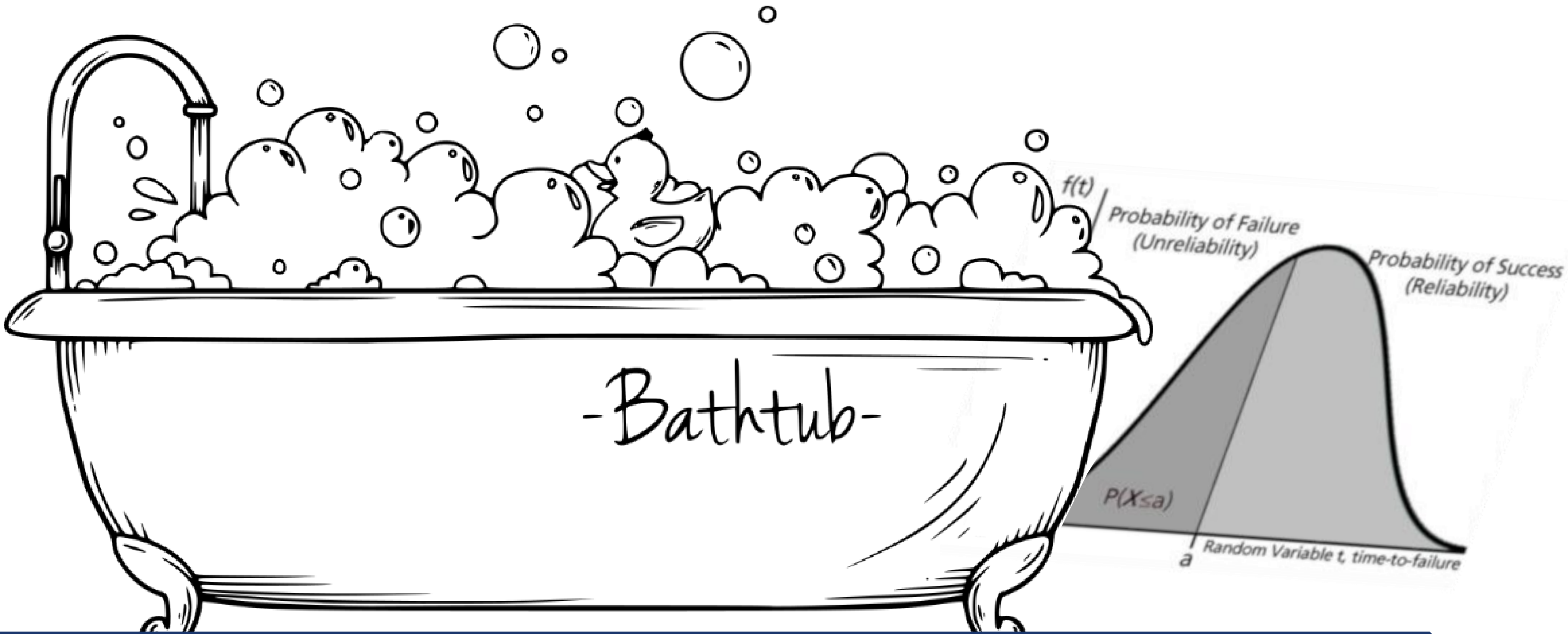


2021

MCTR 701

Master Advanced Mechatronics

Lecturers : Luc Marechal, Christine Barthod, Georges Habchi

UNIVERSITÉ SAVOIE MONT BLANC

POLYTECH ANNECY-CHAMBÉRY

SYMME

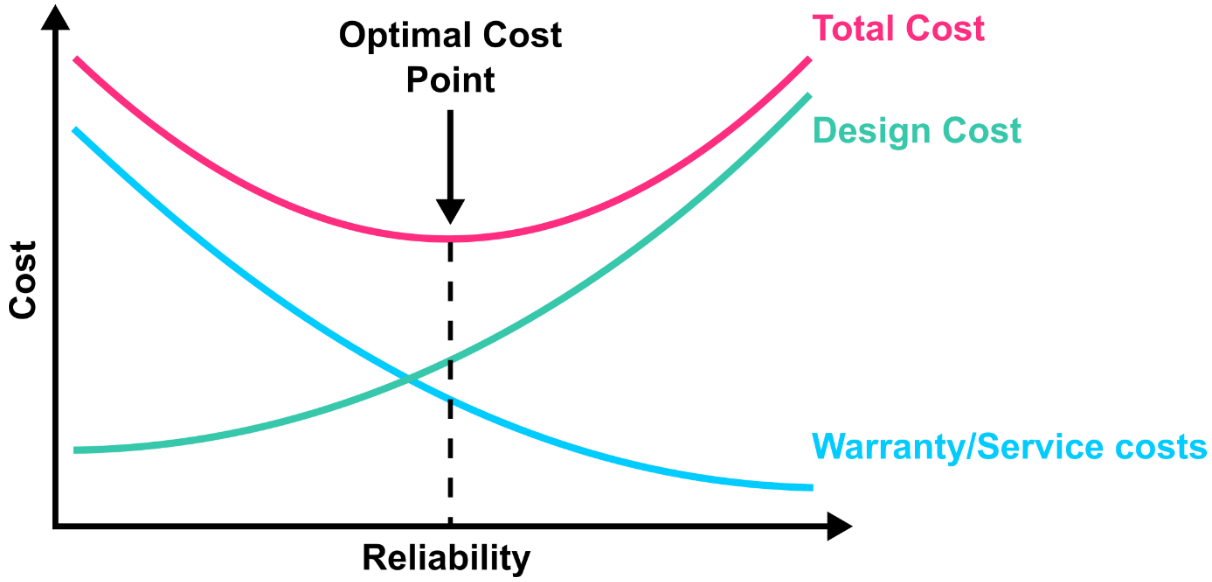
Mechatronics common framework Lecture 3

Contents

Lecture 3

RELIABILTIY

Costs vs Reliability



- Effects of Over-reliability in Development
  - Product is too expensive for target market
  - Product is later getting to market
  - Company is behind technology leaders due to slow program development cycles

- Effects of Under-reliability in Development
  - High field Return Rate
  - High Warranty Cost
  - Loss of product sales once low reliability is known in market
  - Loss of market share in all product lines due to poor brand perception.

1. RELIABILITY FUNCTIONS & ESTIMATORS

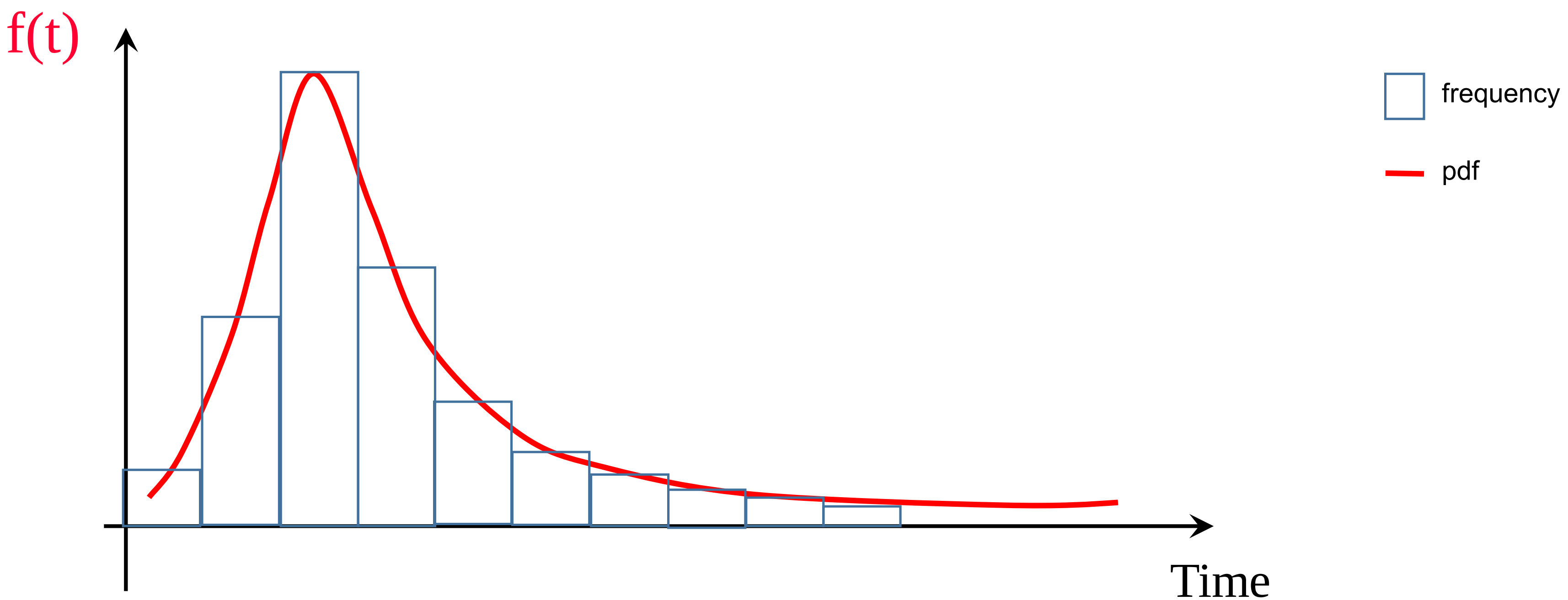
Reliability Functions & Estimators

- Reliability functions
  - The failure function F(t)
  - The reliability function R(t)
  - Complementarity between F(t) and R(t)
  - The failure rate λ(t)
  - The density function f(t)

- Reliability function estimators
  - Estimation of F(t)
  - Estimation of R (t)
  - Estimation of λ(t)
  - Estimation of f (t)
  - Estimation of the mean E(t)

The density function: f(t)

It represents the histogram of the relative failures as a function of time or the probability frequency of the relative failures relative to the unit of time.



