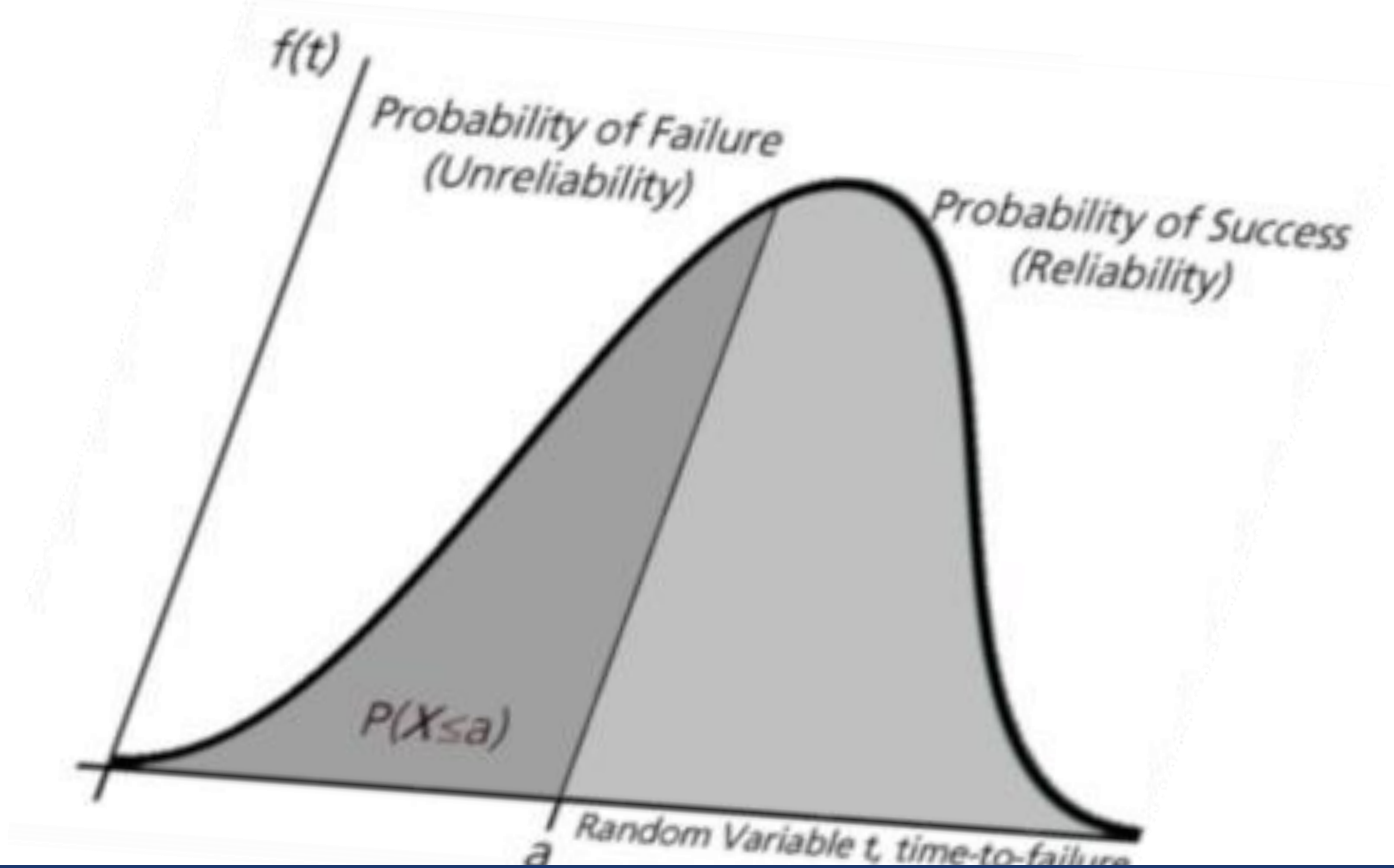

2021

MCTR 702_1

Master Advanced Mechatronics

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UNIVERSITÉ SAVOIE MONT BLANC

