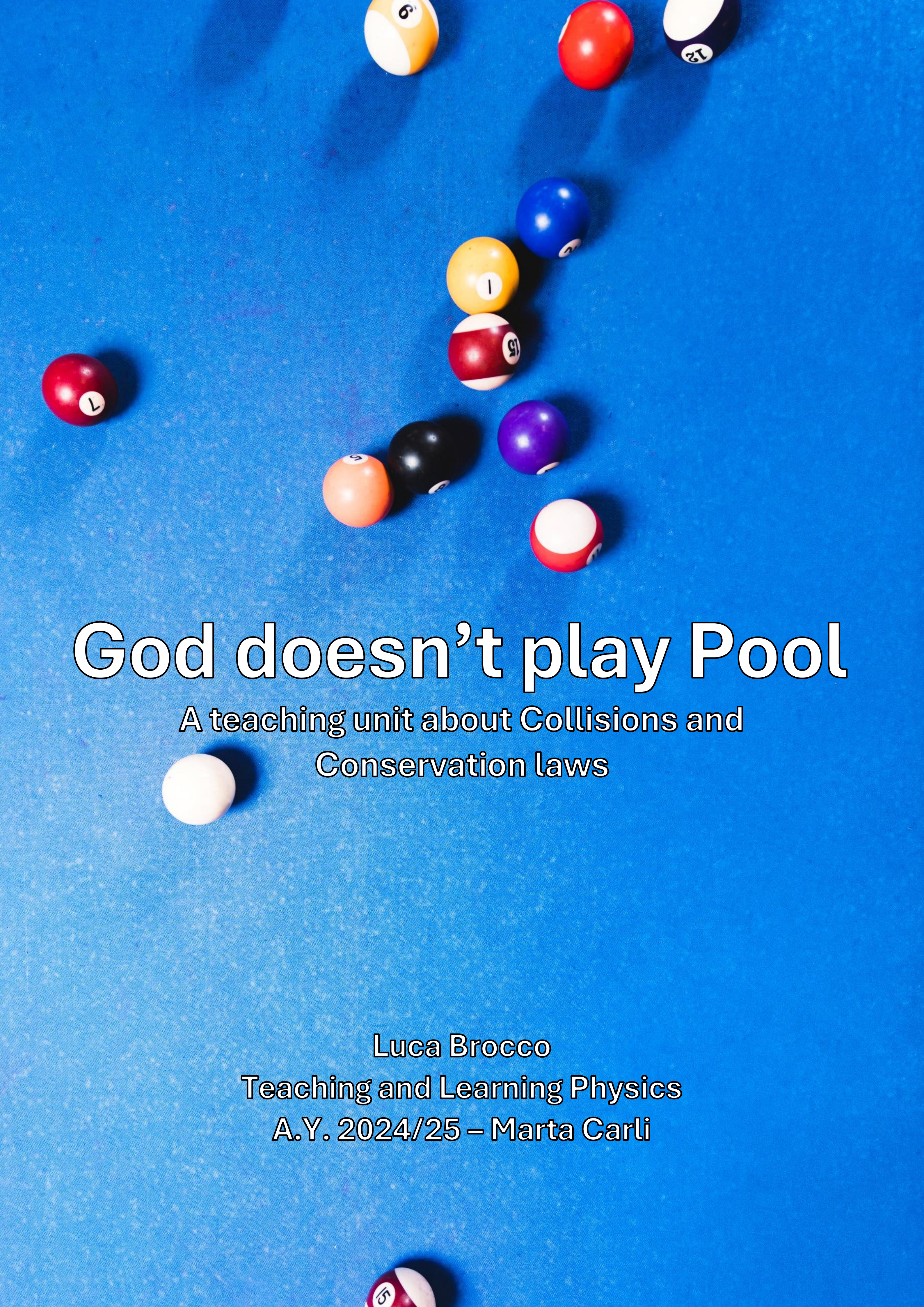

God doesn’t play Pool

A teaching unit about Collisions and Conservation laws

Luca Brocco

Teaching and Learning Physics

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Abstract

Physics, even though it is primarly focused on understanding nature, is often accused of studying phenomena that are too idealistic to be true (e.g. trascurating air friction, approximating objects as material points, and many others impossible hypotesis). This teaching unit aims to introduce some new topics for the students: collisions and their conservation laws, and proposes some experiments that can both produce unexpected results or resemble very well what computed with formulas. The goal is both to work on preconstructed knowledge about Newton’s laws, introducing interactions between bodies, and to demonstrate how Physics laws describe reality through some simple experiments. Focus is mostly on 1D collisions: simplified situations can be more effective in creating students’ awareness of how collisions work, and which elements lead them to behave in non-ideal ways.