The density function: f(t)

Example to better understand PDF

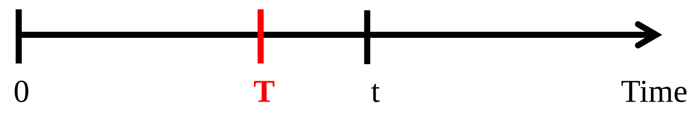
Take the example of heights of students in a class.
Let's say that the height of all the students is between 160 cm and 170 cm.
It makes no sense to ask the probability that the height of the student is EXACTELY 165.84 cm; that probability is zero.

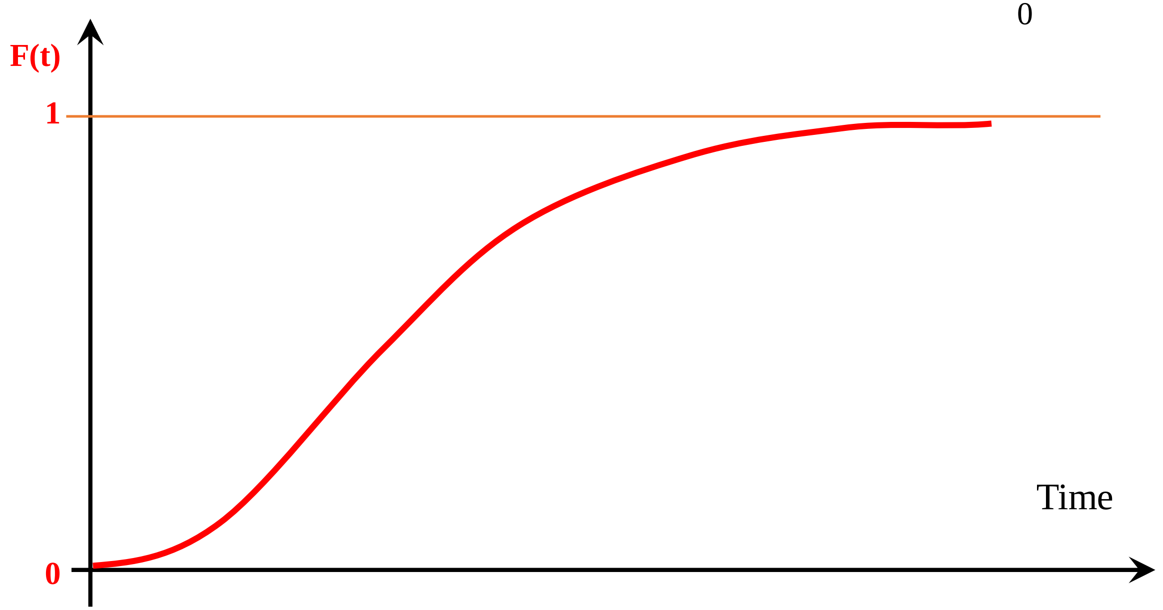
We can, however, define the probability that the height of the student lies in the infinitesimal interval [163;166] cm for example.
The function that gives this probability density is referred to as the probability density function or pdf.

The failure function: F(t)

It is measured by the probability that an entity E fails over the time interval [0, t]:
F(t) = Pr(E failing on [0, t])

That is, if we assume that an entity is failing at a date T:
F(t) = Pr(T≤t)

