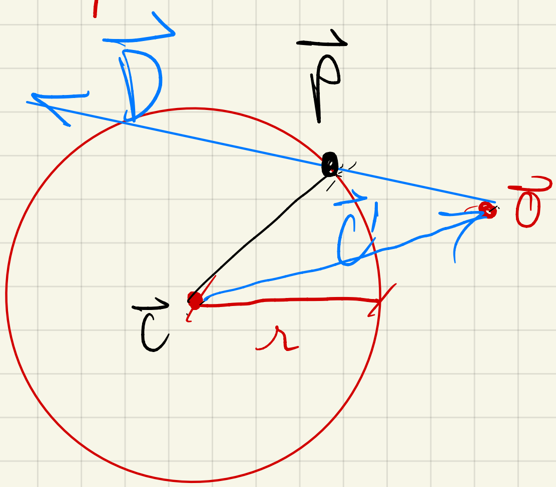
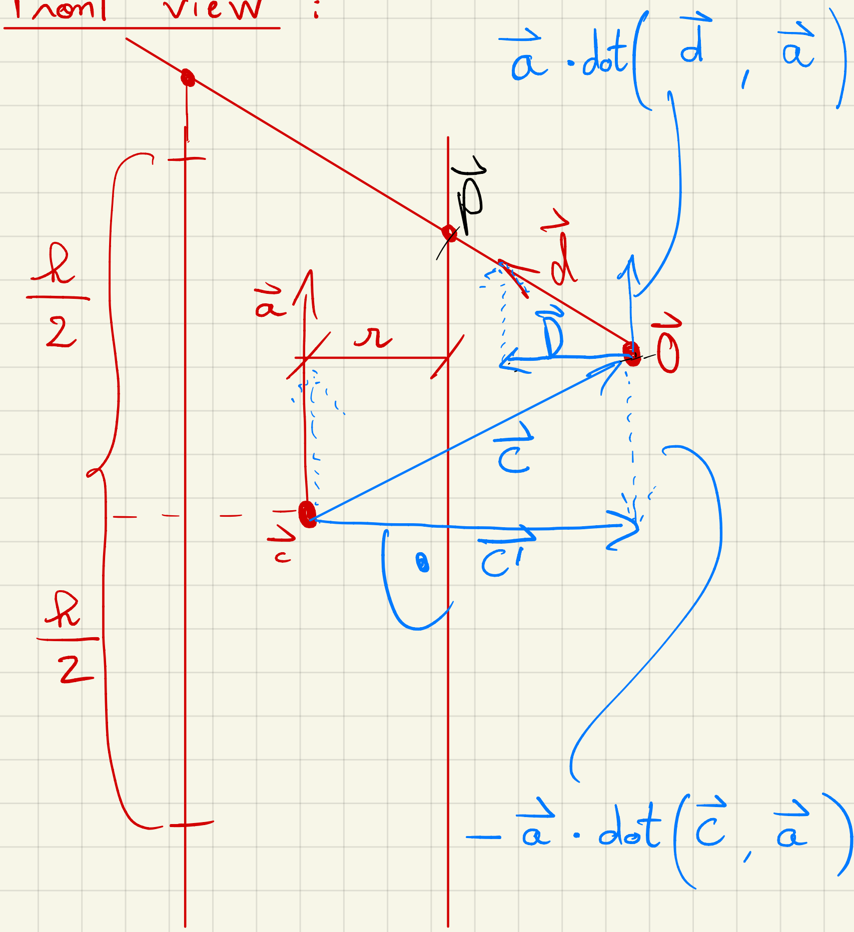
Theory Exercise

Task RT1.2.1 : Derive the expression for a Ray-Cylinder intersection

Top view :



Front view :



Note that :

$$\vec{D} = \vec{d} - \vec{a} \cdot dot(\vec{d}, \vec{a})$$
$$\vec{C}' = \vec{C} - \vec{a} \cdot dot(\vec{C}, \vec{a})$$