POLYTECH ANNECY-CHAMBÉRY

SYMME

Mechatronics common framework Lecture 1

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Lecture 1

RELIABILTIY

Reliability Functions & Estimators

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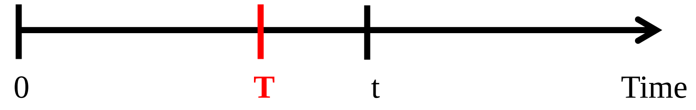
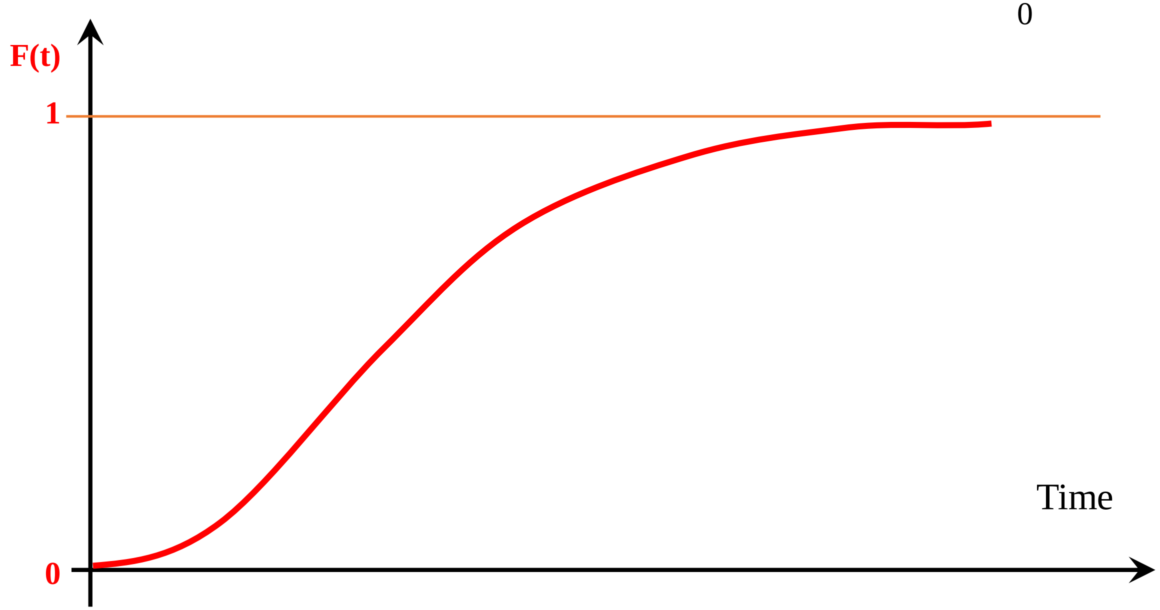
The failure function: F(t)

It is measured by the probability that an entity E fails over the time interval [0, t]:
F(t) = Pr(E failing on [0, t])

That is, if we assume that an entity is failing at a date T:
F(t) = Pr(T≤t)

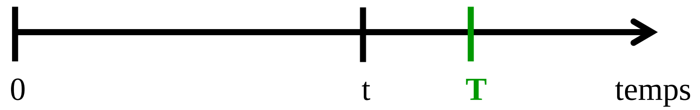


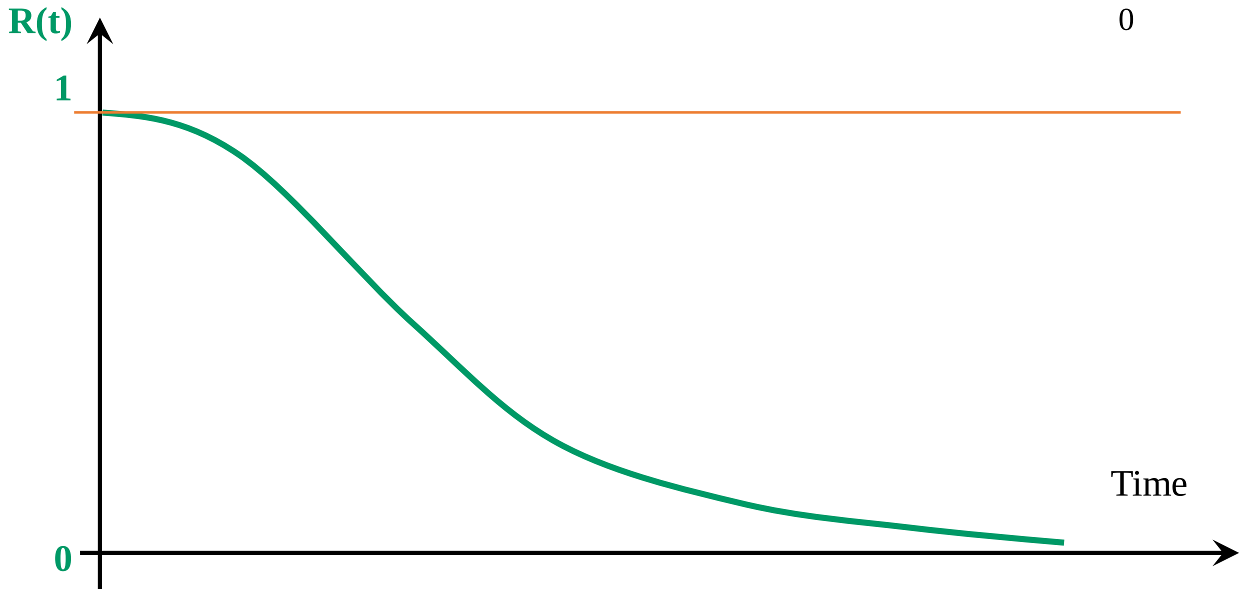


The reliability function: R(t)

