Reduced Fluid-Structure Interaction and non-Newtonian models of blood flows for simulating the aortic valve

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INTRODUCTION

- Clinical motivations
- Goal of the thesis

MATHEMATICAL MODELS

- Reduced FSI model
- Non-Newtonian model
- Numerical method

NUMERICAL RESULTS

- Numerical setting
- Healthy valve
- Pathological cases
- Newtonian vs non-Newtonian

CONCLUSIONS

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Introduction

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Clinical motivations

World Health Organization

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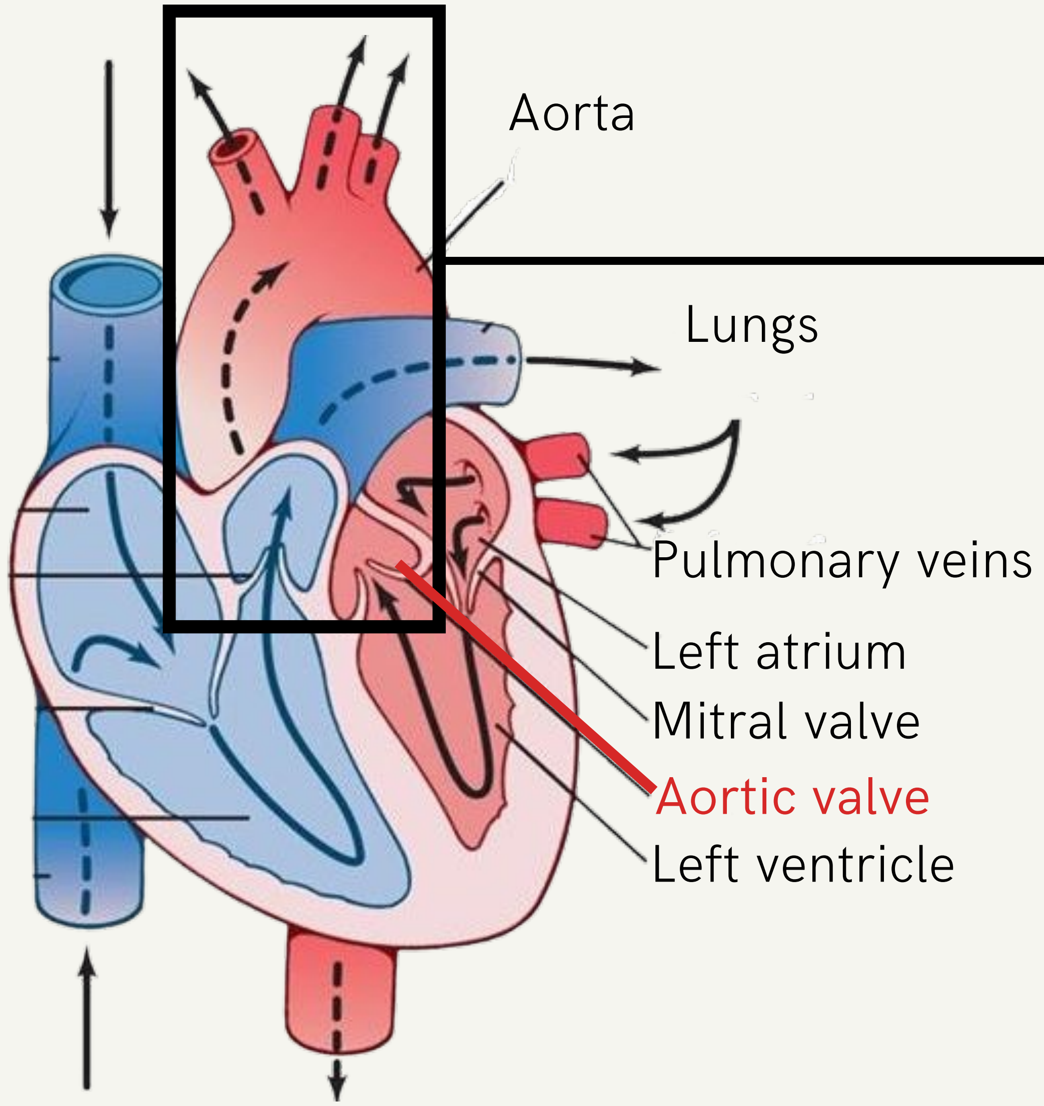
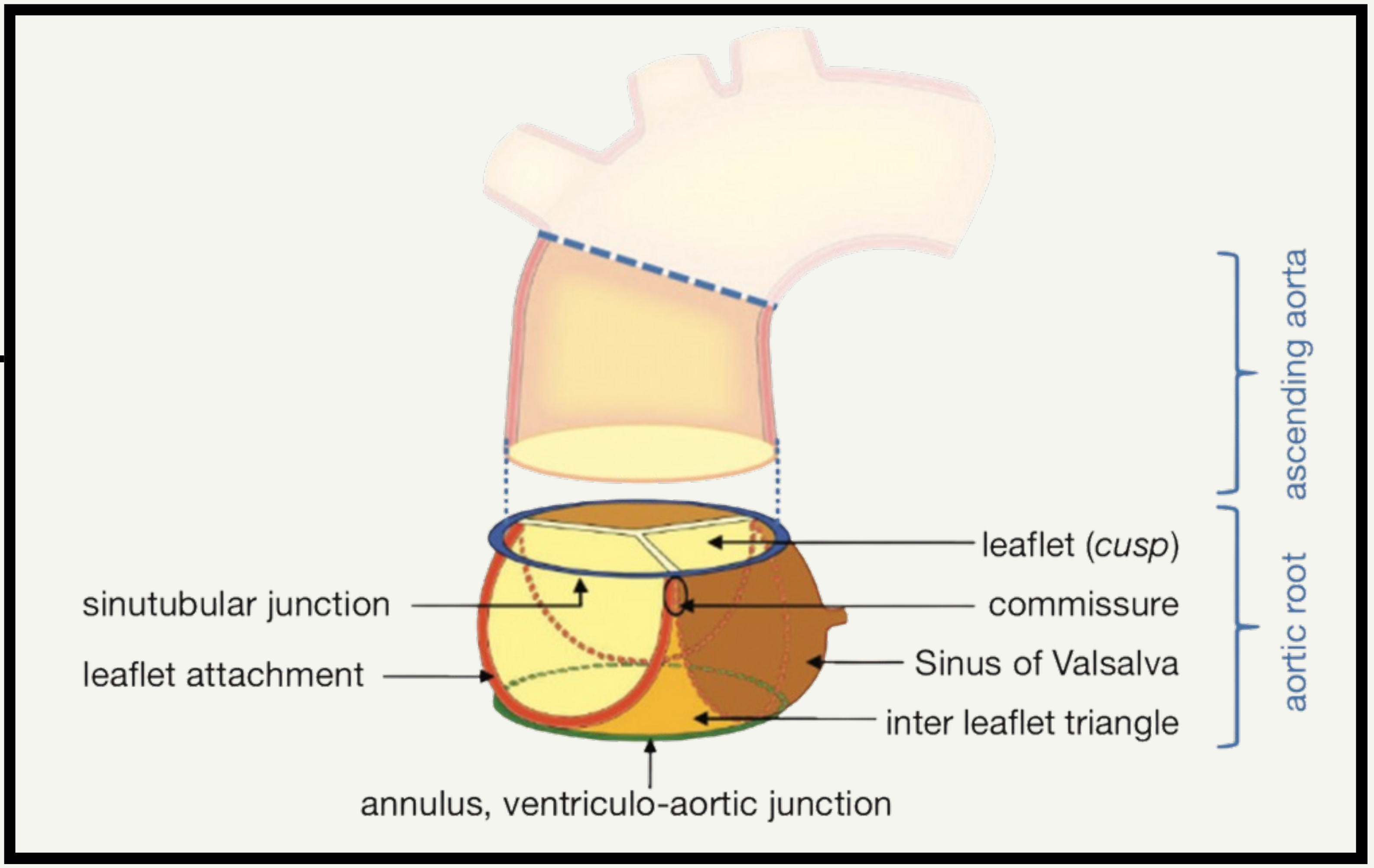


Hall, 2015



Charitos and Sievers, *Annals of cardiothoracic surgery*, 2015

Goal of the thesis

Fluid-Structure Interaction problem

REDUCED VALVE MODEL

- Increased **computational performance**
- **Physical meaning** for the parameters

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CALIBRATE THE PARAMETERS TO REPRODUCE DIFFERENT PHYSIOLOGICAL SCENARIOS

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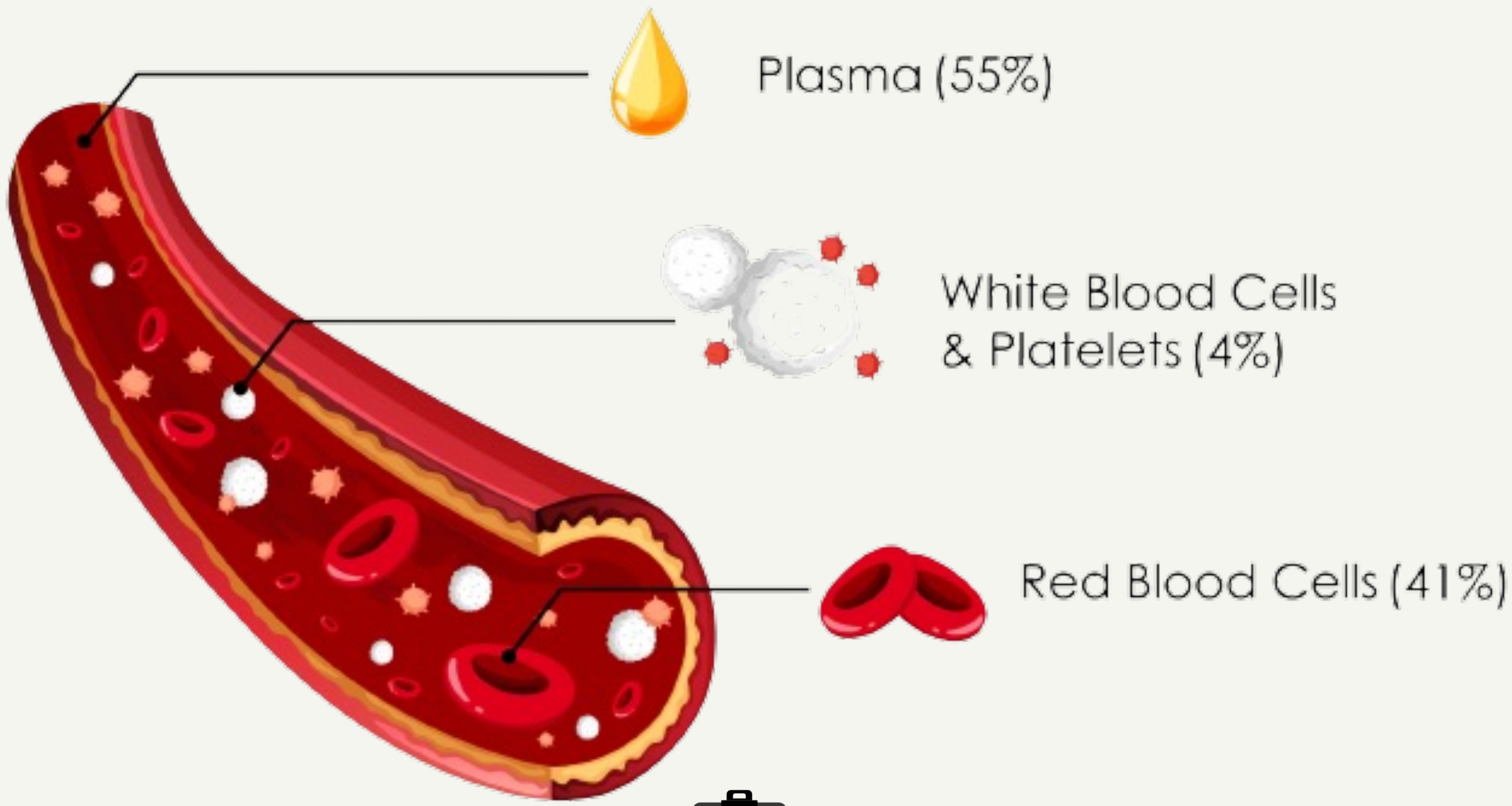
Study of blood's rheology

NON-NEWTONIAN MODEL



https://it.depositphotos.com

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- **Implementation** of the model
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UNDERSTAND THE EFFECTS OF NON-NEWTONIAN RHEOLOGY IN PROXIMITY OF THE AORTIC VALVE

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Mathematical models and methods

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RIIS method

$$\boxed{\Gamma_t = \{\mathbf{x} \in \Omega : \varphi_t(\mathbf{x}) = \mathbf{0}\}}$$

Find the velocity $\boldsymbol{u}$ and the pressure p such that:

$$\begin{cases} \rho \dfrac{\partial \boldsymbol{u}}{\partial t} - \nabla \cdot \sigma + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} & \\ \quad + \nabla p + \boxed{\delta_{t,\varepsilon} \dfrac{R}{\varepsilon} (\boldsymbol{u} - \boldsymbol{u}_\Gamma)} = \boldsymbol{f} & \text{in } \Omega \times (t_0, T)\,, \\ \nabla \cdot \boldsymbol{u} = 0 & \text{in } \Omega \times (t_0, T)\,, \\ \mathbf{u} = \mathbf{0} & \text{on } \Sigma_{\text{wall}} \times (t_0, T)\,, \\ \sigma \mathbf{n} = p_{\text{in}} \mathbf{n} & \text{on } \Sigma_{\text{in}} \times (t_0, T)\,, \\ \sigma \mathbf{n} = p_{\text{out}} \mathbf{n} & \text{on } \Sigma_{\text{out}} \times (t_0, T)\,, \\ \mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}) & \text{in } \Omega \times \{t_0\} \end{cases}$$

Fedele et al, *Biomechanics and modeling in mechanobiology*, 2015

- **Immersed implicit** surface
- **Weak** imposition of **valve's velocity**

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Stress tensor

RIIS method

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Find the velocity $\boldsymbol{u}$ and the pressure p such that:

Smoothed Dirac delta

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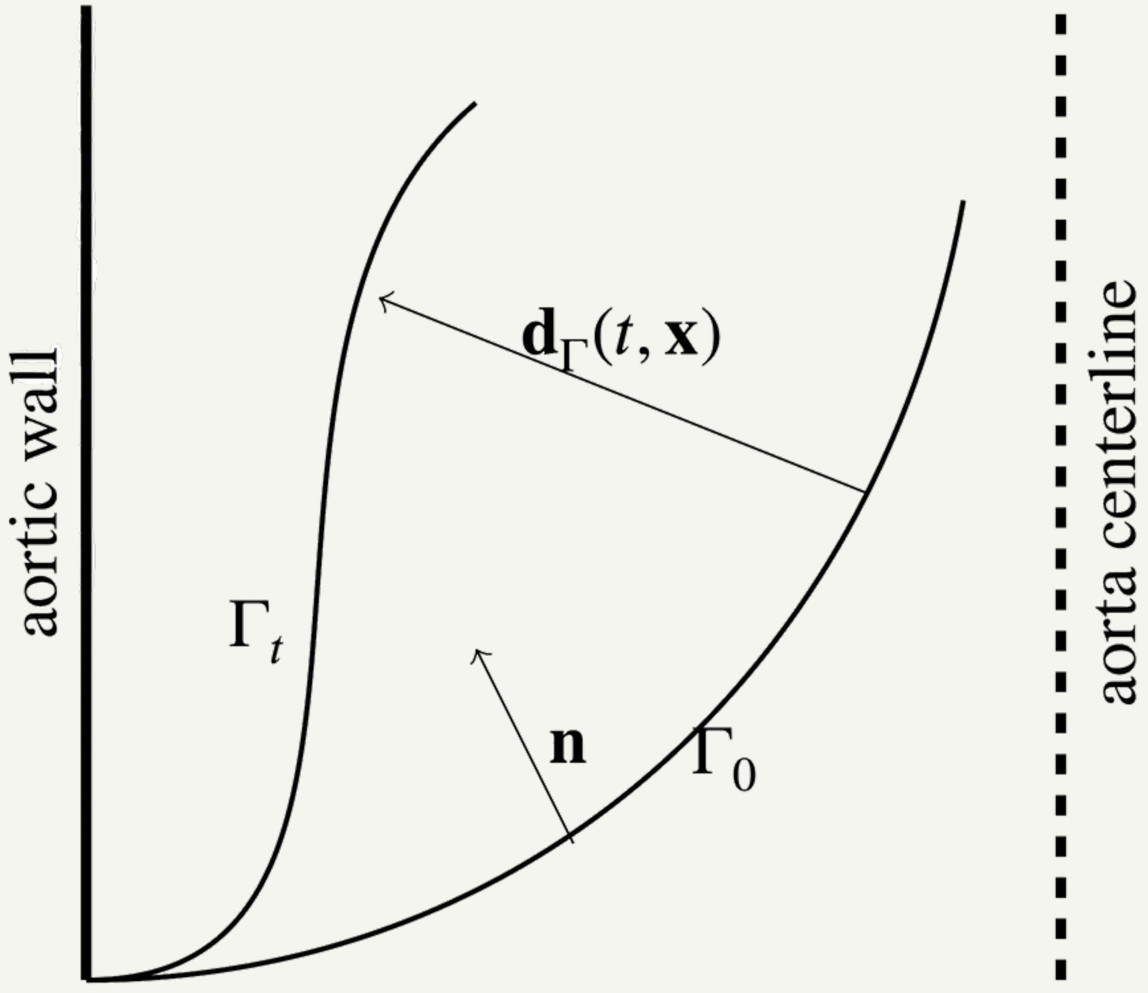
Leaflets' velocity

