Analyse a set of outputs of ARWV with machine learning:
Can Deep Learning help us see order in chaos?

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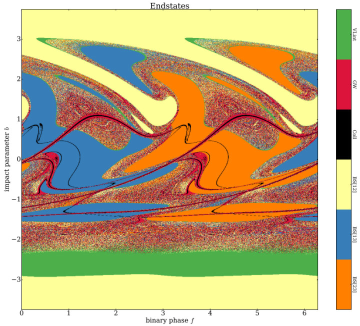
Sections

1. Introduction
2. Data analysis and manipulation
3. Neural Network and training
4. Output analysis

- The 3 body problem has puzzled physicists and mathematicians for centuries.
- Differential equations that describe gravity have no general analytical solutions.
- Numerical integration is required to solve the equations
- **High dependence on initial condition** leads to chaotic behaviours.

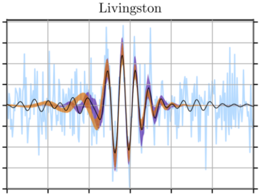
- Computational Complexity rises fast.

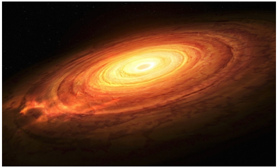


Introduction : Modern Challanges in science

Gravitational Waves astronomy

- The LIGO-VIRGO collaboration has observed many Binary Black Holes (BBH) mergers for years
- The formation of these massive BBH system (like GW190521) is theorized to occur mainly in the densest parts of Star Clusters (SC)
- **Many close gravitational and chaotic interactions** occur in SCs
- Single-binary BHs scattering can lead to formations of tightly bounded high mass systems.
- But are **Hard to integrate**





Introduction : When does ML comes in

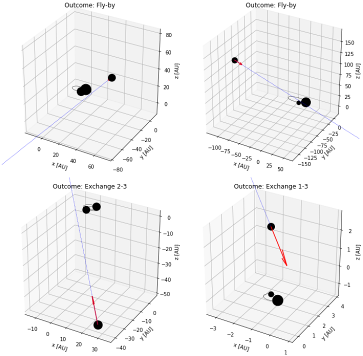
The aim of this project is to build a **Fully connected Deep NN** that can **predict the topology** (general outcome) of an encounter.

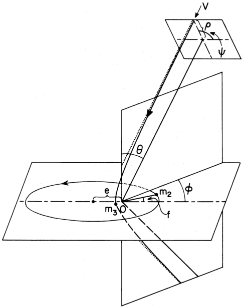
- By taking the output of a large set of 3-body interactions (BBH-BH) computed by ARWV (a higly accurate nbody relativistic integrator) we can **train a neural network on it**
- once trained(and if it works) it computes the outcome in a fraction of the time reqiured by ARWV, skipping the numerical integration alltoghether

This network can be used:

- to study the probability distribution of the outcomes in general
- To predict the outcome of just close encounters in larger SCs simulations, by trading precision with less computational requirements.

Main focus on a dataset of 1e5 simulations performed with ARWV , with 18 columns that describe initial conditions of each simulations and 4 columns that descibe the output.





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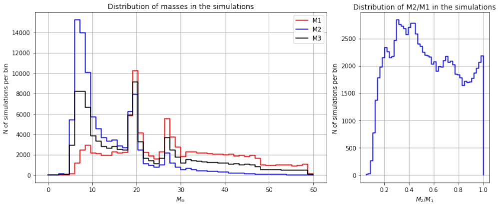
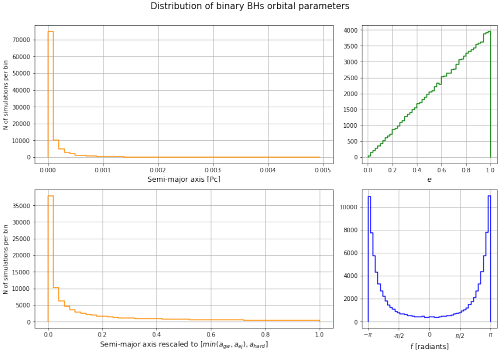
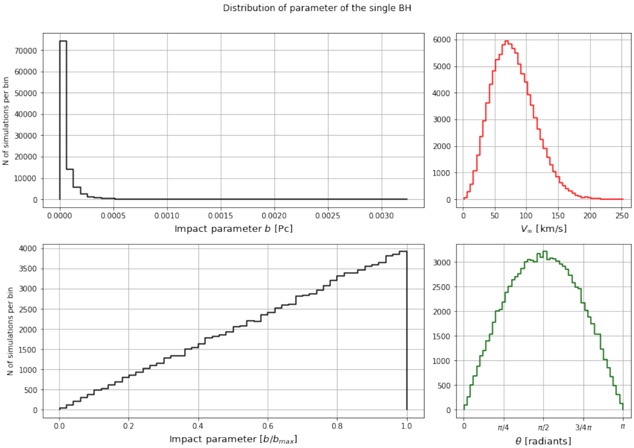
Dataset: Masses and BBH orbital parameters





