

Modelling and Optical Flow

Incremental 2-D motion fields represent the 3D translation vector of a pixel projected onto the 2D image plane.

Rigid Motion:

$$P' = RP + T$$

Rotation Matrices Approximations:

$$R = R_X R_Y R_Z \text{ For small } \theta : \sin(\theta) \approx \theta \cos(\theta) \approx 1 \text{ Therefore for small } \theta_X, \theta_Y, \theta_Z : R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

3-D Motion Field:

$$V = \lim_{\Delta t \rightarrow 0} \{P' - P = (R - I)P + T\}$$



Figure 1: Untitled

Hence for small angles: $V_X = \theta_Y Z - \theta_Z X + T_X V_Y = \theta_Z X - \theta_X Z + T_Y V_Z = \theta_X Y - \theta_Y X + T_Z(\theta_X, \theta_Y, \theta_Z) = \text{Angu}$

2-D Motion Field Equations:

$$\text{For image point } p = (x, y, f) \quad \text{Motion field } v = (v_x, v_y) v_x = \frac{dx}{dt} = \frac{d}{dt} \frac{fX}{Z} = f \frac{V_X Z - X V_Z}{Z^2} \quad x = \frac{fX}{Z}$$

A rotation without translation is independent of the depth.

The diagram illustrates the decomposition of camera projection equations. Two equations are shown within a light blue rounded rectangle:

$$v_x = (fT_x - xT_z) / Z + f\theta_y - \theta_z y - (\theta_x xy - \theta_y x^2) / f$$

$$v_y = (fT_y - yT_z) / Z - f\theta_x + \theta_z x + (\theta_y xy - \theta_x y^2) / f$$

A red circle highlights the first term of each equation, $(fT_x - xT_z) / Z$ and $(fT_y - yT_z) / Z$. A red arrow points from this circle to the text "Translational – dependent on scene depth Z ". A blue oval highlights the remaining terms of each equation. A blue arrow points from this oval to the text "Rotational – independent of scene depth Z ".

Figure 2: Untitled