Modelling and Optical Flow

Incremental 2-D motion fields represent the 3D translation vector of a pixel projected onto the 2D image plane.

Rigid Motion:

$$P' = RP + T$$

Rotation Matricies Approximations:

$R = R_X R_Y R_Z$For small θ : $\sin(\theta) \approx \theta \cos(\theta) \approx 1$Therefore for small $\theta_X, \theta_Y, \theta_Z : R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$

3-D Motion Field:

$$V = \lim_{\Delta t \to 0} \{P' - P = (R - I)P + T\}$$

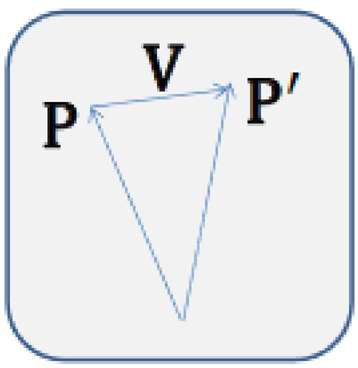

Figure 1: Untitled

Hence for small angles: $V_X = \theta_Y Z - \theta V_X + T_X V_Y = \theta_Z X - \theta_X Z + T_Y V_Z = \theta_X Y - \theta_Y X + T_Z (\theta_X, \theta_Y, \theta_Z) =$ Angu

2-D Motion Field Equations:

For image point p $= (x, y, f)$ Motion field v $= (v_x, v_y) v_x = \dfrac{dx}{dt} = \dfrac{d}{dt}\dfrac{fX}{Z} = f\dfrac{V_X Z - X V_Z}{Z^2}$ $x = \dfrac{fX}{Z}$

A rotation without translation is independent of the depth.

$$v_x = (fT_X - xT_Z)/Z + f\theta_Y - \theta_Z y - (\theta_X xy - \theta_Y x^2)/f$$

$$v_y = (fT_Y - yT_Z)/Z - f\theta_X + \theta_Z x + (\theta_Y xy - \theta_X y^2)/f$$

Translational – dependent on scene depth Z

Rotational – independent of scene depth Z

Figure 2: Untitled