Introduction

In February 2023, renowned strength sports athlete Vispy Kharadi broke the world record for most iron rods bent over the head in one minute, achieving an impressive total of 24. You can watch an overly dramatic video of this feat here: [Most Iron Bars Bent](https://www.youtube.com/watch?v=2sdtRG2OBks&t=2s). While one might first wonder what would possess a person to take on such a challenge, or if bending metal rods over your head is medically advisable (it’s not), as mathematicians, we ask a different question: “How do metals bend?”. A physicist might quickly produce an answer such as: “Well, the man’s head is exerting a shear stress force on the rod, causing defects in the crystalline structure of the metal, called *dislocations*,to move. This results in the atoms rearranging themselves into the shape of a bent iron rod.”. We will go one step further. In this project, we formally introduce a model for the movement of a line dislocation, analysing the existence of solutions to the following linearised PDE. We then finish with a discussion about extending this model to include regions of different materials, simulating an alloy or a composite. Before we embark, let’s give a summary of what dislocations are, and how they were discovered.

Geometry

Throughout this chapter, there will be numerous references to [Bacon Hull], which can be regarded as the definitive source for background information regarding dislocations. Their precise formulation of crystallographic defects’ geometric structure and rich exploration of observational techniques are a fantastic way to immerse yourself in this theory.

*Dislocations* are the linear defects in crystalline structures responsible for plastic deformation. All crystalline materials can have these defects, but it is in metals that most of the interesting behaviour occurs; other materials such as coal and ceramics either have few dislocations, or crack more easily than their dislocations are able to move. There are two types of dislocation: *edge* and *screw*. Let’s outline how this looks at the atomic level.

Imagine a simple cubic structure of atoms, as in figure [diagram 1a], where we think of the vertical and horizontal lines as bonds between atoms, which lie at each intersection. Let’s assume we may model the bonds as flexible springs between adjacent atoms, avoiding the complexity of how bonding works in real solids. Now we describe how a dislocation can be formed with the following sequence of operations:

1. Break all the bonds intersecting the half-plane defined by ABCD. CD is the *leading-edge,* where our dislocation is to be positioned, and the half-plane extends upwards in the direction of CB(->).
2. For an edge dislocation, insert a half-plane of atoms where the bonds have just been broken. This is shown in figure [diagram 1b].

For a screw dislocation, shift all the atoms on one side of ABCD by one bond length in the direction AB(->). This can result in one of two chiral structures depending on which direction the shift is in; a shift by +AB(->) is the mirror image of a shift by -AB(->). One of these is illustrated in figure [diagram 1c].

(reference diagram style as being similar to [Bacon Hull])

Note that both types of dislocation distort the bonds close to the leading-edge CD, and that this distortion decreases with distance. This will be relevant in section [??] when discussing line tension.

Furthermore, we can see how these structures allow for the easy rearrangement of atomic bonds under shear. Figure [diagram 2] shows the cross section of a pure crystal on the left, and an edge dislocation on the right. If we apply a shear force to the pure crystal, we will need to break every bond along EF before shifting the top layer one to the left and reforming the bonds (note it causes the position of the dislocation to move – segway to slip). Such an action would require an immense force, much more than is observed in practice. In contrast, applying the same to the crystal with an edge dislocation is relatively easy. We can just break and reform bonds one at a time to “fill the gap” caused by the dislocation, requiring orders of magnitude less force. In fact, this phenomenon is exactly how dislocations were discovered. (Segway to history)

History

hi [Bacon Hull section 1.4]