⇒ algèbre

2a) $\frac{4^{-2}}{2^{-5}} = \frac{2^5}{4^2} = \frac{2^5}{(2^2)^2} = \frac{2^5}{2^4} = 2^{5-4} = 2^1 = 2$ //

3a) $\frac{(-3x^4)(-2x)^6}{(6x)^2} = \frac{-3x^4 \cdot 2^6 x^6}{6^2 x^2} = \frac{-3 \cdot 2^6 \cdot x^{10}}{(3 \cdot 2)^2 x^2} = -\frac{3 \cdot 2^6 \cdot x^{10}}{3^2 \cdot 2^2 \cdot x^2}$

$= -3^{1-2} \cdot 2^{6-2} \cdot x^{10-2} = -3^{-1} \cdot 2^4 \cdot x^8 = -\frac{2^4 x^8}{3} = -\frac{16x^8}{3}$

3b) $\frac{x^{-4} y^3}{2^{-3} x^{-3} y^{-5}} = 2^3 \cdot x^{-4-(-3)} \cdot y^{3-(-5)}$

$= 2^3 \cdot x^{-1} \cdot y^8 = \frac{8y^8}{x}$ //

13d) $\frac{x}{4} - \frac{5}{3}(x-5) \leq \frac{x}{3} - 7\left(\frac{x}{4} - 3\right)$

$\frac{x}{4} - \frac{5x}{3} + \frac{25}{3} \leq \frac{x}{3} - \frac{7x}{4} + 21 \quad \Big| \cdot 12$

$\frac{12x}{4} - \frac{12 \cdot 5x}{3} + \frac{12 \cdot 25}{3} \leq \frac{12x}{3} - \frac{12 \cdot 7x}{4} + 21 \cdot 12$

$3x - 20x + 100 \leq 4x - 21x + 252$

$-17x + 21x - 4x \leq 252 - 100$

$0 \leq 152$ Vrai

donc, $S = \mathbb{R}$

$$\frac{3x-2}{3} - \frac{3x-5}{2} \leq \frac{x}{2} \quad \Big| \text{ même dénominateur}$$

$$\frac{2(3x-2)}{6} - \frac{3(3x-5)}{6} \leq \frac{3x}{6} \quad \Big| \cdot 6$$

$$2(3x-2) - 3(3x-5) \leq 3x$$
$$6x - 4 - 9x + 15 \leq 3x$$
$$-3x - 3x \leq -11$$
$$-6x \leq -11 \quad \Big| \cdot -1$$
$$6x \geqslant 11$$
$$x \geqslant \frac{11}{6} \qquad S = \left[\frac{11}{6} ; +\infty\right[$$

Produit mixte – ex 3.28 Support Didier Muller

Calculez le volume du tétraèdre ABCD dont les coordonnées des quatre sommets sont

$A(2;-1;1)$, $B(5;5;4)$; $C(3;2;-1)$ et $D(4;1;3)$

$$\overrightarrow{AB} = B - A = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \quad \overrightarrow{AC} = C - A = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \quad \overrightarrow{AD} = D - A = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$\begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 2 \\ 3 & -2 & 2 \end{vmatrix} = 3\begin{vmatrix} 3 & 2 \\ -2 & 2 \end{vmatrix} - 6\begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$
$$= 3(6-(-4)) - 6(2-(-4)) + 3(2-6)$$
$$= 3 \cdot 10 - 6 \cdot 6 + 3 \cdot (-4)$$
$$= 30 - 36 - 12$$
$$= -18$$

$$V = \frac{|-18|}{6} = \frac{18}{6} = 3 \ u^3$$

Logarithmes - fiche

$$C_n = C_0\left(1+\frac{t}{100}\right)^n$$

$$4000 = 3300\cdot\left(1+\frac{2{,}6}{100}\right)^n$$

$$\frac{4000}{3300} = \left(1+\frac{2{,}6}{100}\right)^n$$

$$\frac{40}{33} = \left(\frac{100}{100}+\frac{2{,}6}{100}\right)^n \Longrightarrow 1{,}026^n = \frac{40}{33}$$

$$\Longrightarrow \log_{1{,}026}\left(\frac{40}{33}\right) = n$$

$$\Longrightarrow n = \frac{\log\left(\frac{40}{33}\right)}{\log(1{,}026)} \simeq 7{,}5$$