Dataset: Single BH parameters



Problem: Neural networks work best when data is roughtly in the range $[-1, 1]$:

- Solution for $\{m_1, m_3, a, e, v_\infty, \theta\}$:

$$\mathrm{x_{NN}} = \frac{\mathrm{x_0} - \overline{x_0}}{std(\mathrm{x_0})}$$

- Solution for $\{\psi, \phi, f\}$

$$\mathrm{x_{NN}} = \{sin(x_0), cos(x_0)\}$$

(To impose periodicity of the output function)

- Solution for b:

$$\mathrm{b_{NN}} = \frac{\mathrm{b_0}}{\mathrm{b_{max}}} - \overline{\left(\frac{\mathrm{b_0}}{\mathrm{b_{max}}}\right)}$$

Dataset manipulation

- Solution for a :

$$a_{NN} = \frac{a - min(a_{gw}, a_{ej})}{a_{hard} - min(a_{gw}, a_{ej})},$$

- Solution for m2 :

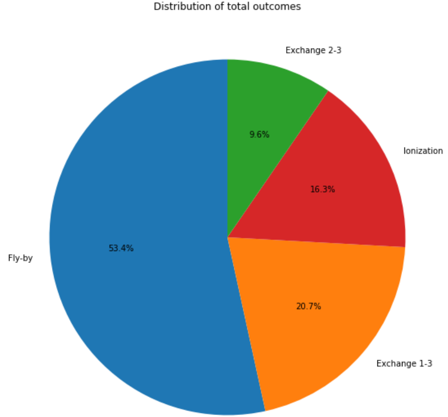
$$M_{2NN} = \frac{M_2}{M_1}$$

- Additional parameter: v_{crit} indicator is 1 if

$$v_{\infty} > v_{crit} = \sqrt{\frac{m_1 m_2 (m_1 + m_2 + m_3)}{m_3 a (m_1 + m_2)}}$$

Dataset: Outcomes

Index	Outcome	Description
0	Fly-by	The binary BHs remain as a binary at the end of the simulation
1	Exchange 1-3	The smaller BH in the binary is replaced by the single BH
2	Exchange 2-3	The bigger BH in the binary is replaced by the single BH
3	Ionization	No binaries remain at the end of the simulation



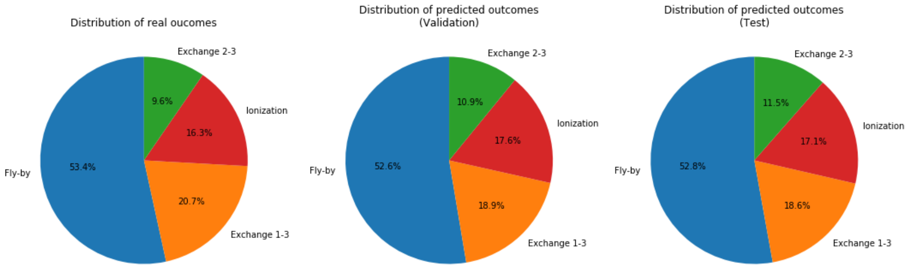
Neural network

Neural network constructed with `pytorch`

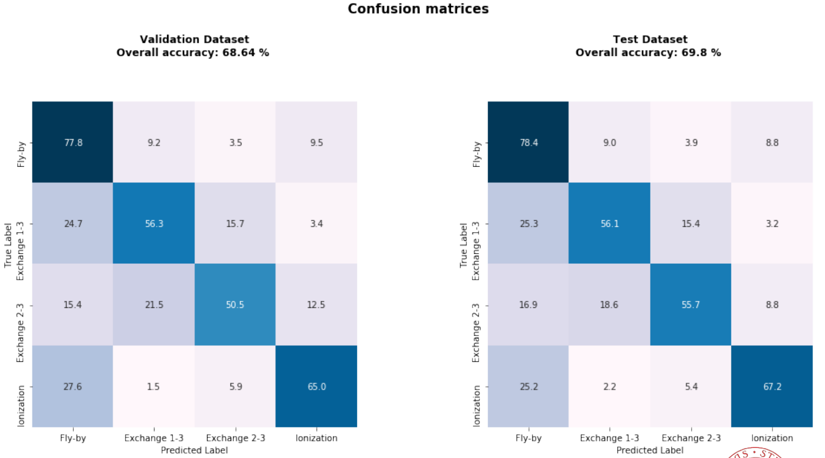
- 4 hidden layers, with 256 nodes each.
- 15 nodes as inputs (8 "normal" parameters + 3 sines + 3 cosines + v_{crit}), 4 nodes as outputs (4 classes)
- ReLU as activation function
- Optimizer: Adamax with lr = 2e-3
- minibatch learning with sample shullfe and 256 as batch size
- Dropout 0.3 and L2 regularization used to avoid overfitting (weight decay = 2e-5)
- loss function: Cross entropy (weighted classes: [0.21 , 0.21 , 0.31 , 0.27])