It is measured by the probability that an entity E is non-faulty over the time interval [0, t]:
R(t) = Pr(E not failing on [0, t])

That is, if we assume that an entity is failing at a date T:
R(t) = Pr(T>t)

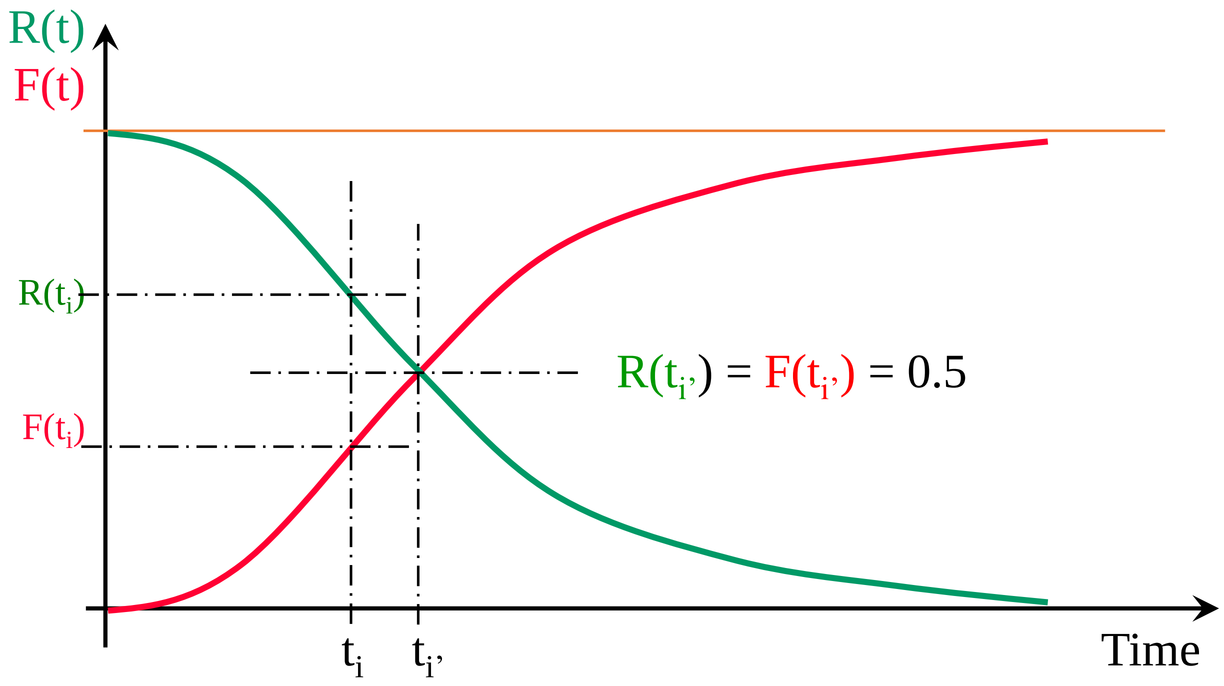




Complementarity between F(t) and R(t)