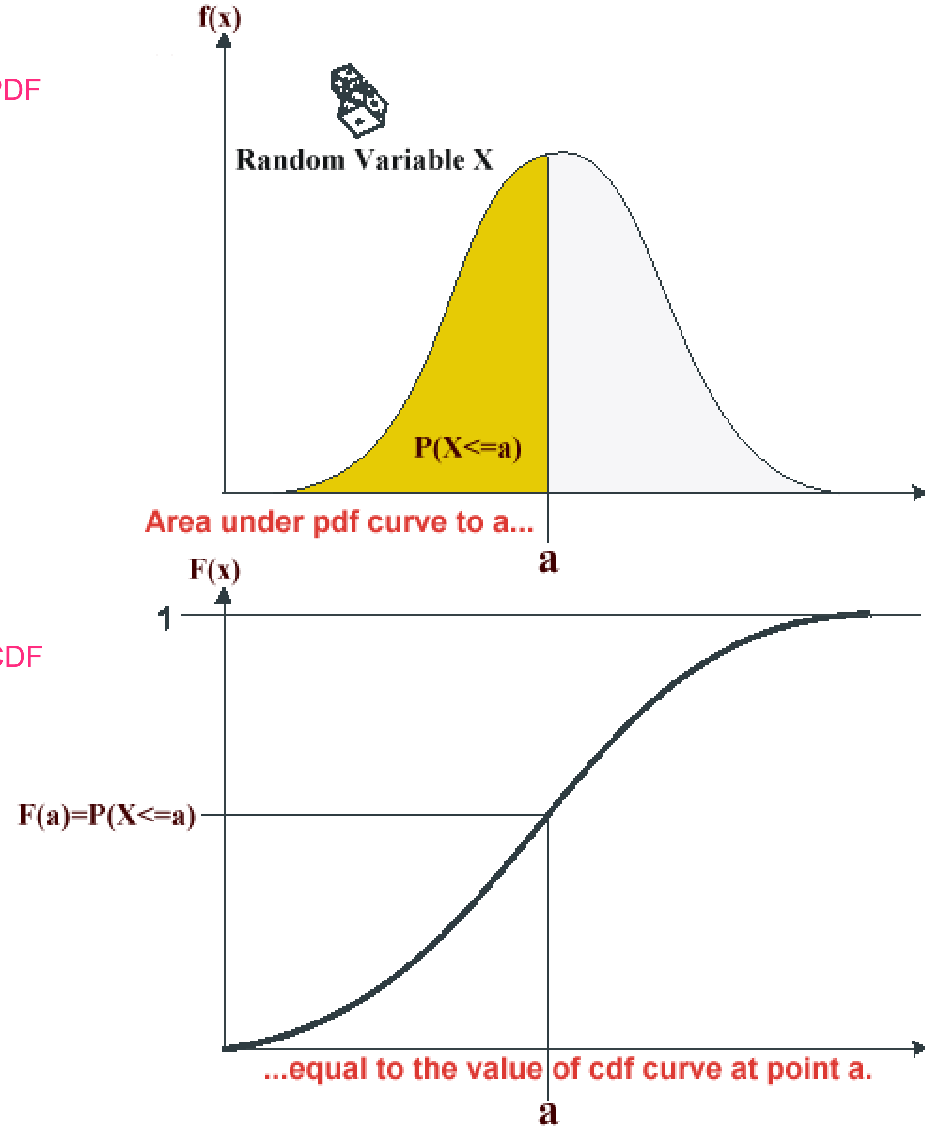


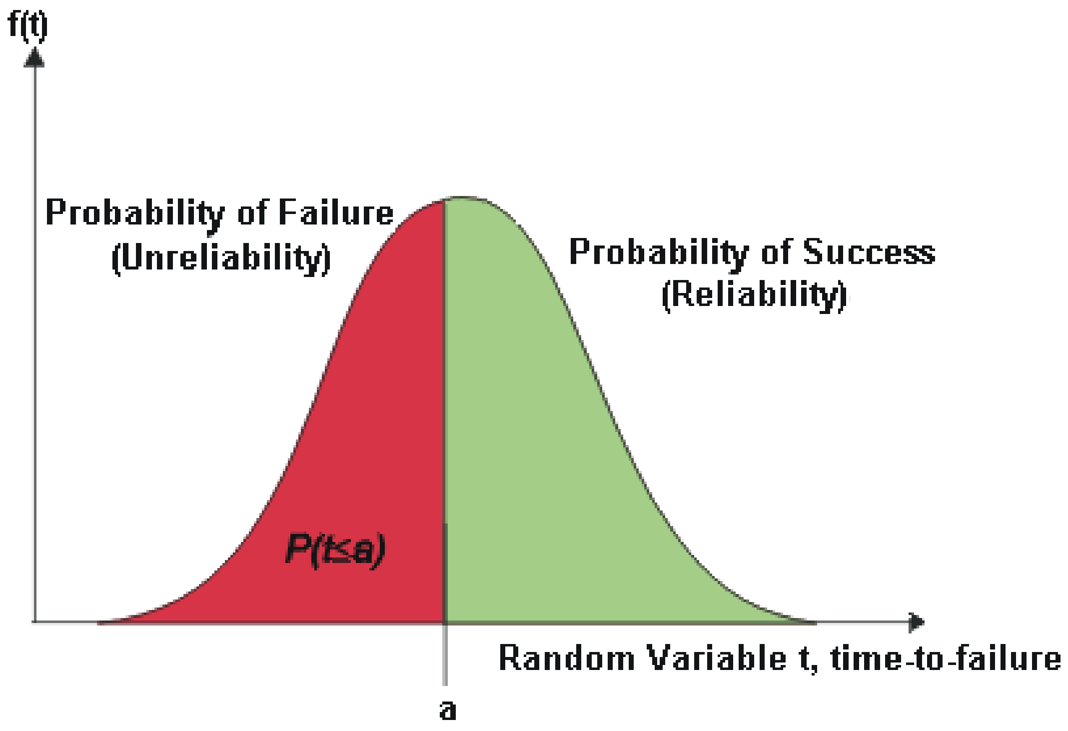


The density function f(t) and Failure function F(t)

PDF

CDF

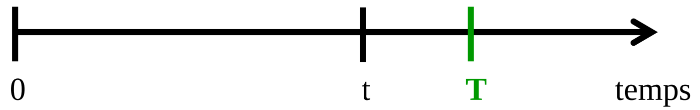


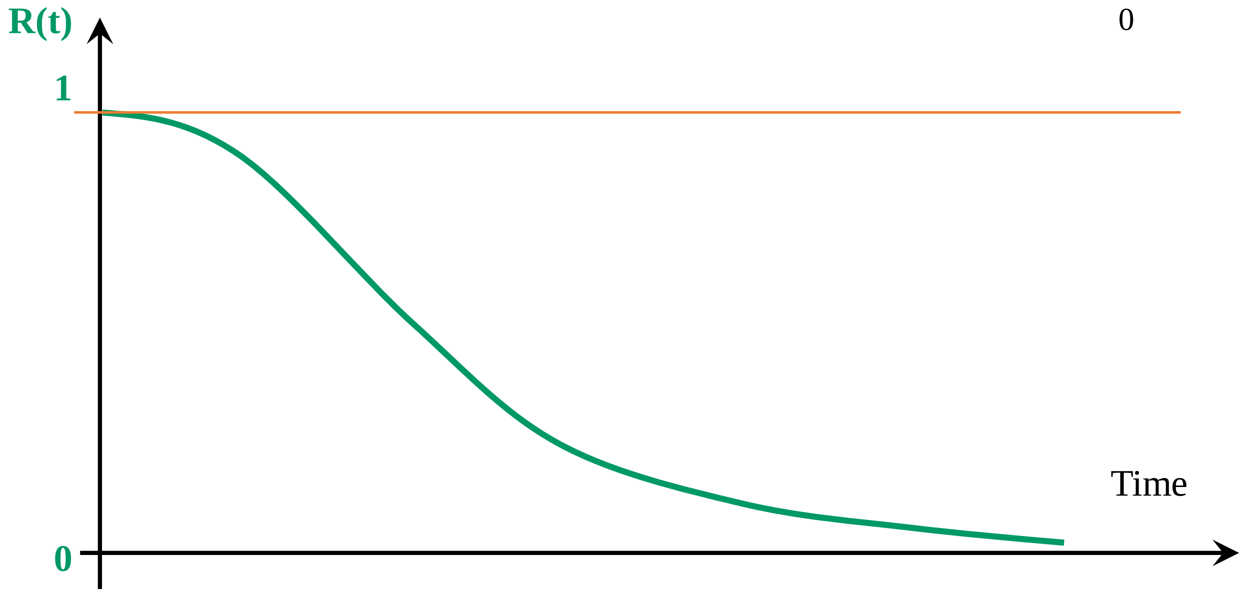


The reliability function: R(t)

It is measured by the probability that an entity E is non-faulty over the time interval [0, t]:
R(t) = Pr(E not failing on [0, t])

That is, if we assume that an entity is failing at a date T:
R(t) = Pr(T>t)

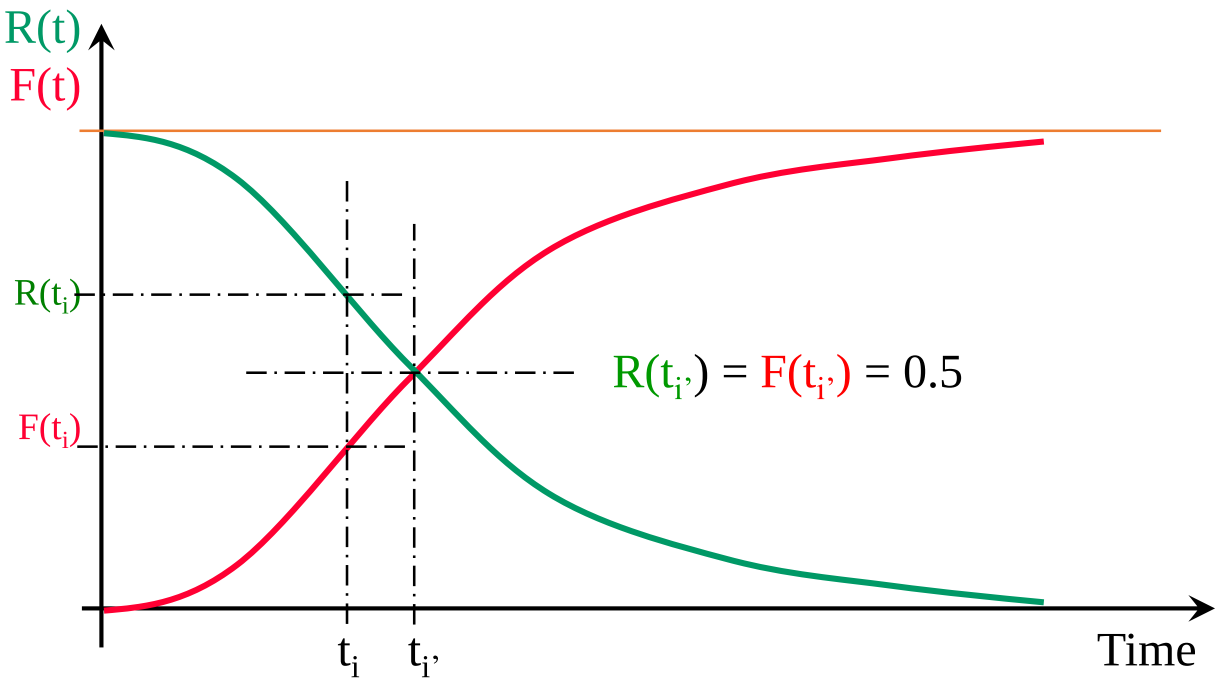




Complementarity between F(t) and R(t)

For a given system, ∀t = ti: we have

$R(ti) + F(ti) = 1$



Reliability Function

Several distributions defined by parameters are commonly used for reliability models, including:

- Exponential
- Weibull
- Gamma
- normal (Gaussian)
- log-normal
- log-logistic.

The choice of parametric distribution for a particular application can be made using graphical methods or using formal tests of fit.

The exponential law is well adapted when the failure rate (λ) is constant, (i.e. independent of time).

The normal law represents behaviours where the lifetime of the population is homogeneous, the probability of failure is centred and symmetrical. It can be used to model maturity behaviour (without breakdowns) and then rapid wear and tear.

The Weibull's law models each of the three phases of a material's life. It generalises the two previous laws but is more difficult to use and interpret.

