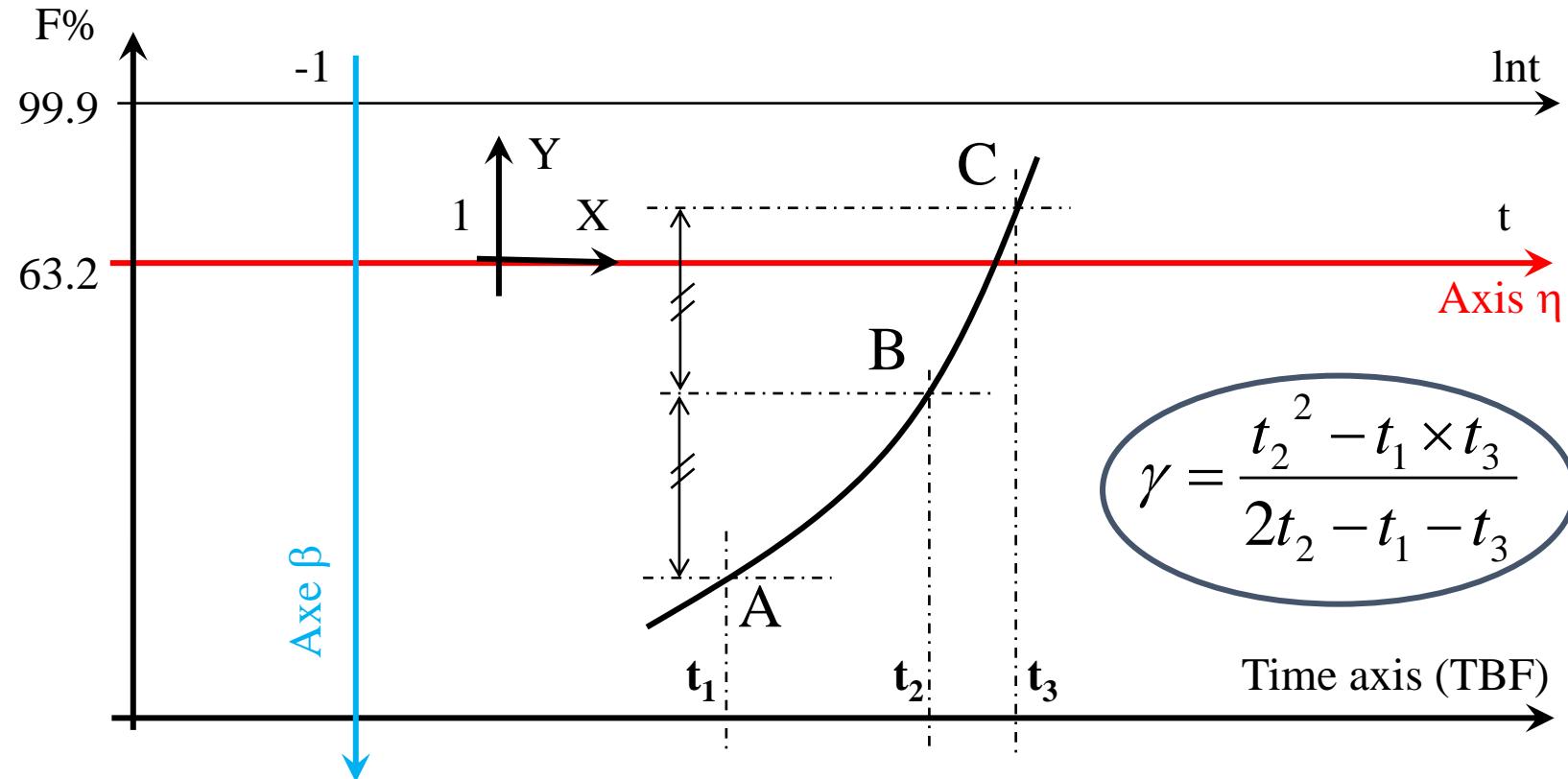
# The Weibull model

## Determination of parameters (case of curves)



# The Weibull model

## Determination of the position parameter $\gamma$



## 4. ESTIMATORS

- Estimation of  $F(t)$
- Estimation of  $R(t)$
- Estimation of  $I(t)$
- Estimation of  $f(t)$
- Estimation of the mean  $E(t)$