For a given system, ∀t = ti: we have

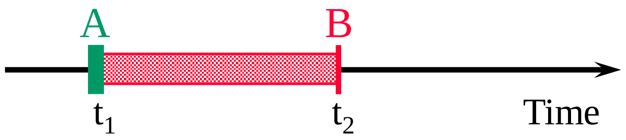
R(ti) + F(ti) = 1

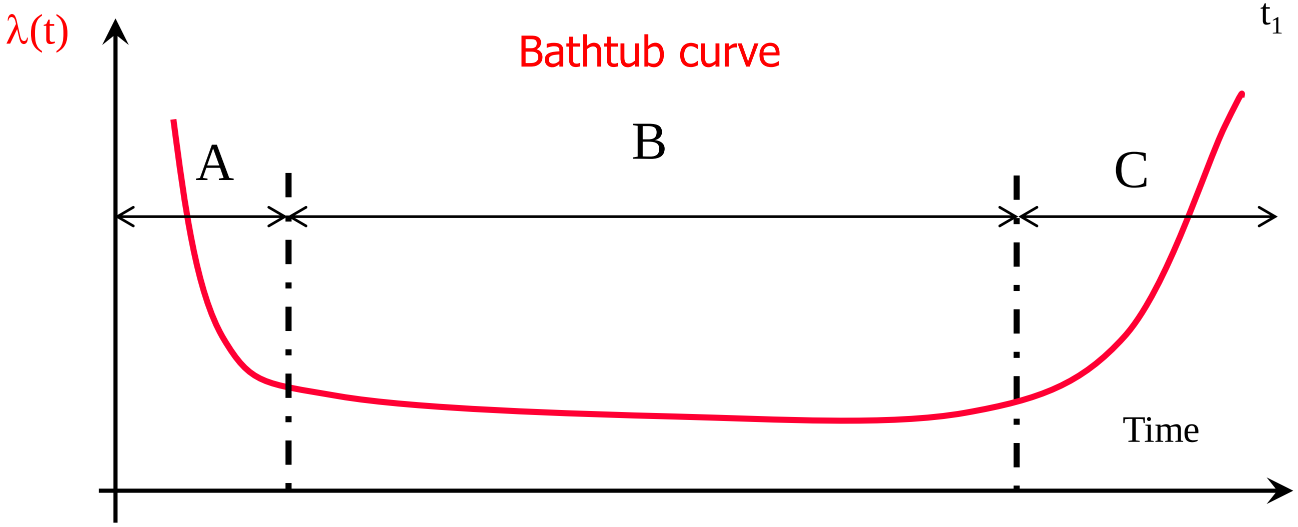


The failure rate: $\lambda(t)$

It expresses the **speed of arrival of failures** at time t, or
The **evolution of the conditional probability of failure** during the life of the material (failure/product and unit of time)

A: Event "product runs at t1"
B: Event "product fails between t1 and t2"
$\lambda(t) = Pr(B/A) / (t2 - t1)$

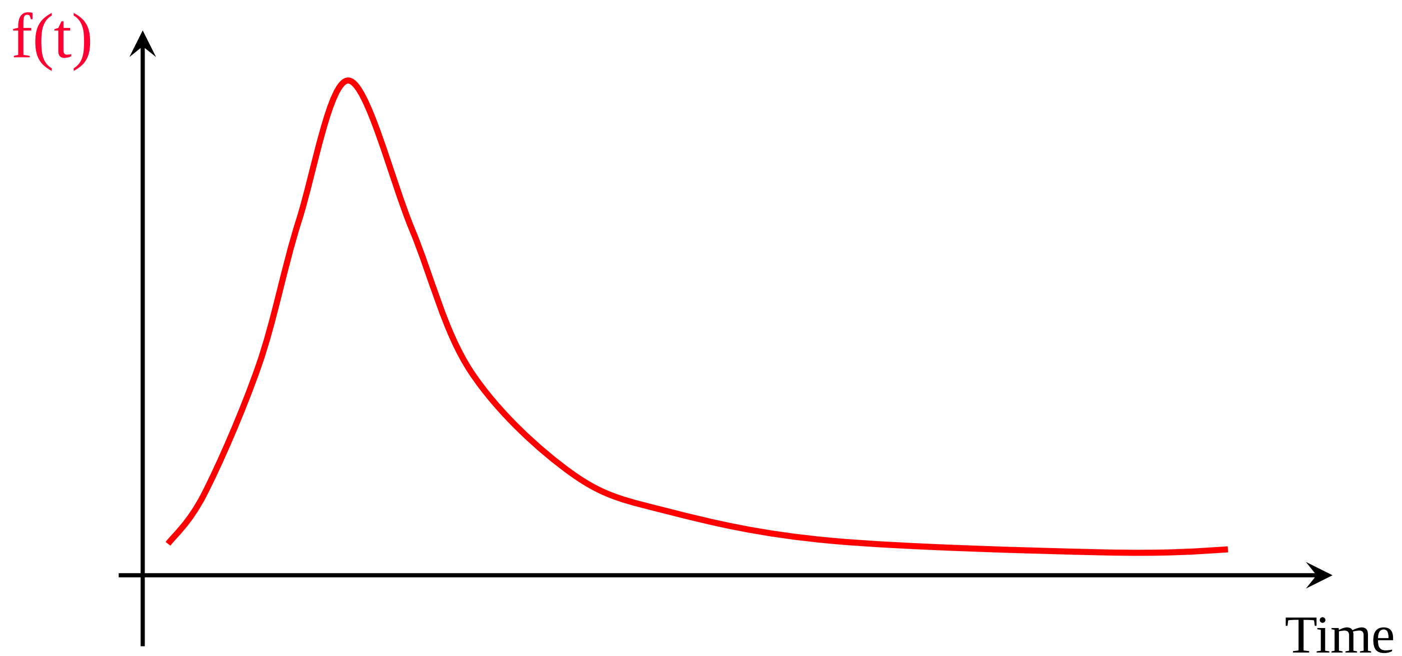




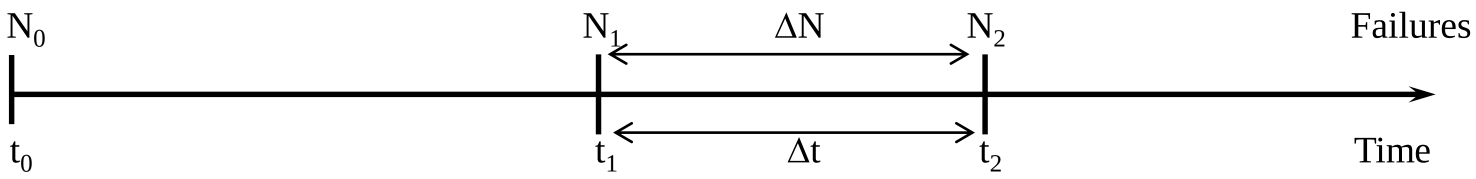
A: Youth
B: Maturity period or useful life
C: Period of aging or wear