Context/target students:

Target students are those of a 3^{rd} year class, Liceo Scientifico, opz. Scienze Applicate, with option to move up to the end of 2^{nd} year if progression is anticipated.

Reference to national guidelines:

From the Italian guidelines for Liceo Scientifico [1]:

- Competenze disciplinari: osservare e identificare fenomeni, formulare ipotesi utilizzando modelli e leggi
- Conoscenze: quantità di moto, energia cinetica, urti elastici e anelastici
- Abilità: descrivere il moto degli oggetti coinvolti in un urto, determinare le componenti di non idealità in un esperimento

Broad physics topic:

Collisions and conservation laws

Specific physics topics and ideas targeted:

- Linear momentum, impulse and their relationship with force
- Kinetic energy and momentum’s conservation laws
- Elastic and unelastic collisions
- Non-ideality elements: friction, non-1-D collisions

Scientific practices targeted:

- Construct Explanations
- Engage in Argument from Evidence + Students Dialogue

These scientific practices are addressed via proposing group experiments whose results are very dependent on the settings chosen. Moreover, reflecting on results and argumenting with own or other groups is a key element of proposed homework and post-labs activities.

Prerequisites:

Students are familiar with Newton’s laws, and the physical quantities involved in Dynamics problems, in particular with mass, forces and friction. Moreover, they know how to describe a motion using Kinematics. In addition, they know how to compute the work of a force and the kinetic energy of an object, over and above their relationship via the work-energy theorem.

Learning goals:

Students should be able to:

- Outline differences between the types of collisions, addressing their attention to the physical quantities that should be conserved
- Determine the outcome of a collision by knowing the initial state of the particles involved and the type of collision
- Distinguish between an ideal case and a "true life" situation, identifying which elements may be influencing the outcome of an experiment
- Solve a numerical problem regarding a 1D collision, knowing if it is elastic or inelastic

Overall duration:

6 hours + homework (two weeks in Liceo Scientifico)

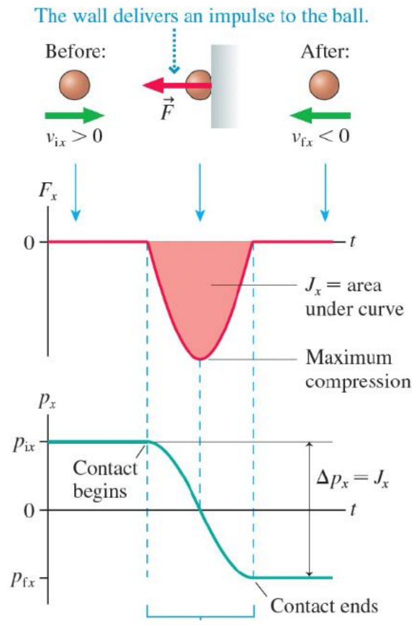
Activities

Lesson 1: first approach with linear momenta and its conservation

Students are introduced to linear momenta as a new Physics quantity, derived from what they already know.
Then they are shown the relationship between it and Newton's second law. The focus is on what the word "collision" physically means. Impulse is defined, with graphical explanation as area under the curve.