# Estimators

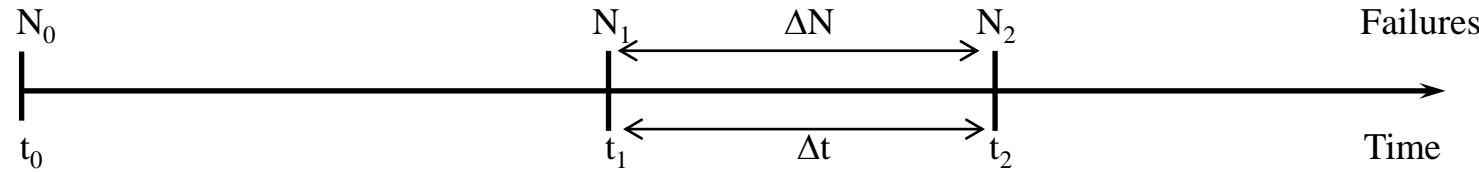
It is obviously not realistic to calculate failure rates by **building many units and running them for many hours**, under expected operating conditions.

This is especially true for well-designed and properly built supplies, with **extremely low failure rates**, where the number of supplies and hours required to get valid results would be in the thousands.

Instead based on representative samples of a population statistical analysis techniques can be used to estimate failure rates.

⚠️ IF YOU DON'T KNOW WHICH LAW (i.e exponential, Normal, Weibull....) the failure function of your component follow, you must use an estimator

# Estimation of the reliability functions



N: total number of products put into operation at time  $t_0$  (sample size)

$N_0$ : number of failures at  $t_0$  (generally equal to 0)

$N_1$ : number of failures at  $t_1$

$N_2$ : number of failures at  $t_2$

$\Delta N$ : number of failures on the interval  $[t_1, t_2]$

$\Delta t$ : time interval  $[t_1, t_2]$

*Reliability function estimators depend on the value of N (number of products put into operation)*

N	$1 < N \leq 20$	$20 < N \leq 50$	$N > 50$
Estimator	Median Ranks	Average Ranks	Cumulative Frequencies

# Failure estimators: F(t)

Point estimator at time t

Median ranks if  $1 < N \leq 20$

$$F(t) = \frac{N_t - 0.3}{N + 0.4}$$

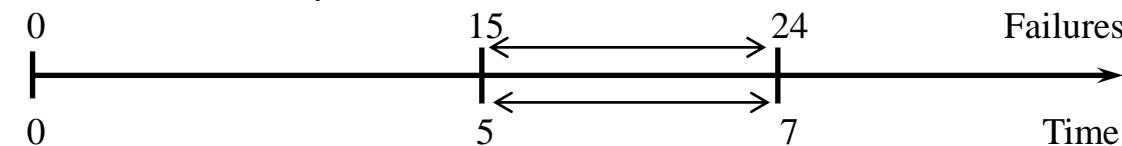
Average ranks if  $20 < N \leq 50$

$$F(t) = \frac{N_t}{N + 1}$$

Cumulative frequencies if  $N > 50$

$$F(t) = \frac{N_t}{N}$$

Example ( $N = 100$ ) → Cumulative frequencies



$$F(5) = \frac{15}{100} = 0.15$$

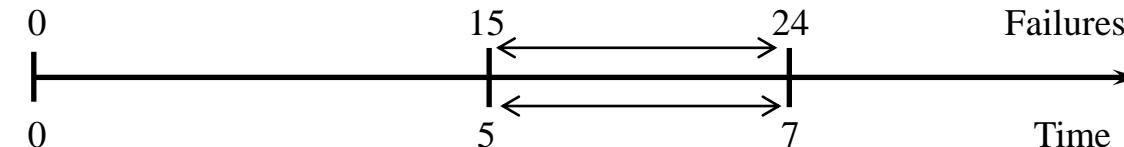
$$F(7) = \frac{24}{100} = 0.24$$

# Reliability estimators: $R(t)$

Point estimator at time t

Whatever the value of N, first compute the estimator of F and then apply the complementarity relation  
 $R(t) = 1 - F(t)$