Reduced structure model

Fumagalli, *arXiv*, 2021



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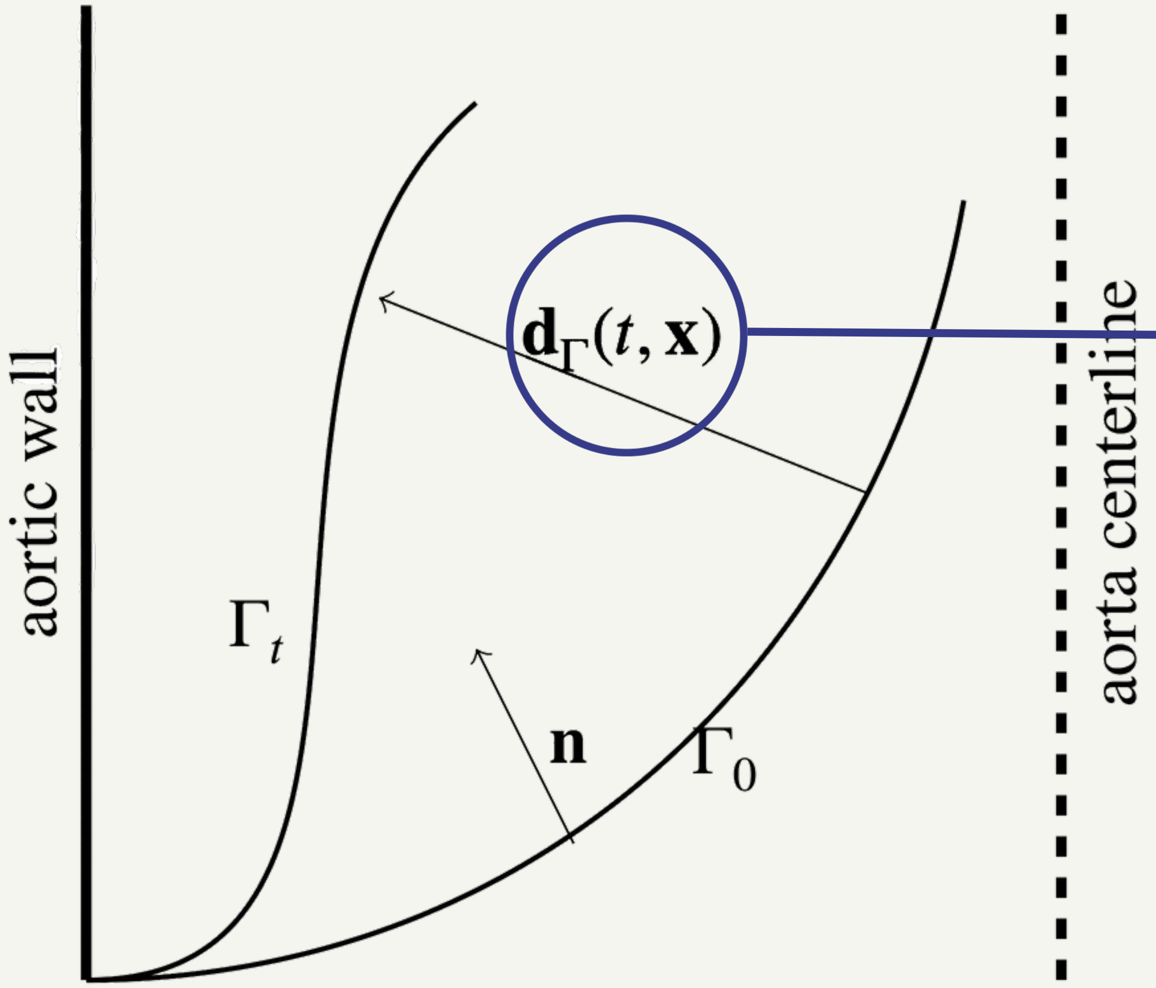
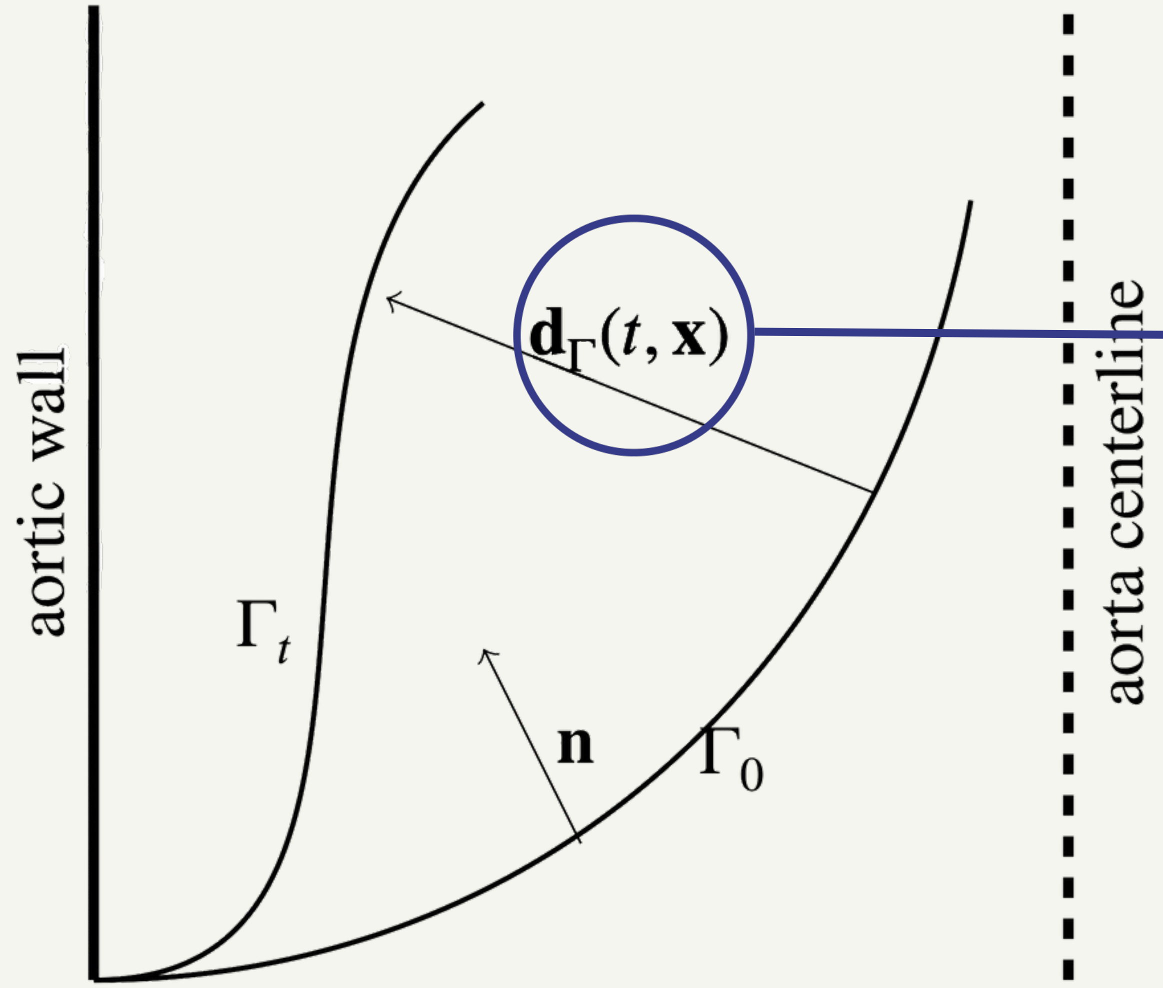


$$\mathbf{d}_\Gamma(t, \widehat{\mathbf{x}}) = c(t)\mathbf{g}(\widehat{\mathbf{x}})$$

Separation of variables

Reduced structure model

Fumagalli, *arXiv*, 2021



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$$\ddot{c} + \beta\dot{c} + \eta(c,\mathbf{f}) = 0$$

LOCAL FORCE BALANCE ON THE LEAFLETS

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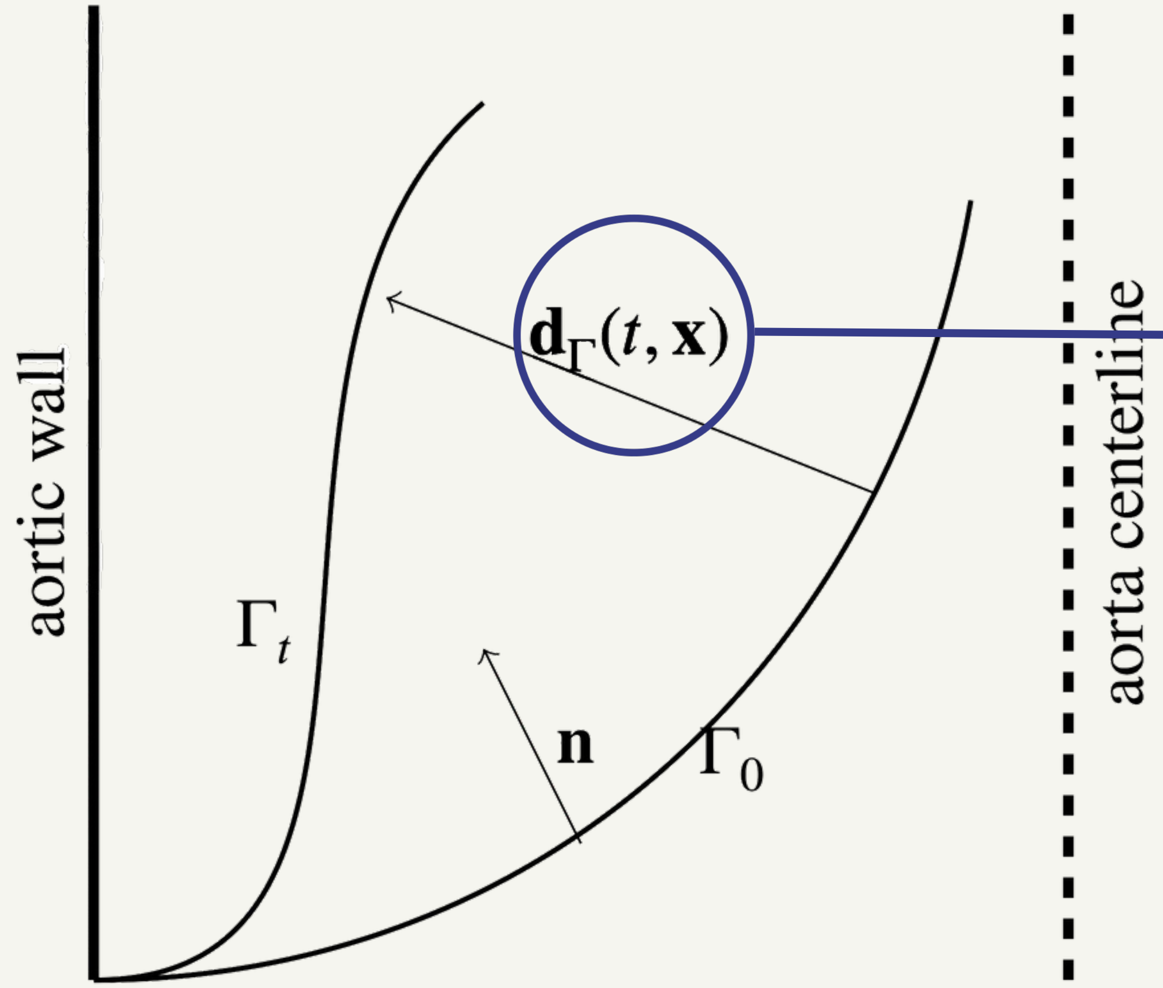

$$\mathbf{d}_\Gamma(t, \widehat{\mathbf{x}}) = c(t)\mathbf{g}(\widehat{\mathbf{x}})$$

Separation of variables

FORCE EXERTED ON THE VALVE DUE TO THE SURROUNDING FLUID

$$\ddot{c} + \beta\dot{c} + \eta(c, \mathbf{f}) = 0$$

LOCAL FORCE BALANCE ON THE LEAFLETS

Non-Newtonian model

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\mu(\dot{\gamma})\varepsilon(\boldsymbol{u})$$

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Non-Newtonian model

$$\dot{\gamma} = \sqrt{2\,\mathrm{tr}\left(\varepsilon(\boldsymbol{u})^2\right)}$$

SHEAR RATE

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\mu(\dot{\gamma})\varepsilon(\boldsymbol{u})$$

Non-Newtonian model

$$\dot{\gamma} = \sqrt{2\,\mathrm{tr}\left(\varepsilon(\boldsymbol{u})^2\right)}$$

SHEAR RATE

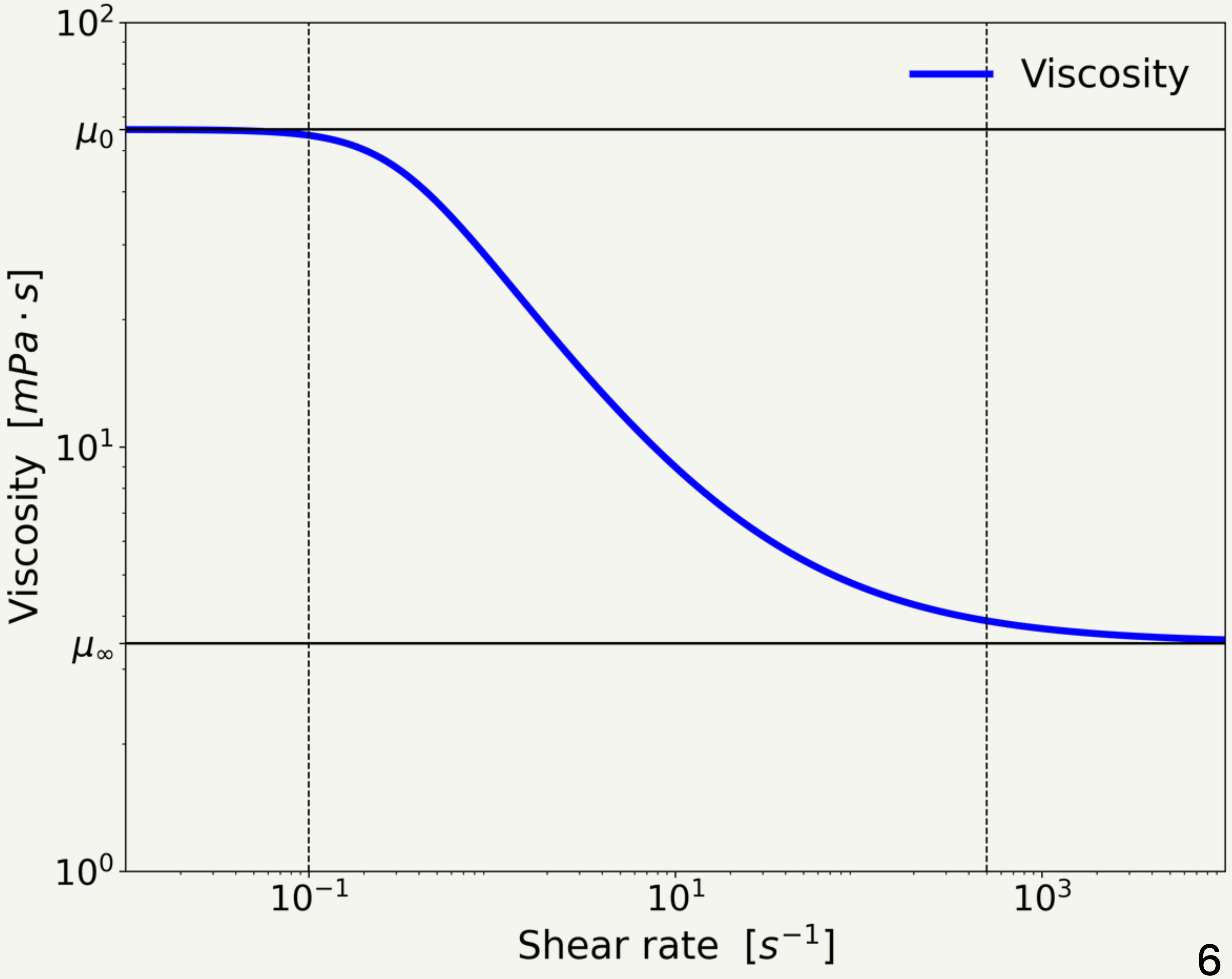
$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\mu(\dot{\gamma})\varepsilon(\boldsymbol{u})$$

$$\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty)\left[1 + (\lambda\dot{\gamma})^2\right]^{\frac{n-1}{2}}$$

CARREAU MODEL

$\mu_\infty\ [Pa \cdot s]$	$\mu_0\ [Pa \cdot s]$	$n\ [\cdot]$	$\lambda\ [s]$
3.45×10^{-3}	5.6×10^{-2}	0.3568	3.313

Cho and Kensey, *Biorheology*, 1991



Numerical methods

FLUID

- **Semi-implicit** Euler scheme
- **$\mathbb{P}1$-$\mathbb{P}1$** velocity-pressure finite elements
- **SUPG-PSPG** stabilization
- **Semi-implicit** treatment of the non-linearity introduced by the **non-Newtonian model**

VALVE

- **Explicit Runge-Kutta** of the fourth order for the 0D equation

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FSI: SEGREGATED EXPLICIT COUPLING

Numerical methods

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FSI: SEGREGATED EXPLICIT COUPLING

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Numerical results

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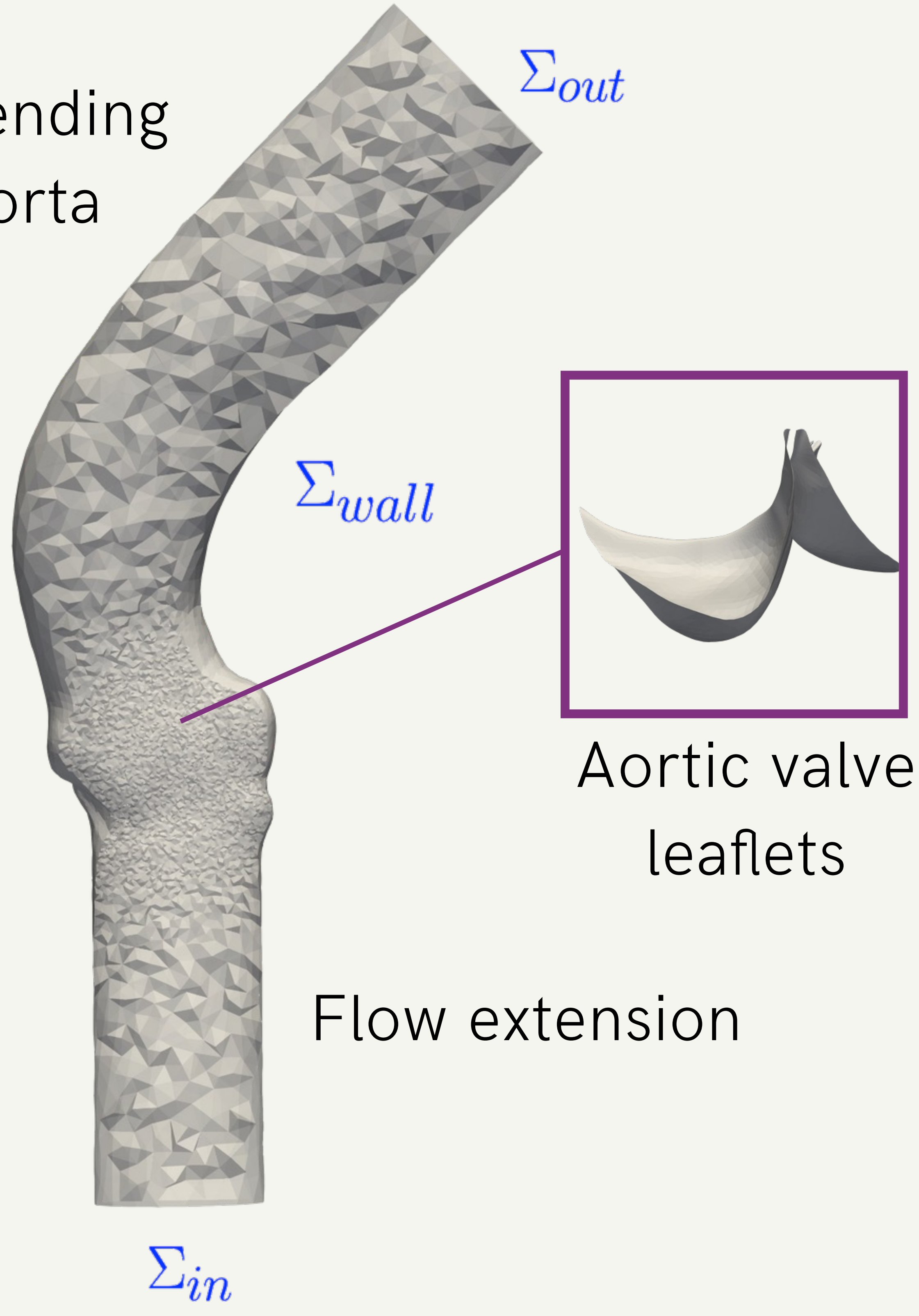
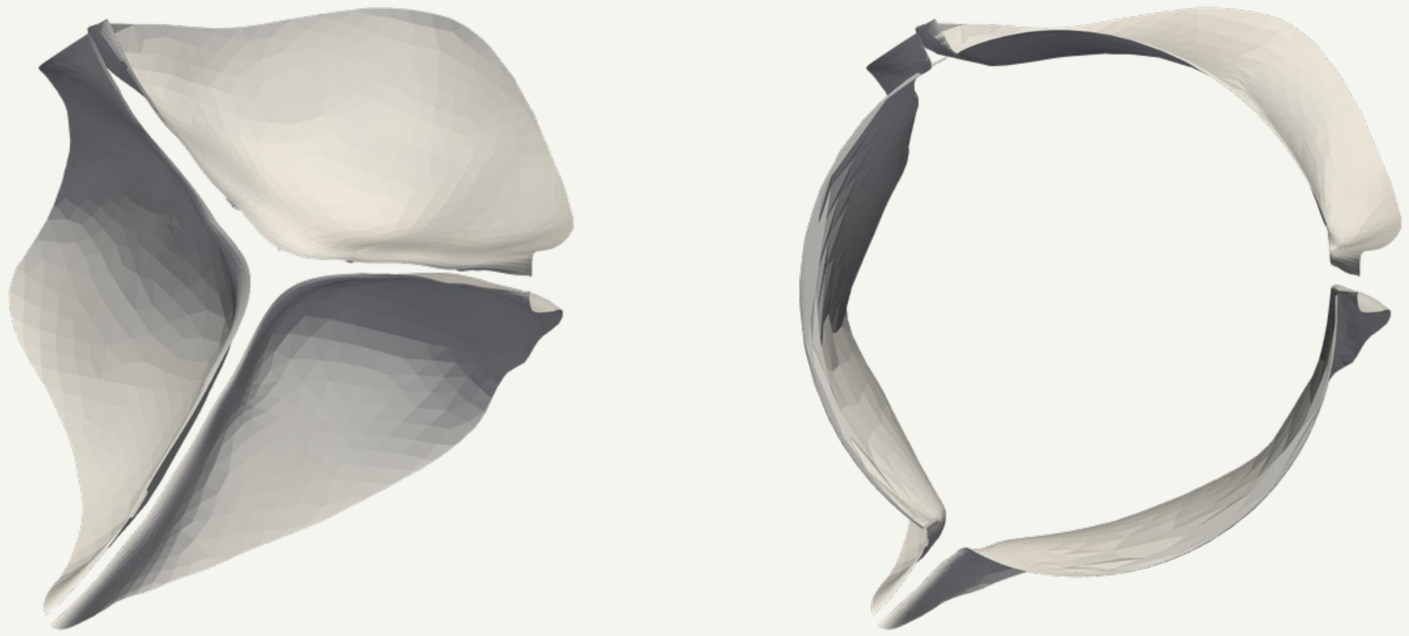
Numerical setting

