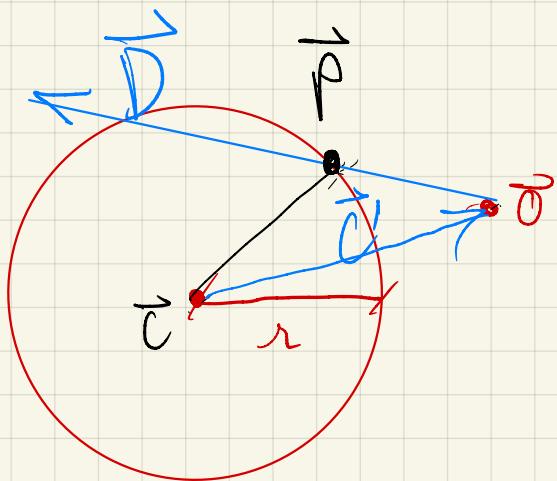

Theory Exercise

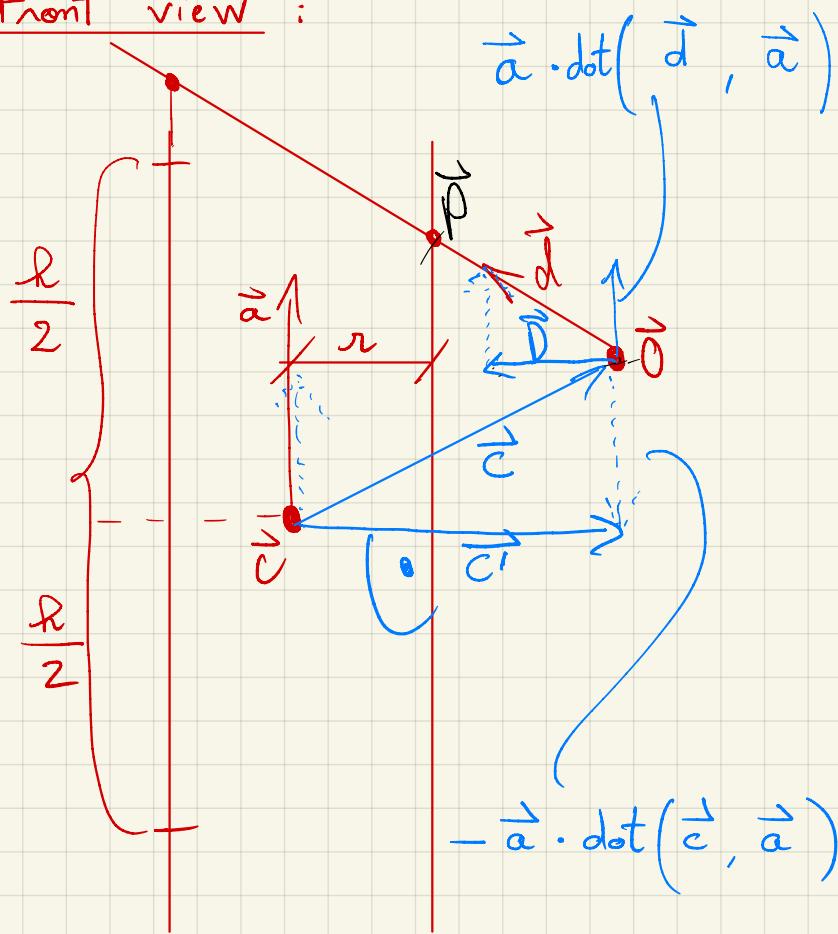


Task RT 1.2.1 : Derive the expression for a Ray-Cylinder intersection

Top view :



Front view :



We have :

$$(\vec{D}t + \vec{c}') \cdot (\vec{D}t + \vec{c}') = \pi^2 \quad (1)$$

$$\Leftrightarrow |\vec{D}t + \vec{c}'|^2 = |\vec{r}|^2$$

$$\Leftrightarrow (\vec{D} \cdot \vec{D}) t^2 + 2(\vec{D} \cdot \vec{c}') t + \vec{c}' \cdot \vec{c}' - \pi^2 = 0$$

$$\Leftrightarrow |\vec{D}|^2 t^2 + 2(\vec{D} \cdot \vec{c}') t + |\vec{c}'|^2 - \pi^2 = 0$$

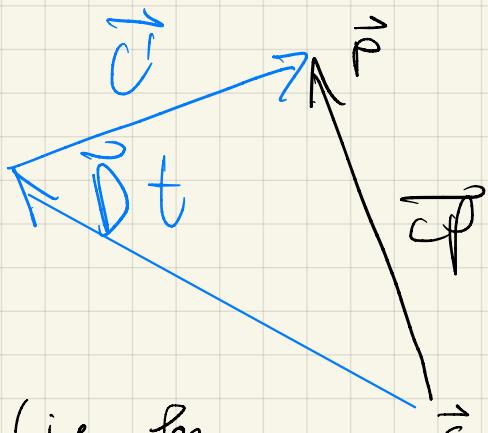
$$\Leftrightarrow at^2 + bt + c = 0 \quad (2)$$

We need to solve this quadratic equation for t.

$$\Rightarrow \begin{cases} a = \vec{D} \cdot \vec{D} = |\vec{D}|^2 \\ b = 2(\vec{D} \cdot \vec{C}') \\ c = |\vec{C}'|^2 - r^2 \end{cases}$$

a , b and c are used as arguments for the quadratic equation.

$$\begin{aligned}\vec{cp} &= -\vec{c} + \vec{p} \quad \text{and} \quad |\vec{cp}| = |r| \\ \vec{cp} &= \vec{C}' + \vec{D}t\end{aligned}$$



To find t , we can use the quadratic equation solutions of (1).

Once we've found the t for the top view (i.e. for \vec{C}' and \vec{D}), we can use the same t for the front view to find the actual intersections.

As we find t with the quadratic equation, we get up to 2 solutions.

This makes sense, since there are up to 2 intersections between the ray and the cylinder.

Now that we found solutions to our quadratic equation, we need to check if those solutions at point \vec{p} are within the height of the actual cylinder:

We will show the calculation for one t (if we have 2 solutions for t , we just check the following for both) :

Intersection point : $\vec{p} = \vec{o} + \vec{d}t$

\vec{p} is a valid intersection point if
the distance along the axis (*)

is inferior or equal to $\frac{h}{2}$

i.e. $\vec{cp} \cdot \vec{a} \leq \frac{h}{2}$.

If both solutions for t are valid, we take
the one that yields a \vec{p} that is in front of the cylinder.