The density function: f(t)

It represents the histogram of the relative failures as a function of time or the probability frequency of the relative failures relative to the unit of time.



Estimation of the reliability functions



N: total number of products put into operation at time t0 (sample size)
N0: number of failures at t0 (generally equal to 0)
N1: number of failures at t1
N2: number of failures at t2
ΔN: number of failures on the interval [t1, t2]
Δt: time interval [t1, t2]

Reliability function estimators depend on the value of N (number of products put into operation)

N	1<N≤20	20<N ≤50	N>50
Estimator	Median Ranks	Average Ranks	Cumulative Frequencies

Failure estimators: F(t)

Point estimator at time t

Median ranks if 1<N≤20

$$F(t)=\frac{N_t-0.3}{N+0.4}$$

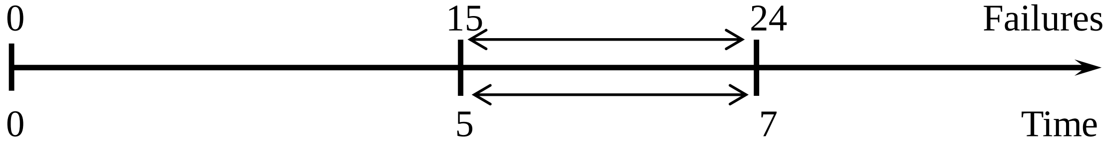
Average ranks if 20<N≤50

$$F(t)=\frac{N_t}{N+1}$$

Cumulative frequencies if N>50

$$F(t)=\frac{N_t}{N}$$

Example (N = 100) ➔ Cumulative frequencies



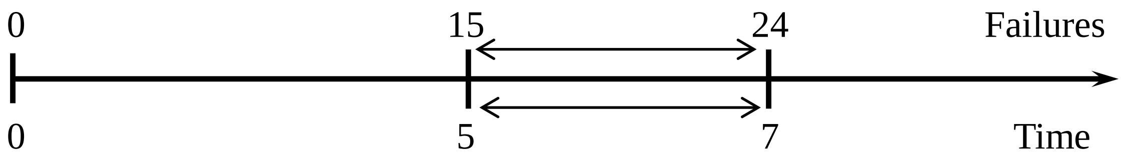
$$F(5)=\frac{15}{100}=0.15 \qquad F(7)=\frac{24}{100}=0.24$$

Reliability estimators: R(t)

Point estimator at time t

Whatever the value of N, first compute the estimator of F and then apply the complementarity relation R(t) = 1 - F(t)

Example (N = 100)



$$R(5) = 1 - 0.15 = 0.85 \qquad R(7) = 1 - 0.24 = 0.76$$

$$R(5) = \frac{100-15}{100} = 0.85 \qquad R(7) = \frac{100-24}{100} = 0.76$$

Density estimators: f(t)

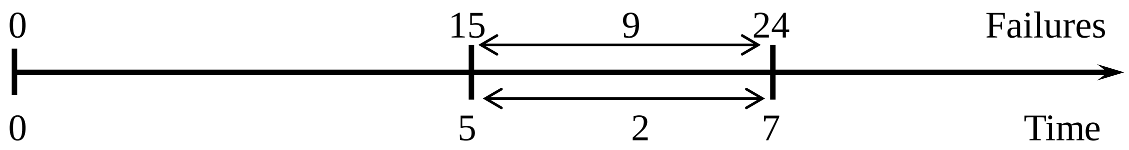
Interval estimator (failure/product and unit of time)

Median ranks if 1<N≤20 $$f_{[t1;t2[} = \frac{\Delta N}{(N+0.4)\times \Delta t}$$

Average ranks if 20<N≤50 $$f_{[t1;t2[} = \frac{\Delta N}{(N+1)\times \Delta t}$$

Cumulative frequencies if N>50 $$f_{[t1;t2[} = \frac{\Delta N}{N\times \Delta t}$$

Example (N = 100)



$$f_{[5;7[} = \frac{9}{100\times 2} = 0.045\ failures/product\ \&\ t.u.$$

Failure rate estimators: λ(t)

