Distance Measures

P-Norm Distances:

$$D(x, y) = (\sum_{i=1}^{n} |x_i - y_i|^p)^{\frac{1}{p}}$$

n = dimensions

Chebyshev Distance:

$$D_{chebyshev}(x, y) = max_i^n(x_i, y_i)$$

Hamming Distance:

For equal-length strings, the hamming distance is the number of pairs of characters (a, b) where a != b and a and b have the same index in their respective strings

Edit Distance:

The number of substitutions, deletions or additions required to make two strings match.

Nyquist Sampling Rate Theorem:

The sampling rate must be at least twice the analogue wave frequency. A naive method is to find the wave's highest frequency (shortest period) and double it for the sampling rate.

Mean and Variance:

u = mean

o = standard deviation

$$o^2 = \frac{1}{N-1} \sum_{i=1} (v_i - \mu)^2$$

For multidimensional data:

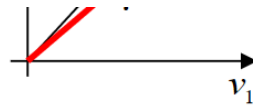
A covariance matrix is always square and symmetric, with variance on the diagonal, covariance between each pair of dimensions is stored on the non-diagonal elements.

The eigenvectors of a covariance matrix define the principal axis of the spread, a larger eigenvalue indicates a larger variance. The major axis corresponds to the larger eigenvalue.

Definition:

Mean vector:
Computed independently for each dimension

$$\boldsymbol{\mu} = \frac{1}{N}\sum_i \mathbf{v}_i$$



Covariance:
Gives both spread and correlation

$$\mathbf{C} = \frac{1}{N-1}\sum_i (\mathbf{v}_i - \boldsymbol{\mu})^2$$

$$\mathbf{C} = \frac{1}{N-1}\sum_i (\mathbf{v}_i - \boldsymbol{\mu})^{\mathrm{T}}(\mathbf{v}_i - \boldsymbol{\mu})$$

$$\mathbf{C} = \frac{1}{N}\sum_i \begin{bmatrix} (v_{i1} - \mu_1)^2 & (v_{i1} - \mu_1)(v_{i2} - \mu_2) \\ (v_{i1} - \mu_1)(v_{i2} - \mu_2) & (v_{i2} - \mu_2)^2 \end{bmatrix}$$

Figure 1: Untitled

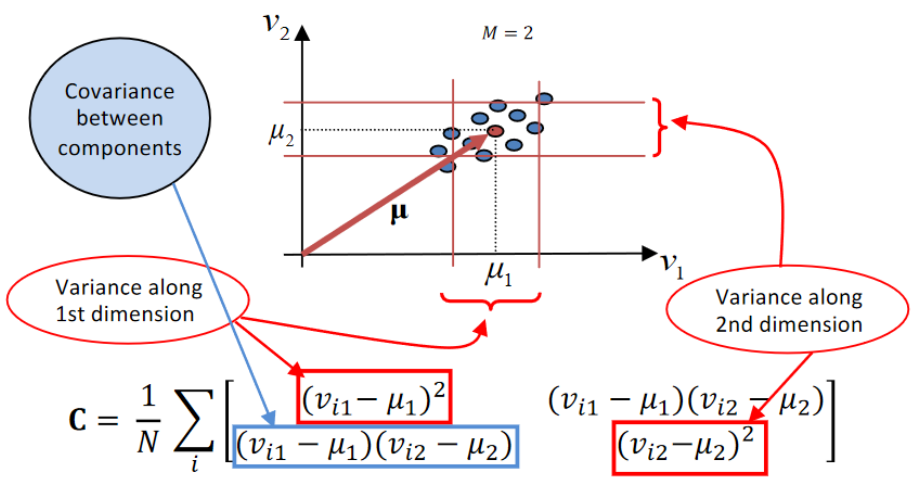


Figure 2: Untitled

for a square matrix C, if there exists a non-zero column vector v where

$$Cv = \lambda v$$

then v = eigenvector and lambda = eigenvalue

$$det(A - \lambda I) = 0$$

Gaussian Distribution:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

For multi-dimensional normal distribution $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, the probability density function (pdf) can be calculated as

$$p(\mathbf{x}) = \frac{1}{2\pi \, \| \boldsymbol{\Sigma} \|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \, \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$

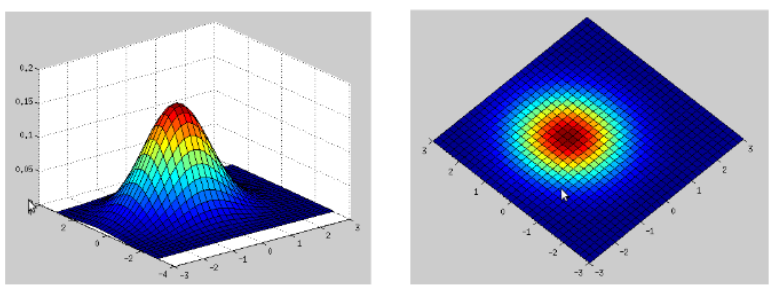


Figure 3: Untitled

Data Normalisation:

- Rescaling

$$x' = \frac{x - min(x)}{max(x) - min(x)}$$

- Standardisation (z-score)

$$x' = \frac{x - \mu}{\sigma}$$

- Scale to unit length

$$x' = \frac{x}{||x||}$$

Outliers:

A small number of points with values significantly different from that of the other points, usually due to a fault in measurement.