R: En environ 7,5 années.//

Support logarithmes

Ex 50. $2\log(x-5) = 1+\log x$

$$D = \{x\in\mathbb{R} : x-5>0 \wedge x>0\} =]5;+\infty[$$
$$x>5$$

$$\log(x-5)^2 - \log(x) = 1$$

$$\log\left(\frac{(x-5)^2}{x}\right) = 1$$

$$10^1 = \frac{(x-5)^2}{x}$$

$$10\cdot x = x^2-10x+25$$

$$x^2-20x+25 = 0$$

$$\Delta = 400-4\cdot1\cdot25 = 300$$

$$x_{1,2} = \frac{+20\pm\sqrt{300}}{2} = \frac{20\pm10\sqrt{3}}{2} = 10\pm5\sqrt{3}$$

$$x_1 = 10+5\sqrt{3} \in D$$
$$x_2 = 10-5\sqrt{3} \notin D$$

Donc,

$$S = \{10+5\sqrt{3}\}$$

19. $\ln(2x+5) = 2\ln(x-5)$

$D = \{x \in \mathbb{R} : 2x+5 > 0 \wedge x-5 > 0\} =]5; +\infty[$

$x > -\frac{5}{2}$ $\quad x > 5$

$\ln(2x+5) = \ln(x-5)^2$ $\quad$ ln bijective

$2x+5 = (x-5)^2$

$x^2 - 10x + 25 = 2x + 5$

$x^2 - 12x + 20 = 0$ $\quad$ ou $\quad (x-10)(x-2) = 0$

$x = 10$ ou $x = 2$

$\in D$ $\quad \notin D$

$\Delta = (-12)^2 - 4 \cdot 1 \cdot 20$

$= 144 - 80$

$= 64$

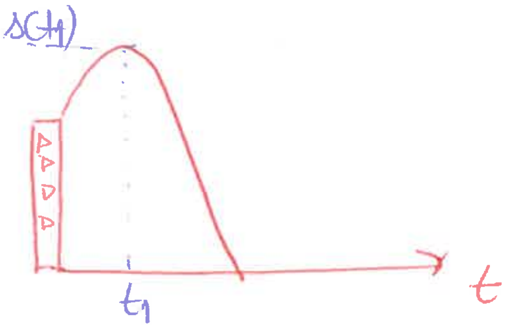
$x_{1,2} = \frac{12 \pm 8}{2} = \begin{cases} + & \frac{20}{2} = 10 \\ - & \frac{4}{2} = 2 \end{cases}$

Donc,

$S = \{10\}$

Paraboles

1.



$s(t) = -4{,}9t^2 + 44t + 30$

On cherche le maximum.

$h = x_S = \frac{-b}{2a} = \frac{-44}{2(-4{,}9)} \approx 4{,}49\ m$

$k = y_S = \frac{-\Delta}{4a} = \frac{-2524}{4(-4{,}9)} \approx 128{,}78\ m$

$\Delta = 44^2 - 4(-4{,}9) \cdot 30$

$= 2524$

R: la distance maximale est d'environ 128,78 m

2. $s(t) = -4{,}9t^2 + v_0 t$

l'objet est au sol quand $s(t) = 0$

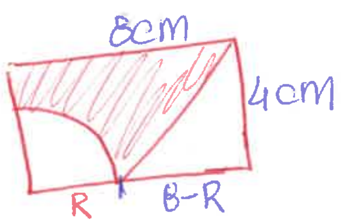
Alors,
$$-4{,}9t^2 + v_0 t = 0$$
$$t(-4{,}9t + v_0) = 0$$
$$t = 0 \quad \text{ou} \quad -4{,}9t + v_0 = 0$$
$$t = \frac{-v_0}{-4{,}9} = \frac{v_0}{4{,}9}$$

ce n'est pas nécessaire

$$x_s = \frac{-b}{2a} = \frac{-v_0}{2(-4{,}9)} = \frac{v_0}{9{,}8}$$
$$y_s = s\left(\frac{v_0}{9{,}8}\right) = -4{,}9\left(\frac{v_0}{9{,}8}\right)^2 + v_0\left(\frac{v_0}{9{,}8}\right)$$
$$= \frac{-4{,}9\,v_0^2}{9{,}8^2} + \frac{v_0^2}{9{,}8}$$
$$= \frac{-v_0^2}{19{,}6} + \frac{v_0^2}{9{,}8} = \frac{2v_0^2 - v_0^2}{19{,}6} = \frac{v_0^2}{19{,}6}$$

La distance maximale est donné par $\frac{v_0^2}{19{,}6}$ m //

5. a)

