

Stereo and Motion

Objects appear in different positions from different viewpoints - parallax

Disparity = Position difference, inversely proportional to depth

If we know disparity and viewpoints we can infer 3D scene structure.

Epipolar Geometry:

Geometry depends on position and orientation of cameras.

Camera Model:

Principal Point - Centre of the image plane (0, 0, f)

Focal Length - f

Optical/Principal Axis - Z

Point on image plane - P(x, y, f)

3D point on surface of object - P(X, Y, Z) projects to P(x, y, f)

$$x = \frac{fX}{Z} \quad y = \frac{fY}{Z} \quad p = \frac{fP}{Z}$$

Simple Two-View Stereo:

Coplanar image planes, T = Baseline distance between the centre of projection (COP) of each camera

$$\frac{T}{Z} = \frac{T - x_L + x_R}{Z - f} \quad x_L = \text{Point on left camera} \quad x_R = \text{Point on right camera} \quad \text{depth: } Z = \frac{fT}{x_L - x_R} =$$

General Two-View Stereo:

Coordinate Transformations:

$$\text{Rotate coordinate frame clockwise by } \theta : \quad v' = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix} v \quad \text{Rotation and Translation:} \quad v' =$$

$$T = \text{3D Camera Position Vector} \quad R = \text{3D Camera Rotation Matrix} \quad \text{Vector defining P in camera coordinates}$$

$$\text{Note: For rotation matrices: } R^T = R^{-1}$$

Homogenous Coordinates:

We can define rotation and translation in a single 4x4 matrix

$$\text{Camera To World} = H_{CW}P'_W = \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = \begin{bmatrix} P_W \\ 1 \end{bmatrix} = \begin{bmatrix} R^T & T \\ 0 & 1 \end{bmatrix} = H_{CW}P'_c \implies P_W = R^T P_C + T$$

$$H_{WC} = \begin{bmatrix} R & -RT \\ 0 & 1 \end{bmatrix} \implies P_C = R(P_W - T)$$

Stereo Coordinate Systems:

$$P_L = R^T P_R + T P_R = R(P_L - T)T = \text{Translation from left to right camera} \quad R = \text{Rotation applied to right camera}$$

Epipolar Geometry:

$$p_L = \begin{bmatrix} X_L \\ Y_L \\ f \end{bmatrix} = \frac{f P_L}{Z_L} \quad p_R = \begin{bmatrix} X_R \\ Y_R \\ f \end{bmatrix} = \frac{f P_R}{Z_R}$$

Vectors $P_L - T$ and $P_L - T$ all lie in an epipolar plane.

$$(P_L - T)^T (T * P_L) = 0 \text{ Cross product can be defined by } S = \begin{bmatrix} 0 & -T_Z & T_Y \\ T_Z & 0 & -T_X \\ -T_Y & T_X & 0 \end{bmatrix} P_R = R(P_L - T)R^T P_R = (P_L - T)^T (T * P_L) = 0$$

All points in the view plane satisfy the constraint above. RS is known as the essential matrix = E

$$E = RS \quad P_R^T E P_L = 0 \quad P_L = \frac{Z_L p_L}{f} \quad P_R = \frac{Z_R p_R}{f} \quad p_R^T E p_L = 0 \text{ Therefore the two D points in the view plane}$$