New formulas (lesson 1):

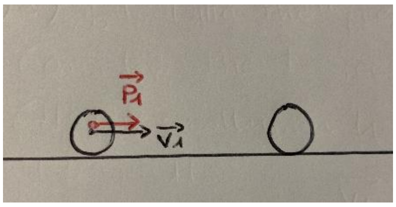
$$p = mv$$
$$\Delta p = m\Delta v = J_x = F\Delta t$$
$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

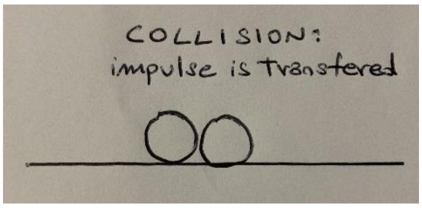


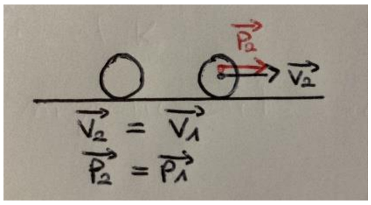
The students are then shown a video of two bowling balls colliding [10].
Lastly, they are explained that the balls collide in the seen manner because the linear momentum is conserved. Particular focus is used here: the sum of all the momenta involved is conserved, not each momentum taken alone [8].
Some "toy" exercises are proposed to them [2]:

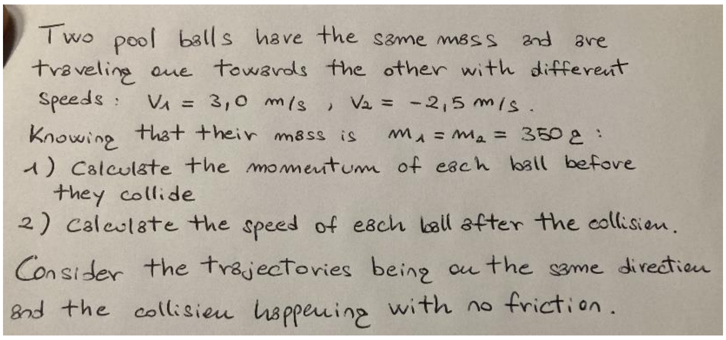
Q6: A 0.1 kg pool ball traveling 2.5 m/s hits another 0.1 kg at rest. If the first ball stops after colliding, how fast is the second now moving?

▾ See Solution

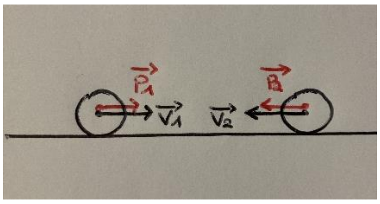


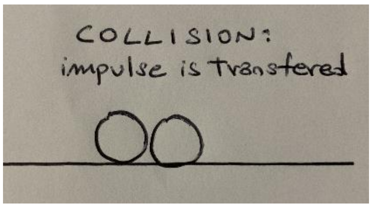


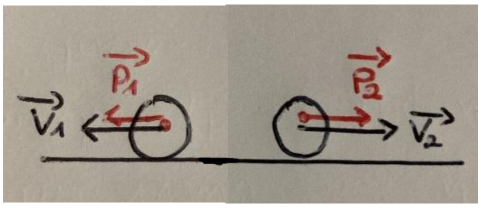




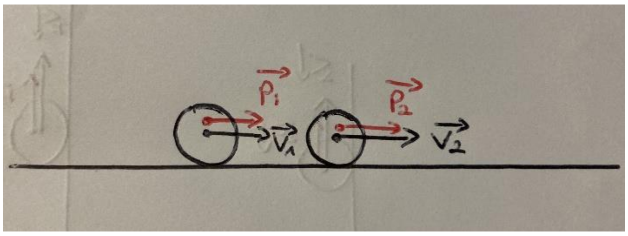
Two pool balls have the same mass and are traveling one towards the other with different speeds: $V_1 = 3,0$ m/s, $V_2 = -2,5$ m/s.
Knowing that their mass is $m_1 = m_2 = 350$ g:
1) Calculate the momentum of each ball before they collide
2) Calculate the speed of each ball after the collision.
Consider the trajectories being on the same direction and the collision happening with no friction.