We have :

$$(\vec{D}t + \vec{C}') \cdot (\vec{D}t + \vec{C}') = \vec{r}^2 \qquad (1)$$

$$\Leftrightarrow \quad |\vec{D}t + \vec{C}'| = |\vec{r}|$$

$$\Leftrightarrow (\vec{D} \cdot \vec{D})\, t^2 + 2(\vec{D} \cdot \vec{C}')\, t + \vec{C}' \cdot \vec{C}' - \vec{r}^2 = 0$$

$$\Leftrightarrow \quad |\vec{D}|^2 t^2 + 2(\vec{D} \cdot \vec{C}')\, t + |\vec{C}'|^2 - \vec{r}^2 = 0$$

$$\Leftrightarrow \quad at^2 + bt + c = 0 \qquad (2)$$

We need to solve this quadratic equation for t.

$$\Longrightarrow \begin{cases} a = \vec{D} \cdot \vec{D} = |\vec{D}|^2 \\ b = 2(\vec{D} \cdot \vec{C}') \\ c = |\vec{C}'|^2 - r^2 \end{cases}$$

a, b and c are used as arguments for the quadratic equation.

To find t, we can use the quadratic equation solutions of (1).

Once we've found the t for the top view (i.e. for $\vec{C}'$ and $\vec{D}$), we can use the same t for the front view to find the actual intersections.

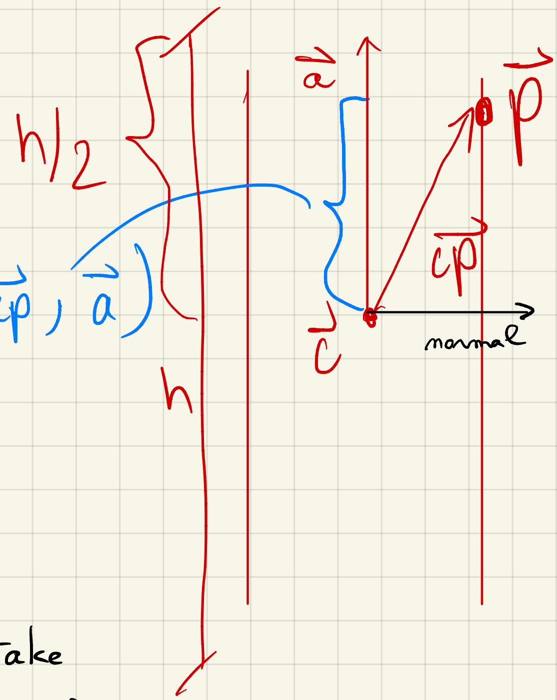
As we find t with the quadratic equation, we get up to 2 solutions.

This makes sense, since there are up to 2 intersections between the ray and the cylinder.

Now that we found solutions to our quadratic equation, we need to check if those solutions at point $\vec{p}$ are within the height of the actual cylinder:

We will show the calculation for one t (if we have 2 solutions for t, we just check the following for both):

Intersection point: $\vec{p} = \vec{o} + \vec{d}\,t$



$\vec{p}$ is a valid intersection point if the distance along the axis (*) is inferior or equal to $\frac{h}{2}$

(*) $\mathrm{dot}(\vec{cp}, \vec{a})$

i.e. $\vec{cp} \cdot \vec{a} \leqslant \frac{h}{2}$.

If both solutions for t are valid, we take the one that yields a $\vec{p}$ that is in front of the cylinder.

Finally, we need to find the normal vector at point $\vec{p}$.
To find the normal vector, we just have to get rid of the vertical component of the vector $\vec{cp}$ by substracting its projection on the cylinder axis from itself.

Lastly, we still need to normalize the result:

$$\mathrm{normal} = \mathrm{normalize}\left(\underbrace{\vec{p} - \vec{c}}_{= \vec{cp}} - \underbrace{\mathrm{dot}(\vec{p} - \vec{c}, \vec{a})}_{\text{distance of } \vec{cp} \text{ along the cylinder axis}} \cdot \vec{a} \right)$$