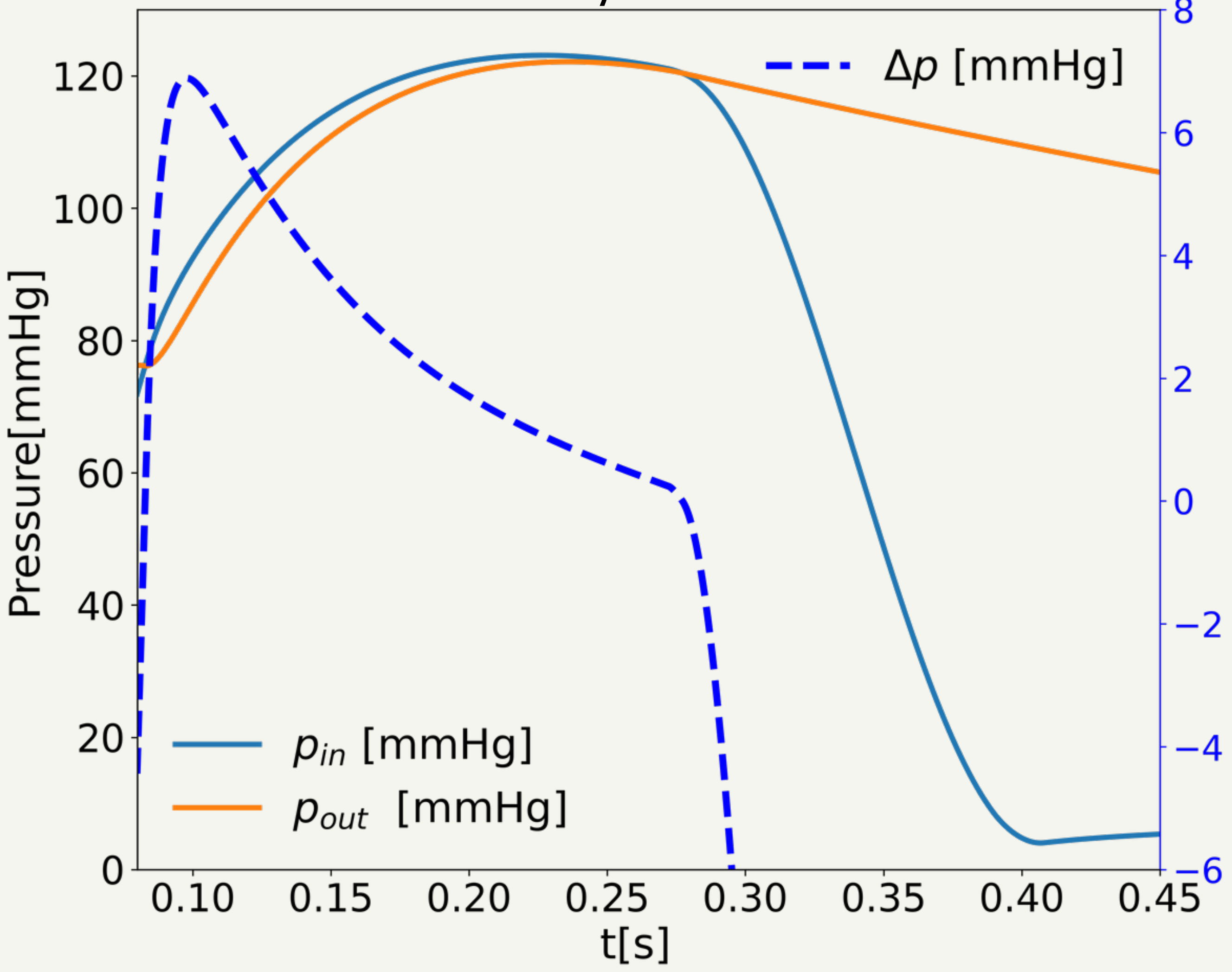


Closed configuration

Open configuration



Boundary conditions



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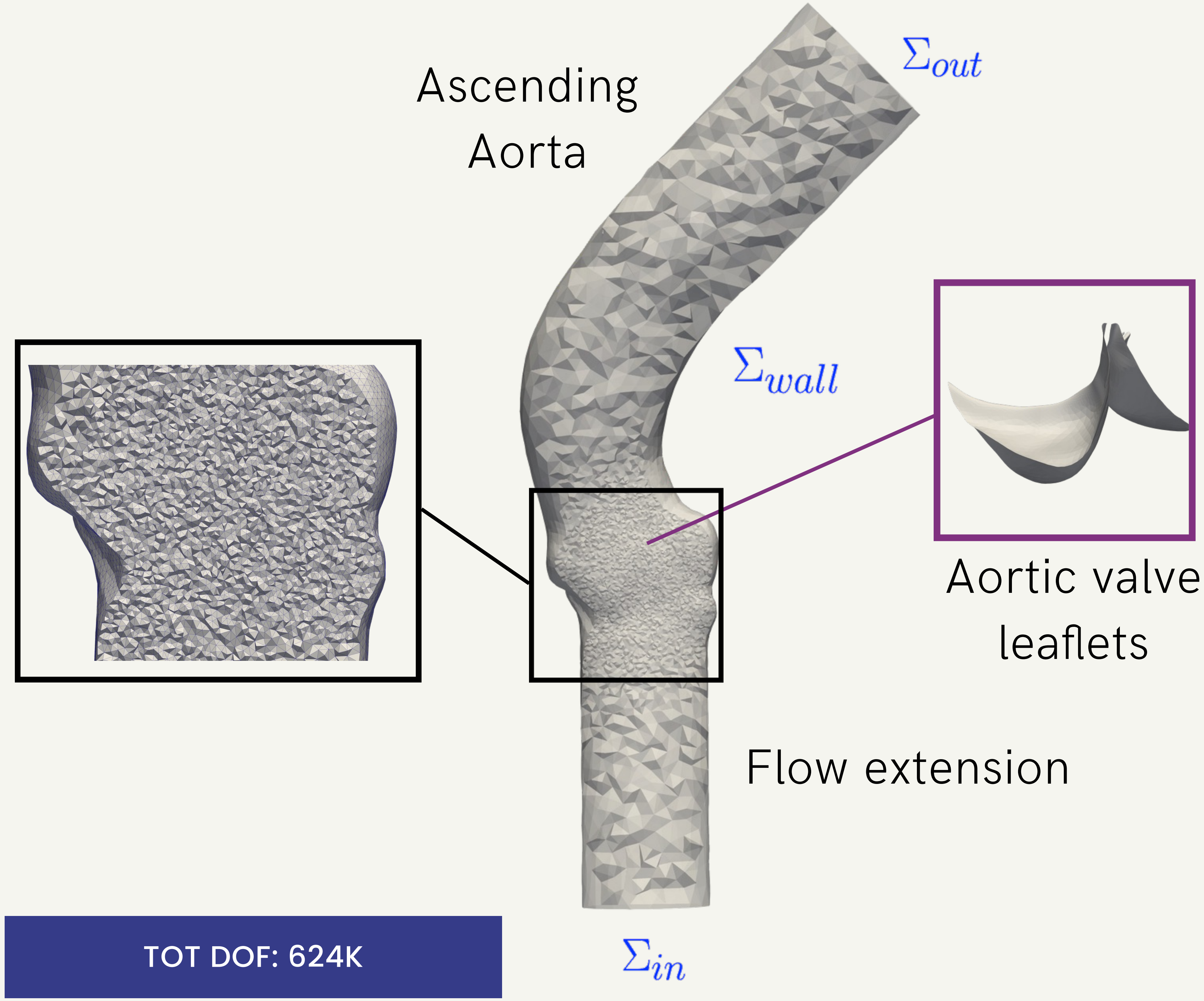
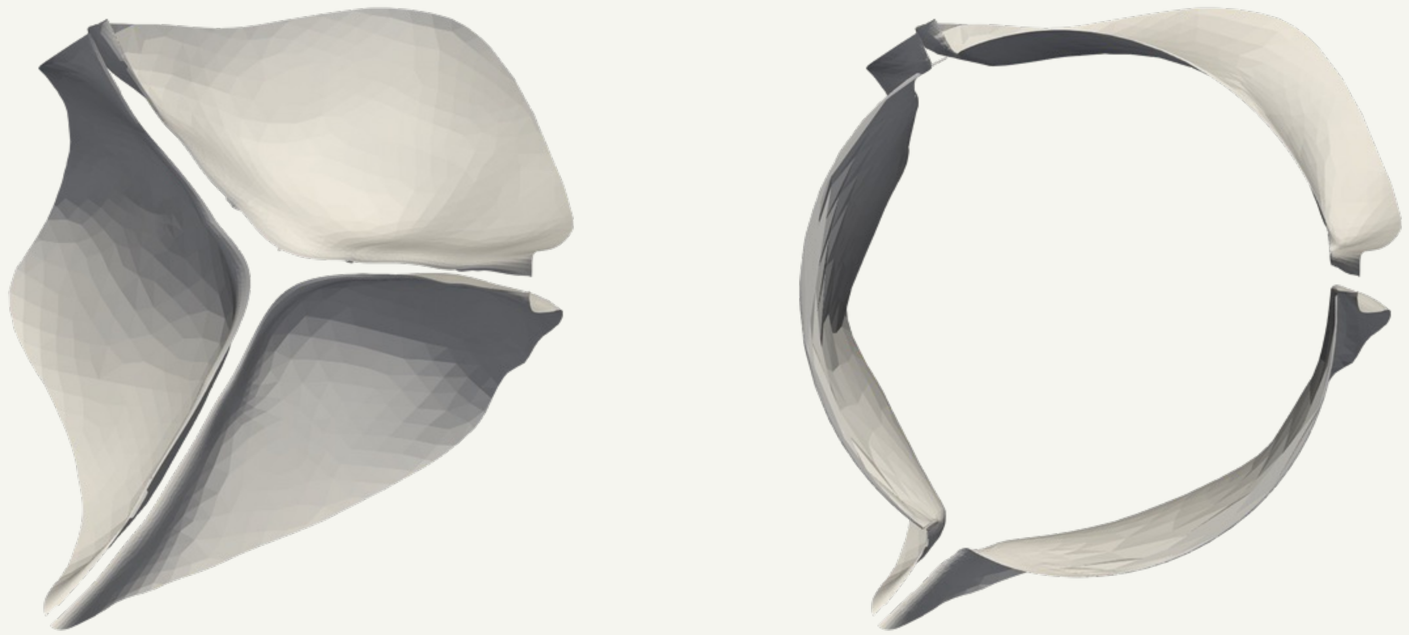


TOT DOF: 624K

Closed configuration

Open configuration



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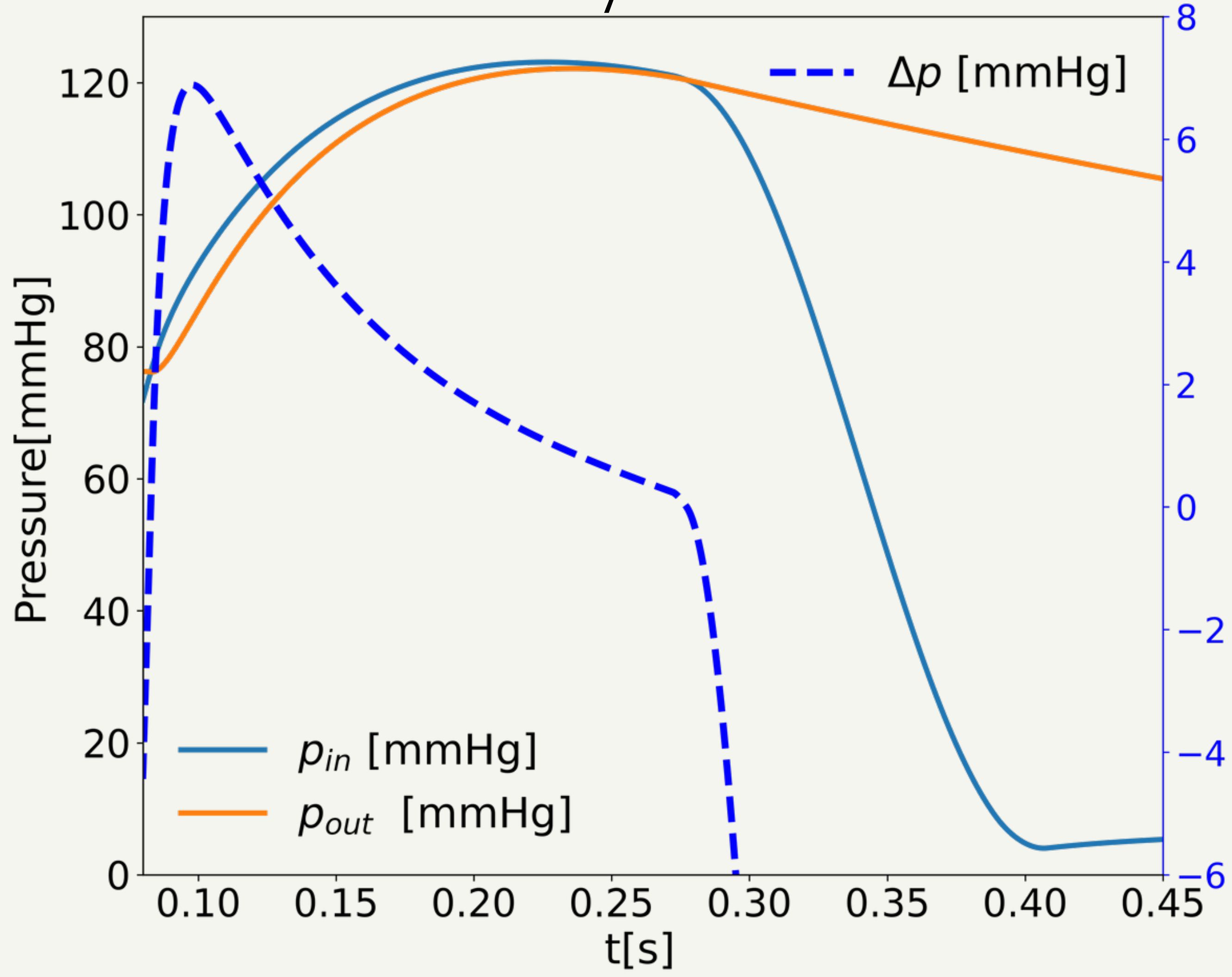
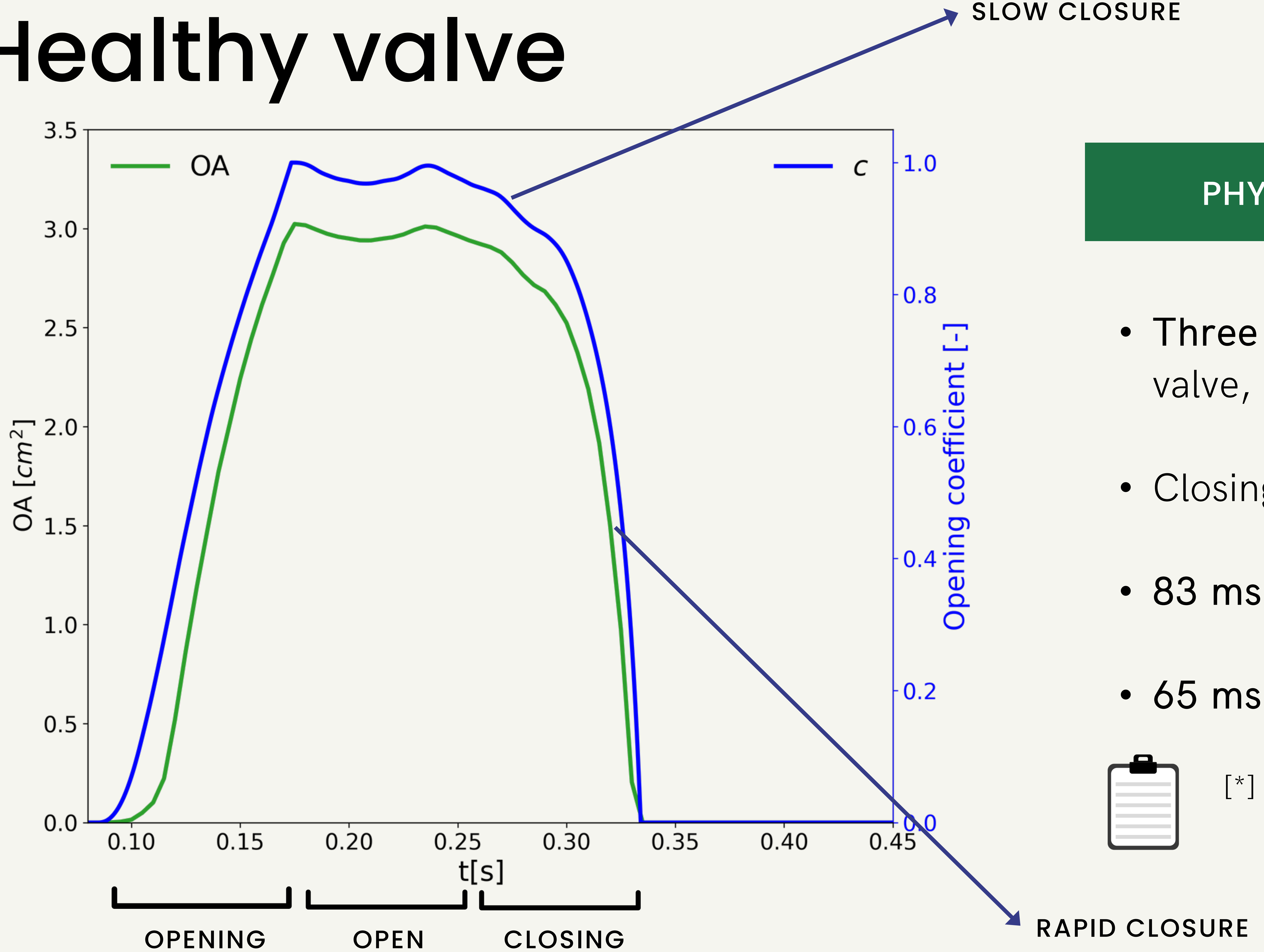