Exponential Law

Exponential distribution

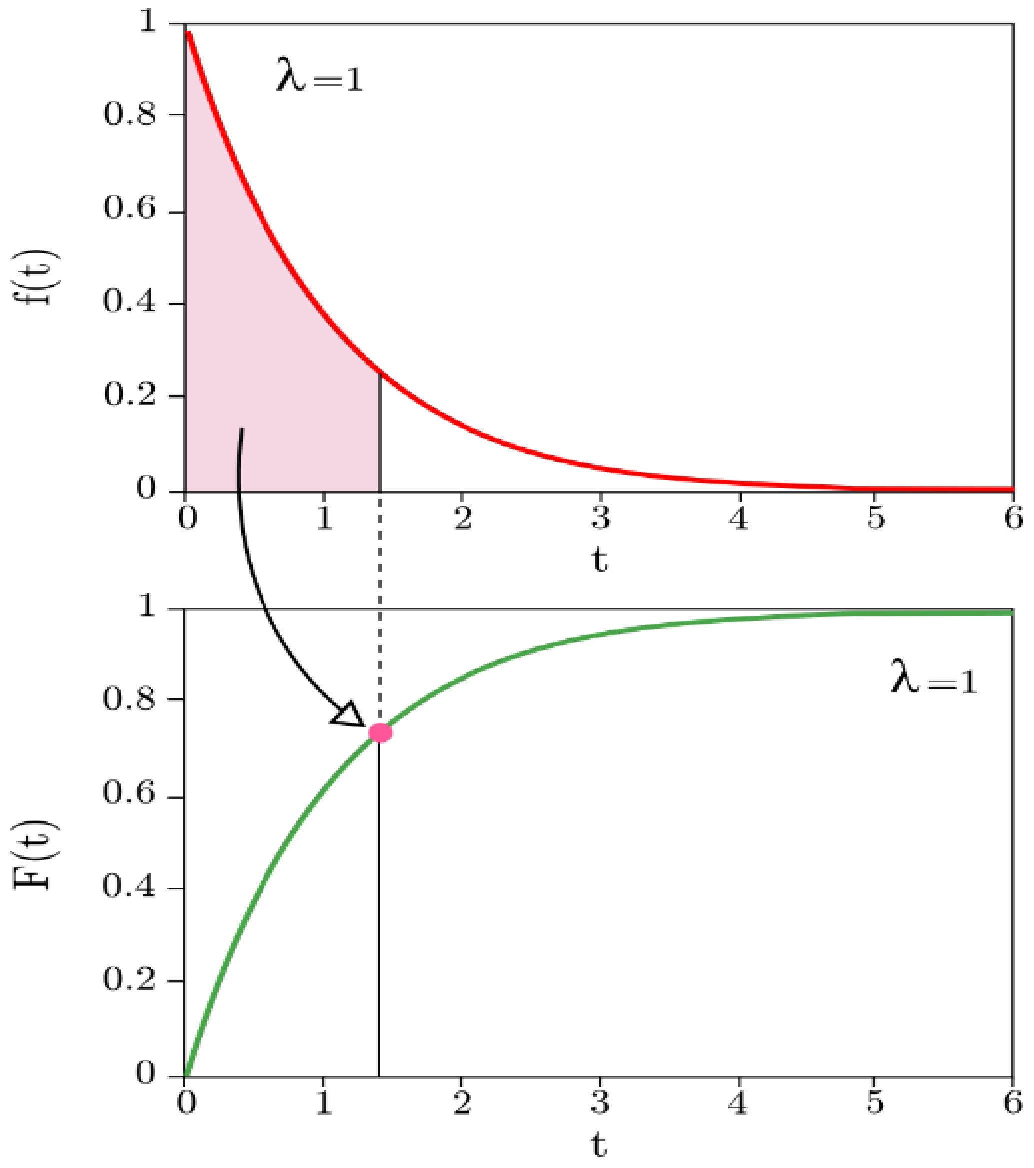
F(t) in this particular case with the exponential distribution, is the Failure function of a product when the failure rate λ is constant. The failure density is linked to the failure rate by the following relation:

$$f(t) = \lambda.e^{(-\lambda.t)}$$

$$\mathbf{F(t) = 1 - e^{(-\lambda.t)}}$$

$$\boldsymbol{MTTF : E(t) = 1/\lambda}$$

$$\mathbf{R(t) = e^{(-\lambda.t)}}$$

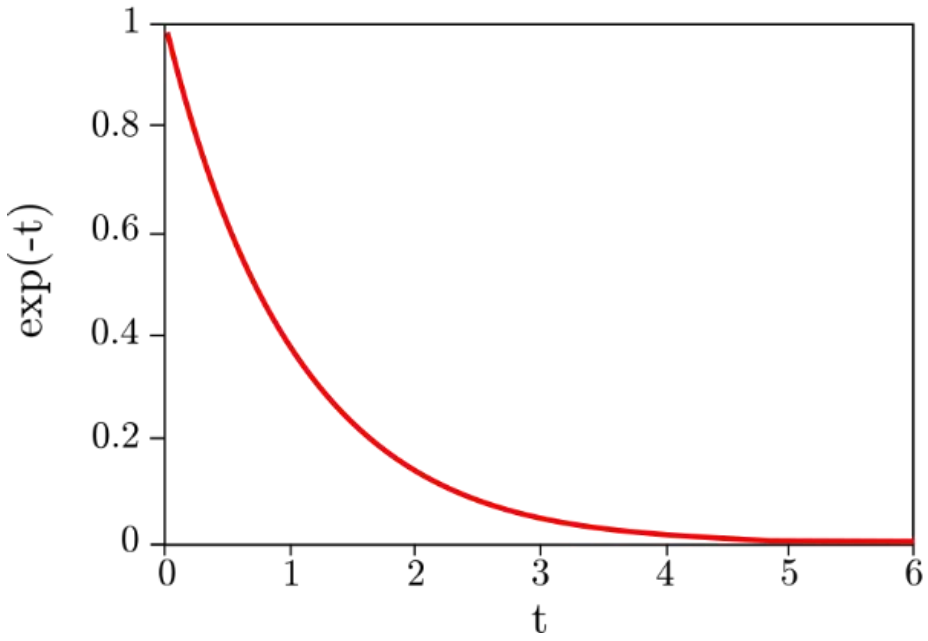


Exponential distribution

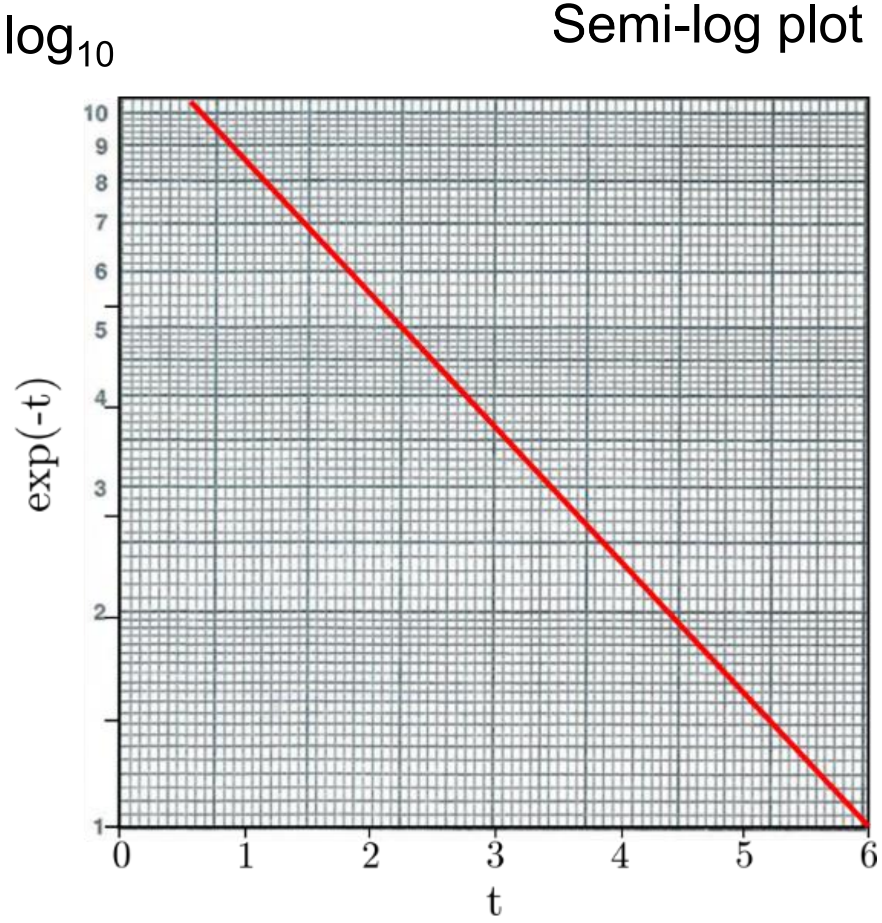
Recall

$$R(t) = e^{(-\lambda . t)} \Leftrightarrow ln(R(t)) = ln(e^{(-\lambda . t)})$$

$$\Leftrightarrow ln(R(t)) = -\lambda . t$$



Semi-log plot



Exponential distribution

Example

Assume that the mean time to failure of an engineering system is 1500 hours.

