Motion Estimation

The relationship between varaition in pixels - known as apparent motion or optical flow - and the true motion is not trivial.

- Apparent motion/Optical Flow - Perceived mtoion in the video sequence caused by changes in pixel values.
- Sometimes its not possible to determine 2-D motion field without additional constraints or assumptions

Optical Flow:

- Assume optical flow results from a **brightness constancy contraint** - A moving pixel retains its value between frames
- For continuous video I(x, y, t):

$$I(x+\Delta x, y+\Delta y, t+\Delta t) = I(x, y, t)\text{Using Taylor Series (Higher Order Terms} \rightarrow 0 \text{ for small delta)}I(x+\Delta x, y+\Delta$$

The rate of change of x and y with time:

$$\text{Optical flow field u} = (u_x, u_y)\text{Spatial and temperal gradients} = (I_x, I_y, I_t)$$

Normal Flow:

$$\text{OFE for u} = (u_x, u_y) \text{ and } \nabla I = (I_x, I_y) I_x u_x + I_y u_y + I_t = 0 \rightarrow \nabla I.u + I_t = 0$$

Therefore the OFE alone is not sufficient to estime motion. We can only estimate normal flow, i.e the flow in the direction of the spatial graident normal, u_n is the motion field in the direction of the normal.

$$\nabla I.u + I_t = \nabla I.u_n + I_t = 0 \qquad ||U_n|| = \frac{-I_t}{||\nabla I||} \angle u_n = \angle \nabla I$$

This means that good motion estimations depend on having sufficient varitation in spatial gradients.

Constraining the OFE:

OFE is under-constrained, can only estimate normal flow. We must add extra constraints. For example we can assume parametric form of motion field in regions.

For example linear (Affine):

$$u_x = ax + by + cu_y = dx + ey + f$$

Constant Velocity Model:

For a region, find the velocity u which minimises:

$$\epsilon(u_x, u_y) = \sum_{\text{region}} (I_x u_x + I_y u_y + I_t)^2 \frac{\delta \epsilon}{\delta x} = 2 \sum_{\text{region}} (I_x u_x + I_y u_y + I_t) I_x = 0 \frac{\delta \epsilon}{\delta y} = 2 \sum_{\text{region}} (I_x u_x + I_y u_y + I_t) I_y = 0 \text{We ca}$$

Spatial and Temporal Gradients:

$$I_x = \frac{\delta I}{\delta x} \approx I(x+1, y, t) - I(x, y, t) \text{i.e. assume } \delta x = 1$$

Luckas and Kanade Algorithm:

- For a pair of frames at time t and time t+1
- For each pixel x and y in the first frame
- For each pixel the region about x and y
- Compute the spatial gradients, compute A and B sum for each pixel x and y.
- Use A and B to get the motion field.