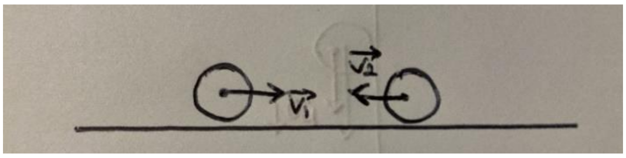




Students are encouraged to represent graphically the various instances of collisions, and to draw speed and momentum for each ball. Some common errors based on their everyday life knowledge may be represented in similar drawings:



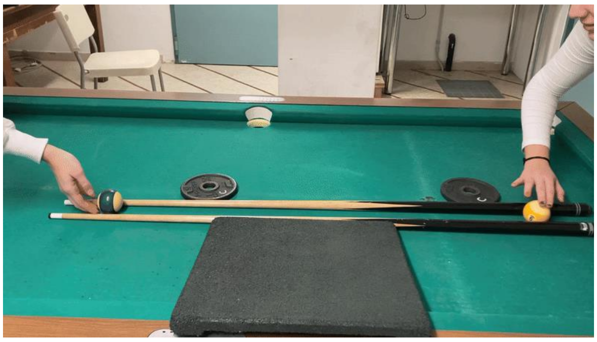
Or, according to [3], mistakenly interpretating momentum as a scalar and not representing it:



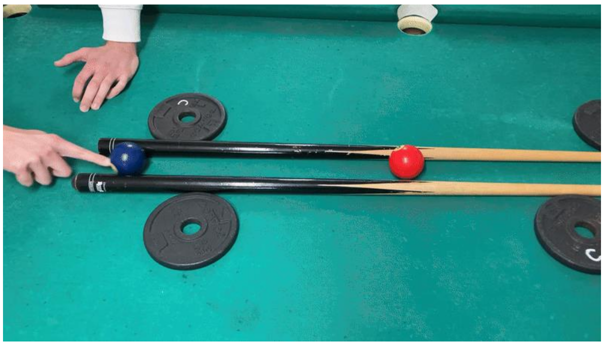
The solution to both exercises is also given by simulating with PhET Colorado’s Collision Lab [19]. Students are shown how to use this website so that they can simulate the outcome of their future experiments.

Lesson 2 (lab session 1): recreating ball shots

In this lab session students are invited to recreate the “toy problems” situations in a real-life experiment. Students are divided into small groups (3-4 students each) and each group has to take a video of both ball shots to analyze at home, comparing them to simulations. They can try out different different experimental conditions (different balls, surfaces, launch styles, camera views...) to achieve the goal. The use of different tools to adjust the shot and to measure it, e.g. wooden sticks, ropes or books, is also allowed.



These GIFs represent our attempt to perform the second exercise in real life. We used billiard cues and some gym weights to adjust the shots. From up-left: we experimented with a low friction surface (billiard bed), very low friction (table tennis table, plastic) and high friction surface (gym bench). We also tried different camera angles. From a qualitative point of view, it looks like the best video is obtained adopting a from-above perspective, and the best surface is one with low friction on it.

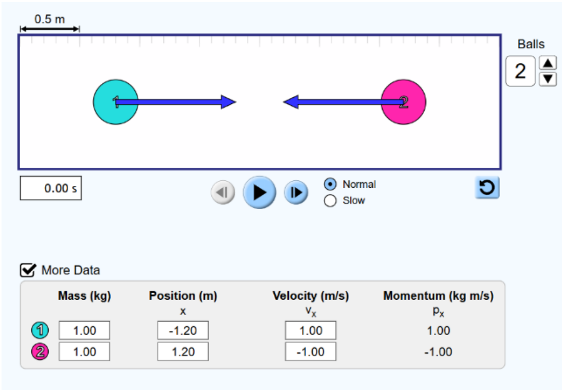


We also tried the case with a ball still. It is hard to have the first ball stop at the moment of impact, but with enough trial and error it is possible to get a good shot.

The teacher should guide students towards getting an (as far as possible) elastic 1D shot, avoiding non ideal conditions. A brief ending discussion can be performed: has everybody got a video of a good shot? If not, what was the problem?
During this lab session students are invited to “play out” [20] with physics, as they will settle knowledge via comparing videos with simulations [4,6] and having confrontation with their own and other groups.

