2 EAD Model

2.1 Implied Volatility

We started from the data available in the "Market.data.xlsx" related to the option's prices at different strike prices for different maturities for each stock, particularly Salzburg Bank, Bank of Cluj. We computed then the implied volatility using the Black and Scholes model. We did it since the stock price volatility is one parameter in the Black–Scholes–Merton pricing formulas that cannot be directly observed. Implied volatility is forward-looking and usually less variable than option prices. For this reason, it can be considered a good proxy for our volatility estimation.

The results are summarized in the table below:

Strike	T = 3/12	T = 6/12	T = 1	T = 2
80	0,2301	0,2602	0,28	0,2999
90	0,2103	0,2202	0,25	0,2701
110	0,1903	0,2001	0,24	0,2499
120	0,1795	0,1897	0,2199	0,23

Table 5: *Implied volatility for each strike*

From these values we perform a model calibration, where the purpose is to find a parameter which minimizes the so-called error function between the observed market price of the option and the price of the option derived from the selected model all divided by the number of the option used for calibration. The function will be the Root Mean Squared Error:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} |C_i^{(m)} - C_i^*(\theta)|^2}$$

Equation 1*: RMSE*

Considering this formula we ended up with the following values: 0.20255 for the options of Salzburg Bank and 0.245 for the options of Cluj Bank.

2.2 Salzburg Bank Portfolio

The portfolio of Salzburg bank is composed of two European options:

- One call option with a maturity of one year with a strike price of 95 USD and having as underlying SPDRM.
- One put option with a maturity of two years with a strike price of 115 USD and have the same underlying.

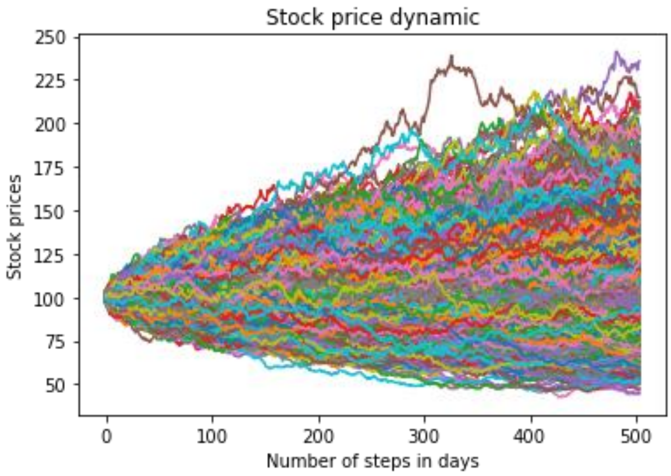
2.2.1 Trade Level

In order to simulate the stock prices of the underlying we first compute Monte Carlo Simulation. The simulation has been built with the volatility that minimizes the RMSE equal to 0.20255 and with the initial stock price S0 = 100. The formula for deriving the 5000 simulated paths for the stock price is given by:

$$S(t_{i+1}) = S(t_i)\exp\left\{\left(r - \frac{1}{2}\sigma^2\right)(t_{i+1} - t_1) + \sigma\sqrt{t_{i+1} - t_i Z_{i+1}}\right\}$$

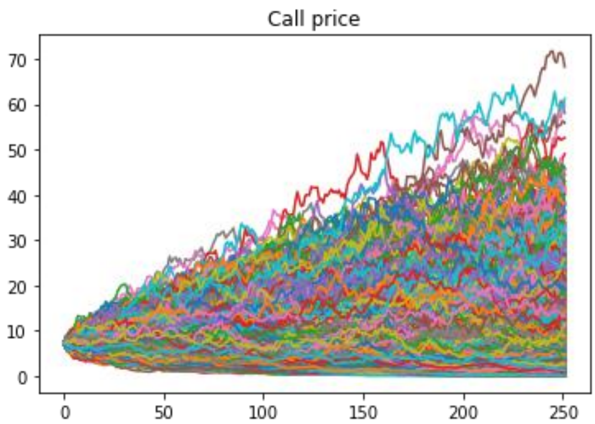
Equation 2: *Monte Carlo Simulation*

The simulation of the stock prices for the 5000 paths is given by:



We replicated the Monte Carlo Simulation to simulate the Call and Put option prices for the 5000 paths. The following inputs give the simulation for the call option:

- Initial price = 100
- Strike price = 95
- Risk free rate = 0.05
- Volatility = 0.20255
- Time period = 1
- Number of path = 5000
- Number of steps = 252

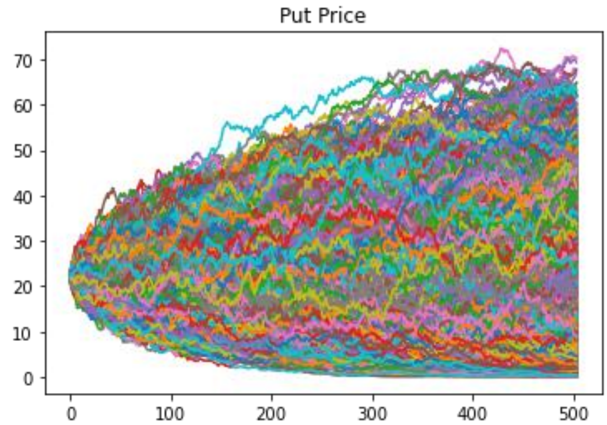


We did the same replication also for the put option keeping all the variables fixed except:

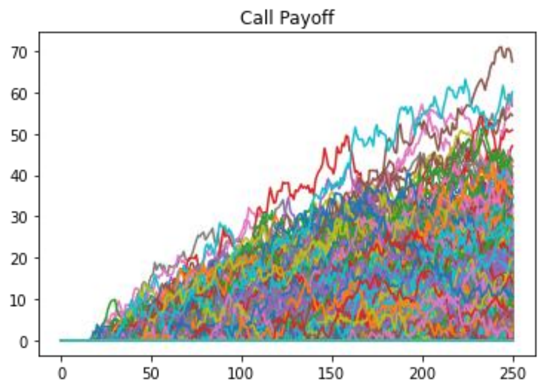
Strike price = 115

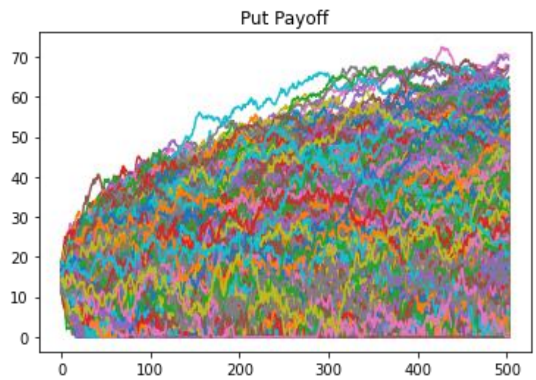
Time period = 2

Number of steps = 504



We then calculated also the payoffs of the call and put option and the result is:





After using Monte Carlo, we focused on calculating all the variables needed to pursue the Exposure at default and, consequently, the Risk Weighted Asset.

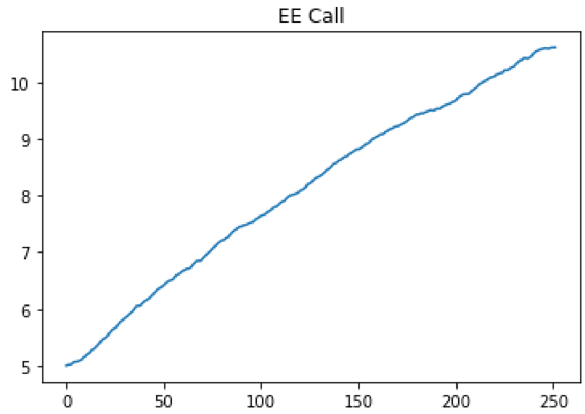
We calculated the Expected Exposure for both call and put options. The formula to derive the EE is given:

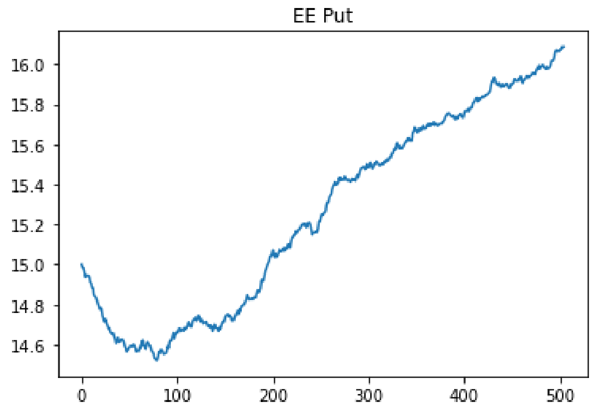
$$EE_t = \mathbb{E}(\max(V_{t,0}))$$

Equation 3: *Expected Exposure*

Where Vt = payoff of the option. In the case of a call option, the payoff is given then by (St – K), whereas instead, the payoff for the put option is given by (K – St).

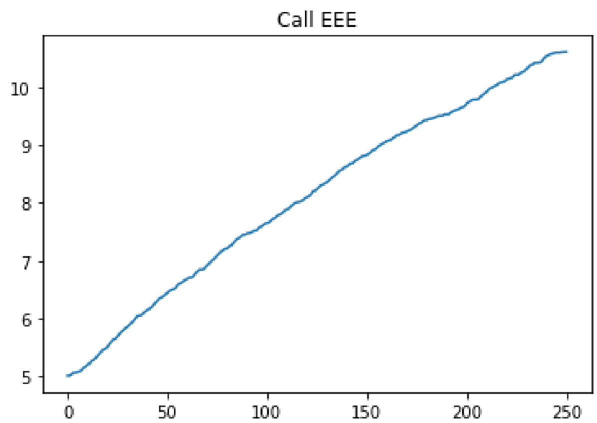
In particular, the graphs for both call and put EE are:

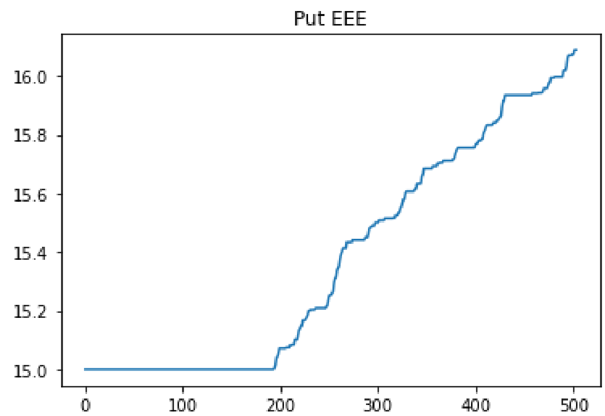




The EE is calculated to represent the cases where the bank will make a loss if their counterparty default. Of course, the EE will always be greater than the expected MtM since it concerns only the positive Mtm. The increase in the exposure is also consistent with the theory behind this argument. The exposure tends to grow due to the increased possibility of being high in money. The fact that the put option will decrease at the beginning of the period could be given by the fact that the option is in, out or at the money, but the basic shape is the same.