Healthy valve



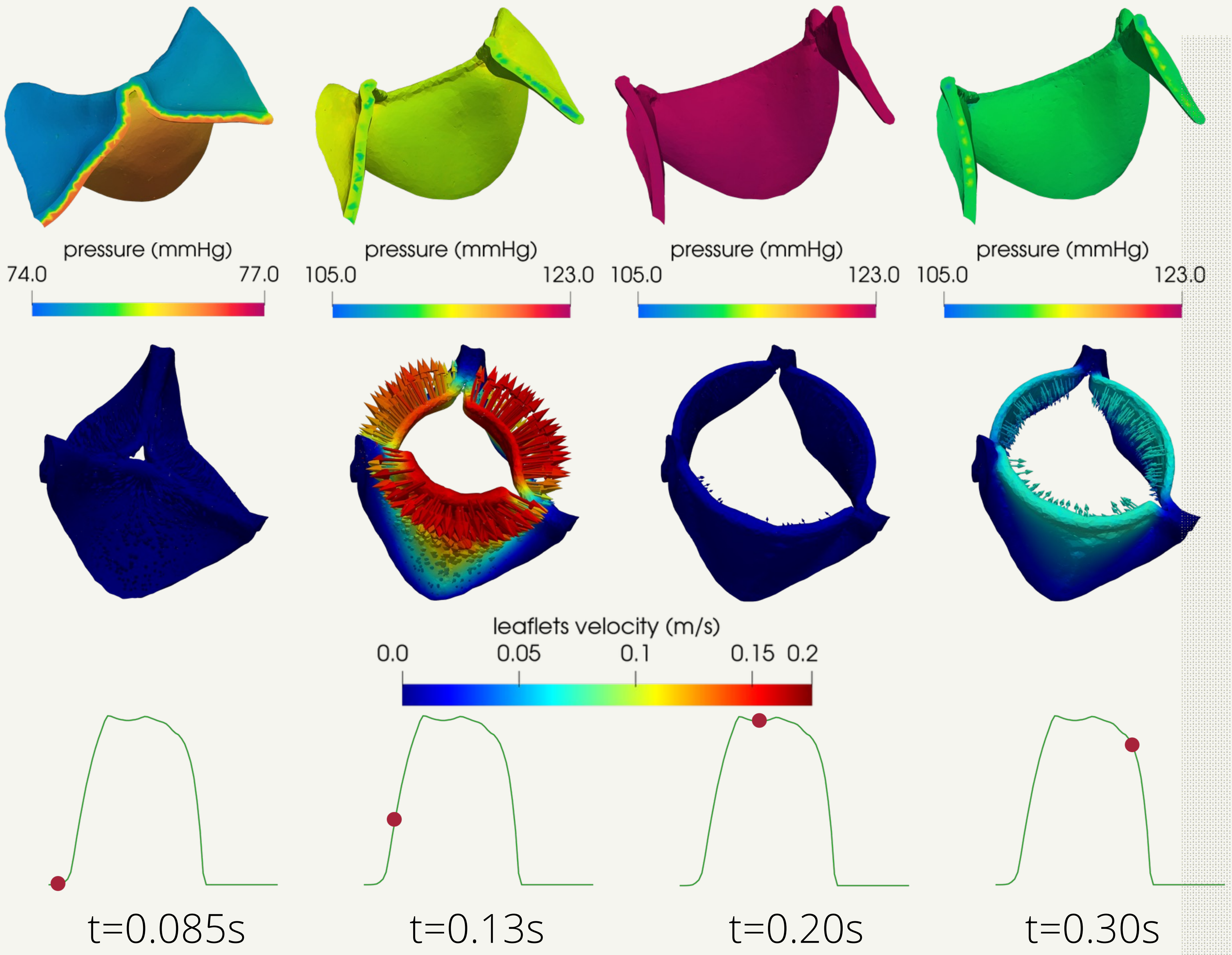
PHYSIOLOGICAL PARAMETERS

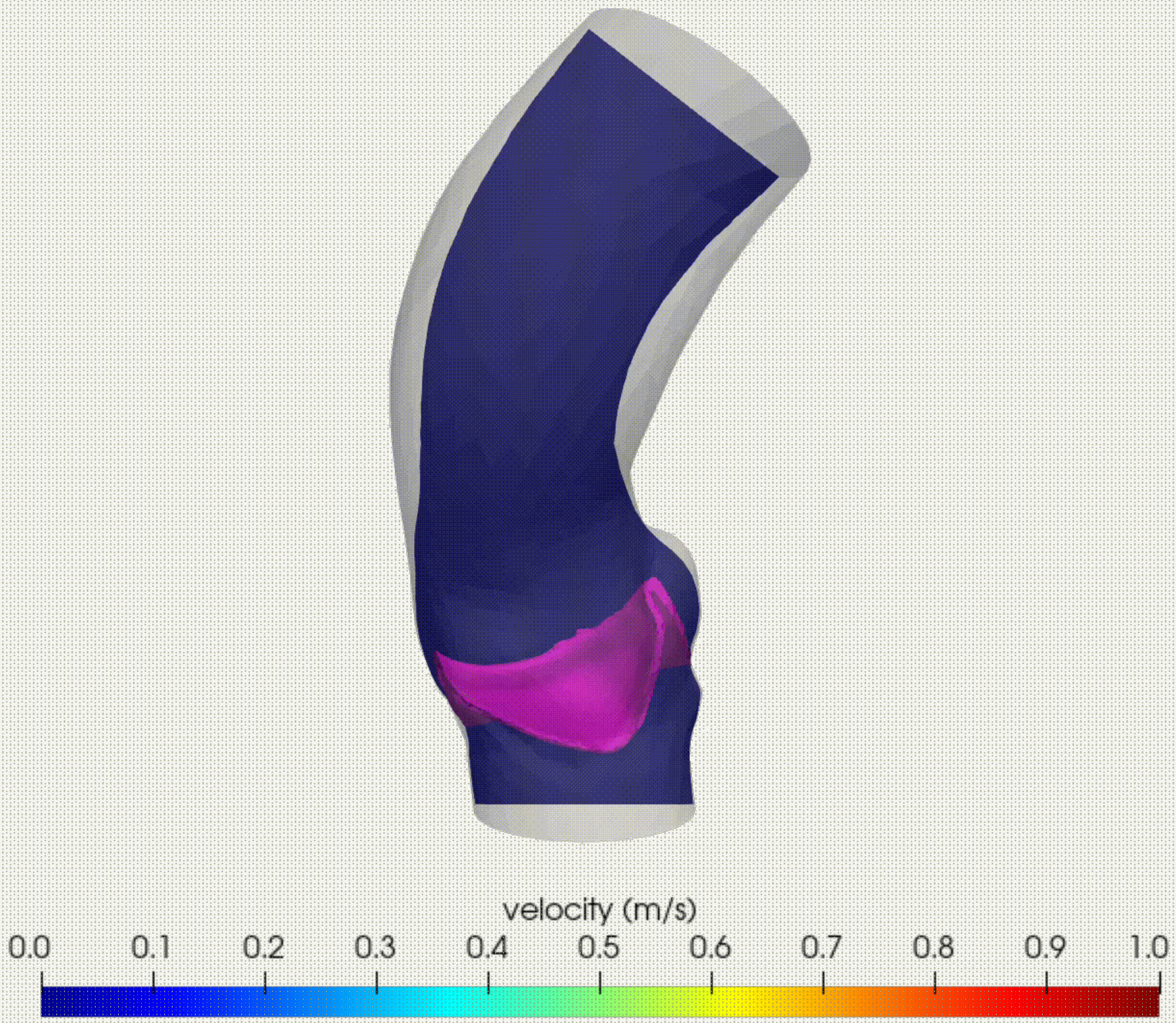
- **Three stages**: opening phase, open valve, closing phase
- Closing phase: **slow** and **rapid** phase
- **83 ms** opening phase (76±30 ms [*])
- **65 ms** closing phase (42±16 ms [*])

[*] Handke et al, *The Journal of Thoracic and Cardiovascular Surgery*, 2003

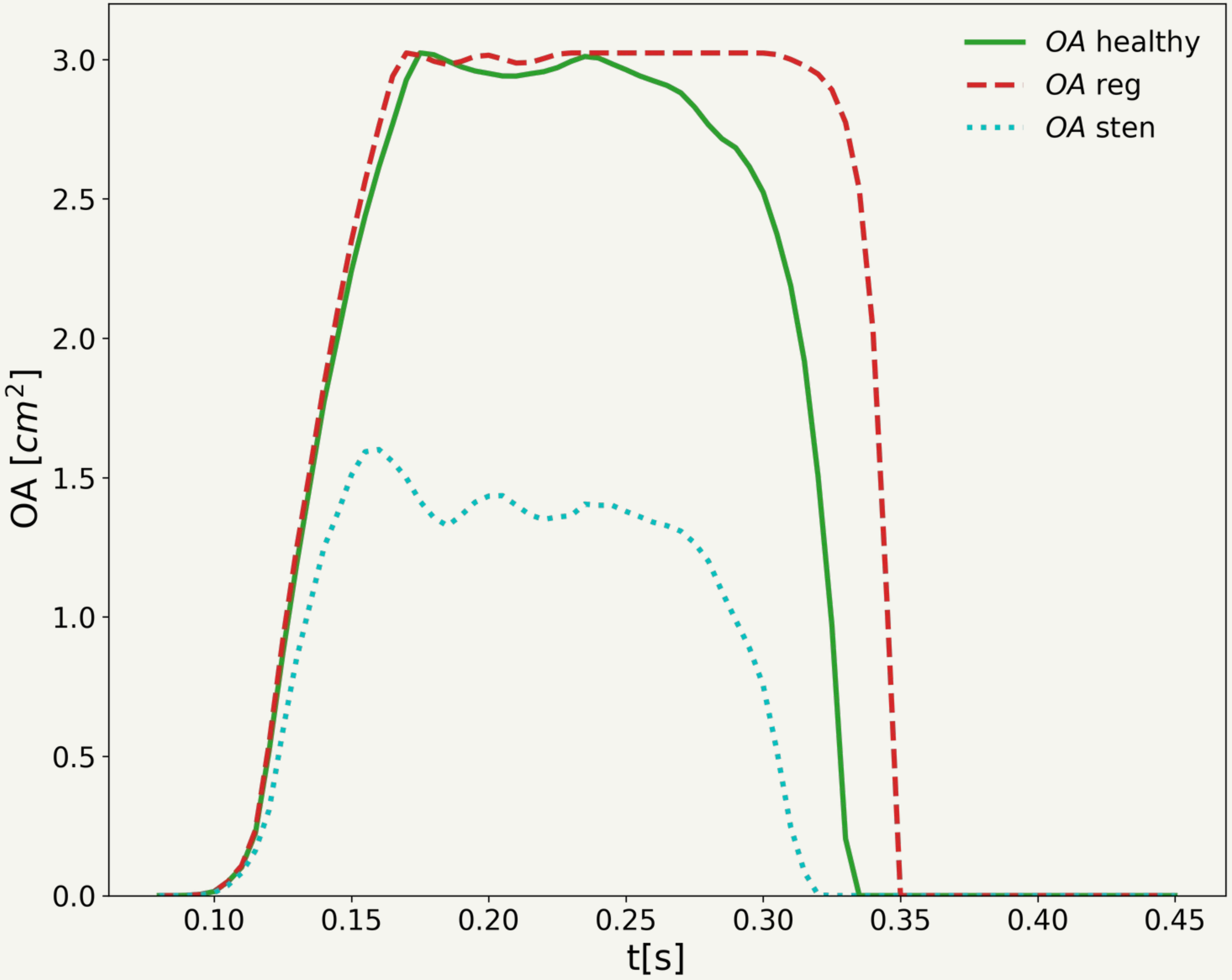
Healthy valve

- Valve is closed until **3 mmHg** of transvalvular **pressure drop** develops
- During the opening **progressive** development of **aortic jet flow**





Pathological cases



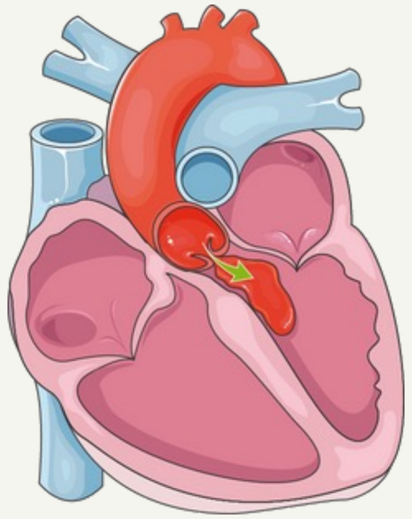
STENOTIC CASE

Narrowing of the **orifice** that limits the **anterograde flow** through the valve

https://www.heart-valve-surgery.com

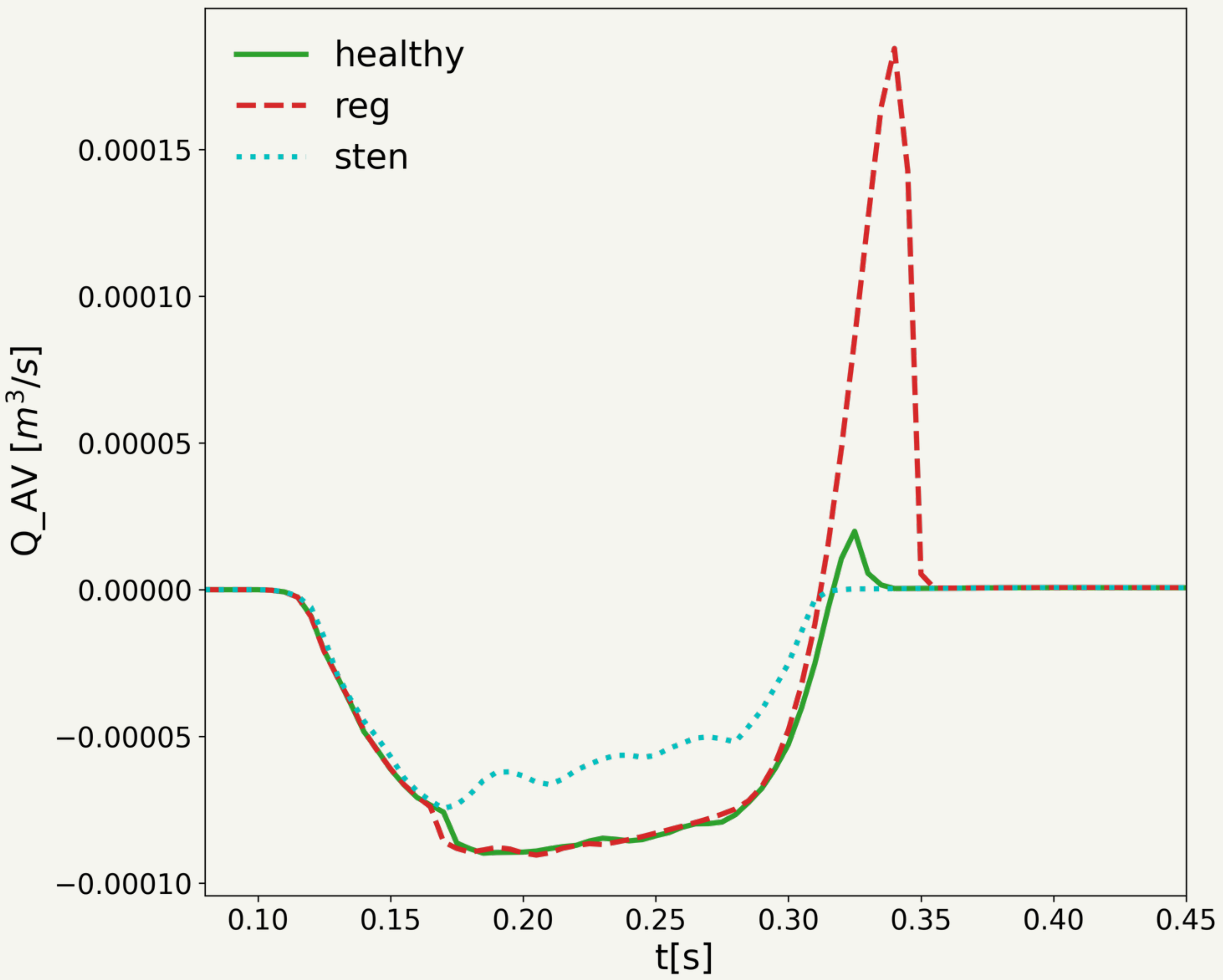
REGURGITANT CASE

Delayed closing that results in **reverse flow** occurring while the valve closes