Homework:
Students analyse the videos taken from a qualitative point of view. The teacher should provide them with some questions to answer to guide the analysis:

- What difficulties did you run into when trying to recreate the shots?
- Have you tried different points of view for your camera? Why or why not?
- If one ball was still before the collision, what did you expect would happen after the collision? Were your predictions correct?
- You may compare your predictions to PhET Colorado's Collision Lab [19] simulations. Do those agree?
- In a whole, do pool balls behave like those in theoretical problems and simulations or not? In which aspects behave similarly? In which differently?



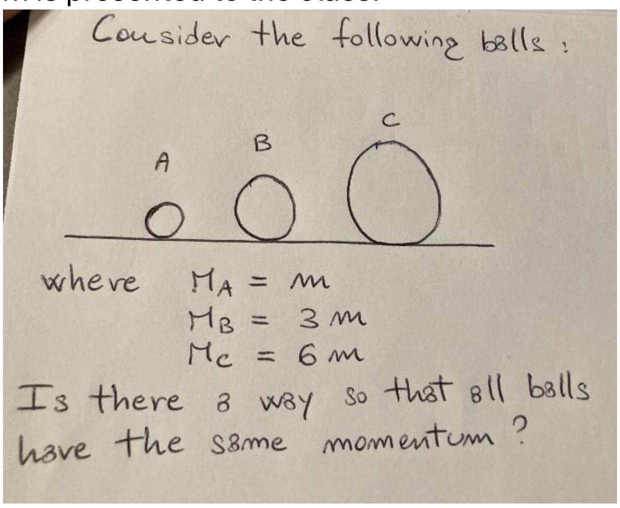
Answers to questions should be written by the group after having a discussion on the videos as a discursive draft of a lab report. The draft will be assessed and account for 20% of the final score, as outlined in the "Assesment" section.

Lesson 3: results discussion

During this lesson students submit their results to the class via a brief presentation of the videos and their answers to the questions. If a group succeeded in replicating exactly the experiments, they share what operative settings they have chosen and why.
Some elements of non-ideality may emerge from the discussion, such as:

- balls spinning [11]
- eccessive or too low friction [12]
- 2D shots [13], if the ball isn't hit on its baricenter

After the discussion some simulations with PhET collisions lab are run trying out different values for masses and speeds [5]. The class is now invited to reflect collectively on which elements can influence the real-life experiment and are not present in the simulations. After that the following problem is presented to the class:



And later the following question is posed:

Imagine a collision between A and B having the same momentum. How would it look like?

The problem is ment to be "solved" via brainstorming, in a progression of ideas towards abstraction. A numerical exercise on the same thought line is then given as homework, which will be assessed, as will be all "on paper" exercises proposed from now on. Assessment details are exposed in "Assessment" section.

Homework:

Consider the 3 balls of the previous exercise, being $m = 1{,}0$ kg. At the beginning, B and C are still, while A is moving towards B with $\vec{V}_A = 20{,}0$ m/s.

0) Draw a sketch of the problem before any collision occurs

1) What's the speed of B after colliding with A?
2) What's the speed of C after colliding with B?
3) Let $\Delta t_{AB} = 0{,}01$ s and $\Delta t_{BC} = 0{,}04$ s being collision times of A-B collision and B-C collision respectively. What's the value of the force released in each collision?

4) Draw a sketch of the problem after B has collided with C

Lesson 4: introducing kinetic energy's conservation law

In this lesson a new conservation law is introduced: the one relative to kinetic energy.
The focus is now on definition of kinetic energy: it's a scalar quantity and it's an energy, so it must not be represented in diagrams as a vector. Those concepts should be solidly acquired by the students.
The students are told that this conservation law works for specific types of collisions: those which do not deform the objects involved (particular focus is used at this point).