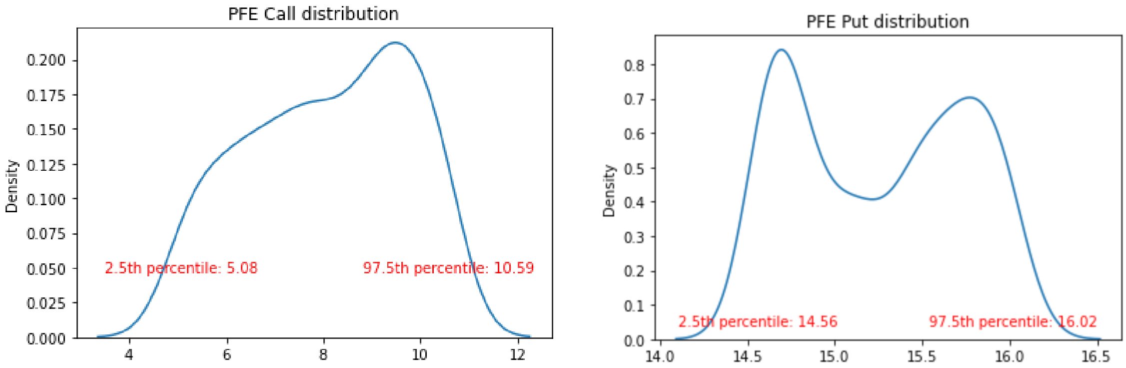
We then calculated the Effective Expected Exposure (EEE) given by the effective expected exposure is the maximum expected exposure that occurs over the exposure horizon time interval. Effective EE is equal to nondecreasing EE. We plotted the graphs for EEE again, both for call and put options, and the result is:





As we can see, the result is in line with our expectations. The Call EEE was almost everywhere positive already before, and since that, the graph does not change that much. We can instead see big differences for the put EEE at the beginning of the path. This is why considering the non-decreasing EE, the EEE will remain steady at the initial level of fifteen until a new max is reached.

We calculated then our Potential Future Exposure both for call and put options. The PFE is the worst exposure we could have at a certain time in the future. We calculated the PFE for the percentiles of 97.5 % and 2.5 % meaning that for the first one it will define the exposure that would be exceeded with probability of no more than 2.5%, for the latter is the same thing but with the data reversed.



The next step of computing the Exposure at Default procedure is to calculate the Effective Expected Positive Exposure defined as the average EEE through time. It could be useful for having a single number representation of exposure. The two results are

- Call EEPE: 8.11
- Put EEPE. 15.38

Having these two numbers to calculate the Exposure at default, we can use this formula:

$$EAD = \alpha EEPE$$

Equation 4:*Exposure at default I*

We also compare the result obtained with the other formula to have again the Exposure at the Default

$$EAD = \alpha \times \sum_{k=1}^{n} (\forall t_k < 1 : (t_k - t_{k-1}) EEE_{t_k})$$

Equation 5: *Exposure at Default II*

As expected, the result is the same, and it is equal to:

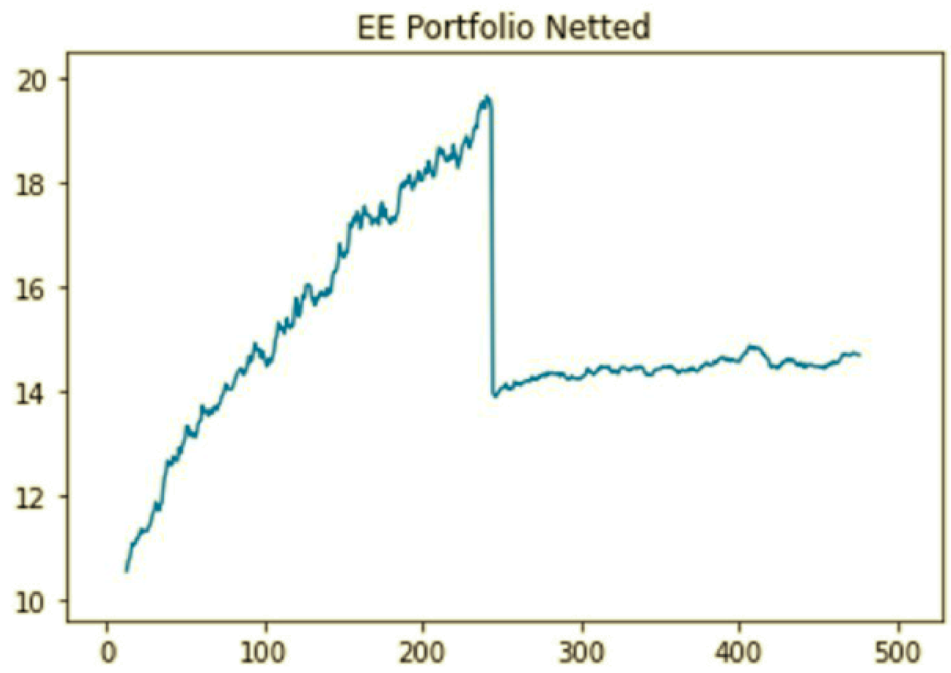
- Call EAD = 11.39
- Put EAD = 21.58

Having computed the EAD, to calculate the Risk Weighted Asset at each trade, we need to compute K. In our calculation, K = 0.06679 and then our Risk Weighted Asset is:

- Call RWA = 5.49
- Put RWA = 10.41

PORTFOLIO LEVEL

At portfolio level we considered the total effect of the two positions on exposure, so instead of considering the EE of the two positions, we only consider their Mtm. This is because, the EE is the maximum between the Mtm and 0. Therefore, only positive values will be considered. For there to be a benefit of netting, there must be Mtm with negative values to offset the exposure. So in determining the exposure, we have used the sum of the Mtm with the following results:



as we can see there is a sudden drop in the exposure after that the first option reach its maturity.
We then compute the Effective Expected Positive Exposure as the mean of the EEE, and the result is:

- 16.44

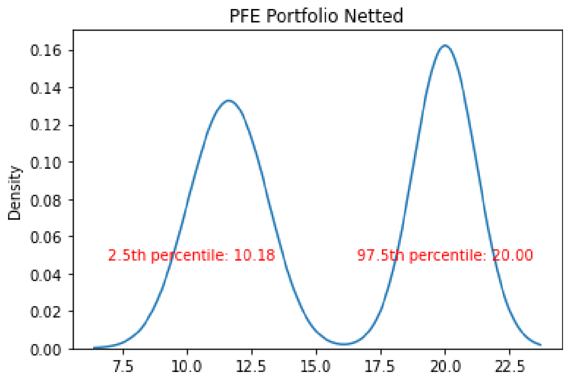
As we did for the trades, we calculated the Risk Weighted Asset, and the result is

- RWA of portfolio = 11.07

As we can see it is smaller than the sum of the RWA of each option.

The Potential future exposure of the portfolio is:

- 97.5 % of PFE = 10.18
- 2.5% of PFE = 20.0



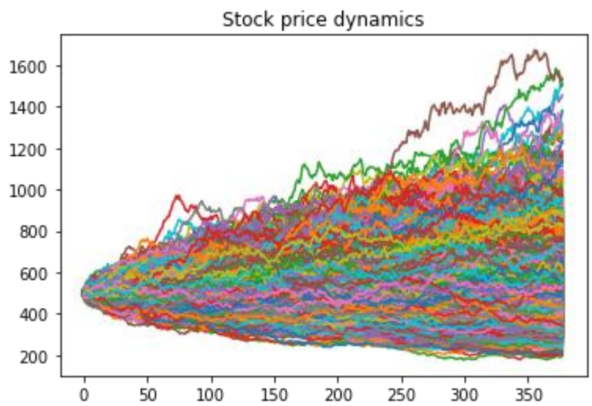
2.3 BANK OF CLUJ

The calculations and methodologies performed in this section are the same as those performed in the first portfolio, which is why, to avoid repetition, we will only report the data without additional explanations of the variables. The portfolio of Cluj bank is composed of two European options:

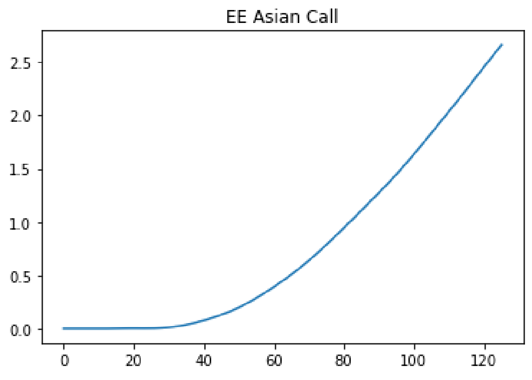
- One call option with a maturity of 6 months with a strike price of 570 USD and having as underlying IRNMN.
- One put option with a maturity of one year and a half with a strike price of 450 USD and have the same underlying.