Estimator on interval (failure/product and unit of time) (f/R)

Median ranks if 1<N≤20

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N + 0.7 - N_1)\times \Delta t}$$

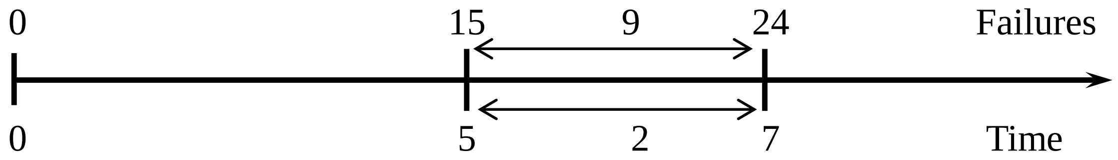
Average ranks if 20<N≤50

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N + 1 - N_1)\times \Delta t}$$

Cumulative frequencies if N>50

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N - N_1)\times \Delta t}$$

Example (N = 100)



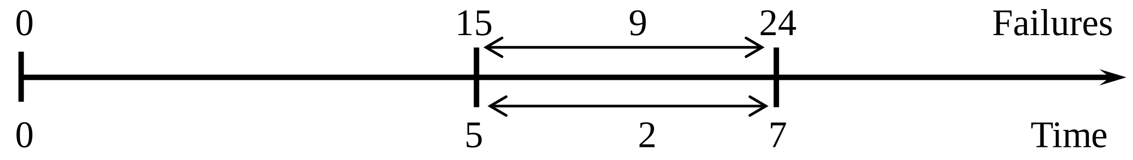
$$\lambda_{[5;7[} = \frac{9}{85\times 2} = 0.053 \; \textit{failures/product} \;\; \& \;\; t.u.$$

Average estimators: E(t)

Interval Estimator

Whatever the value of N, first calculate the estimator of λ and then apply the inverse relation
E(t) = 1/λ(t)

Example (N = 100)



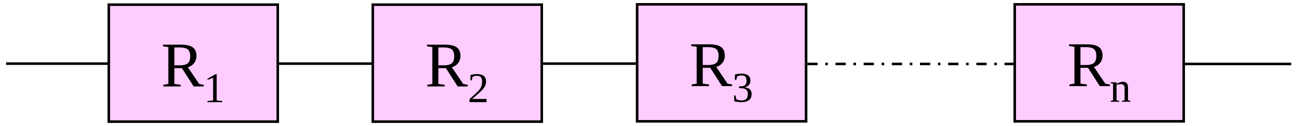
$$E(t)_{[5;7[} = \frac{1}{0.053} = \frac{85 \times 2}{9} = 18.89\ u.t.$$

Predictive Reliability

- Definition
  - Predictive reliability (predicted or theoretical) is calculated on the basis of a mathematical model defined from the functional decomposition of the system into subsets, components, etc. and the estimated or predicted reliability of its components

- Analysis and calculation procedure
  - Decompose the system into components (parts, subsystems ...) and establish the functional links between the components
  - Identify component reliability models or collect reliability at a given time
  - Search for a model of the system and calculate its reliability

Reliability of the serial system (S)



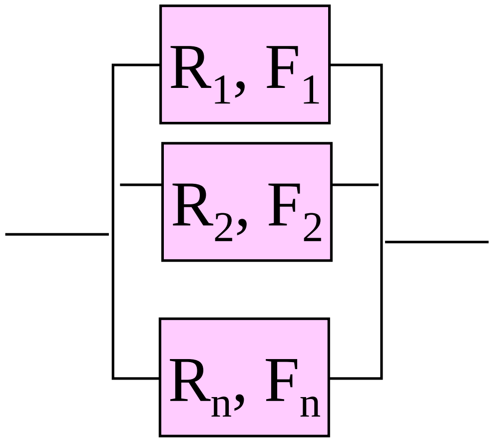
$$R(S) = R_1 \times R_2 \times R_3 \times \ldots \times R_n$$

$$R(S) = \prod_{1}^{n} R_i$$

- R1, R2, R3, ... Rn are the elementary reliabilities of the system components at a given time

Reliability of the parallel system (P)

- 1 only component on "n" must operate (system 1/n by default)



$$F(P) = F_1 \times F_2 \times F_3 \times \ldots \times F_n$$

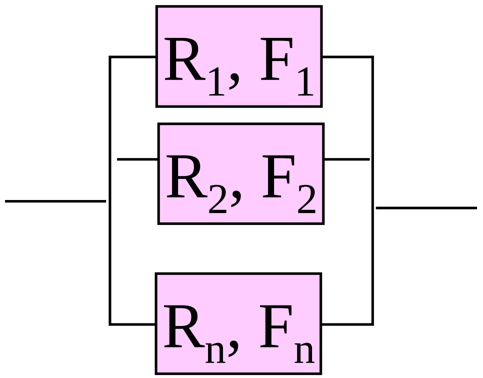
$$F(P) = \prod_1^n F_i$$

$$R(P) = 1 - \prod_1^n F_i = 1 - \prod_1^n (1 - R_i)$$

- R1, R2,... Rn and F1, F2,... Fn are respectively the elementary reliabilities and failures of the components of the system at a given time

