3D Reconstruction

Epipolar Lines

$p_R^T E p_L = 0 \qquad u_L = Ep_L p_R^T u_L = x_R u_{L1} + y_R u_{L2} + f u_{L3} = 0 \qquad = \text{Epipolar Line}$

Pixel Coordinates

$S_x, S_y = \text{Width and height of each pixel}(\hat{x}, \hat{y}) = \text{Pixel Coordinate}(\hat{o}_x, \hat{o}_y) = \text{Principal Point (Where the princi}$

Fundamental Matrix:

$$p_L = \begin{bmatrix} x_L \\ y_L \\ f \end{bmatrix} = M_L \begin{bmatrix} \hat{x}_L \\ \hat{y}_L \\ f \end{bmatrix} = M_L \hat{p}_L$$

$p_R^T E p_L = 0 \implies \hat{p}_R^T M_R^T E M_L \hat{P}_L = 0 \implies \hat{p}_R^T F \hat{p}_L F = M_R^T E M_L = \text{The fundamental matrix}$

In order to find the actual point P in the scene we can find the intersection of the epipolar lines, however they will usually never intersect so we can pick the closest point on each line and take the average of those points:

$ap_L = \text{Closest point to right epipolar line on left line} bp_R = \text{vice versa} bR^T p_R + T \text{ is } bp_R \text{ wrt left camera} c(p_L \times R$

Given corresponding points, we know: $ p_L, p_R $

Given calibrated views, we know: $ R, T $

$\hat{p} = \frac{(ap_L + bR^T p_R + T)}{2} = \text{Point in the scene at the average of the closest points}$

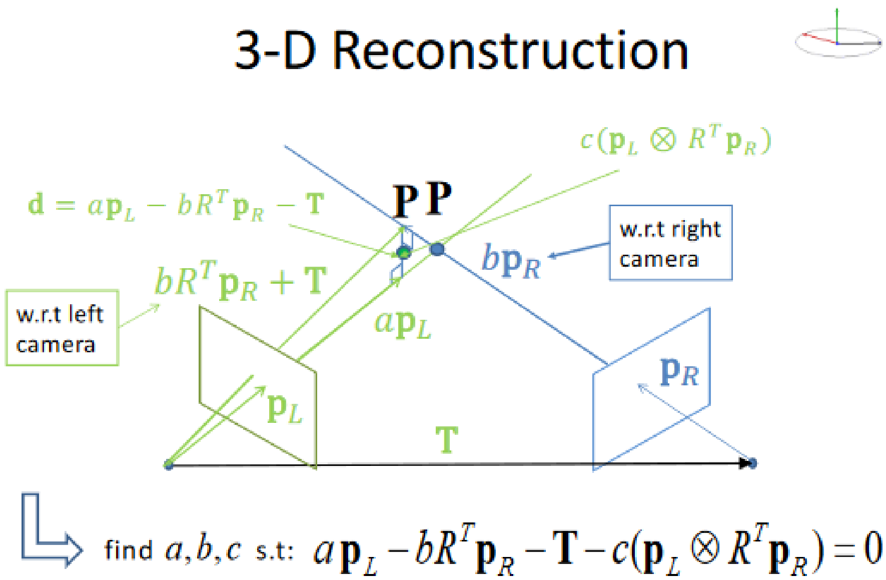

Figure 1: Untitled

$$
a\underbrace{\begin{bmatrix} \bullet \\ \mathbf{p}_L \\ \bullet \end{bmatrix}}_{3\text{x}1} - b\underbrace{\begin{bmatrix} \\ R^T\mathbf{p}_R \\ \\ \end{bmatrix}}_{3\text{x}1} - c\underbrace{\begin{bmatrix} \\ \mathbf{p}_L \otimes R^T\mathbf{p}_R \\ \\ \end{bmatrix}}_{3\text{x}1} = \underbrace{\begin{bmatrix} \\ \mathbf{T} \\ \\ \end{bmatrix}}_{3\text{x}1}
$$

$$
\Rightarrow \quad H\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{T} \quad \Rightarrow \quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = H^{-1}\mathbf{T}
$$

Figure 2: Untitled