Example (N = 100)



$$R(5) = 1 - 0.15 = 0.85$$

$$R(5) = \frac{100 - 15}{100} = 0.85$$

$$R(7) = 1 - 0.24 = 0.76$$

$$R(7) = \frac{100 - 24}{100} = 0.76$$

# Density estimators: $f(t)$

**Interval** estimator (failure/product and unit of time)

Median ranks if  $1 < N \leq 20$

$$f_{[t1;t2[} = \frac{\Delta N}{(N + 0.4) \times \Delta t}$$

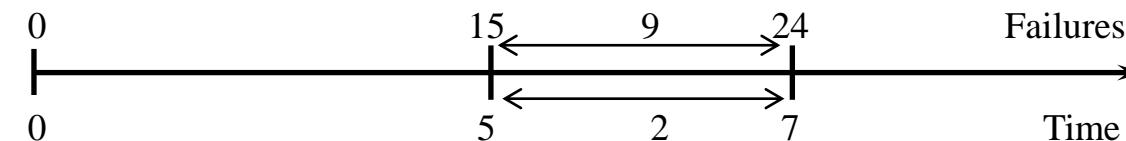
Average ranks if  $20 < N \leq 50$

$$f_{[t1;t2[} = \frac{\Delta N}{(N + 1) \times \Delta t}$$

Cumulative frequencies if  $N > 50$

$$f_{[t1;t2[} = \frac{\Delta N}{N \times \Delta t}$$

Example ( $N = 100$ )



$$f_{[5;7[} = \frac{9}{100 \times 2} = 0.045 \text{ failures/product \& t.u.}$$

# Failure rate estimators: $\lambda(t)$

Estimator on **interval** (failure/product and unit of time) (f/R)

Median ranks if  $1 < N \leq 20$

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N + 0.7 - N_1) \times \Delta t}$$

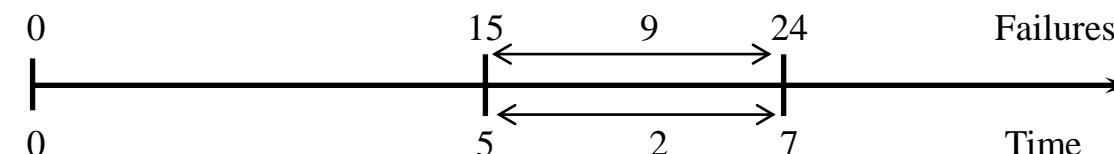
Average ranks if  $20 < N \leq 50$

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N + 1 - N_1) \times \Delta t}$$

Cumulative frequencies if  $N > 50$

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N - N_1) \times \Delta t}$$

Example ( $N = 100$ )



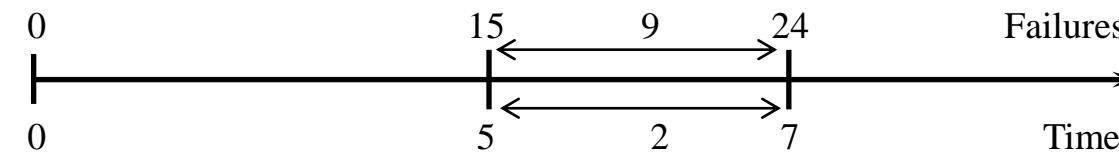
$$\lambda_{[5;7[} = \frac{9}{85 \times 2} = 0.053 \text{ failures/product \& t.u.}$$

# Average estimators: $E(t)$

## Interval Estimator

Whatever the value of N, first calculate the estimator of  $\lambda$  and then apply the inverse relation  
 $E(t) = 1/\lambda(t)$

Example ( $N = 100$ )