Calculate the probability of failure of the engineering system during a 500-hour mission.

$$\mathrm{MTTF} = 1500 \text{ hours}$$

$$\lambda = \frac{1}{\mathrm{E(t)}} = \frac{1}{1500} = 0.00066 \text{ failures per hour}$$

$$F(500) = 1 - \mathrm{e}^{(-0.00066 \times 500)} = 0.2834 \rightarrow 28.3\%$$

Example

The failure time (T) of an electronic circuit board follows exponentially distribution with failure rate $\lambda = 10^{-4}$ /h.

a) What is the probability that it will fail before 1000 h ?

b) What is the probability that it will survive at least 10,000 h

c) What is the probability that it will fail between 1000 and 10,000 h

a) For exponential distribution $F(T) = 1 - e^{-\lambda t}$ $\quad$ $F(T = 1000) = 0.09516$

b) For exponential distribution $R(T) = e^{-\lambda t}$ $\quad$ $R(T = 10,000) = 0.3678$

c) $F(10,000) - F(1000) = [1 - R(10,000)] - F(1000) = [1 - 0.3678] - 0.09516 = 0.537$

Exponential distribution

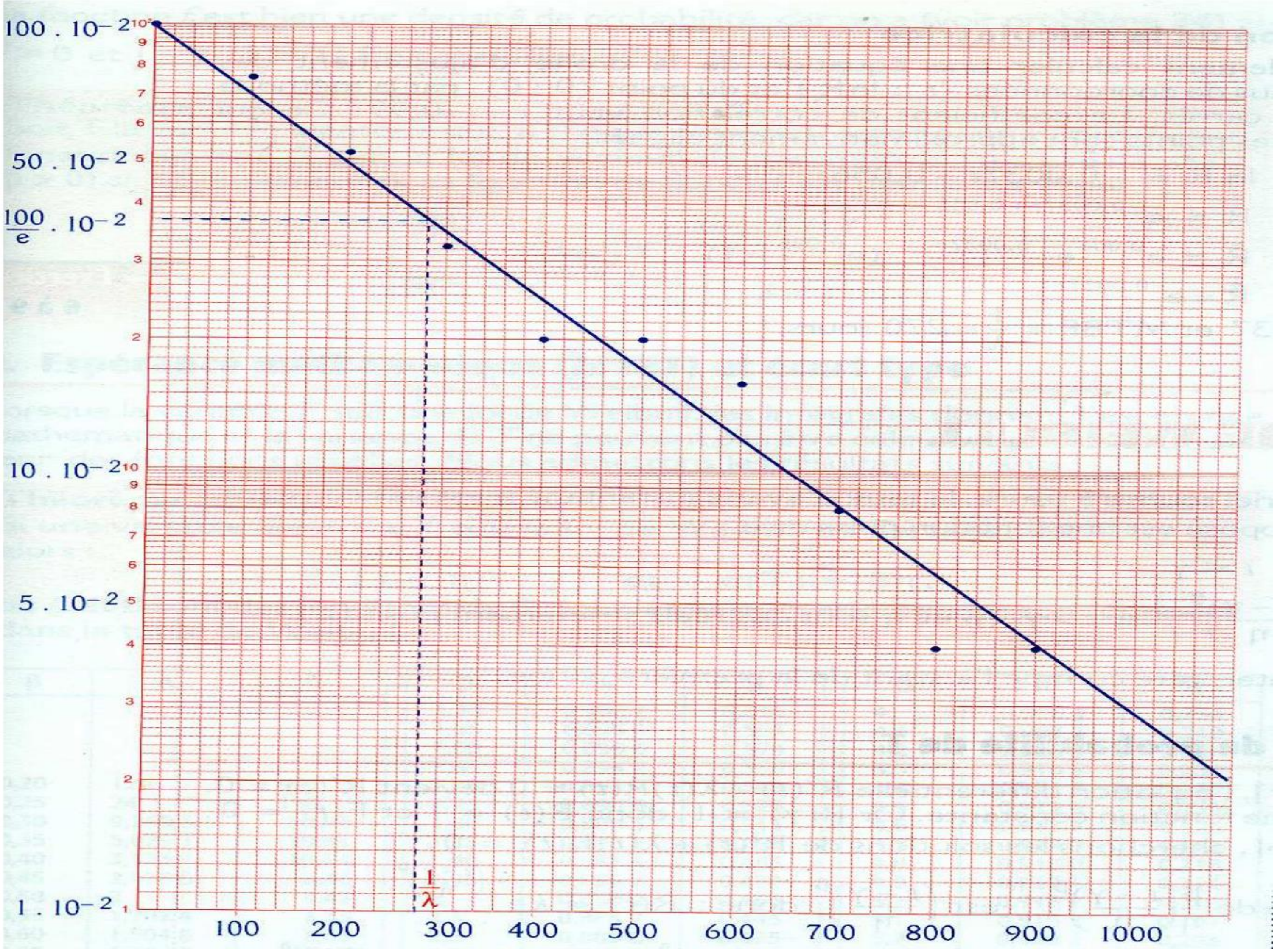
Example

The reliability data of a machine is given :

Determine if this lifetime law follows an exponential distribution?

TTF (day)	0	100	200	300	400	500	600	700	900
R(t)	1	0,76	0,52	0,32	0,20	0,20	0,16	0,08	0,04
R(t)	$100.\ 10^{-2}$	$76.\ 10^{-2}$	$52.\ 10^{-2}$	$32.\ 10^{-2}$	$20.\ 10^{-2}$	$20.\ 10^{-2}$	$16.\ 10^{-2}$	$8.\ 10^{-2}$	$4.\ 10^{-2}$

Exponential distribution



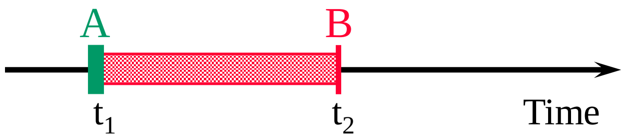
$$\frac{1}{\lambda} \approx 280 \text{ , d'où } \lambda \approx 0{,}0036.$$

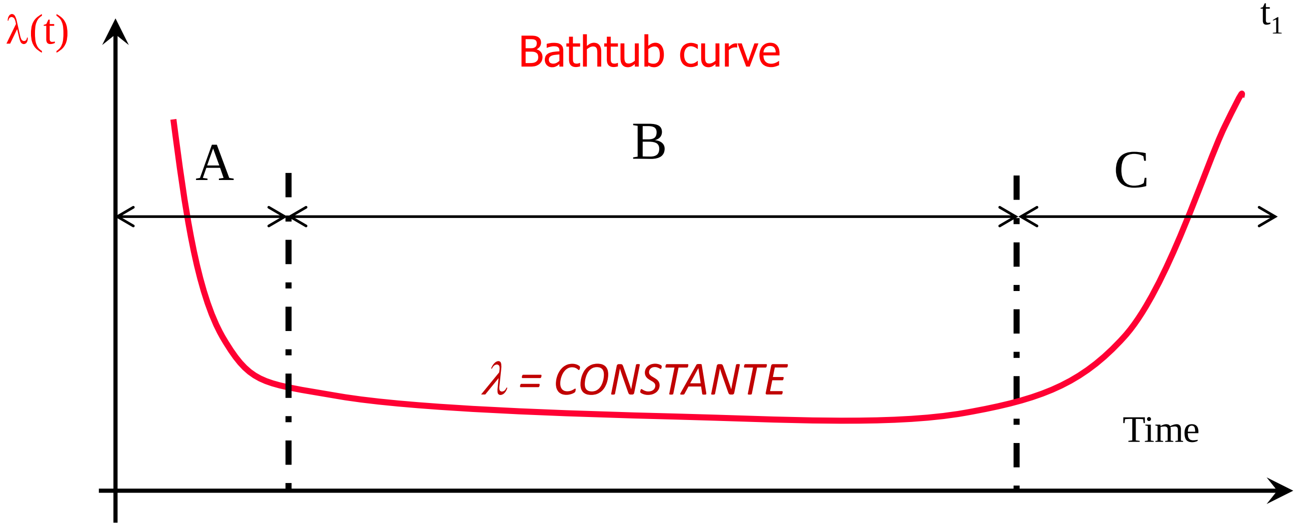
Weibull’s Law

The failure rate: $\lambda(t)$

It expresses the speed of arrival of failures at time t, or the evolution of the conditional probability of failure during the life of the syst. (failure/product and unit of time)

