

## Modelling and Optical Flow

Incremental 2-D motion fields represent the 3D translation vector of a pixel projected onto the 2D image plane.

**Rigid Motion:**

$$P' = RP + T$$

**Rotation Matrices Approximations:**

$$R = R_X R_Y R_Z \text{ For small } \theta : \sin(\theta) \approx \theta \cos(\theta) \approx 1 \text{ Therefore for small } \theta_X, \theta_Y, \theta_Z : R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

**3-D Motion Field:**

$$V = \lim_{\Delta t \rightarrow 0} \{P' - P = (R - I)P + T\}$$



Figure 1: Untitled

Hence for small angles:  $V_X = \theta_Y Z - \theta_Z X + T_X V_Y = \theta_Z X - \theta_X Z + T_Y V_Z = \theta_X Y - \theta_Y X + T_Z(\theta_X, \theta_Y, \theta_Z) = \text{Angular velocity}$

**2-D Motion Field Equations:**

$$\text{For image point } p = (x, y, f) \quad \text{Motion field } v = (v_x, v_y) v_x = \frac{dx}{dt} = \frac{d}{dt} \frac{fX}{Z} = f \frac{V_X Z - X V_Z}{Z^2} \quad x = \frac{fX}{Z}$$

A rotation without translation is independent of the depth.

The diagram shows two equations for camera projection,  $v_x$  and  $v_y$ , enclosed in a light blue rounded rectangle. A red circle highlights the first term of each equation,  $(fT_x - xT_z)/Z$  and  $(fT_y - yT_z)/Z$ , with a red arrow pointing to the text 'Translational – dependent on scene depth Z'. A blue oval highlights the second term of each equation,  $f\theta_y - \theta_z y - (\theta_x xy - \theta_y x^2)/f$  and  $-f\theta_x + \theta_z x + (\theta_y xy - \theta_x y^2)/f$ , with a blue arrow pointing to the text 'Rotational – independent of scene depth Z'.

$$v_x = (fT_x - xT_z)/Z + f\theta_y - \theta_z y - (\theta_x xy - \theta_y x^2)/f$$

$$v_y = (fT_y - yT_z)/Z - f\theta_x + \theta_z x + (\theta_y xy - \theta_x y^2)/f$$

Translational – dependent on scene depth  $Z$

Rotational – independent of scene depth  $Z$

Figure 2: Untitled