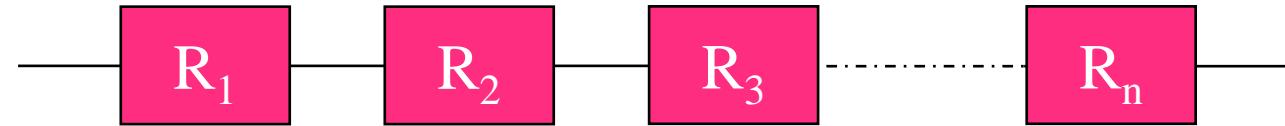
$$E(t)_{[5;7]} = \frac{1}{0.053} = \frac{85 \times 2}{9} = 18.89 \text{ u.t.}$$

# 5. RELIABILITY MODELING

# Predictive Reliability

- Definition
  - Predictive reliability (predicted or theoretical) is calculated on the basis of a mathematical model defined from the functional decomposition of the system into subsets, components, etc. and the estimated or predicted reliability of its components
- Analysis and calculation procedure
  - Decompose the system into components (parts, subsystems ...) and establish the functional links between the components
  - Identify component reliability models or collect reliability at a given time
  - Search for a model of the system and calculate its reliability

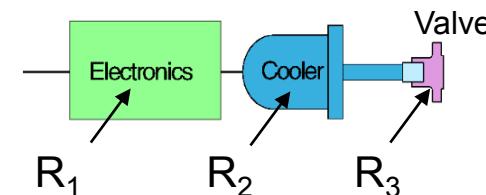
# Reliability of the serial system (S)



$$R(S) = R_1 \times R_2 \times R_3 \times \dots \times R_n$$

$$R(S) = \prod_{i=1}^n R_i$$

$R_1, R_2, R_3, \dots R_n$  are the elementary reliabilities of the system components at a given time



# Redundancy

In active parallel, both units are in use, and therefore subject to failure.

In standby parallel, the second unit is only brought into service when the first fails.

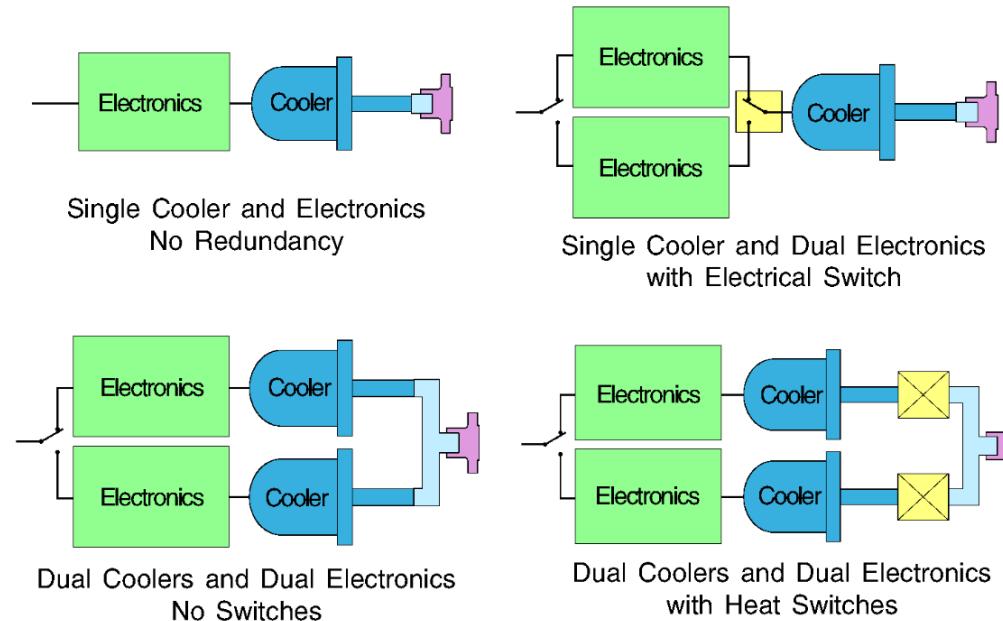
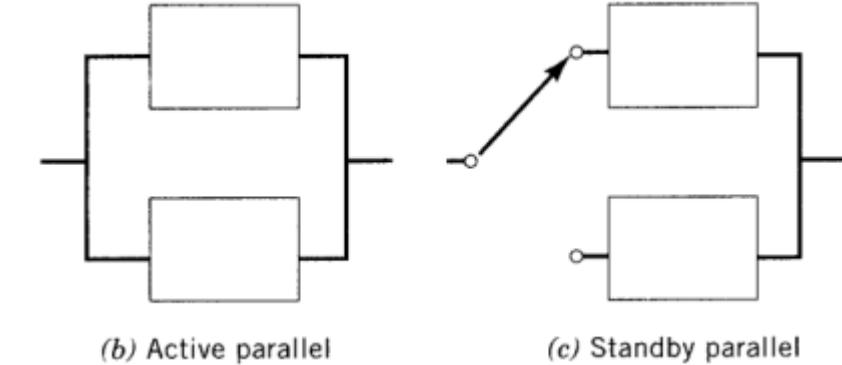
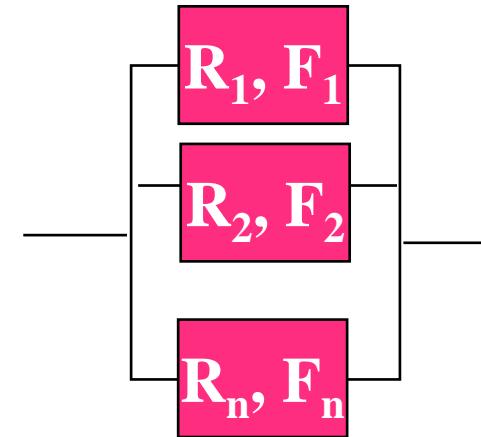