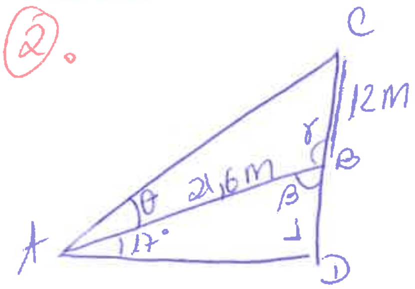


$$A_G = A_{\square} - \frac{A_{\circ}}{4} - A_{\triangle}$$
$$= 8 \cdot 4 - \frac{\pi R^2}{4} - \frac{(8-R)4}{2}$$
$$= 32 - \frac{\pi R^2}{4} - 2(8-R)$$
$$= 32 - \frac{\pi R^2}{4} - 16 + 2R$$
$$= -\frac{\pi R^2}{4} + 2R + 16$$

b) $R_S = \frac{-b}{2a} = \frac{-2}{2\left(-\frac{\pi}{4}\right)} = \frac{2}{\frac{\pi}{2}} = \frac{4}{\pi}$

Trigonométrie

② .



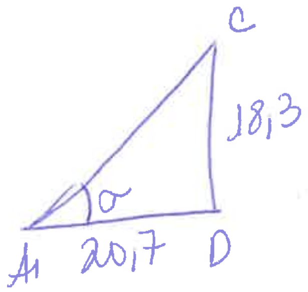
On cherche $\overline{AC}$

$\sin(17°) = \frac{\overline{BD}}{21,6} \Rightarrow \overline{BD} = 21,6 \cdot \sin(17°) \simeq 6,3\ m$

$\beta = 180 - 90 - 17° = 73°$

$\gamma = 180 - 73 = 107°$

$\cos(17°) = \frac{\overline{AD}}{21,6m} \Rightarrow \overline{AD} = 21,6 \cdot \cos(17) \simeq 20,7\ m$



$\tan\sigma = \frac{18,3}{20,7} \Rightarrow \sigma = \tan^{-1}\left(\frac{18,3}{20,7}\right)$

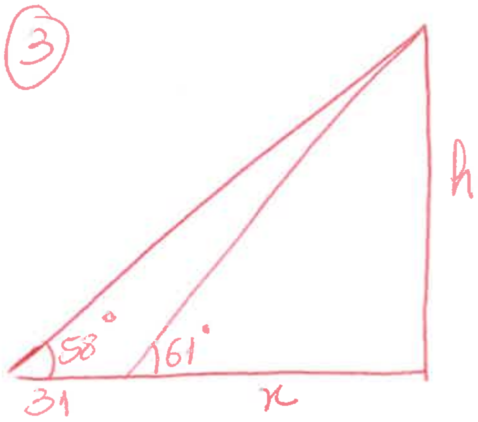
$\sigma \simeq 41,5°$

$\theta = 41,5° - 17° = 24,5°$

$\frac{12}{\sin(24,5)} = \frac{\overline{AC}}{\sin(107°)} \Rightarrow \overline{AC} = \frac{12 \cdot \sin(107)}{\sin(24,5°)}$

$\overline{AC} \simeq 27,7\ m$

3



$$\tan(58) = \frac{h}{31+x} \implies h = (x+31)\cdot\tan(58^\circ)$$

$$\tan(61^\circ) = \frac{h}{x} \implies h = x\cdot\tan(61^\circ)$$

$$(x+31)\tan(58^\circ) = x\tan(61)$$

$$x\tan(58^\circ) - x\tan(61) = -31\tan(58^\circ)$$

$$x(\tan(58) - \tan(61)) = -31\tan(58^\circ)$$

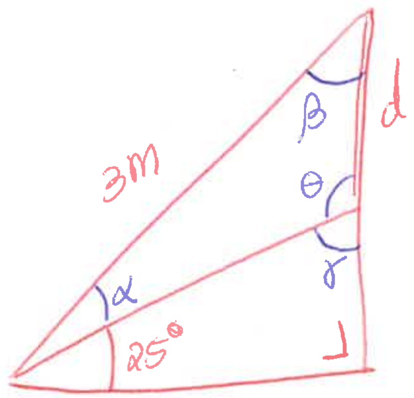
$$x = \frac{-31\tan(58)}{\tan(58)-\tan(61)} \simeq 243{,}53\ m$$

Alors, $h = x\cdot\tan(61^\circ)$

$$= 243{,}53\cdot\tan(61)$$

$$\simeq 439{,}34\ m$$

4



$\alpha = 45 - 25 = 20°$

$\beta = 45°$

$\gamma = 180 - 90 - 25 = 65°$

$\theta = 180 - 65 = 115°$

Alors,

$$\frac{d}{\sin(20°)} = \frac{3}{\sin(115°)}$$

$$d = \frac{3 \cdot \sin(20°)}{\sin(115°)} \approx 1,13 \text{ m}$$