Theory Exercise

Derive iterative formula : Task RT2.3.1

We need to show that we have the following equation :

$$c_b = \sum_{i=0}^{+\infty} (1-\alpha_i)\left(\prod_{k=0}^{i-1} \alpha_k\right) c_i$$

We have that :

$$c_b = (1-\alpha_0)\, c_0 + \alpha_0 c_1$$
$$c_1 = (1-\alpha_1)\, c_1 + \alpha_1 c_2$$
$$\vdots$$
$$c_i = (1-\alpha_i)\, c_i + \alpha_i c_{i+1}$$

If ray i doesn't intersect , $c_i = 0$

So here , $i \to +\infty$ and first non-intersecting ray is $i+1$

Therefore :

$$c_b = (1-\alpha_0) c_0 + \alpha_0 \Big[(1-\alpha_1) c_1 + \alpha_1 \big[(1-\alpha_2) c_2 + \dots + \alpha_{i-1} [(1-\alpha_i) c_i] \big] \Big]$$

If we expand this, we get :

$$c_b = (1-\alpha_0) c_0 + \alpha_0 (1-\alpha_1) c_1 + \alpha_0 \alpha_1 (1-\alpha_2) c_2 + \dots + \alpha_0 \alpha_1 \dots \alpha_{i-1} (1-\alpha_i) c_i$$
$$= \sum_{i=0}^{+\infty} (1-\alpha_i)\left(\prod_{k=0}^{i-1} \alpha_k\right) c_i$$

For simplification , the above formula can be written as iterative formula for limit N . We note :

$$\Pi(0) = 1 \quad \text{and} \quad \Pi(i) = \alpha_i \cdot \Pi(i-1) \qquad \text{for } \forall i \geqslant 1$$

So we have :

$$c_b = \sum_{i=0}^{N} (1 - \alpha_i) \pi(i) \, c_i$$

In algorithm form, we would get :

$p\_i = 1, \quad c = 0$

for $i = 0$ to N :

$\quad c \mathrel{+}= (1 - \alpha_i) \cdot p\_i \cdot c_i$

$\quad p\_i \mathrel{*}= \alpha_i$