Figure 1. Example cryocooler redundancy options.

One key means of improving the reliability of systems required to provide continuous cooling during multi-year space missions is to incorporate redundant components to protect against individual failures. There are many options for incorporating redundancy; four common ones, highlighted in Fig. 1, have been analyzed previously by this author with respect to their total systems advantages and disadvantages.<sup>1</sup> Although most space cryocooler missions to date have not incorporated redundancy, the 'dual coolers with dual electronics and no switches' approach in the lower left corner of Fig. 1 was adopted by the NASA AIRS mission, which was launched in May 2002.<sup>2,3</sup> The original analysis of the reliability of that configuration utilized the classic equations for the reliability ( $R_{clsys}$ ) of a two-parallel redundant system as noted in Eq. 1. This classical equation describes the reliability ( $R_{clsys}$ ) of the two-cooler system over ( $T$ ) years of operation as:

# Reliability of the parallel system (P)

- 1 only component on "n" must operate (system 1/n by default)



$$F(P) = F_1 \times F_2 \times F_3 \times \dots \times F_n$$

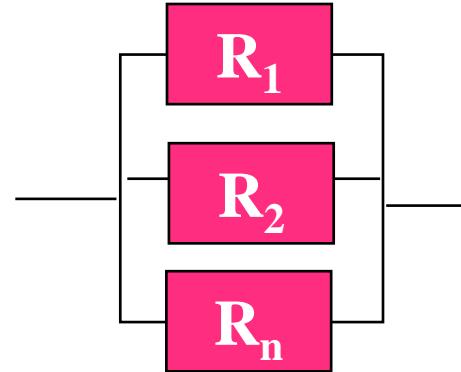
$$F(P) = \prod_{i=1}^n F_i$$

$$R(P) = 1 - \prod_{i=1}^n F_i = 1 - \prod_{i=1}^n (1 - R_i)$$

- $R_1, R_2, \dots, R_n$  and  $F_1, F_2, \dots, F_n$  are respectively the elementary reliabilities and failures of the components of the system at a given time

# Reliability of the parallel system (k/n)

"k" components on "n" must operate simultaneously (system k/n notation required)



$$R_S = \sum_{i=k}^n C_n^i R^i (1-R)^{n-i}$$

$$C_n^k = \frac{n!}{k!(n-k)!}$$

$n \rightarrow$  total number of components

$p \rightarrow$  probability of success (reliability of a component)