Reliability of the parallel system (k/n)

“k" components on "n" must operate simultaneously (system k/n notation required)



$$B(p,k,n) = C_n^k\, p^k (1-p)^{n-k} \quad binomial$$

$$with: C_n^k = \frac{n!}{k!(n-k)!}$$

$$R(P) = B(p,k,n) + B(p,k+1,n) + B(p,k+2,n) + \ldots + B(p,n,n)$$

$$R(P) = \sum_{i=k}^{n} C_n^i\, p^i (1-p)^{n-i}$$

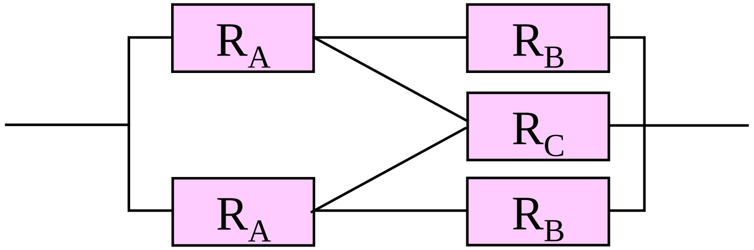
$n \rightarrow$ *total number of components*

$p \rightarrow$ *probability of success (reliability of a component)*

$k \rightarrow$ *number of components to operate simultaneously*

R1, R2, ... Rn and F1, F2, ... Fn are respectively the elementary reliabilities and failures of the components of the system at a given time (binomial law applicable only if the R are identical, otherwise, application of the truth table)

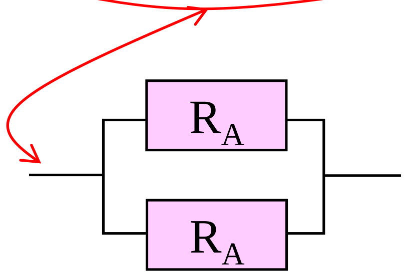
Reliability of a complex system

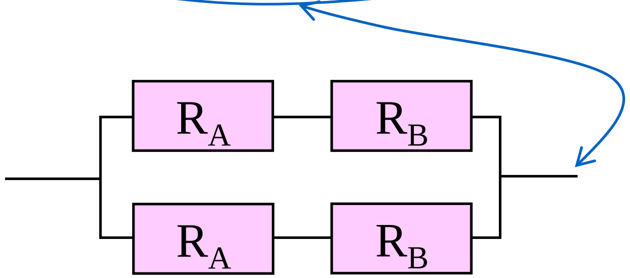


- Baye's theorem

$$\Pr(S)=\Pr(S/C)\times\Pr(C)+\Pr(S/\overline{C})\times\Pr(\overline{C})$$

$$R(S)=\left(1-(1-R_A)^2\right)\times R_C+\left(1-(1-R_AR_B)^2\right)\times(1-R_C)$$





Application to exponential model

R(t) = $e^{-\lambda t}$ and F(t) = 1-$e^{-\lambda t}$
with: λ = 1/E(t) (failure rate)

The serial system

$$R(t) = e^{(-t\sum_{i=1}^{n}\lambda_i)} = e^{(-t\lambda_S)}$$

The parallel system (n components)

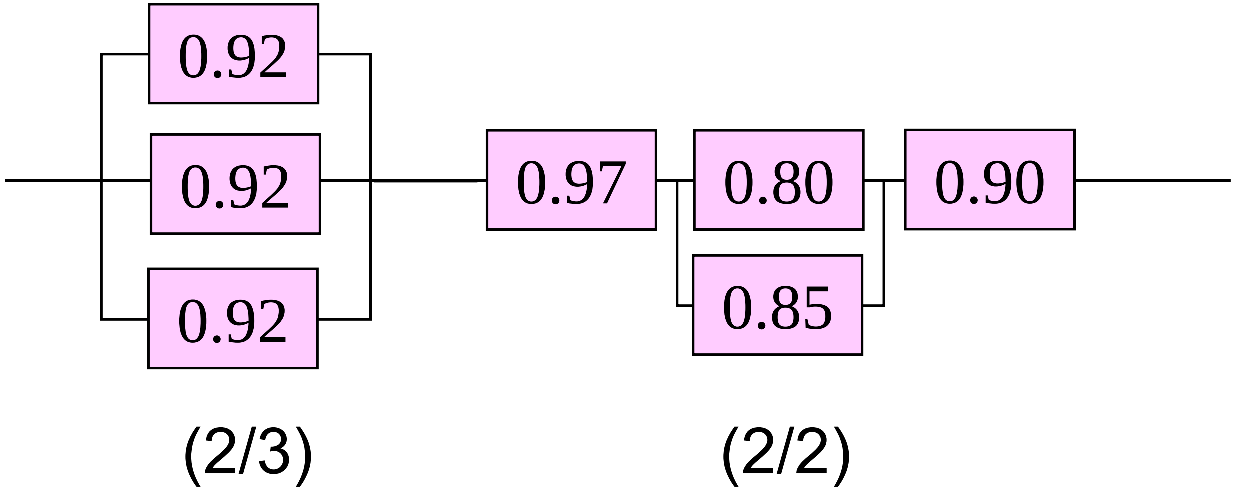
$$R(t) = 1 - \prod_{i=1}^{n}(1 - e^{(-\lambda_i t)})$$

