

# Kernels

## Kernel Approach:

Choose a kernel function  $k$  which measures the similarity between raw data vectors  $k(x_m, x_n)$ . Learn parameters using the values and class labels. When making a prediction for a test data vector use the learned parameters to make a prediction.

Design Matrix = Data after performing feature mapping, where the  $n$ th row is the  $n$ th data point.

$$w = \Phi^T a \quad a = \text{N-dimensional parameter vector} \quad \Phi = \text{Design Matrix}$$

$$y(x) = w^T \phi(x) = a^T \Phi \phi(x)$$

$$k(x, x') = \phi(x)^T \phi(x')$$

The kernel function represents the degree of similarity between its two arguments (since it's the dot product of two feature vectors).

In addition to this most of the parameters in  $a$  will be close enough to zero that they can be discarded.

## Prediction:

$$y(x) = a^T \begin{bmatrix} k(x_1, x) \\ k(x_2, x) \\ k(x_3, x) \end{bmatrix} = a_1 k(x_1, x) + a_2 k(x_2, x) + a_3 k(x_3, x) \quad x = \text{test data point}$$

We must learn the parameter vector  $a$  without having to evaluate the feature vectors or we lose the benefit of using kernels. Kernel methods are 'non-parametric' because there are the same number of parameters as data points. In addition to this, we only have to care about parameters that have non-zero values, these are the dual coefficients and the corresponding data points are the support vectors.

## The Gram Matrix

$$\text{Gram Matrix: } K = \Phi \Phi^T$$

$$\text{Learning: } a = (K + \lambda I_N)^{-1} t$$

**Classification:**

During classification, there will be many different hyperplanes that will linearly separate the data. We can use a maximum margin classifier to choose the hyperplane that leads to the largest distance between the closest data point to the plane. This will help to reduce overfitting.

$$\operatorname{argmax}_{w,b} \left\{ \frac{1}{\|w\|} \min [t_n (w^T \phi(x_n) + b)] \right\}$$