$k \rightarrow$  number of components to operate simultaneously

$R_i$  is the elementary reliability of the components  $i$  at a given time

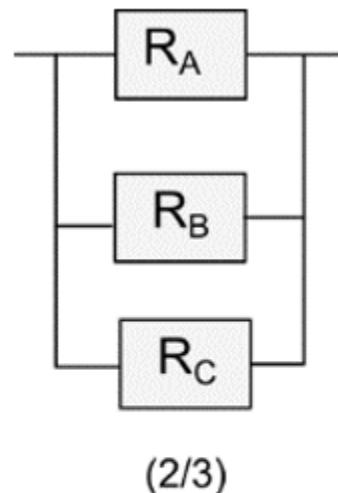


( Only if  $R_1 = R_2 = \dots = R_n$ , otherwise, application of the truth table )

# Reliability of the parallel system (k/n)

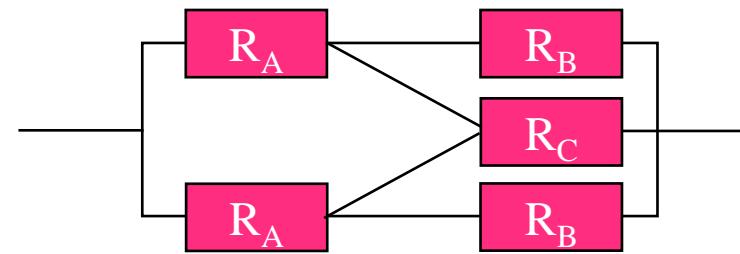
Truth table

exemple for (k=2/n=3)



A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	$F_A \times R_B \times R_C = (1-R_A) \times R_B \times R_C$
1	0	0	0
1	0	1	$R_A \times F_B \times R_C = R_A \times (1-R_B) \times R_C$
1	1	0	$R_A \times R_B \times F_C = R_A \times R_B \times (1-R_C)$
1	1	1	$R_A \times R_B \times R_C$
$\sum$		$R_s = (1-R_A).R_B.R_C + R_A.(1-R_B).R_C + R_A.R_B.(1-R_C) + R_A.R_B.R_C$	

# Reliability of a complex system



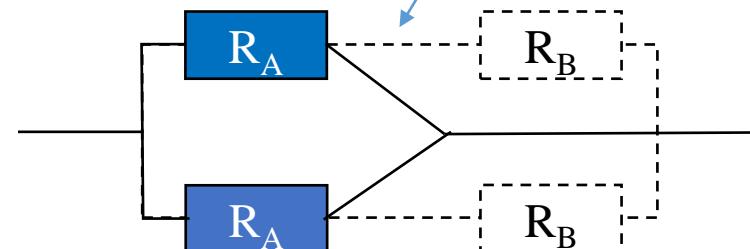
Component C is functioning only if the two components B are not working

Decomposition Method (Baye's theorem)

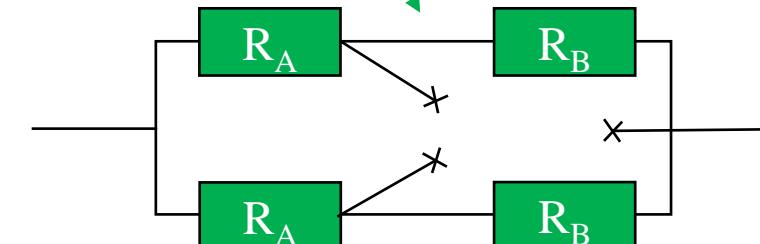
$$P\{S\} = P\{S|C\} \times P\{C\} + P\{S|\bar{C}\} \times P\{\bar{C}\}$$

$$R(S) = \underbrace{(1 - (1 - R_A)^2)}_{\text{We path through } C, \text{ thereby bypassing B.}} \times R_C + \underbrace{(1 - (1 - R_A \cdot R_B)^2)}_{\text{We disconnect all the paths leading through C}} \times (1 - R_C)$$

We path through  
C, thereby  
bypassing B.