Servier medical art

Pathological cases



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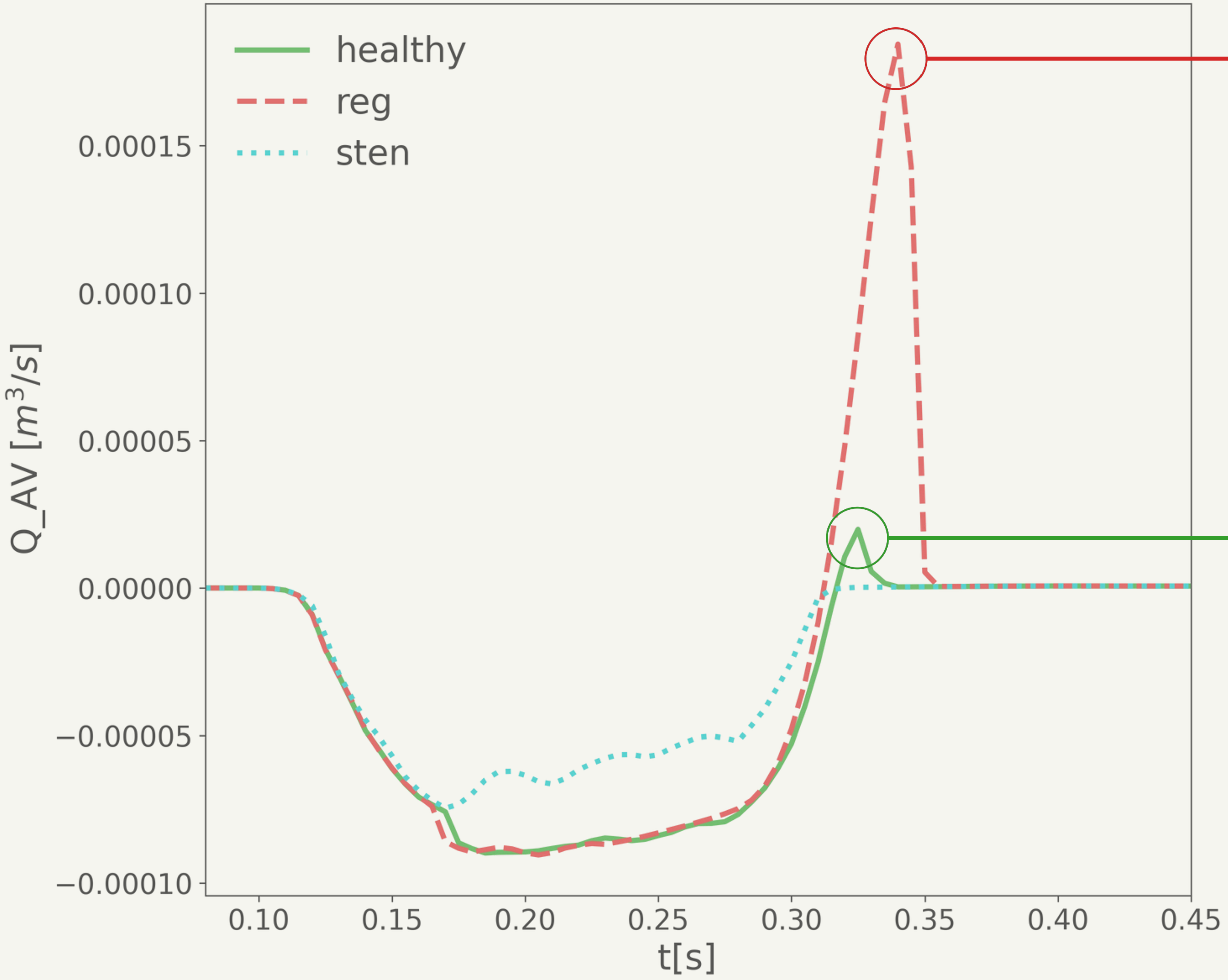

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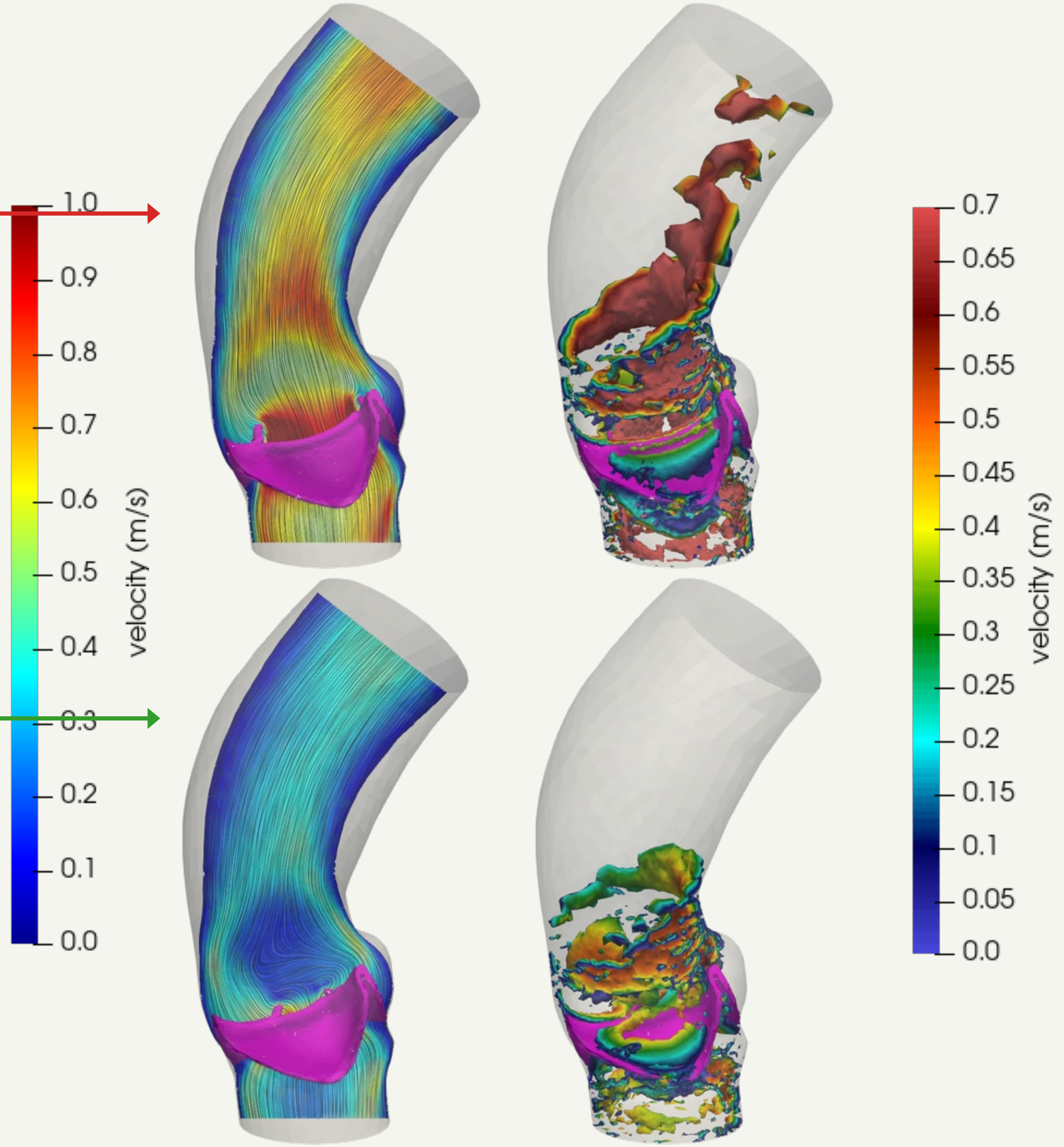
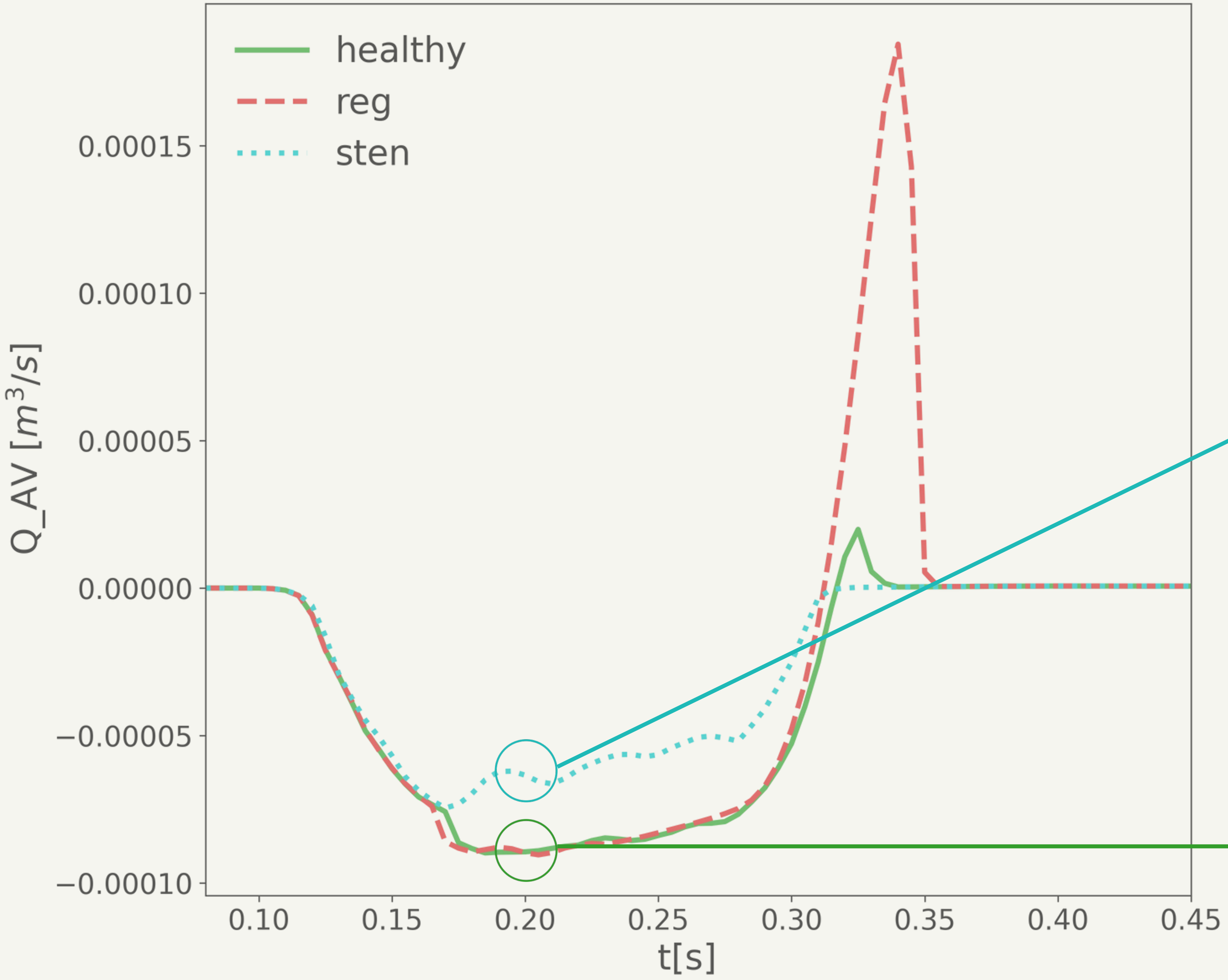
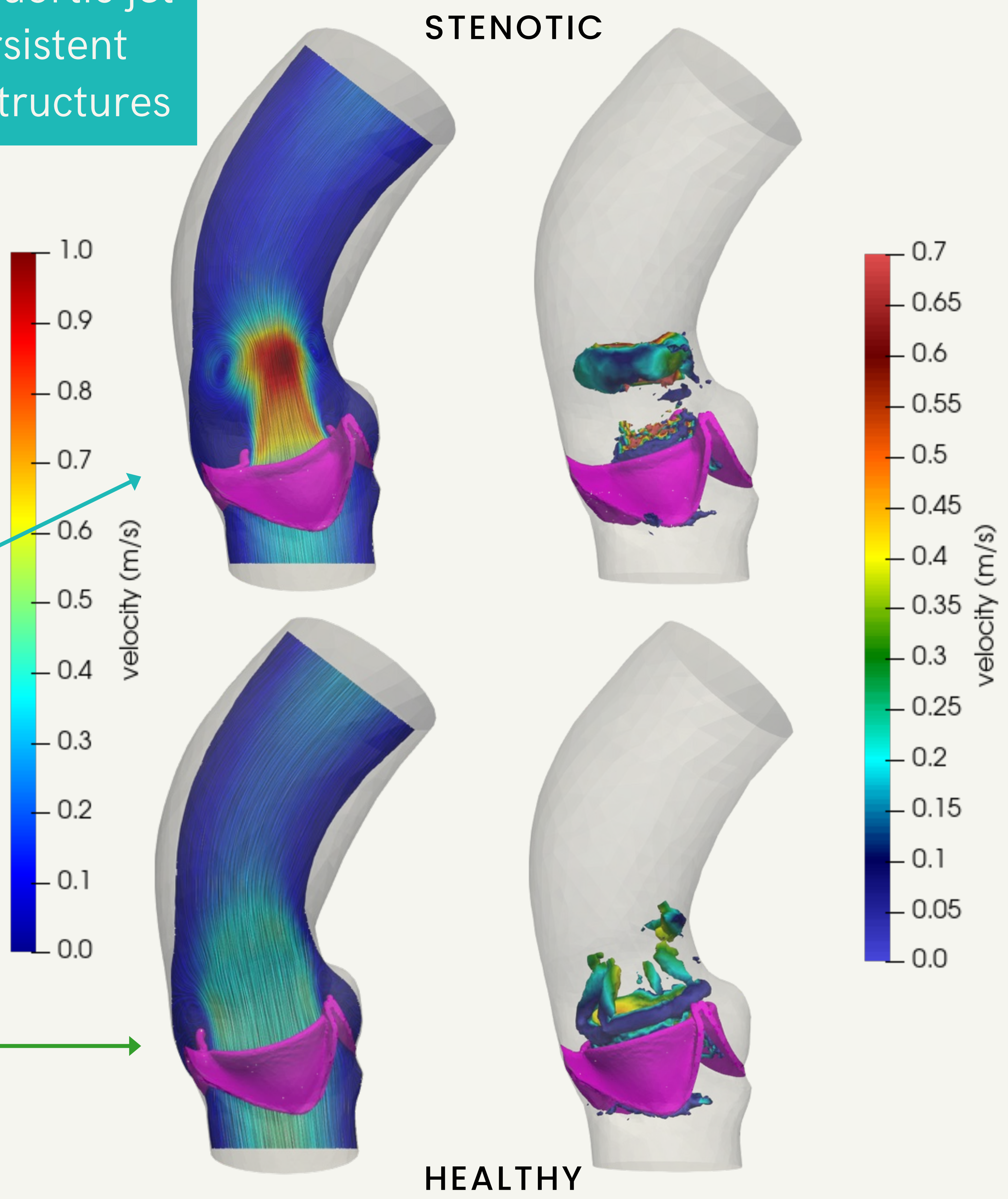


HEALTHY

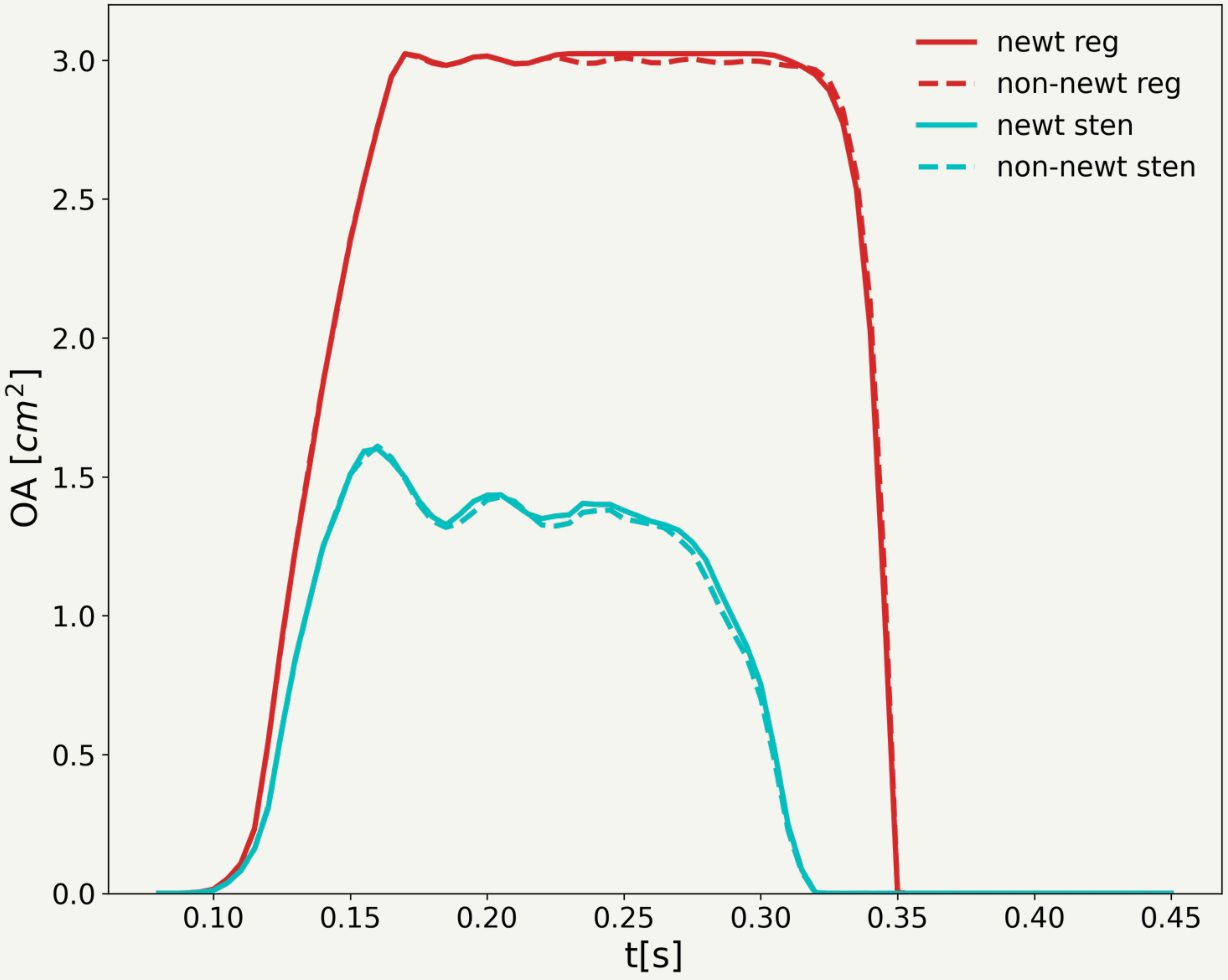
Pathological cases

- Stronger aortic jet
- More persistent vortical structures





Newtonian vs Non-Newtonian

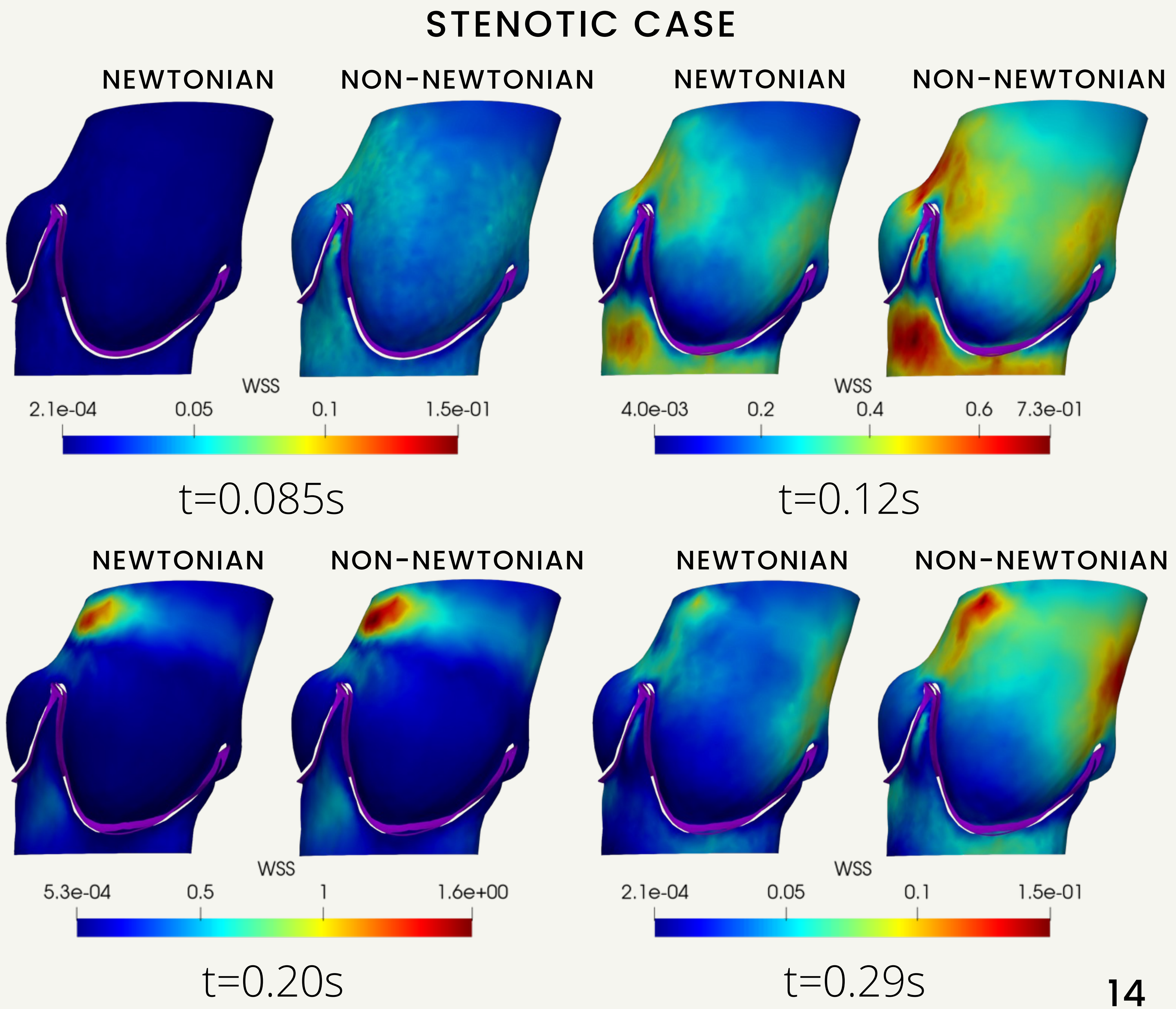


LITTLE INFLUENCE ON:

- **Valve dynamics**
- Transvalvular **pressure drop**
- **Large** flow structures

Newtonian vs Non-Newtonian

$$\mathbf{WSS} = \mu \frac{\partial \mathbf{u_t}}{\partial \mathbf{n}}\bigg|_{\text{wall}}$$



CAN INFLUENCE:

- Atherosclerotic plaques
- Wall elasticity

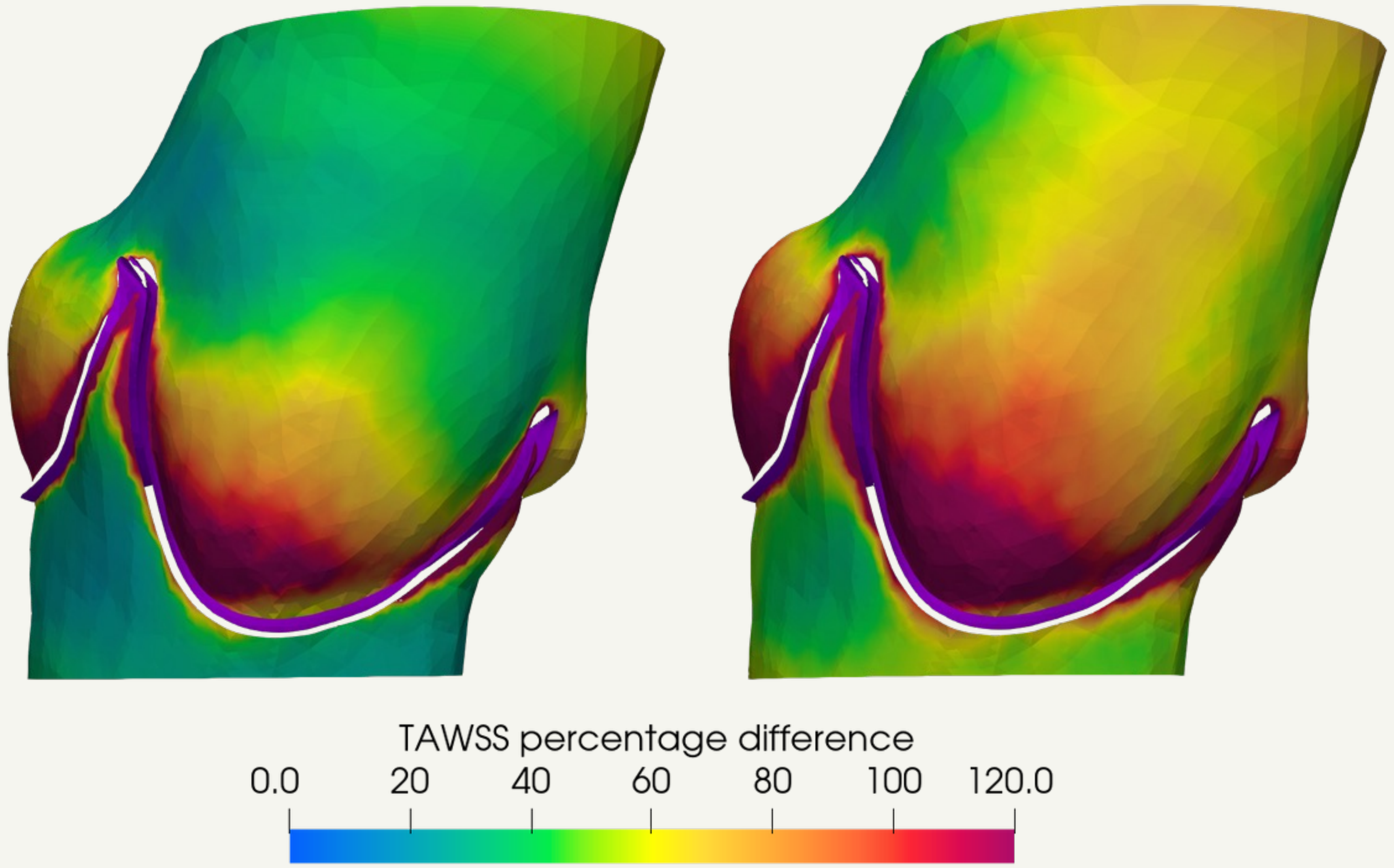
Newtonian vs Non-Newtonian

$$\mathbf{WSS} = \mu \frac{\partial \mathbf{u_t}}{\partial \mathbf{n}}\bigg|_{\text{wall}}$$

$$\text{TAWSS} = \frac{1}{T}\int_0^T |\mathbf{WSS}(\mathbf{s}, t)|dt$$

SIGNIFICANT DIFFERENCES

REGURGITANT STENOTIC



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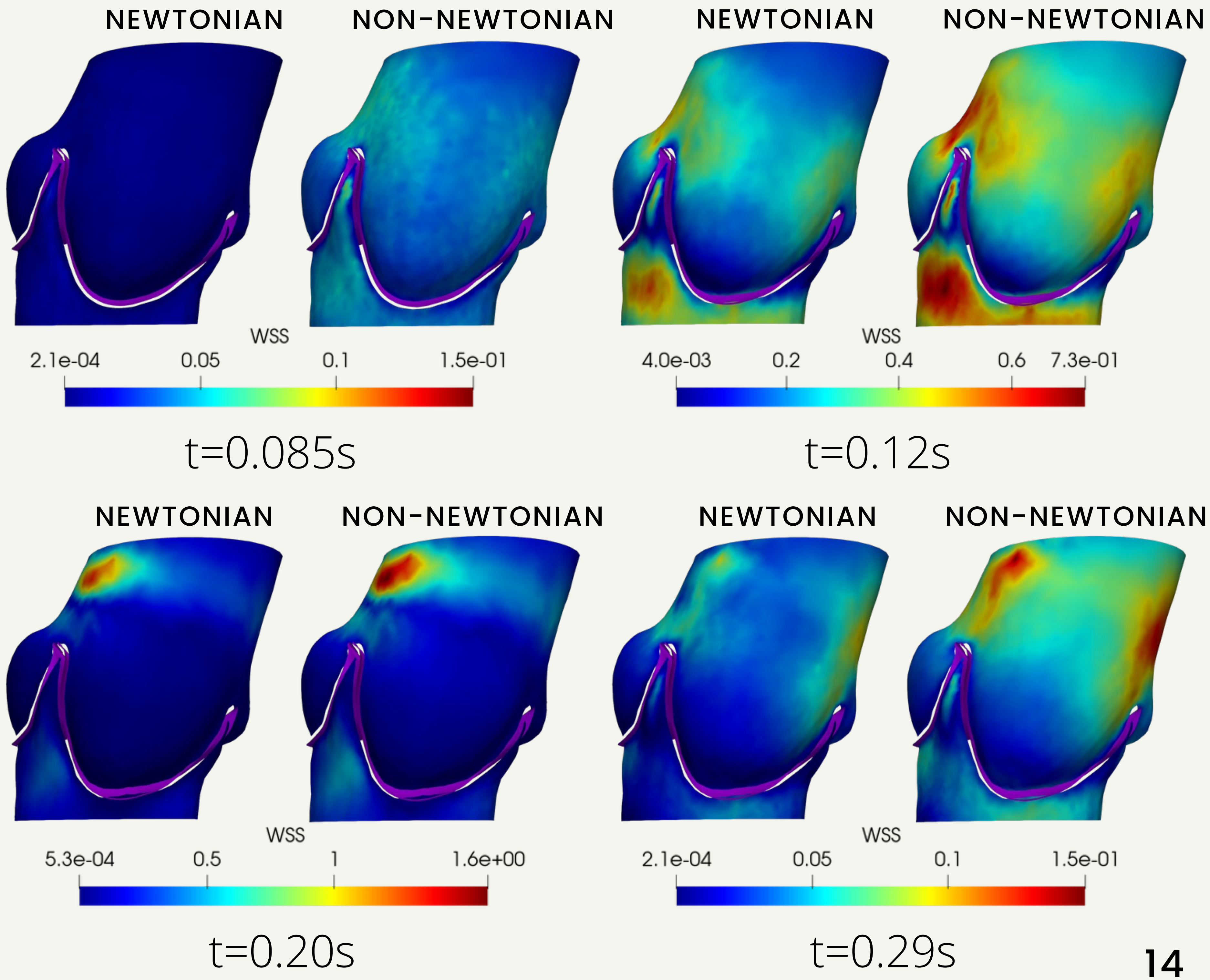

Conclusions

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ACHIEVEMENTS

- **Calibration** of the model in different scenarios

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Significant differences observed only on **wall shear stress**

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Significant differences observed only on **wall shear stress**

FUTURE DEVELOPMENTS

- Investigation of other types of **non-Newtonian models**
- Inclusion of the **left ventricle** in the domain
- Account for the **compliance** of the aortic wall
- Integration of **patient-specific data** in the computational framework

Thank you for your attention!

Essential bibliography

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A. Quarteroni, L. Dede’, A. Manzoni, and C. Vergara. Mathematical Modelling of the Human Cardiovascular System: Data, Numerical Approximation, Clinical Applications. *Cambridge University Press*, 2019.

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Additional material

Reduced structure model

$$\Gamma_t = \left\{ \mathbf{x} \in \mathbb{R}^3 : \mathbf{x} = \mathbf{T}_t(\widehat{\mathbf{x}}) = \widehat{\mathbf{x}} + \mathbf{d}_\Gamma(t, \widehat{\mathbf{x}}) \text{ for some } \widehat{\mathbf{x}} \in \widehat{\Gamma} \right\}$$

$$\rho_\Gamma \ddot{\mathbf{x}} + \beta \rho_\Gamma \dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}) + \gamma (H(\mathbf{x}) - \hat{H}(\widehat{\mathbf{x}})) \mathbf{n}_\Gamma(\mathbf{x})$$

Leaflets' curvature

Surface density

Elasticity coefficient

Damping coefficient

$$\ddot{c} + \beta \dot{c} + \eta(c, \mathbf{f}) = 0, \qquad \text{where}$$

$$\eta(c(t), \mathbf{f}(t)) = \frac{\gamma \int_{\Gamma_t} \left(H(\mathbf{x}) - \hat{H}\left(\mathbf{T}_t^{-1}(\mathbf{x})\right) \right) d\mathbf{x} - \int_{\Gamma_t} \mathbf{f}(t, \mathbf{x}) \cdot \mathbf{n}_\Gamma(\mathbf{x}) d\mathbf{x}}{\int_{\Gamma_t} \rho_\Gamma \mathbf{g}\left(\mathbf{T}_t^{-1}(\mathbf{x})\right) \cdot \mathbf{n}_\Gamma(\mathbf{x}) d\mathbf{x}}$$

Numerical methods

Given $\mathbf{u}_h^n, n = 0, \ldots, s-1$, for each $n = s, \ldots, N$, find $\mathbf{u}_h^n, p_h^n$ such that

$$\left(\rho \frac{\alpha_s \mathbf{u}_h^n - \mathbf{u}_h^{n,\mathrm{BDFs}}}{\Delta t}, \mathbf{v}_h\right) + a^n\left(\mathbf{u}_h^n, \mathbf{v}_h, \mu\left(\mathbf{u}_h^{n,BDFs}\right)\right)$$

$$+ c\left(\mathbf{u}_h^{n,s}, \mathbf{u}_h^n, \mathbf{v}_h\right) + b\left(\mathbf{v}_h, p_h^n\right) - b\left(\mathbf{u}_h^n, q_h\right)$$

$$+ \sum_{K \in \mathcal{T}_h} \left(\tau_{\mathrm{M}}^{n,s} \mathbf{r}_{\mathrm{M}}^n\left(\mathbf{u}_h^n, p_h^n, \mu\left(\mathbf{u}_h^{n,BDFs}\right)\right), \rho \mathbf{u}_h^{n,s} \cdot \nabla \mathbf{v}_h + \nabla q_h\right)_K$$

$$+ \sum_{K \in \mathcal{T}_h} \left(\tau_{\mathrm{C}}^{n,s} r_{\mathrm{C}}^n\left(\mathbf{u}_h^n\right), \nabla \cdot \mathbf{v}_h\right)_K = F\left(\mathbf{v}_h\right)$$

for all $\mathbf{v}_h \in V_h$ and $q_h \in Q_h$

Linear forms

$$a^n(\mathbf{u}, \mathbf{v}, \mu) = \mathcal{D}(\mathbf{u}, \mathbf{v}, \mu) + \left(\frac{R}{\varepsilon} \mathbf{u} \delta_\varepsilon^n, \mathbf{v}\right)$$

$$b(\mathbf{v}, q) = -(\operatorname{div} \mathbf{v}, q)$$

$$c(\mathbf{w}, \mathbf{u}, \mathbf{v}) = (\mathbf{w} \cdot \nabla \mathbf{u}, \mathbf{v})$$

$$F(\mathbf{v}) = \int_\Omega \boldsymbol{f} \cdot \boldsymbol{v} d\Omega + \int_{\partial\Omega_N} \boldsymbol{h} \cdot \boldsymbol{v} d\gamma - \left(\frac{R}{\varepsilon} \mathbf{u}_{\Gamma,h}^n \delta_\varepsilon^n, \mathbf{v}\right)$$

$$\mathbf{r}_{\mathrm{M}}^n\left(\mathbf{u}_h^n, p_h^n, \mu\left(\mathbf{u}_h^{n,BDFs}\right)\right) = \rho \frac{\alpha_s \mathbf{u}_h^n - \mathbf{u}_h^{n,\mathrm{BDFs}}}{\Delta t} - \mu\left(\mathbf{u}_h^{n,BDFs}\right) \Delta \mathbf{u}_h^n + \rho \mathbf{u}_h^{n,s} \cdot \nabla \mathbf{u}_n^n + \nabla p_h^n + \frac{R}{\varepsilon} \delta_\varepsilon^n \left(\mathbf{u}_h^n - \mathbf{u}_\Gamma^n\right)$$

VMS-inspired coefficient

$$r_{\mathrm{C}}^n\left(\mathbf{u}_h^n\right) = \nabla \cdot \mathbf{u}_h^n$$

$$\tau_{\mathrm{C}}^{n,s} = \left(\tau_{\mathrm{M}}^{n,s} \mathfrak{g} \cdot \mathfrak{g}\right)^{-1}$$

$$\tau_{\mathrm{M}}^{n,s} = \frac{1}{\sqrt{\frac{\rho^2 \alpha_s^2}{\Delta t^2} + \rho^2 \mathbf{u}_h^{n,s} \cdot \mathfrak{G} \mathbf{u}_h^{n,s} + C_{\mathrm{r}} \mu^2 \mathfrak{G} : \mathfrak{G} + \frac{R^2}{\varepsilon^2} \left(\delta_\varepsilon^n\right)^2}}$$

$$\boxed{\mathbf{u}_{\Gamma,h}^n = \frac{c^n - c^{n-1}}{\Delta t} \widetilde{\mathbf{g}}_h^n}$$

First order approximation

Numerical methods

NUMERICAL SOLUTION SCHEME FOR THE NON-NEWTONIAN 3D-0D FSI MODEL

1: Given $\mathbf{u}_h^n, p_h^n, c^n$ for $n = 0, \dots, s-1$, and compute the functions $\varphi^n, \widetilde{\mathbf{n}}_\Gamma^n, \widetilde{H}^n$ that corresponds to the surface Γ^n and $\mu\left(\mathbf{u}_h^{n,BDFs}\right)$, for $n = 0, \dots, s-1$,

2: **for** $n \leftarrow s$ **do**

3: Compute the integrals appearing in the 0D equation in terms of $\mathbf{u}_h^{n-1}$, $p_h^{n-1}, \Gamma^{n-1}, \varphi^{n-1}$.

4: Find c^n by solving the 0D equation with a step of RK4.

5: Move the valve to its next configuration Γ^n described by $\mathbf{d}_\Gamma^n = c^n \mathbf{g}$ and compute $\mathbf{u}_\Gamma^n = \dfrac{c^n - c^{n-1}}{\Delta t}\tilde{\mathbf{g}}$.

6: Compute the next distance function φ^n with respect to Γ^n and assemble fields related to the normal and the curvature $\widetilde{\mathbf{n}}_\Gamma^n$ and $\widetilde{\boldsymbol{H}}^n$.

7: Find $(\mathbf{u}_h^n, p_h^n) \in V_h^r \times Q_h^r$ by solving the linear problem related to the fluid dynamics problem.