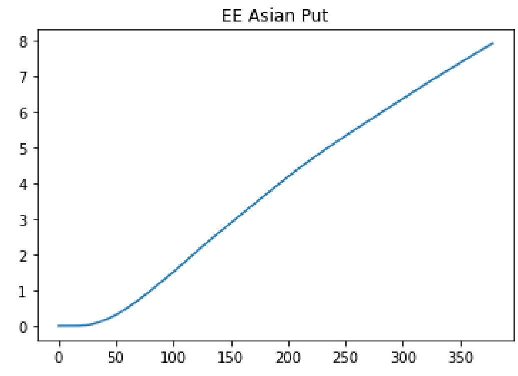
2.3.1 Trade level

In order to simulate the stock prices of the underlying, we first compute Monte Carlo Simulation. The simulation has been built with the volatility that minimizes the RMSE equal to 0,245% and the initial stock price S0 = 500. The graph is:



We calculated the EE for both the Call option and the Put option, and the graphs are the following:





We then calculated the Effective Expected Exposure as non-decreasing EE. The graphs are the following:

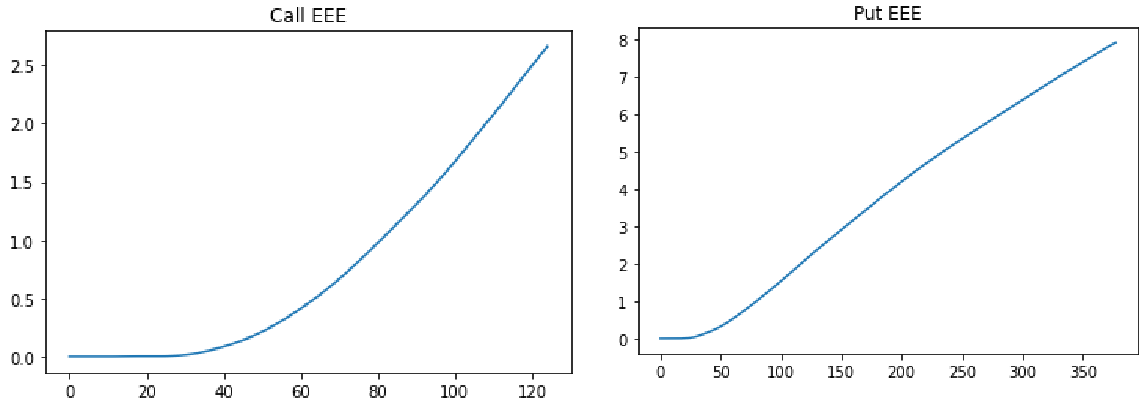


Illustration 3:*EEE of both options*

We calculated the Effective Expected Exposure as the mean of the EEE for both call and put options, and the results are:

- EEPE for Call Option = 0.7857
- EEPE for Put Option = 3.7702

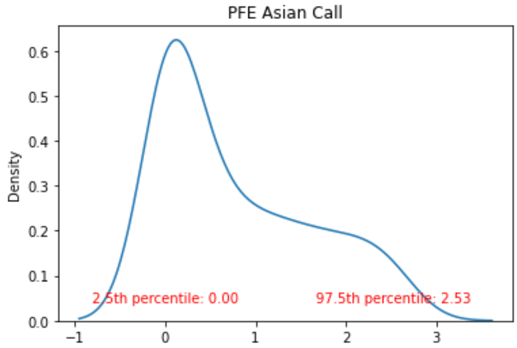
The Exposures at Default are then:

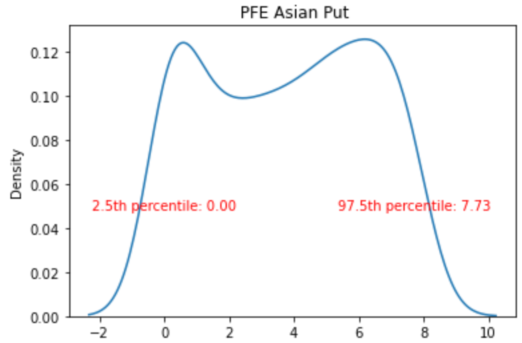
- EAD for Call Option = 1.0999
- EAD for Put Option = 5.2784

We calculate the Potential Future Exposure and the result is:

- PFE for Call Option at 2.5 %= 0.0
- PFE for Call Option at 97.5% = 2.53
- PFE for Put Option 2.5% = 0.0
- PFE for Call Option 97.5% = 7.73

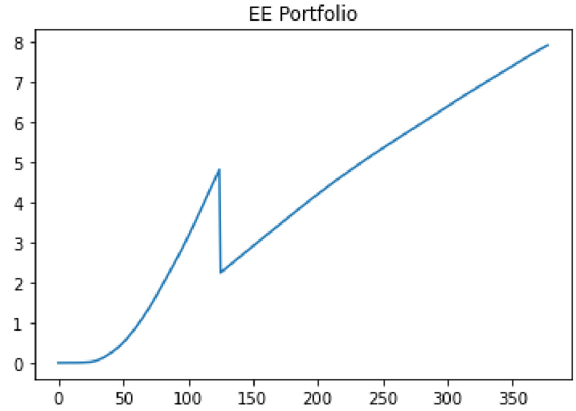
We plotted then the graphs of the PFE as a distribution and we plotted them, the result is:



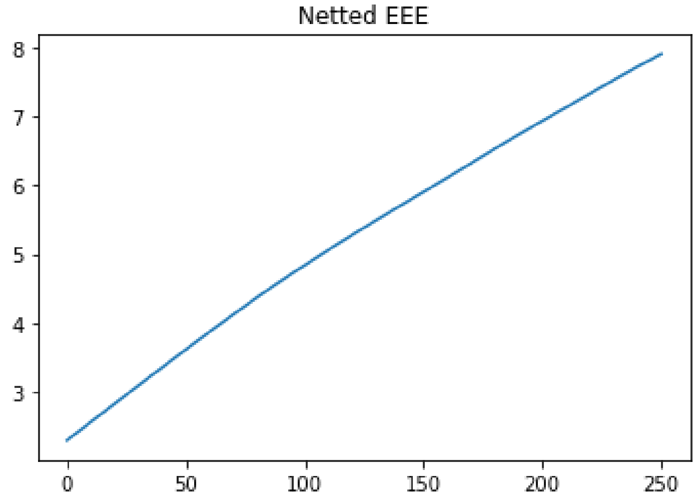


2.3.2 PORTFOLIO LEVEL

Since It is not a netted portfolio the portfolio exposure is simply the sum of the two portfolios. We calculated the Expected Exposure at a portfolio level and we plotted it

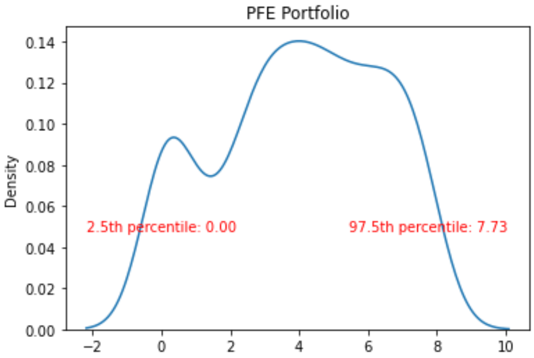


We then calculated the EEE of the portfolio (non decreasing EE)



We computed the EEPE for the portfolio as the mean of EEE and the result is: 5.28
Having all these values the Exposure at Default is then equal to 7.40
The potential future Exposure for the portfolio are:

- PFE at 2.5% = 0.0
- PFE at 97.5% = 7.73


PFE Portfolio
Density
2.5th percentile: 0.00
97.5th percentile: 7.73