New formulas (lesson 4):

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

As done for momentum conservation, the teacher underlines how this formula works: it's the total kinetic energy that is conserved, not each object's energy. To give a real-life visual representation [6], two videos are shown to the students:

- A pool shot is performed [14]
- Two students are ice skating (from 1:06) [15]

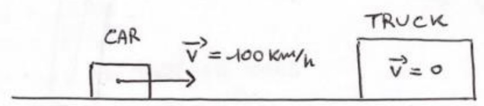
The students discuss on what "deform objects" means. In which video are the objects involved more deformed by the collision? They should grasp differences between elastic and inelastic collisions [8], without knowing their definitions.
One more video is shown to the students, involving a car crashing into a wall [16]. A class discussion is now opened with some reflection hint given by the teacher:

- In which videos can the formula for kinetic energy conservation be used? In which ones would lead to wrong results?
- Can you think about real life collisions in which the objects involved are not deformed?
- What about those in which the objects involved are deformed? What happens to the objects? Is there any perceptible change other than deformation?

The teacher guides students towards being aware that most real-life collisions do deform the objects involved, so the conservation of kinetic energy can't be applied, since some kinetic energy is lost as heat.

Homework:
A numerical problem on last lesson's wavelength is given as homework. This problem's solution will be assessed as specified in later sections.

Assume you are on the road and you are seeing the following scenario: a car is heading very fast towards a still truck.



Given the masses: $m_{CAR} = 1000\ \mathrm{kg}$, $m_{TRUCK} = 12\,000\ \mathrm{kg}$
calculate the speed of the car and the truck after the collision if:
1) The car bounces back (behaving like a pool ball)
2) The car and the truck merge together after crashing
Which scenario sounds more realistic to you? Why?
In fashion of the above one, draw a simplified schema of what you expect to happen after the collision in both 1) and 2)

Lesson 5 (lab session 2): collision experiment

For this lab session a Physics lab with the right equipment is needed. Material list:

- Metal track, better if scaled
- Standard carts and magnetic carts
- Cart masses (replacable with other items that fit into the cart and are easy to weigh)

If this material is not available this lab can be executed with PhET's simulations, with the right adjustments on Assessment.

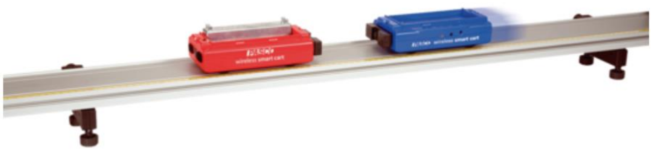
Students are divided into the same groups of the last lab and are asked to recreate the collisions they saw in the last lesson's videos. They have to record a video for two types of collisions: one in which the items are deformed and the other in which they are not. They are free to use the materials given as they wish, trying out different configurations and picking the ones that underline best what they want to show. They can also try out phyphox on their phone for getting data with carts.

LAB WORKSHEET: collision lab
Goals:

- Realize two different kinds of collision using the same items
- Compute the physical quantities useful to determine the output of the collision
- Explain why each collision behaves in its own manner

Materials:

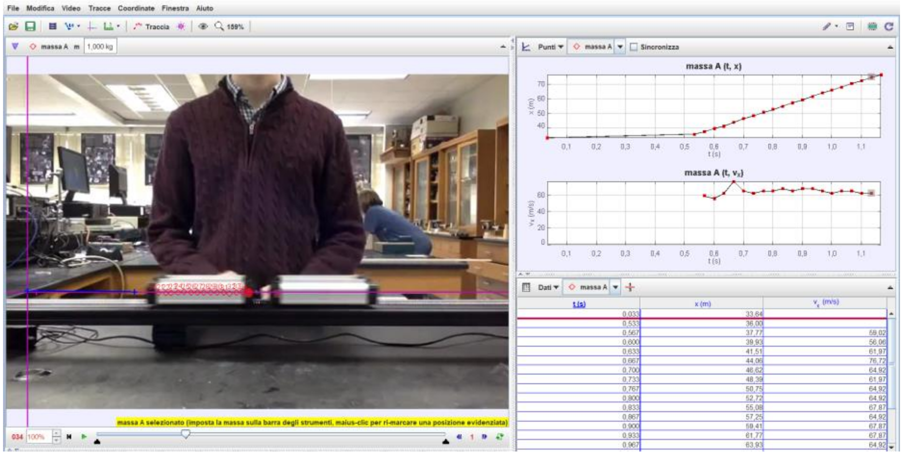
- Magnetic carts and tracks
- Phone to record video or with pyphox installed
- Masses or other objects to put onto carts