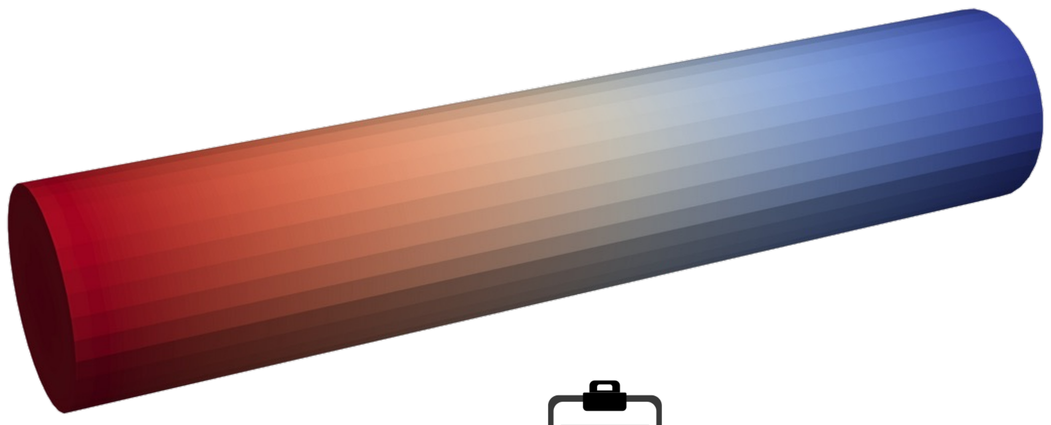
8: **end for**

Validation of the non-Newtonian model

COMPARISON WITH THE LITERATURE

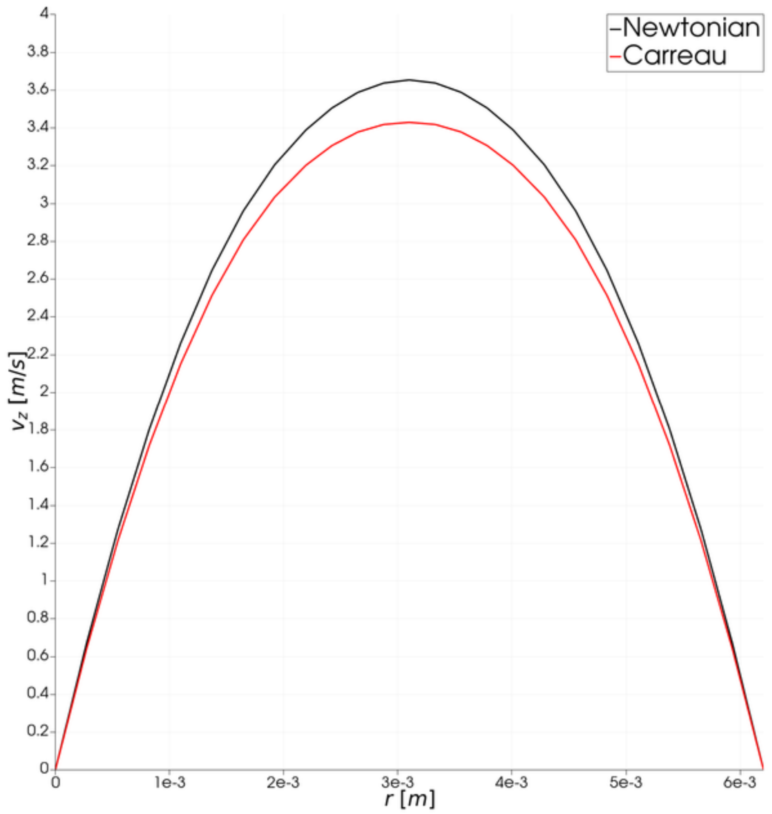
- Semi-analytical procedure in the reference
- Steady and pulsatile pressure gradient for two different radii

	Reference	life^x
Newtonian flux (m^3/s)	$6.30 \cdot 10^{-5}$	$5.72 \cdot 10^{-5}$
Carreau flux (m^3/s)	$5.98 \cdot 10^{-5}$	$5.43 \cdot 10^{-5}$
Ratio flux	5.08%	5.03%
Newtonian WSS (Pa)	9.3	9.31
Carreau WSS (Pa)	9.3	9.26



Tabakova et al, *AIP conference proceedings*, 2014

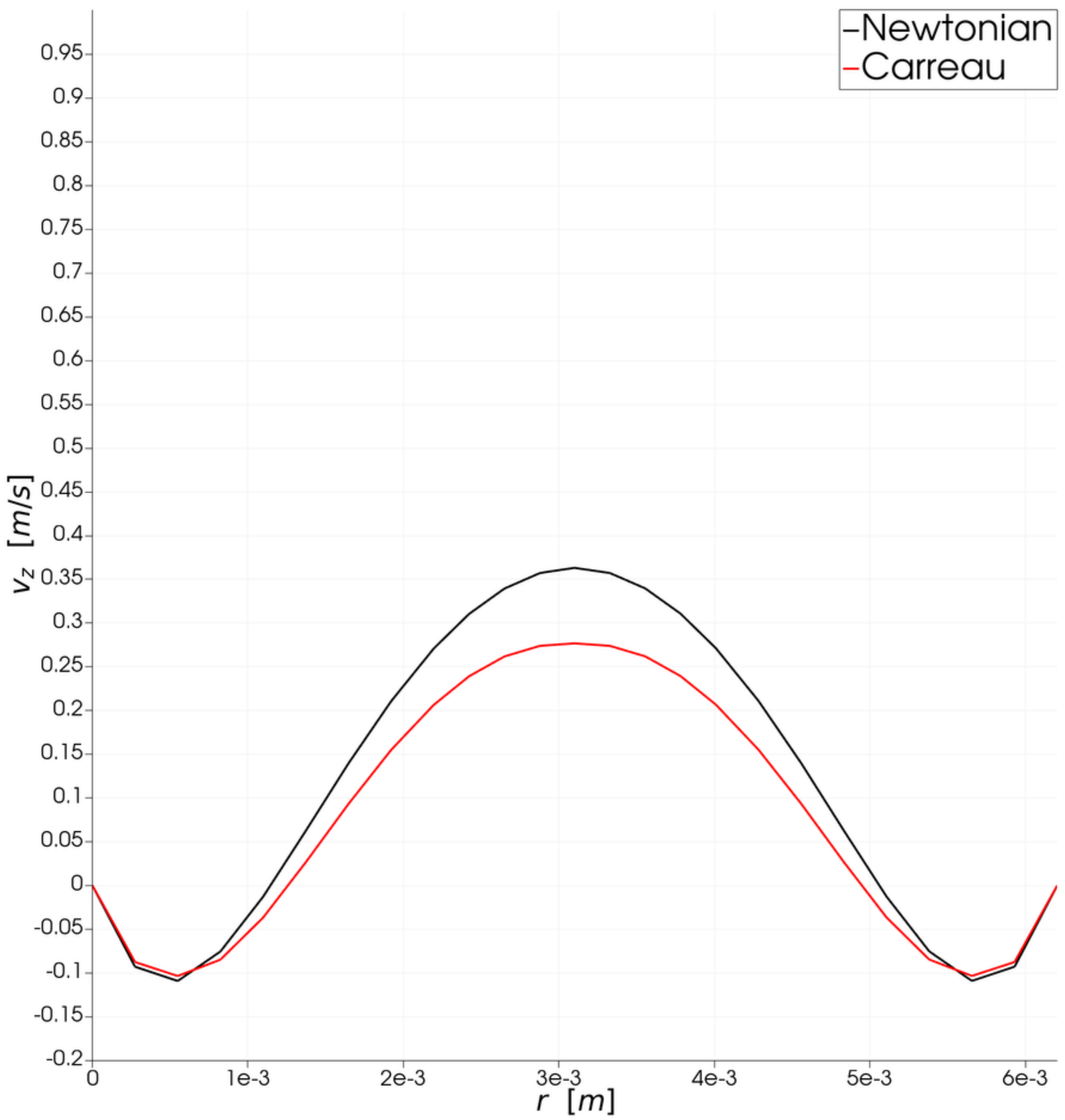
STEADY CASE



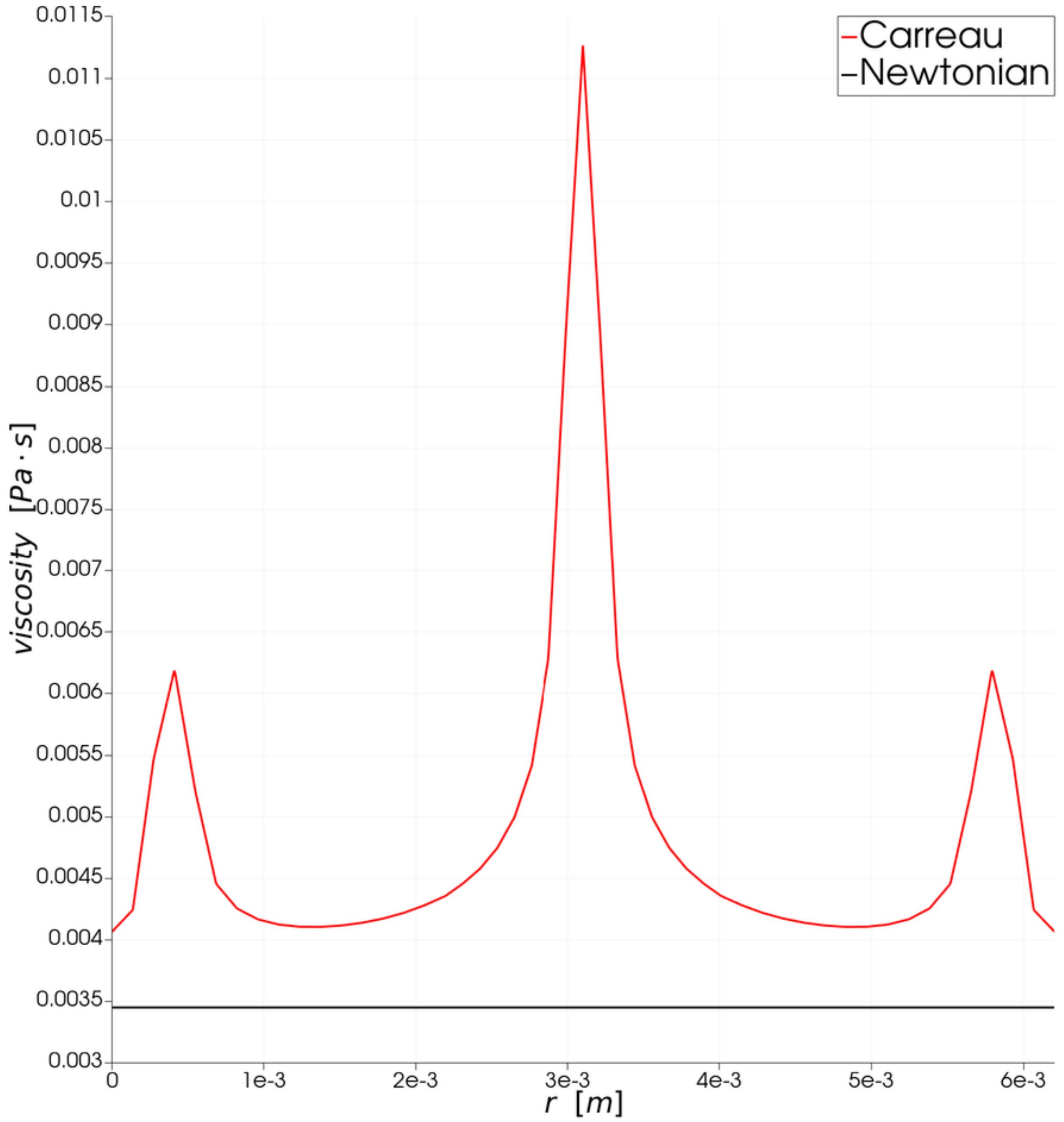
Validation of the non-Newtonian model

PULSATILE CASE

MINIMUM FLUX



VELOCITY PROFILE



VISCOSITY DISTRIBUTION

Numerical setting

Parameter	Value	Description
ρ $[\mathrm{Kg/m^3}]$	1060	Density of blood
μ_{new} $[\mathrm{Pa \cdot s}]$	$3.5 \cdot 10^{-3}$	Newtonian viscosity of blood
ε $[\mathrm{m}]$	$5 \cdot 10^{-4}$	Half-thickness of the valve
R $[\mathrm{Pa \cdot s}]$	1000	Resistance parameter in the RIIS method
β $[\mathrm{s^{-1}}]$	0.2	Damping parameter
ρ_Γ $[\cdot]$	0.1	Scaling factor for density in the 0D valve model
$\widehat{\widetilde{H}}$ $[\mathrm{m^{-1}}]$	0.04	Initial curvature offset
t_0 $[\mathrm{s}]$	0.08	Initial time of the simulation
T $[\mathrm{s}]$	0.45	Final time of the simulation
Δt $[\mathrm{s}]$	$1 \cdot 10^{-4}$	Time step