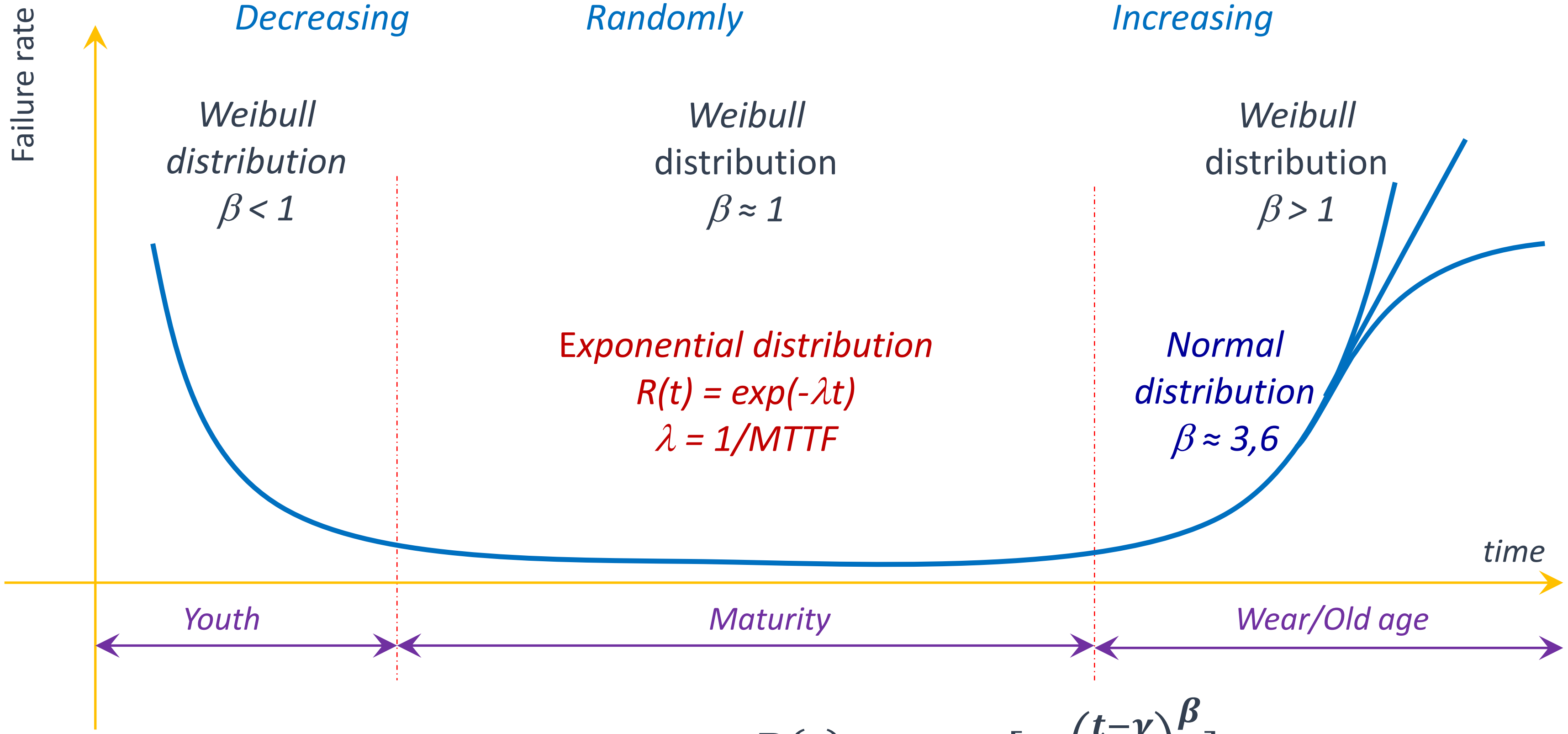
A: Event "product runs at t1"
B: Event "product fails between t1 and t2"
$\lambda(t) = Pr(B/A) / (t2 - t1)$





A: Youth
B: Maturity period or useful life
C: Period of aging or wear

Reliability distribution and parameters (Weibull's Law)



$$\text{Weibull Distribution : } \boldsymbol{R(t) = exp}\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]$$

β : shape parameter ; η : scale parameter ; γ : position parameter

Reliability distribution and parameters

Technology	Comments	Reliability distribution and parameters
Electronics	Known distribution and parameters available in Database	Exponential : **MTTF*** or λ (failure rate)
Mechanics	• Distribution are known for few standard elements • To find for most of specific components	Weibull : ▪ β (shape parameter) ▪ η (scale parameter)

*MTTF : Mean Time To Failure

FIDES 2009
(reliability database for electronic *components)*

Feedback or estimation of reliability parameters for mechanical components

FIDES database

FIDES

The expression of the failure rate or the MTTF of electronic components depends on several factors that themselves depend on 3 steps:

- design technology
- manufacturing process
- environmental operation

Design Technology

Electronic Reliability

Manufacturing *Process*

Environment Conditions of use

https://www.fides-reliability.org/

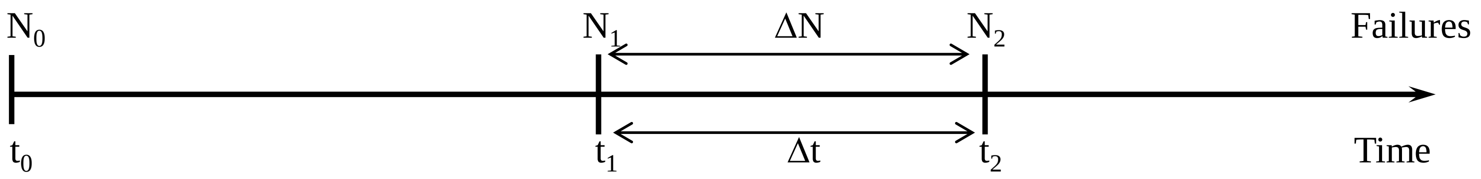
Estimators

It is obviously not realistic to calculate failure rates by building many units and running them for many hours, under expected operating conditions.

This is especially true for well-designed and properly built supplies, with extremely low failure rates, where the number of supplies and hours required to get valid results would be in the thousands.

Instead based on representative samples of a population statistical analysis techniques can be used to estimate failure rates.

Estimation of the reliability functions



N: total number of products put into operation at time t0 (sample size)
N0: number of failures at t0 (generally equal to 0)
N1: number of failures at t1
N2: number of failures at t2
ΔN: number of failures on the interval [t1, t2]
Δt: time interval [t1, t2]

Reliability function estimators depend on the value of N (number of products put into operation)

N	$1<N\leq20$	$20<N\leq50$	$N>50$
Estimator	Median Ranks	Average Ranks	Cumulative Frequencies

Failure estimators: F(t)

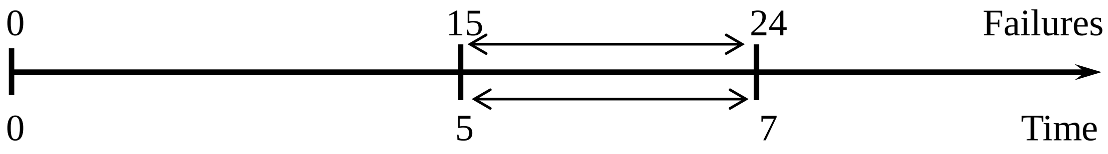
Point estimator at time t

Median ranks if 1<N≤20 $$F(t)=\frac{N_t-0.3}{N+0.4}$$

Average ranks if 20<N≤50 $$F(t)=\frac{N_t}{N+1}$$

Cumulative frequencies if N>50 $$F(t)=\frac{N_t}{N}$$

Example (N = 100) ➔ Cumulative frequencies



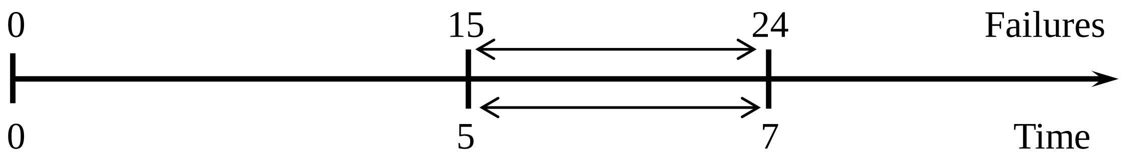
$$F(5)=\frac{15}{100}=0.15 \qquad F(7)=\frac{24}{100}=0.24$$

Reliability estimators: R(t)

Point estimator at time t

Whatever the value of N, first compute the estimator of F and then apply the complementarity relation
R(t) = 1 - F(t)

Example (N = 100)



$$R(5) = 1 - 0.15 = 0.85 \qquad R(7) = 1 - 0.24 = 0.76$$

$$R(5) = \frac{100 - 15}{100} = 0.85 \qquad R(7) = \frac{100 - 24}{100} = 0.76$$

Density estimators: f(t)

Interval estimator (failure/product and unit of time)

Median ranks if 1<N≤20

$$f_{[t1;t2[} = \frac{\Delta N}{(N+0.4)\times \Delta t}$$

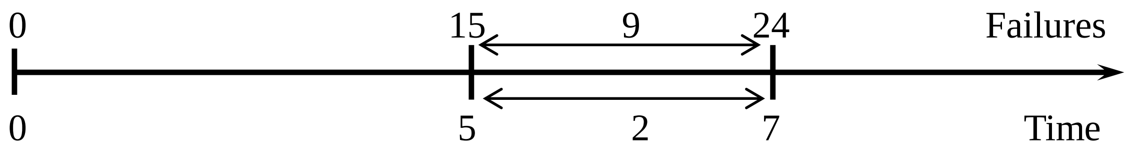
Average ranks if 20<N≤50

$$f_{[t1;t2[} = \frac{\Delta N}{(N+1)\times \Delta t}$$

Cumulative frequencies if N>50

$$f_{[t1;t2[} = \frac{\Delta N}{N\times \Delta t}$$

Example (N = 100)



$$f_{[5;7[} = \frac{9}{100\times 2} = 0.045 \; failures/product \,\&\, t.u.$$

Failure rate estimators: λ(t)