We disconnect all  
the paths leading  
through C



# Application to exponential model

$$R(t) = e^{-\lambda t} \quad \text{and} \quad F(t) = 1 - e^{-\lambda t}$$

with:  $\lambda = 1/E(t)$  (failure rate)

The serial system

$$R(t) = e^{(-t \sum_{i=1}^n \lambda_i)} = e^{(-t \lambda_s)}$$

The parallel system (n components)

$$R(t) = 1 - \prod_{i=1}^n (1 - e^{(-\lambda_i t)})$$

The parallel system (2 components)

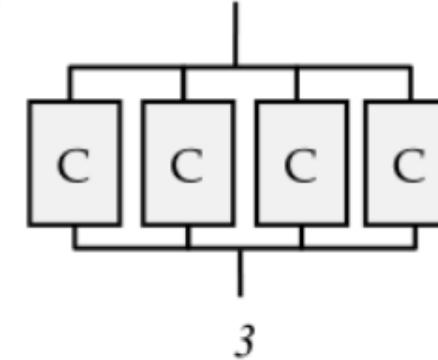
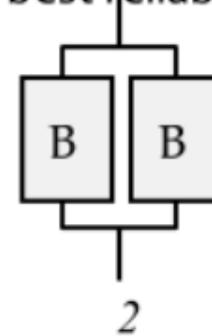
$$R(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

## Example (exercise 5)

**Example 1** A personal computer consists of four basic sub systems: motherboard (MB), hard disk (HD), power supply (PS) and processor (CPU). The reliabilities of four subsystems are 0.98, 0.95, 0.91 and 0.99 respectively. What is the system reliability for a mission of 1000 h?

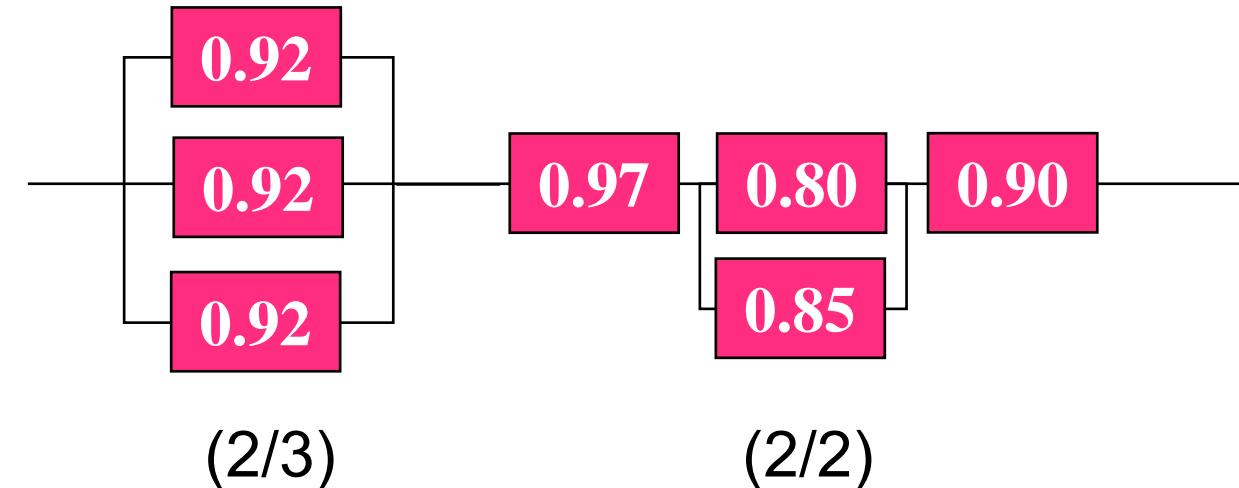
## Example (exercise 6)

- We consider three components A, B and C of respective prices 150, 75 and 45 €. Be at a given moment,  $F_A=0.1$ ,  $F_B=0.3$  and  $F_C=0.4$  the respective failures of the three components.
  - Which of the following three systems has the highest reliability?
  - Which of the three systems has the best reliability/price ratio?



