

## Motion Estimation

The relationship between variation in pixels - known as apparent motion or optical flow - and the true motion is not trivial.

- Apparent motion/Optical Flow - Perceived motion in the video sequence caused by changes in pixel values.
- Sometimes it's not possible to determine 2-D motion field without additional constraints or assumptions

### Optical Flow:

- Assume optical flow results from a **brightness constancy constraint** - A moving pixel retains its value between frames
- For continuous video  $I(x, y, t)$ :

$$I(x+\Delta x, y+\Delta y, t+\Delta t) = I(x, y, t) \text{ Using Taylor Series (Higher Order Terms} \rightarrow 0 \text{ for small delta)}$$

The rate of change of  $x$  and  $y$  with time:

$$\text{Optical flow field } u = (u_x, u_y) \text{ Spatial and temporal gradients} = (I_x, I_y, I_t)$$

### Normal Flow:

$$\text{OFE for } u = (u_x, u_y) \text{ and } \nabla I = (I_x, I_y) \quad I_x u_x + I_y u_y + I_t = 0 \rightarrow \nabla I \cdot u + I_t = 0$$

Therefore the OFE alone is not sufficient to estimate motion. We can only estimate normal flow, i.e. the flow in the direction of the spatial gradient normal,  $u_n$  is the motion field in the direction of the normal.

$$\nabla I \cdot u + I_t = \nabla I \cdot u_n + I_t = 0 \quad \|u_n\| = \frac{-I_t}{\|\nabla I\|} \angle u_n = \angle \nabla I$$

This means that good motion estimations depend on having sufficient variation in spatial gradients.

### Constraining the OFE:

OFE is under-constrained, can only estimate normal flow. We must add extra constraints. For example we can assume parametric form of motion field in regions.

For example linear (Affine):

$$u_x = ax + by + cu_y = dx + ey + f$$

### Constant Velocity Model:

For a region, find the velocity  $u$  which minimises:

$$\epsilon(u_x, u_y) = \sum_{\text{region}} (I_x u_x + I_y u_y + I_t)^2 \frac{\delta \epsilon}{\delta x} = 2 \sum_{\text{region}} (I_x u_x + I_y u_y + I_t) I_x = 0 \frac{\delta \epsilon}{\delta y} = 2 \sum_{\text{region}} (I_x u_x + I_y u_y + I_t) I_y = 0$$

We can solve for  $u_x$  and  $u_y$  by setting the derivatives to zero.

### Spatial and Temporal Gradients:

$$I_x = \frac{\delta I}{\delta x} \approx I(x+1, y, t) - I(x, y, t) \text{ i.e. assume } \delta x = 1$$

### Lukas and Kanade Algorithm:

- For a pair of frames at time  $t$  and time  $t+1$
- For each pixel  $x$  and  $y$  in the first frame
- For each pixel the region about  $x$  and  $y$
- Compute the spatial gradients, compute A and B sum for each pixel  $x$  and  $y$ .
- Use A and B to get the motion field.