Modelling and Optical Flow

Incremental 2-D motion fields represent the 3D translation vector of a pixel projected onto the 2D image plane.

Rigid Motion:

$$P' = RP + T$$

Rotation Matricies Approximations:

$$R = R_X R_Y R_Z \text{For small } \theta : \sin(\theta) \approx \theta \cos(\theta) \approx 1 \text{Therefore for small } \theta_X, \theta_Y, \theta_Z : R \approx \begin{bmatrix} 1 & -\theta_Z & \theta_Y \\ \theta_Z & 1 & -\theta_X \\ -\theta_Y & \theta_X & 1 \end{bmatrix}$$

3-D Motion Field:

$$V = \lim_{\Delta t \to 0} \{ P' - P = (R - I)P + T \}$$

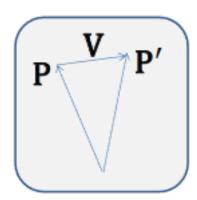


Figure 1: Untitled

Hence for small angles: $V_X = \theta_Y Z - \theta V_X + T_X V_Y = \theta_Z X - \theta_X Z + T_Y V_Z = \theta_X Y - \theta_Y X + T_Z (\theta_X, \theta_Y, \theta_Z) = \text{Angular Angular An$

2-D Motion Field Equations:

For image point
$$p = (x, y, f)$$
 Motion field $v = (v_x, v_y)v_x = \frac{dx}{dt} = \frac{d}{dt}\frac{fX}{Z} = f\frac{V_XZ - XV_Z}{Z^2}$ $x = \frac{fX}{Z}$

A rotation without translation is independent of the depth

$$v_{x} = (fT_{X} - xT_{Z})/Z + f\theta_{Y} - \theta_{Z}y - (\theta_{X}xy - \theta_{Y}x^{2})/f$$

$$v_{y} = (fT_{Y} - yT_{Z})/Z + f\theta_{X} + \theta_{Z}x + (\theta_{Y}xy - \theta_{X}y^{2})/f$$

 ${\it Translational-dependent\ on\ scene\ depth\ } Z$

Rotational – independent of scene depth ${\cal Z}$

Figure 2: Untitled