1 Introduction

2 Modelling

In this section, we model the movement of a line of dislocations subjected to an applied shear stress. We imagine this as a curve moving through three-dimensional space in a single *active slip plane* of infinite extent. Furthermore, we intend to capture the characteristics of line dislocations moving through a large solid, imposing a sensible boundary condition that will not impede our analysis. The following assumptions establish a model with this mind.

2.1 Model assumptions

Consider a regular embedded family of curves $\gamma(t)$ in $\mathbb{R}^3$, representing the dislocation curve at time t. Suppose every curve lies in the xy-plane, which we call the *active slip plane*. The aim is to determine the dislocation curve at all times $t \in [0, T]$, given knowledge of $\gamma(0)$.

Now assume that each curve has graphical form. That is, $\gamma(t)$ is parametrised by

$$\phi(x, t) = (x, f(x, t), 0) \quad \text{for } x \in \mathbb{R}.$$

We now aim to determine f given initial condition $g(x) = f(x, 0)$.

Finally, we assert f is L-periodic. This captures the characteristics of movement through a solid of fixed size, and we will naturally consider L as being much larger than "fluctuations" of the curve [1]. Leveraging this periodicity, we can eliminate the need for boundary conditions in our future analysis by defining the problem on the circle, $\mathbb{T} = \mathbb{R}/\mathbb{Z}$. We will make the shift to this viewpoint after Nondimensionalization in section (??). For now, assume $f \in C^1([0, L] \times [0, T])$ with boundary conditions

$$\begin{aligned} f(0, t) &= f(L, t), \\ \partial_x f(0, t) &= \partial_x f(L, t). \end{aligned}$$

This completes setup for the model, illustrated in figure 1.

2.2 Force balancing

With our model in place, we define the motion of dislocations by *quasi-static evolution*. That is, at all times, the sum of forces acting on any individual point vanish. Equivalently, the system has no inertia. For the rest of this subsection, we fix a certain time, t, and omit it's presentation.

Fix $x_0 \in [0, L]$ and let $\delta > 0$ be sufficiently small. We study the forces of friction, line tension and applied shear stress acting on the small section

[1]this assumption is key to the linearisation argument given in (??)

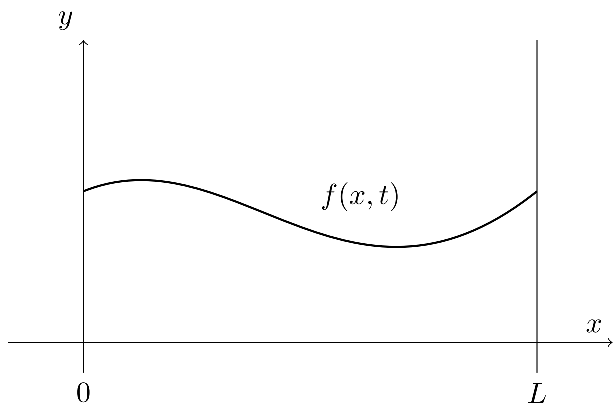

Figure 1: Dislocation curve at time t

of curve from $x_- := x_0 - \frac{\delta}{2}$ to $x_+ := x_0 + \frac{\delta}{2}$, which we'll call γ_δ. In figure 2, this curve section is drawn in red, with unit tangents $-\mathbf{t}(x_-)$ and $\mathbf{t}(x_+)$ showing the direction line tension acts on the ends of the curve. These two forces are equal to $-E_0\mathbf{t}(x_-)$ and $E_0\mathbf{t}(x_+)$ respectively, where E_0 is a known constant.

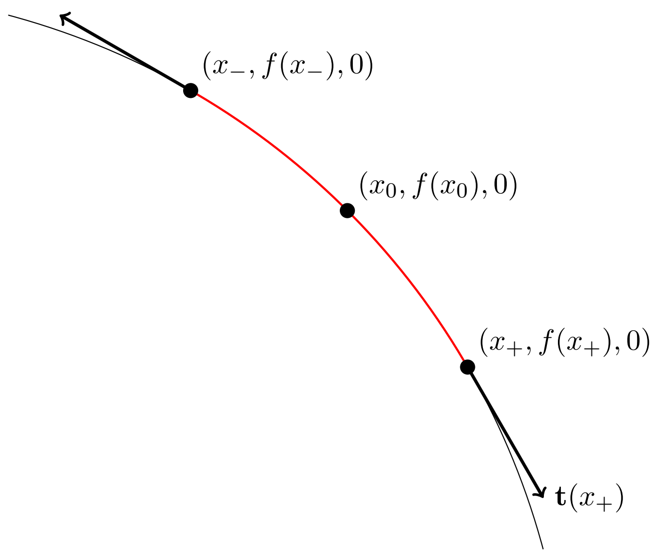

Figure 2: A small section of curve γ

Now, letting $\mu \in \mathbb{R}$ be the coefficient of friction, $\boldsymbol{\sigma} \in \mathbb{R}^{3x3}$ the ?? matrix, $\mathbf{b} \in \mathbb{R}^3$ the burger's vector, and $\Delta(\gamma_\delta)$ the length of γ_δ, we have

$$\begin{aligned} \text{Friction} =& -\mu\mathbf{v}\Delta, \\ \text{Stress} =& (\boldsymbol{\sigma}\mathbf{b}) \times \mathbf{t}\Delta. \end{aligned}$$

The length of the curve can be well approximated as

$$\begin{aligned}\Delta(\gamma_\delta) &\approx \sqrt{(x_+ - x_-)^2 + (f(x_+) - f(x_-))^2} \\ &\approx \delta\sqrt{1 + \left(\frac{f(x_+) - f(x_-)}{\delta}\right)^2},\end{aligned}$$

and we compute the unit tangent vector as

2.3 The non-linear equation