Neural Network: Training for 60 epochs

Network Output Distribution

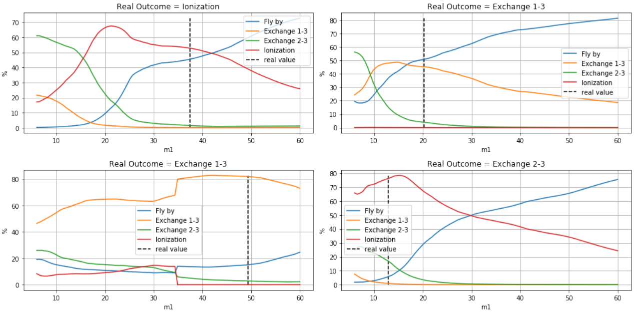


Network output: Confusion matrices



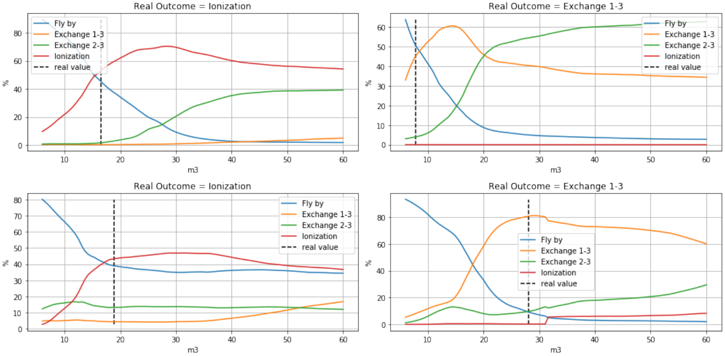
Network output: 1D analysis of M_1

Here we analyze the network output in more specific detail, choosing a random simulation and looking at how the output function changes by varing just one parameter



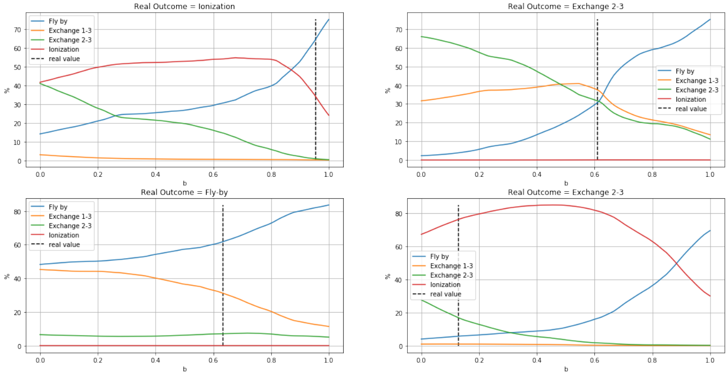
- High M_1 -> high fly-by
- Interesting behaviour with ionization in some cases

Network output: 1D analysis of M_3



- High M_3 -> Low fly-by
- some "trading" exists between exchange probabilities

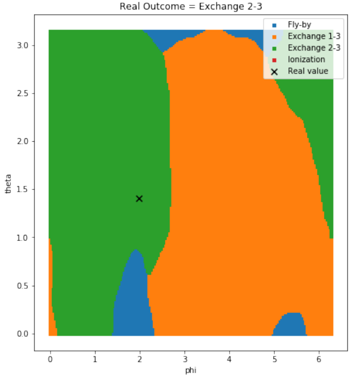
Network output: 1D analysis of b/b_{max}

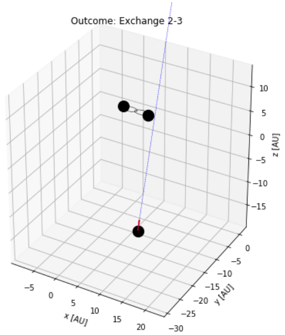


- High b/b_{max} -> high fly-by
- Exchanges follow similar patterns

Network output: 2D analysis of ϕ and θ

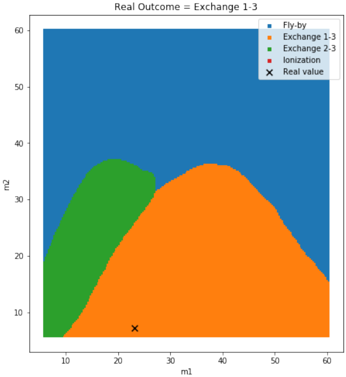
Same concept as before applies, but now 2 parameters are changed at the same time.