Estimator on interval (failure/product and unit of time) (f/R)

Median ranks if 1<N≤20

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N + 0.7 - N_1) \times \Delta t}$$

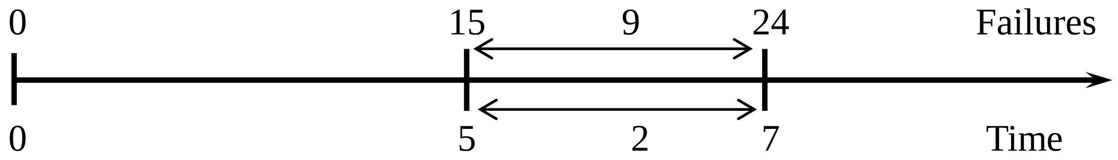
Average ranks if 20<N≤50

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N + 1 - N_1) \times \Delta t}$$

Cumulative frequencies if N>50

$$\lambda_{[t1;t2[} = \frac{\Delta N}{(N - N_1) \times \Delta t}$$

Example (N = 100)



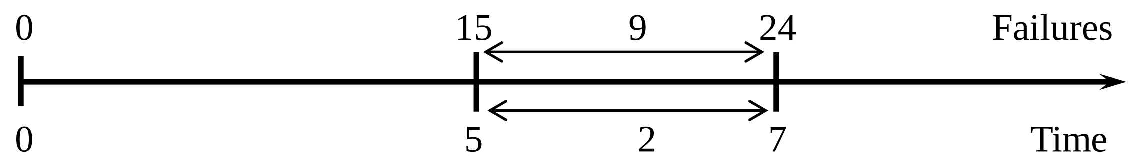
$$\lambda_{[5;7[} = \frac{9}{85 \times 2} = 0.053 \; failures/product \;\; \& \;\; t.u.$$

Average estimators: E(t)

Interval Estimator

Whatever the value of N, first calculate the estimator of λ and then apply the inverse relation $E(t) = 1/\lambda(t)$

Example (N = 100)



$$E(t)_{[5;7[} = \frac{1}{0.053} = \frac{85 \times 2}{9} = 18.89\ u.t.$$

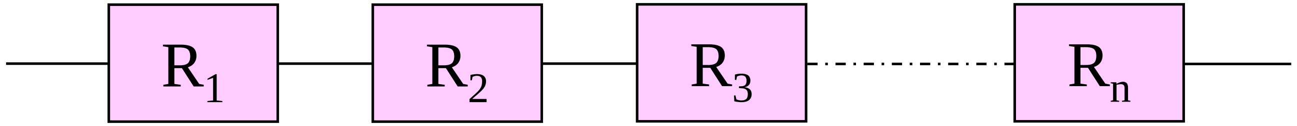
2. PREDICTIVE RELIABILITY

Predictive Reliability

- Definition
  - Predictive reliability (predicted or theoretical) is calculated on the basis of a mathematical model defined from the functional decomposition of the system into subsets, components, etc. and the estimated or predicted reliability of its components

- Analysis and calculation procedure
  - Decompose the system into components (parts, subsystems ...) and establish the functional links between the components
  - Identify component reliability models or collect reliability at a given time
  - Search for a model of the system and calculate its reliability

Reliability of the serial system (S)



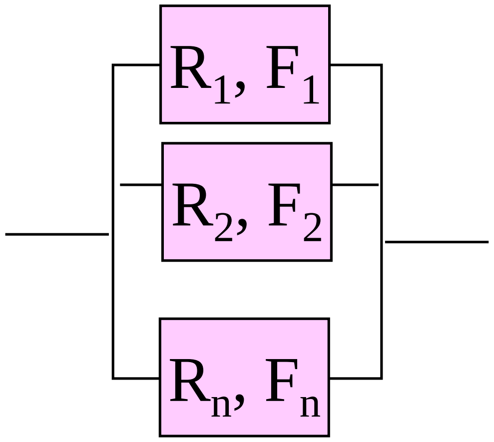
$$R(S) = R_1 \times R_2 \times R_3 \times \ldots \times R_n$$

$$R(S) = \prod_{1}^{n} R_i$$

- R1, R2, R3, ... Rn are the elementary reliabilities of the system components at a given time

Reliability of the parallel system (P)

- 1 only component on "n" must operate (system 1/n by default)



$$F(P) = F_1 \times F_2 \times F_3 \times \ldots \times F_n$$

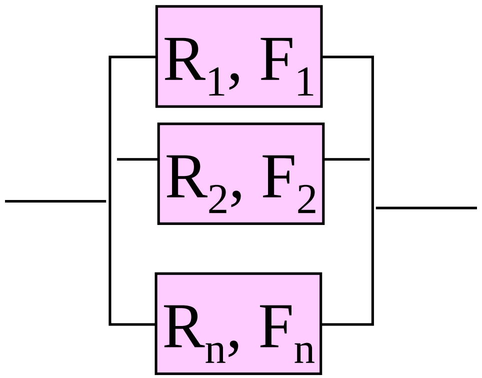
$$F(P) = \prod_1^n F_i$$

$$R(P) = 1 - \prod_1^n F_i = 1 - \prod_1^n (1 - R_i)$$

- R1, R2,... Rn and F1, F2,... Fn are respectively the elementary reliabilities and failures of the components of the system at a given time

Reliability of the parallel system (k/n)

“k" components on "n" must operate simultaneously (system k/n notation required)



$$B(p,k,n) = C_n^k p^k (1-p)^{n-k} \quad binomial$$

$$with: C_n^k = \frac{n!}{k!(n-k)!}$$

$$R(P) = B(p,k,n) + B(p,k+1,n) + B(p,k+2,n) + ... + B(p,n,n)$$

$$R(P) = \sum_{i=k}^{n} C_n^i p^i (1-p)^{n-i}$$

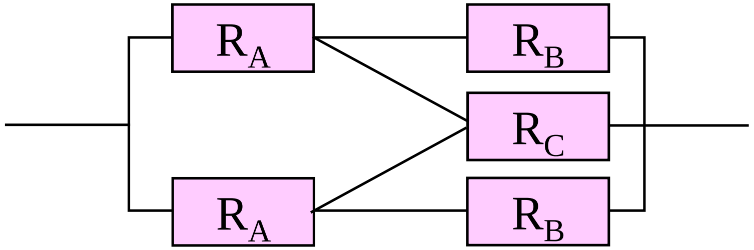
$n \rightarrow$ total number of components

$p \rightarrow$ probability of success (reliability of a component)

$k \rightarrow$ number of components to operate simultaneously

R1, R2, ... Rn and F1, F2, ... Fn are respectively the elementary reliabilities and failures of the components of the system at a given time (binomial law applicable only if the R are identical, otherwise, application of the truth table)

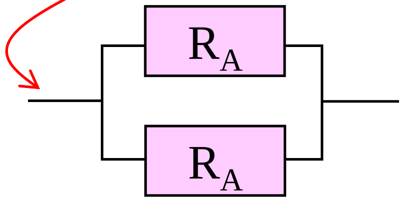
Reliability of a complex system

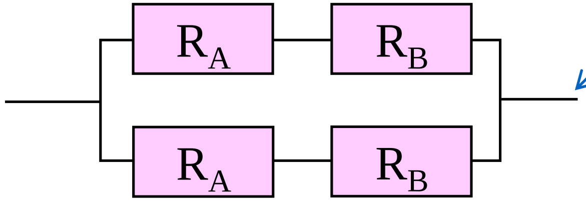


- Baye's theorem

$$\Pr(S) = \Pr(S/C)\times\Pr(C) + \Pr(S/\overline{C})\times\Pr(\overline{C})$$

$$R(S) = \left(1-(1-R_A)^2\right)\times R_C + \left(1-(1-R_AR_B)^2\right)\times(1-R_C)$$





Application to exponential model

R(t) = $e^{-\lambda t}$ and F(t) = 1-$e^{-\lambda t}$
with: λ = 1/E(t) (failure rate)

The serial system

$$R(t) = e^{(-t\sum_{i=1}^{n}\lambda_i)} = e^{(-t\lambda_S)}$$

The parallel system (n components)

$$R(t) = 1 - \prod_{i=1}^{n}(1 - e^{(-\lambda_i t)})$$

The parallel system (2 components)

$$R(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

Example 1 A personal computer consists of four basic sub systems: motherboard (MB), hard disk (HD), power supply (PS) and processor (CPU). The reliabilities of four subsystems are 0.98, 0.95, 0.91 and 0.99 respectively. What is the system reliability for a mission of 1000 h?