Procedure:

- Set up your collisions, deciding how to realize both types. You may choose to use standard carts or magnetic carts. Another idea could be using carts of different masses. Try out different configurations and pick those that in your opinion resemble best those two situations: the items are deformed by the collision and the items are not deformed by it.
- Take note of the objects' masses you are intending to use
- Draw out the schemas of the collisions on paper and try to determine what will happen after the carts collide.
- Try to run the collisions with the carts. Do you see what you expected?
- Acquire data for both types of collision. You have two options:

1. **Phyphox (difficult):** put phones with phyphox installed onto both carts and run the collision you have planned. Run the accelerometer on both devices and acquire the data of your collision. What do you see in the graphs? How can you compute the momenta? It could be useful to take a video of the collision in case you find it hard to analyze the graphs.



2. **Tracker (less difficult) [17]**: take a video recording of the collision you have planned. Measure an object that appears in the video and take note of its length, as it will be used as reference. Can you distinguish well the objects colliding in the video? Is the camera well alligned with the track? What's the problem if it is not?
   In both cases: make sure to have backup videos to analyze if you have trouble with the first take.

- Can you get the collisions to work as you intended? If not, what could be the reason?

Getting the data and analysis:

- Compute the momentum of both carts before the collision. You may do this up to 3-4 times using different frames of the video. What do you expect to get?
- Compute the momenta of both carts after the collision. Do you get the same results in both collision types?
- Using the same data, calculate the kinetic energy before and after the impact. What do you observe?

Final considerations:

- Is any physical quantity conserved before and after the collisions? Why or why not?
- Can you use the same procedure to analyze the collision between objects different than carts? Which objects would you use? What do you expect to see?

Homework:
Students have to analyze both videos getting the data from Tracker or phyphox and answer the following questions, in addition to those proposed in the **Procedure** section, writing a roughly precise lab report that will be assessed and account for 60% of the final score:

- What are the speeds of the carts before and after the collisions?
- What are the differences between the videos? Which physical quantities do remain the same after the collision? Which do not?
- How was your experience in the lab? Was it easier or harder to execute collisions compared to lab 1? Why?

Lesson 6: final recap

As the final lesson of the unit, a recap of the concepts and the formulas covered is done. By this time students have already delivered their "on paper" problems solutions to the teacher, so that the teacher can show their solutions step-by-step, explaining what lines of thought are used to solve the problems, and what to expect in real life when experiencing situations such as those proposed in the problems. This lesson aims to consolidate physical concepts in students' minds, beyond clarifying what elements can "ruin" ideality. Finally, a definition of elastic and inelastic collision is given to the students, and a connection between definitions and what seen in previous lectures is built: no real collision is perfectly elastic, but some almost are. A final video, showing elastic and inelastic collisions [18] with carts, is shown at the end.

Assessment

As previously outlined in the lessons schedule, the drafts lab reports made after lab session 1 are evaluated using the following rubrics (in fashion of [7, 21], given the relative guidelines [1]) and account for 20% of the final score. Rubric A is going to be applied for assessing lab 1, as it is focused on identifying and executing an experiment in a qualitative, meaningful way. Rubric A, B and C are going to be applied for assessing lab 2, as the students handle different instruments, numeric data and build an experiment to achieve a given goal. Homework done after lab 2 will account for 60% of the final score. In the end, "on paper" homework problems will be addressed according to Rubric D, re-adapting a general scoring guide used for undegraduate [22] and will account for 20% of the final score.