The parallel system (2 components)

$$R(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

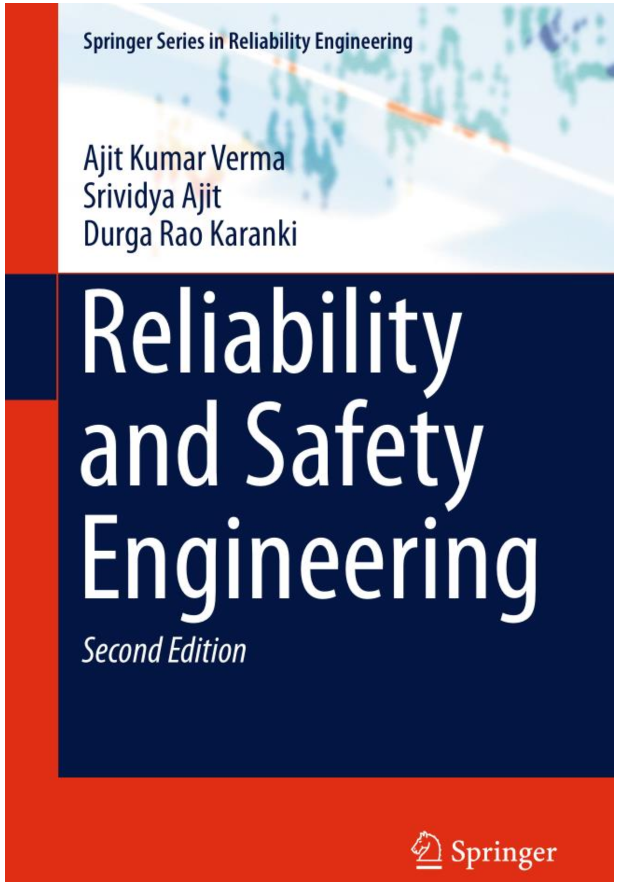
Example

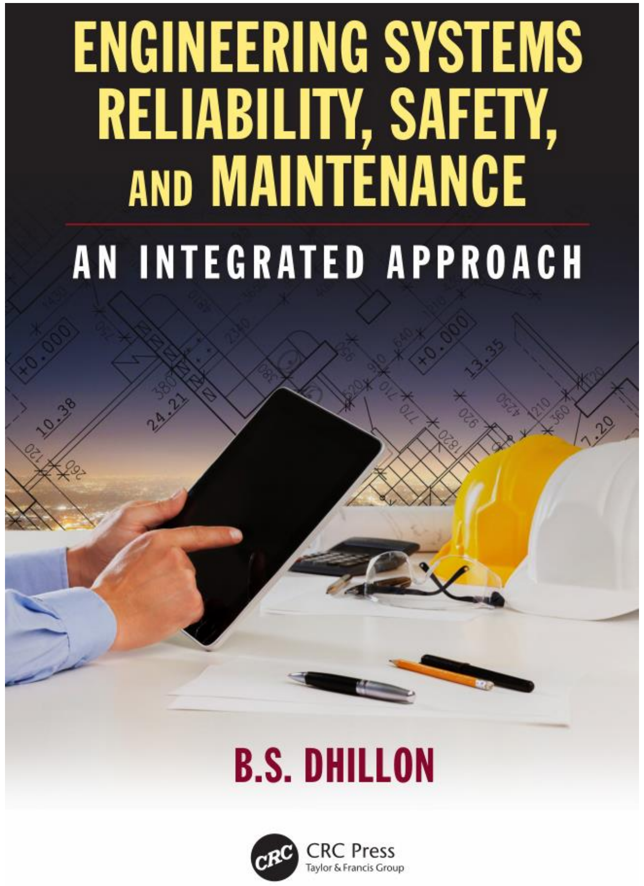
The given values correspond to the reliability of the components at a given time



Calculate the predictive reliability of this system

Relevant books





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