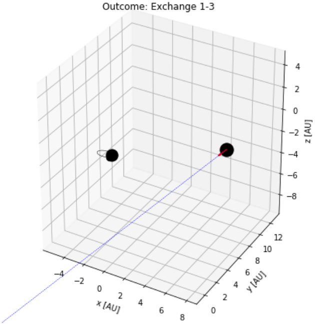




- Highly simmetrical system leads to highly simmetrical output
- fly-by probabilities: net artifacts? Actual property?

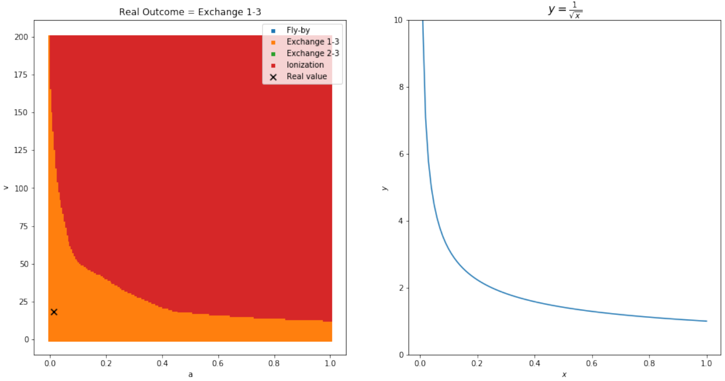
Network output: 2D analysis of M_1 and M_2





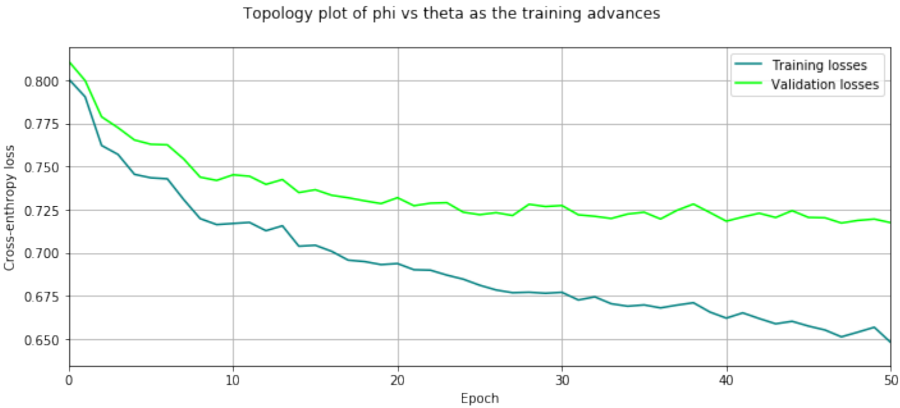
- Expected Simmetry in $M_1 - M_2$ is weakly present, even if the network was not trained on recognising it
- moving far away form actual training data distr makes the accuracy go down

Network output: 2D analysis of v_∞ and Semi-major axis

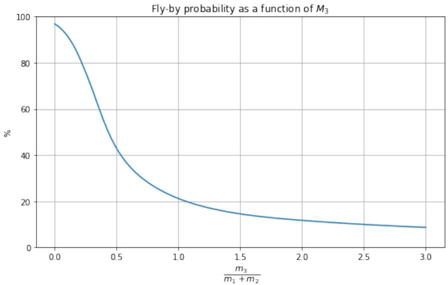


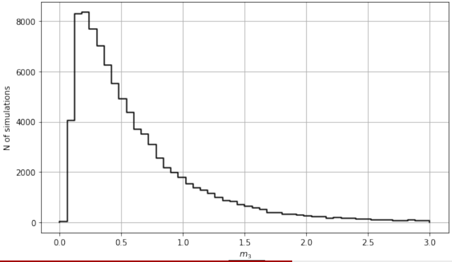
- Expected behaviour from v_{crit} formula

Network output: Complexity as a function of training



Epochs = 10 Epochs = 20 Epochs = 30 Epochs = 40 Epochs = 50





- We know that this relation should hold up in theory, since fly-by should go down as this ratio increases and probability of interaction become more common
- Prediction breaks down when the data lacks (around 0), where we expect 100 % fly by

Conclusions

- A Deep Neural Network can approximate the outcome of 3-body encounters whith $\sim$ 70 % accuracy.
- really good for a chaotic system
- if it can be proven that it does not introduce any bias, it could be possibly implemented in large cluster simulations to cut down time.
- It could be used to study outcome probability distribution in more details, to help us understand chaos of 3-body interactions

Can we improve it?

- More simulations -> possible to expand the network
- Experiment with data augmentation
- More accurate Gridsearch or bayesian search of hyperparameters
- Experiment with letting the network overfit
- Weighting loss function over chaoticness of interaction.

Thanks for your attention!