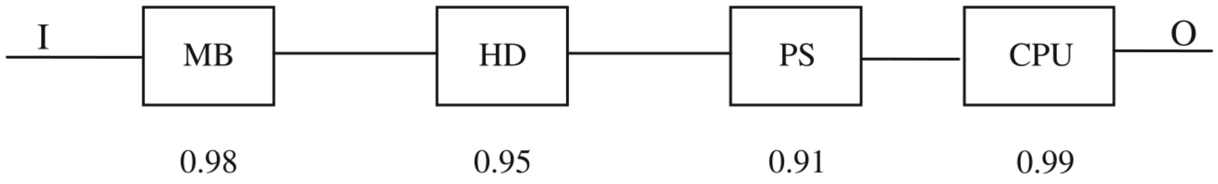
Solution: As all the sub-systems need to be functioning for the overall system success, the RBD is series *configuration*

The reliability of system is

$$R_{sys} = R_{MB} \times R_{HD} \times R_{PS} \times R_{CPU}$$

$$R_{sys} = 0.98 \times 0.95 \times 0.91 \times 0.99$$

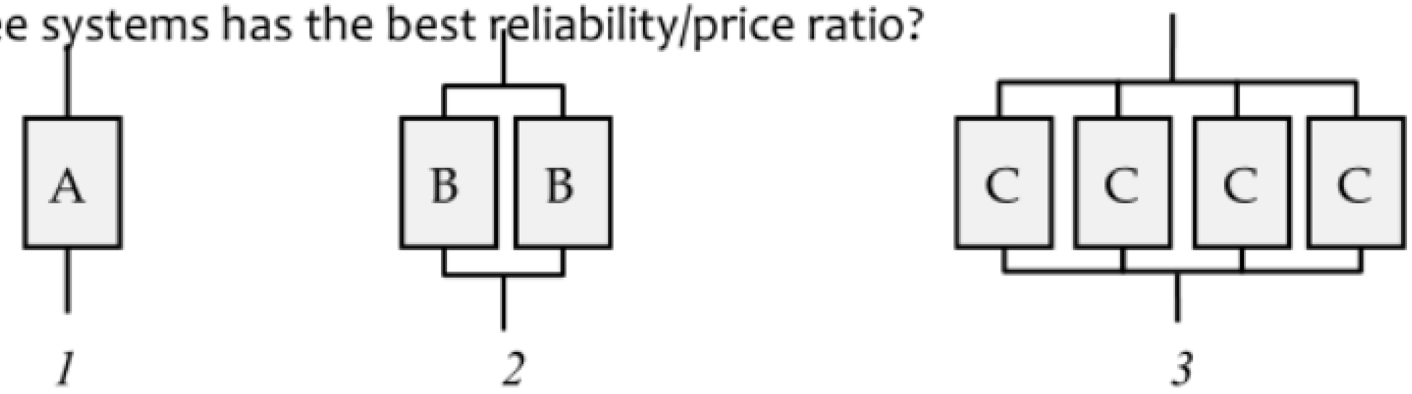
$$R_{sys} = 0.8387$$



3. We consider three components A, B and C of respective prices 150, 75 and 45 €. Be at a given moment, F_A=0.1, F_B=0.3 and F_C=0.4 the respective failures of the three components.

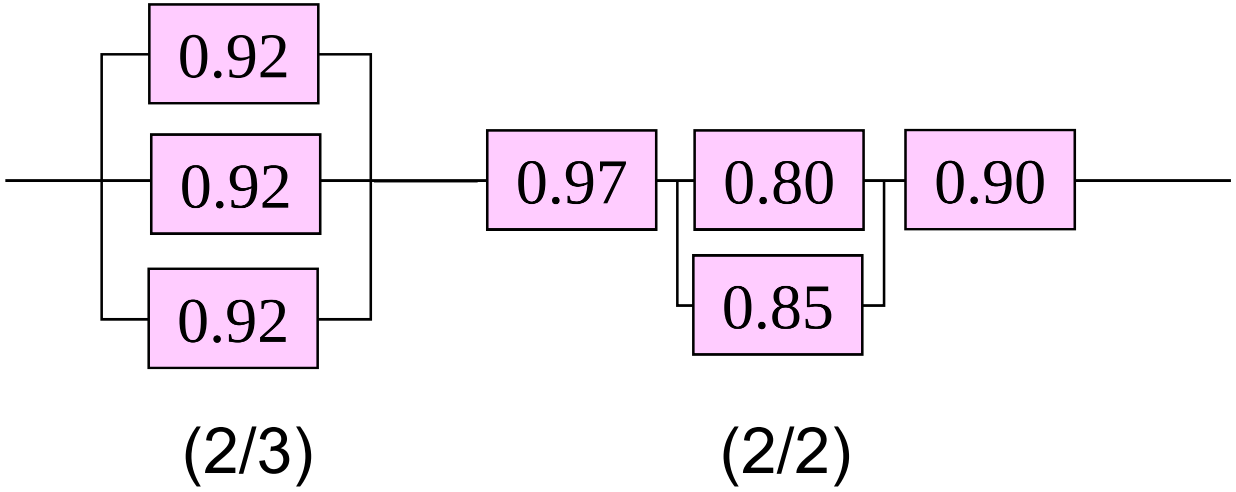
I. Which of the following three systems has the highest reliability?

II. Which of the three systems has the best reliability/price ratio?



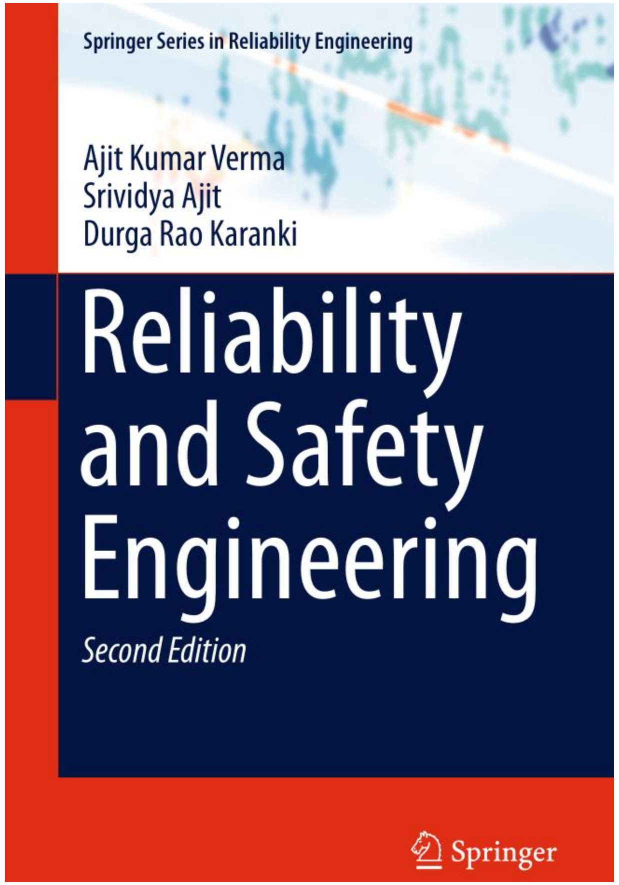
Example

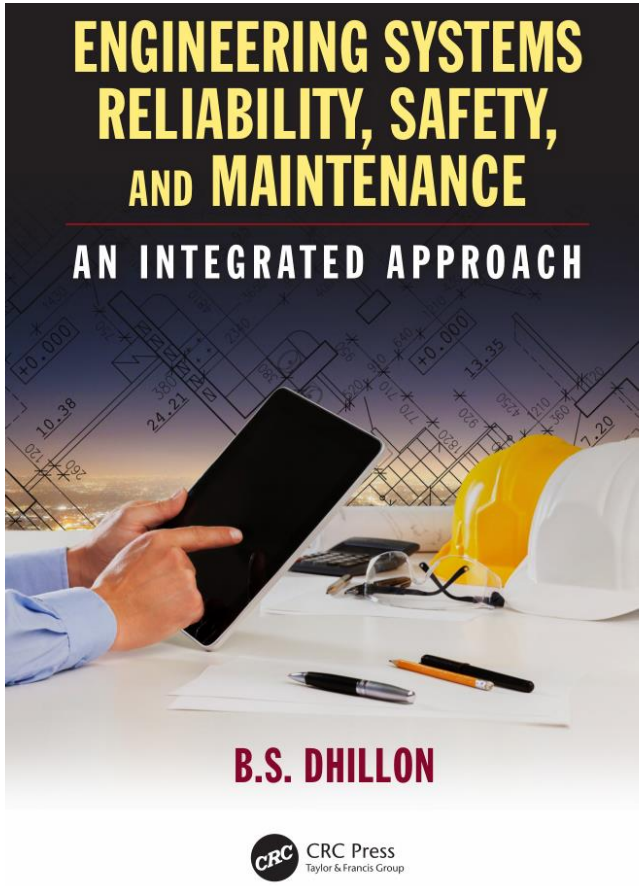
The given values correspond to the reliability of the components at a given time



Calculate the predictive reliability of this system

Relevant books





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