Rubric A: ability to design and conduct a testing experiment				
Scientific ability	0 (Missing)	1 (Inadequate)	2 (Needs improvement)	3 (Adequate)
1) Is able to identify the relationship or explanation to be tested	No mention is made of a realtionship or explanation	An attempt is made to identify the relationship or explanation to be tested but is described in a confusing manner	The relationship or explanation to be tested is described but there are minor omissions or vague details	The relationship or explanation is clearly stated
2) Is able to decide whether or not to confirm the prediction based on the results of the experiment	No decision is made to confirm or disconfirm the prediction	A decision is made but it is not strongly based on the results of the experiment	A decision is made based on the results of the experiment, but the reasoning is flawed	A correct decision is made and is based on the results of the experiment
3) Is able to make a reasonable judgment about the relationship or explanation	No judgment is made about the relationship or explanation, or is not based on the results	A judgment is made but it is based only on the degree of agreement between the results and the prediction	A judgment is made based on the reliability of the experiment and the degree of agreement between the results and the prediction, but the reasoning is flawed	A reasonable judgment is made based on the reliability of the experiment and the degree of agreement between the results and prediction

Rubric B: ability to construct, modify and apply relationships or explanations				
4) Is able to make a reasonable prediction based on a relationship or explanation	No attempt to make a prediction is made. The experiment is not treated as a testing experiment	A prediction is made but it does not follow from the relationship or explanation being teste, or it ignores or contradicts some of the assumptions inherent in the relationship or explanation	A predictionis made that follows from the relationship or explanation, but it does not incorporate the assumptions	A prediction is made that follows from the relationship or explanation and incorporates the assumptions
5) Is able to identify the assumptions made in making the prediction	No attempt is made to identify any assumptions	An attempt is made to identify assumptions, but most are missing, described vaguely, or incorrect	Most assumptions are correctly identified	All assumptions are correctly identified

Rubric C: ability to revise explanations and evaluate results				
6) Is able to revise the explanation of a prediction, based on the results of an experiment	No attempt is made to explain the outcome of the experiment, to revise the previous explanation or assumptions. The difference between the prediction and the outcome of the experiment is not addressed	An attempt is made to explain the outcome and revise the previous explanation or assumptions, but is mostly incomplete and/or based on incorrect reasoning	The revision of the previous explanation or assumptions is partially complete and correct, yet still lacking in some relevant details	The revision of the explanation or assumptions is explained completely and correctly
7) Is able to analyze a relevant special case for a given model, equation or claim	No meaningful attempt is made to analyze a relevant special case	An attempt is made to analyze a special case, but for the identified special case it is not relevant or major steps are missing from the analysis	An attempt is made to analyze a relevant special case, but the student's analysis is flawed, or the student's judgment is inconsistent with their analysis	A relevant special case is correctly analyzed, and a proper judgment is made

Rubric D: ability to solve theoretical problems				
8) Is able to specificly apply Physics	The solution does not indicate an application of Physics and it is necessary	An attempt is made to apply specific Physics, but most of the application is missing or contains errors	The application of specific Physics is mostly correct, but contains some minor omissions or errors	The application of specific Physics is appropriate and complete
9) Is able to make and apply correct logical progression	No logical progression is present and it is necessary	The logical progression proposed is confused, unclear and/or contains major errors	The logical progression proposed is mostly clear and focused with minor inconsistencies	The entire problem solution is clear, focused and logically connected
10) Is able to draw a correct and complete sketch of the problem	No sketch or representation is proposed	A sketch is drawn but most relevant physical quantities are missing or not labeled, or it contains wrong information. Coordinate axes are missing	The sketch proposed contains no incorrect information, but some important information is missing. Most of the items are correctly drawn	The sketch proposed contains all key items. Axes are correctly drawn. All relevant quantities have consistent substripts

Additional comments

This project was structured taking into consideration findings from PER on common students' misconceptions about collisions [6] and difficulties about momentum and kinetic energy concepts and theorems [3, 8, 9]:

- Distinguish scalar quantities from vectorial ones
- Correlate physical quantities to all elements that involve their computation (e.g. understanding that two objects with different masses can have the same momentum)
- Discern elastic from inelastic collisions and recognize their characteristics
- Determine which quantities are conserved in a collision and why

As pointed out in the papers, first person experience in labs may help students create knowledge and awareness on the Physics topics covered here, which in most cases are confusing and usually remain an ensemble of "plug and chug" formulas, without any applicability in real life. This didactic unit is primarily focused on building students' consciousness that Physics really describe phenomena, and it is not just an abstraction hoc-made to get an expected result.

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