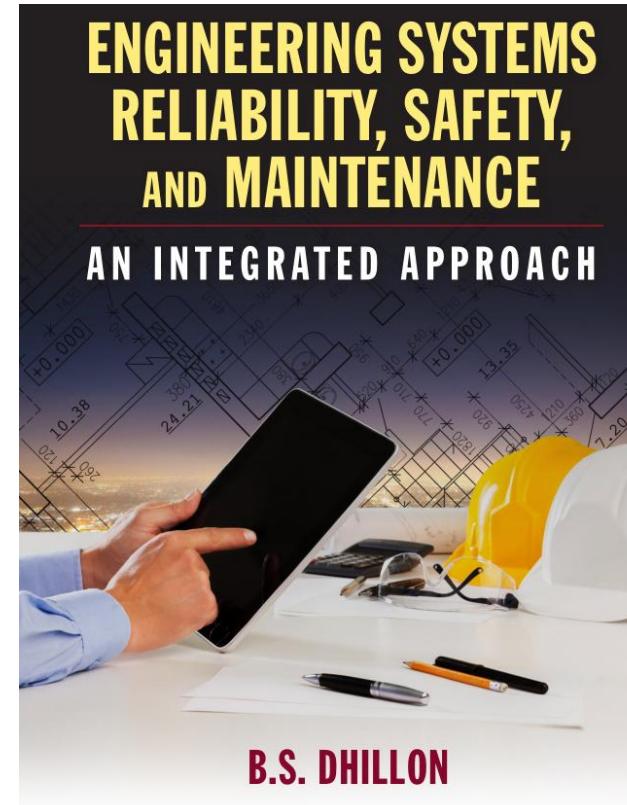
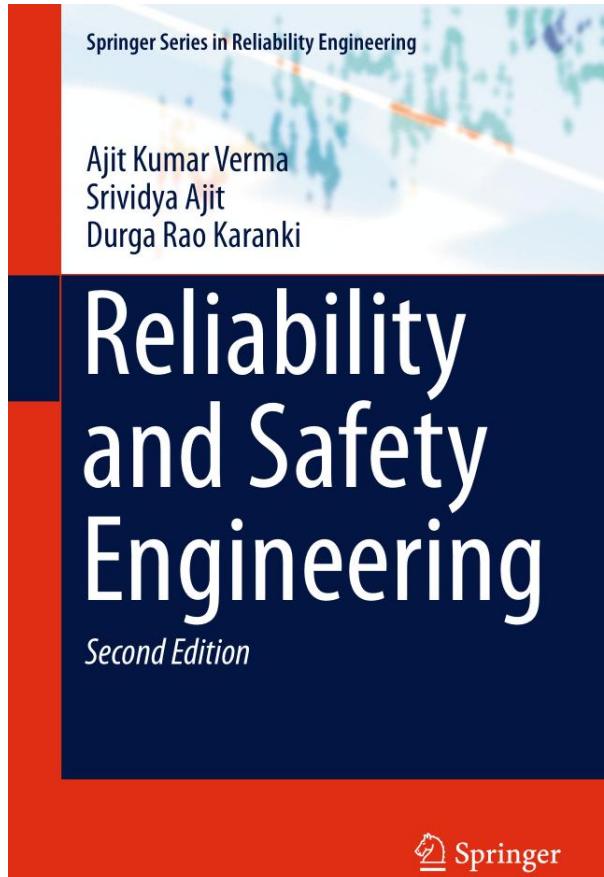
## Example (exercise 7)

The given values correspond to the reliability of the components at a given time



**Calculate the predictive reliability of this system**

## Relevant books



# Contact Information

## Université Savoie Mont Blanc

Polytech' Annecy Chambéry  
Chemin de Bellevue  
74940 Annecy  
France

<https://www.polytech.univ-savoie.fr>

## Lecturer

Dr Luc Marechal ([luc.marechal@univ-smb.fr](mailto:luc.marechal@univ-smb.fr))  
SYMME Lab (Systems and Materials for Mechatronics)



SYMME

## Acknowledgement

Pr Georges Habchi  
Pr Christine Barthod  
SYMME Lab (Systems and Materials for Mechatronics)  
for the original writing of this lecture