Research Project Progress Report

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Title of Project: Analysis of a PDE to model Dislocation Motion

Supervisor: Dr. Thomas Hudson

Progress to Date

Main Sources:

1. Book: Evans, L.C. (2010). *Partial differential equations*. Providence, R.I.: American Mathematical Society. This is the primary source for PDE existence theory, which makes up the bulk of the second half of the project. I have read all the relevant sections.

2. Journal article: Colliander, J., Keel, M., Gigliola Staffilani, Takaoka, H. and Tao, T. (2003). Sharp global well-posedness for KdV and modified KdV on $\mathbb{R}$ and $\mathbb{T}$. *Journal of the American Mathematical Society*, 16(3), pp.705–749. doi: https://doi.org/10.1090/s0894-0347-03-00421-1. Sec. 7 of this article describes a Fourier Analysis approach to construct Sobolev Spaces on the circle with an example application to a periodic KdV equation; this approach will be mimicked the PDE analysis section of the project. I understand the main concept, however details still need to be worked out.

3. Preprint journal article: Hudson, T., Rindler, F. and Rydell, J., 2024. A quantitative model for the Frank-Read dislocation source based on pinned mean curvature flow. *arXiv preprint arXiv:2409.20294*. Contains a similar modelling procedure followed by this project, as well as several links to other sources from which key results may be drawn. Some of these are listed below:

4. Book: Anderson, P.M., Hirth, J.P. and Lothe, J., 2017. *Theory of dislocations*. Cambridge University Press.
5. Journal article: Peach, M. and Koehler, J., 1950. The forces exerted on dislocations and the stress fields produced by them. *Physical Review*, *80*(3), p.436.

6. Book: Hull, D. and Bacon, D.J. (2011). *Introduction to dislocations*. Amsterdam; London: Butterworth-Heinemann. Useful background for understanding the physics of dislocations.

Progress:

I have worked out a suitable model for the motion of line dislocations and derived a non-linear PDE with periodic boundary conditions. The model appears sensible because it intuitively exhibits the right behaviour.

I also understand the main concepts needed to apply existence theory from Evans but still have to work out the details. Perhaps reading ahead in the Advanced Real Analysis module will yield some insight.

Project Plan

Key components of report:

- Introduction: I will include a brief introduction to dislocation theory by explaining what a dislocation is, along with some history and a few applications. Reference 6 above will be used here for concise geometric view of crystalline structures, while providing information on the discovery and observation of dislocations.

- Modelling: I will describe a model for the motion of line dislocations which uses a periodic boundary. Through a force balance argument, and references to 4 and 5, I will derive a non-linear PDE, which can then reasonably be replaced with a one-dimensional periodic heat equation via a linearisation argument.

- Analysis: I plan on stating a few definitions to construct Sobolev Spaces on the circle, along with any preliminary results before passing to Evans for the bulk of the well-known existence theory. I want to explain the intuition behind the main ideas in this section, as there are several rich concepts and clever solutions to be used. At the end of this section, I hope to have shown the existence of a solution to the linearised equation and briefly discussed the long-time behaviour of solutions.

- Extension: The third section may take one of two different directions. One option is a computational simulation of the non-linear equation, verifying the reliability of earlier linearisation and existence theory. Another possibility is to extend the previous model to consider motion through regions of different metals, thus simulating an alloy. In this case I would aim to repeat a similar procedure in proving the existence of solutions, but this would depend on the complexity of the problem.

I hereby confirm that my named supervisor has seen and approved this report.

Signed: Luca Bollini Date: 13/01/2025