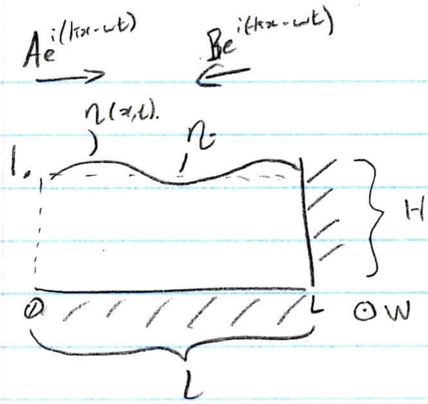
STEELE, LUC

(2)



Standing wave.

$p(L,t) = p_0 \cos\omega t$

at surface

$p(L,H,t) = \rho g \eta(L,t)$

$p_0 \cos\omega t = \rho g \eta(L,t)$

let $t=0$ $\quad p_0 = \rho g \eta_0 \Rightarrow \eta_0 = \dfrac{p_0}{g\rho} \Rightarrow \eta(L,t) = \eta_0 \cos(\omega t)$

$\frac{\omega}{k} = \sqrt{gH}$

$\eta(x,t) = Ae^{i(kx-\omega t)} + Be^{i(-kx-\omega t)}$ $\qquad |A| = |B|$ no energy loss.

As in assignment #2 : $A+B=\eta_0$ from $\eta(0,t) = Re(\eta_0 e^{-i\omega t})$

$A+B=\eta_0$

$u(L,t) = 0 = \dfrac{\partial \eta}{\partial x}(L,t) = ikAe^{i(kL-\omega t)} - ikBe^{i(-kL-\omega t)} = 0$

$A = Be^{-2ikL}$

$B = \dfrac{\eta_0 e^{ikL}}{e^{ikL}+e^{-ikL}} = \dfrac{\eta_0 e^{ikL}}{2\cos kL} \Rightarrow A = \dfrac{\eta_0}{2}\dfrac{e^{-ikL}}{\cos kL}$

$\eta(x,t) = \dfrac{\eta_0}{2}\dfrac{e^{-ikL}}{\cos kL} e^{i(kx-\omega t)} + \dfrac{\eta_0}{2}\dfrac{e^{ikL}}{\cos kL} e^{i(-kx-\omega t)} = \eta_0 \dfrac{\cos(kx)}{\cos(kL)}\cos(\omega t)$

$\eta_0 = \dfrac{p_0}{g\rho}$

2. $\dfrac{\partial u}{\partial t} = -g\dfrac{\partial \eta}{\partial x}$

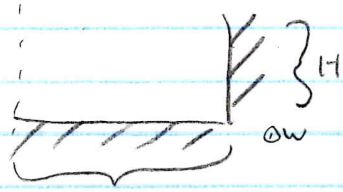
$= gk\eta_0 \dfrac{\sin(kx)}{\cos(kL)}\cos(\omega t)$

$u(x,t) = \dfrac{gk\eta_0}{\omega}\dfrac{\sin(kx)\sin(\omega t)}{\cos(kL)} + C$

3. $u = u_0 \sin(\omega t + \phi)$

∴ energy into basin.

$p = p_0 \cos \omega t$



looking for average so can ignore 2 will average out.

$u : m^3/s$ want $J/s = Nm/s = c\, m^3/s$

$c = N/m^2 = Pa$

$up : \frac{m^3}{s} \frac{N}{m^2} = J/s.$

$$F = \int_0^H \int_0^W up \, dy \, dz = WHup$$

average over one cycle $\frac{2\pi}{\omega}$

$$F_a = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} WH u_0 \sin(\omega t + \phi) p_0 \cos(\omega t) dt = \frac{\omega}{2\pi} WH u_0 p_0 \int_0^{2\pi/\omega} \sin(\omega t + \phi) \cos(\omega t) \, dt$$

integrated with wolframalpha to get

$$F_a = \frac{WH u_0 p_0 \cos(\phi)}{2}$$

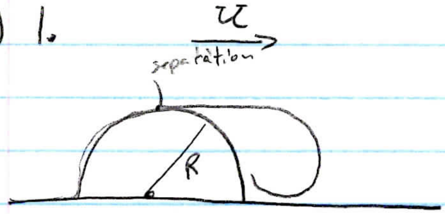
4. $\phi \ll 1$, $p_0 \ll \rho g H$, $L \ll \frac{2\pi}{\omega}\sqrt{gH}$

assuming $\phi > 0$ } η_0 is small

$$F_a = \frac{WH u_0 p_0}{2} \cos(\phi)$$

STEELELUX

③ 1.



assuming same flow as that of a sphere (very wide channel).

$$\psi = \mathcal{U}\left[r - \frac{R^2}{r^2}\right]\sin\Theta \,\Big|_{\emptyset}^{\pi}$$

$$\frac{\partial\psi}{\partial\Theta} = \phi = \mathcal{U}\left[r - \frac{R^2}{r^2}\right]\cos\Theta$$

$$\frac{\partial\phi}{\partial\Theta} = u_\Theta = -\mathcal{U}\left[1 + \frac{R^2}{r^2}\right]\sin\Theta \qquad r = R$$

$$= -2\mathcal{U}\sin\Theta \quad , \quad \text{max at } \frac{\pi}{2}$$

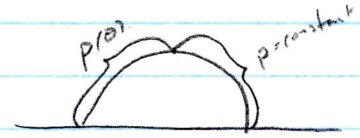
$$= -2\mathcal{U}$$

max speed at separation pt, $2\mathcal{U}$ downstream.

2.

$$D = R\int_0^{\pi} p\cos\Theta\, d\Theta \qquad\qquad P_s = P_{bubble} = p\left(\frac{\pi}{2}\right)$$

$$P_1 = \frac{1}{2}\rho\mathcal{U}^2(1 - 4\sin^2\Theta) \qquad \Theta : [\pi, \pi/2]$$

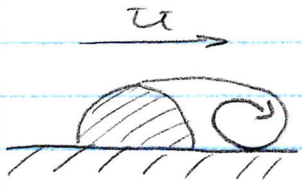


$$P_2 = \frac{-3}{2}\rho\mathcal{U}^2 \qquad \Theta : [\pi/2, 0]$$

$$D_2 = -\frac{3}{2}R\rho\mathcal{U}^2\int_0^{\pi/2}\cos\Theta\, d\Theta = -\frac{3}{2}R\rho\mathcal{U}^2$$

$$D_1 = R\frac{1}{2}\rho\mathcal{U}^2\int_{\frac{\pi}{2}}^{\pi}(1 - 4\sin^2\Theta)\cos\Theta\, d\Theta = \frac{1}{6}R\rho\mathcal{U}^2$$

$$D_{total} = \left(\frac{1}{6} - \frac{3}{2}\right)R\rho\mathcal{U}^2 = -\frac{4}{3}R\rho\mathcal{U}^2$$

3. 

The surface of the water will be a clockwise rotating vortex.
More turbulent near the centre but becomes smoother as you go out from the centre of the vortex.
At $r \gg R$ the flow is dominated by U the channel flow.

4. Once the Flow is stopped the vortex will travel upstream towards the headland at a speed equal to and opposite the initial flow.