

## Distance Measures

### P-Norm Distances:

$$D(x, y) = \left( \sum_{i=1}^n |x_i - y_i|^p \right)^{\frac{1}{p}}$$

n = dimensions

### Chebyshev Distance:

$$D_{chebyshev}(x, y) = \max_i^n(x_i, y_i)$$

### Hamming Distance:

For equal-length strings, the hamming distance is the number of pairs of characters (a, b) where  $a \neq b$  and a and b have the same index in their respective strings

### Edit Distance:

The number of substitutions, deletions or additions required to make two strings match.

### Nyquist Sampling Rate Theorem:

The sampling rate must be at least twice the analogue wave frequency. A naive method is to find the wave's highest frequency (shortest period) and double it for the sampling rate.

### Mean and Variance:

u = mean

o = standard deviation

$$o^2 = \frac{1}{N-1} \sum_{i=1} (v_i - \mu)^2$$

For multidimensional data:

A covariance matrix is always square and symmetric, with variance on the diagonal, covariance between each pair of dimensions is stored on the non-diagonal elements.

The eigenvectors of a covariance matrix define the principal axis of the spread, a larger eigenvalue indicates a larger variance. The major axis corresponds to the larger eigenvalue.

Definition:

Mean vector:  
Computed independently  
for each dimension

$$\boldsymbol{\mu} = \frac{1}{N} \sum_i \mathbf{v}_i$$



Covariance:  
Gives both spread and  
correlation

$$\mathbf{C} = \frac{1}{N-1} \sum_i (\mathbf{v}_i - \boldsymbol{\mu})^2$$

$$\mathbf{C} = \frac{1}{N-1} \sum_i (\mathbf{v}_i - \boldsymbol{\mu})^T (\mathbf{v}_i - \boldsymbol{\mu})$$

$$\mathbf{C} = \frac{1}{N} \sum_i \begin{bmatrix} (v_{i1} - \mu_1)^2 & (v_{i1} - \mu_1)(v_{i2} - \mu_2) \\ (v_{i1} - \mu_1)(v_{i2} - \mu_2) & (v_{i2} - \mu_2)^2 \end{bmatrix}$$

Figure 1: Untitled



Figure 2: Untitled

for a square matrix  $C$ , if there exists a non-zero column vector  $v$  where

$$Cv = \lambda v$$

then  $v$  = eigenvector and  $\lambda$  = eigenvalue

$$\det(A - \lambda I) = 0$$

**Gaussian Distribution:**

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

For multi-dimensional normal distribution  $N(\mu, \Sigma)$ , the probability density function (pdf) can be calculated as

$$p(\mathbf{x}) = \frac{1}{2\pi \|\Sigma\|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)^T \Sigma^{-1}(\mathbf{x} - \mu)\right)$$



Figure 3: Untitled

**Data Normalisation:**

- Rescaling

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

- Standardisation (z-score)

$$x' = \frac{x - \mu}{\sigma}$$

- Scale to unit length

$$x' = \frac{x}{||x||}$$

**Outliers:**

A small number of points with values significantly different from that of the other points, usually due to a fault in measurement.