HEALTHY

$$\gamma = 0.2\,\mathrm{N/m}$$

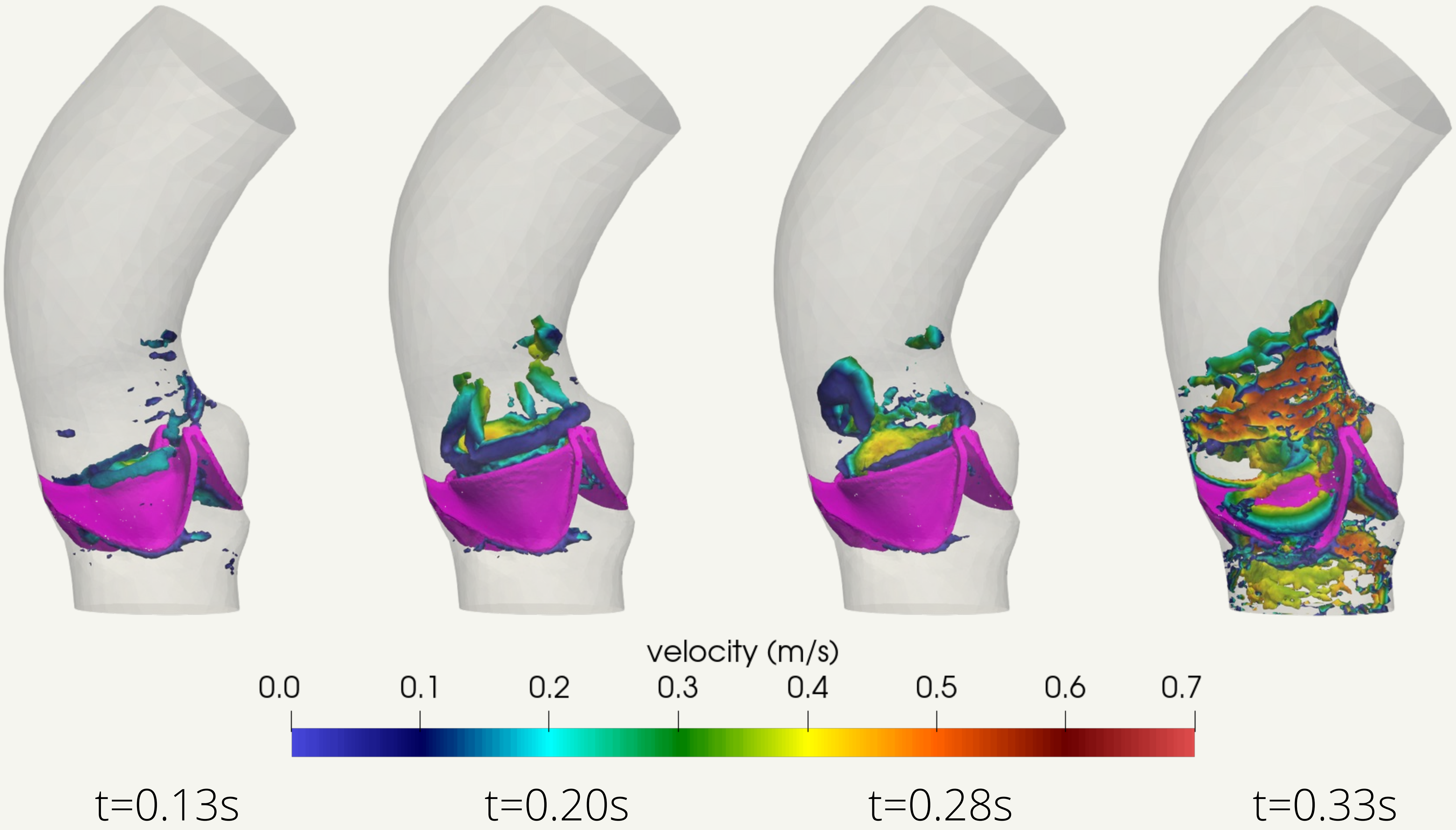
STENOTIC

$$\gamma = 1.0\,\mathrm{N/m}$$

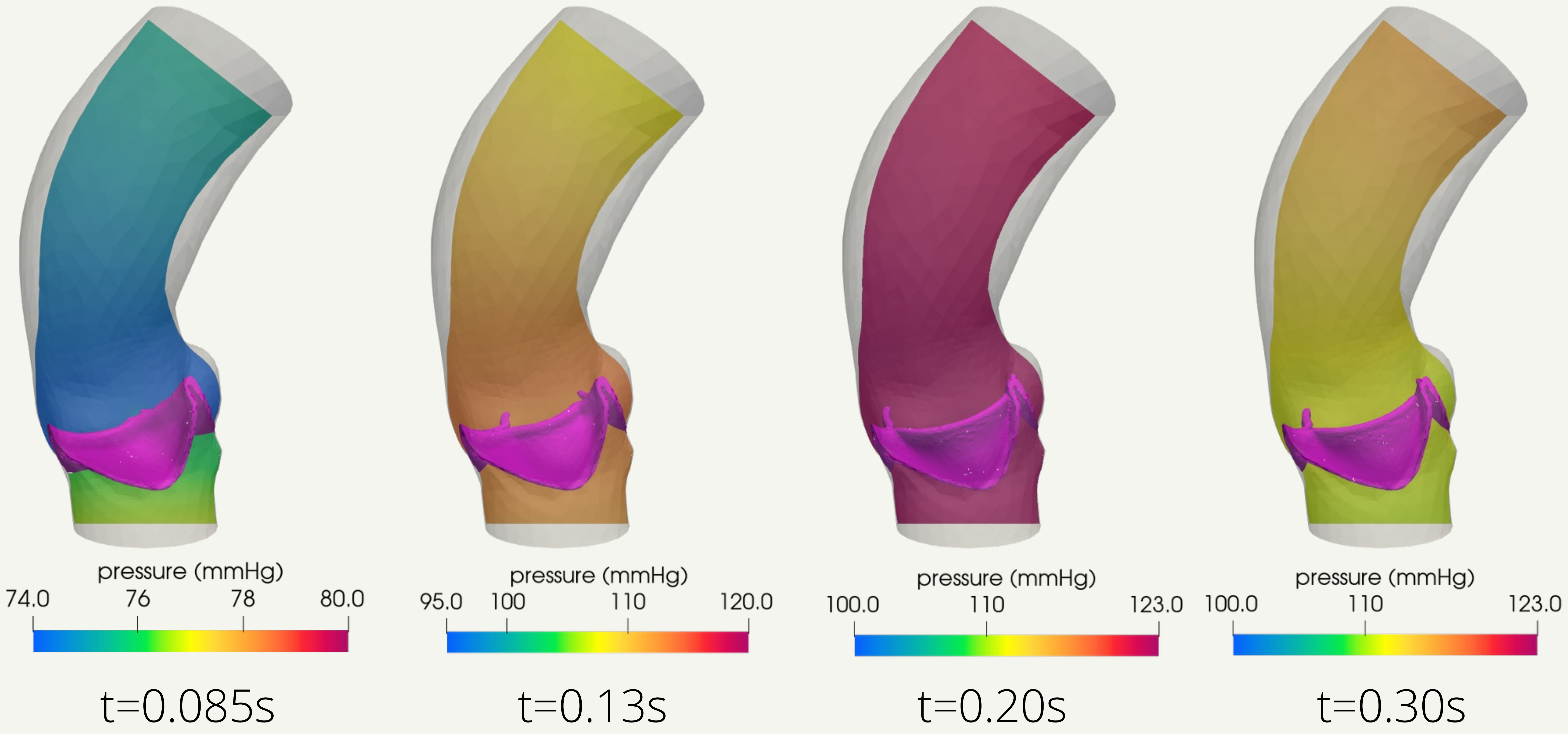
REGURGITANT

$$\gamma = 0.1\,\mathrm{N/m}$$

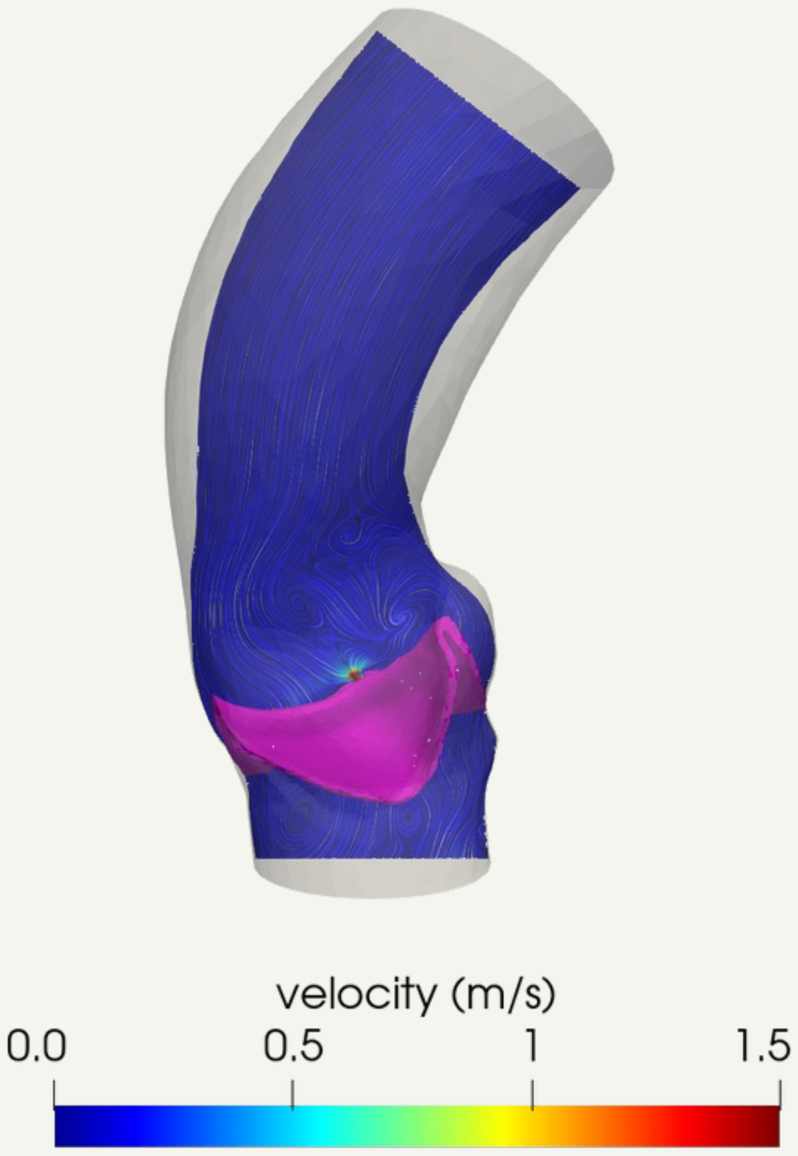
Healthy valve



Healthy valve



Healthy valve: limitation

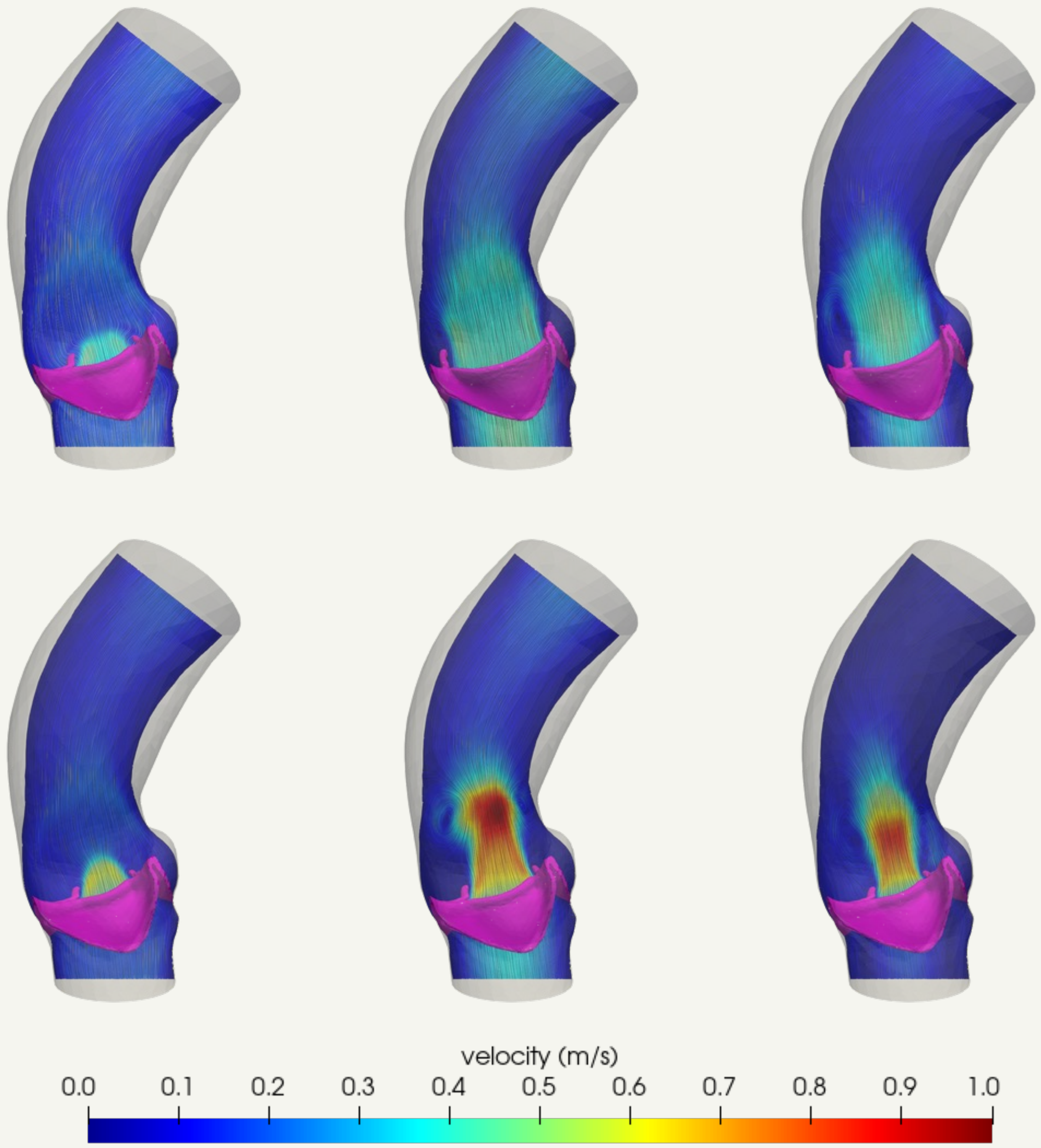


t=0.42s

- Reverse flow when the valve should be closed
- Geometrical inconsistency

- Don't affect the behavior during opening and closing
- Possible solution: manually adjust the geometry

Pathological cases

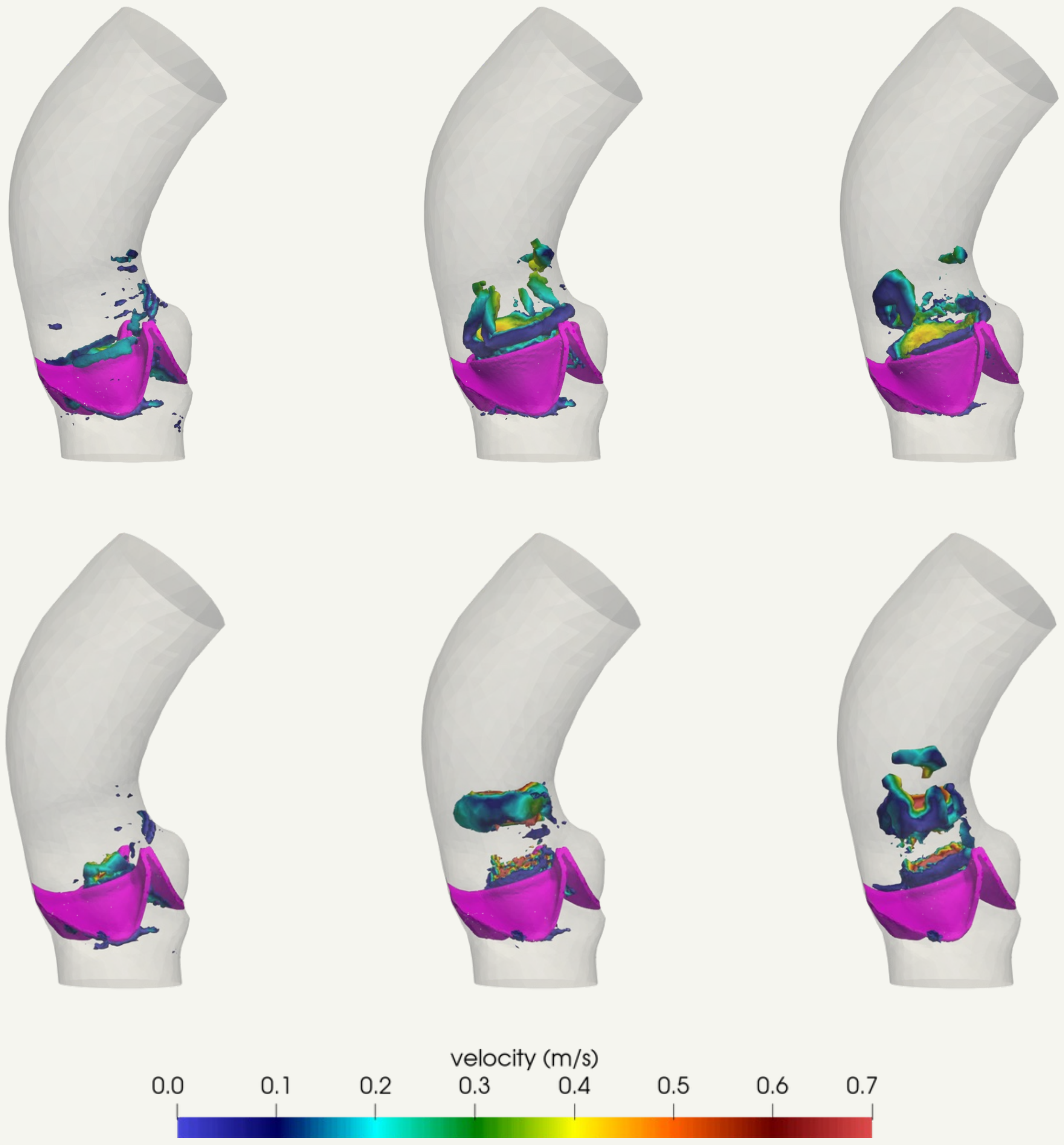


t=0.13s t=0.20s t=0.30s

HEALTHY

STENOTIC

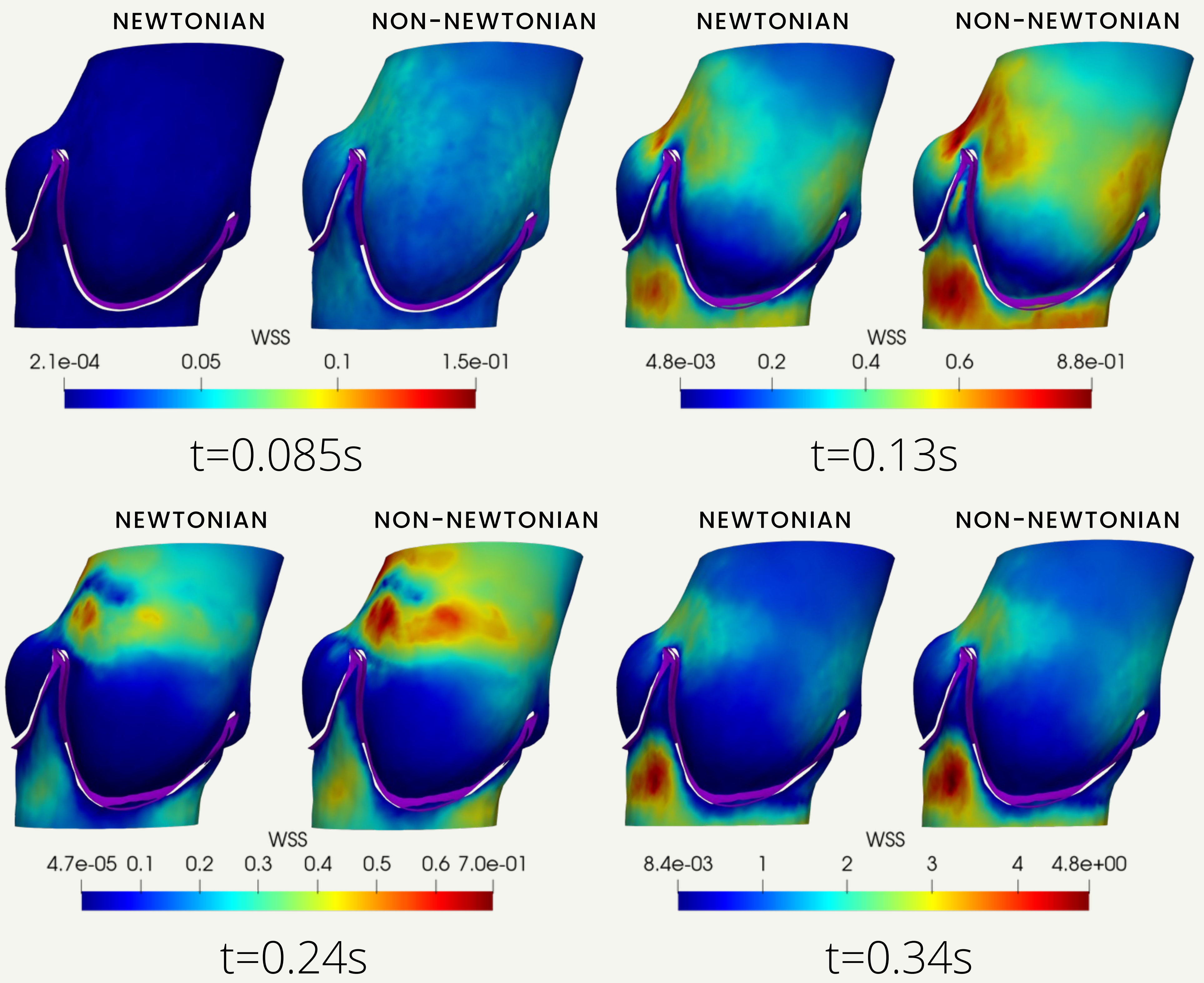
- Stronger aortic jet
- More persistent vorticity structures



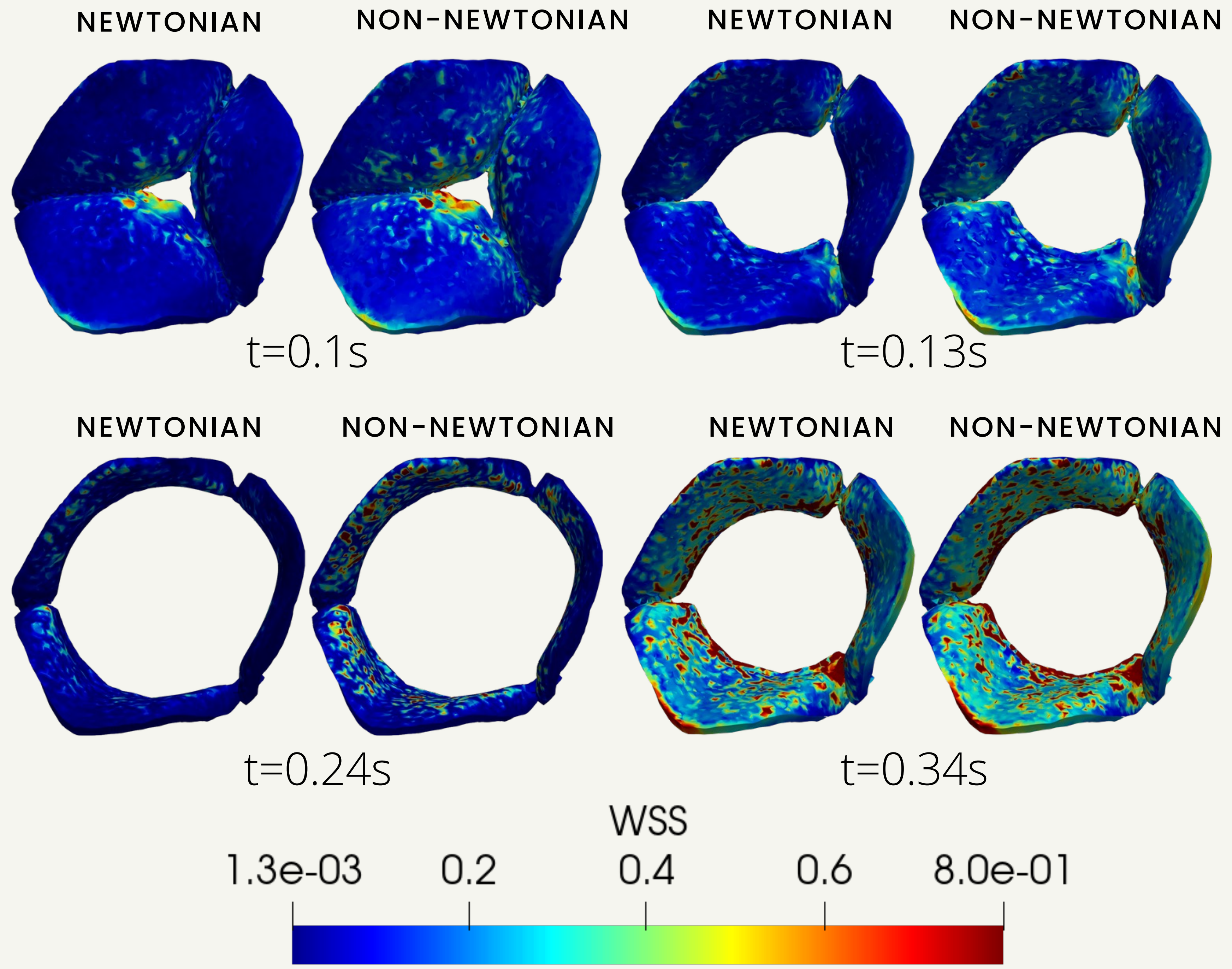
t=0.13s t=0.20s t=0.28s

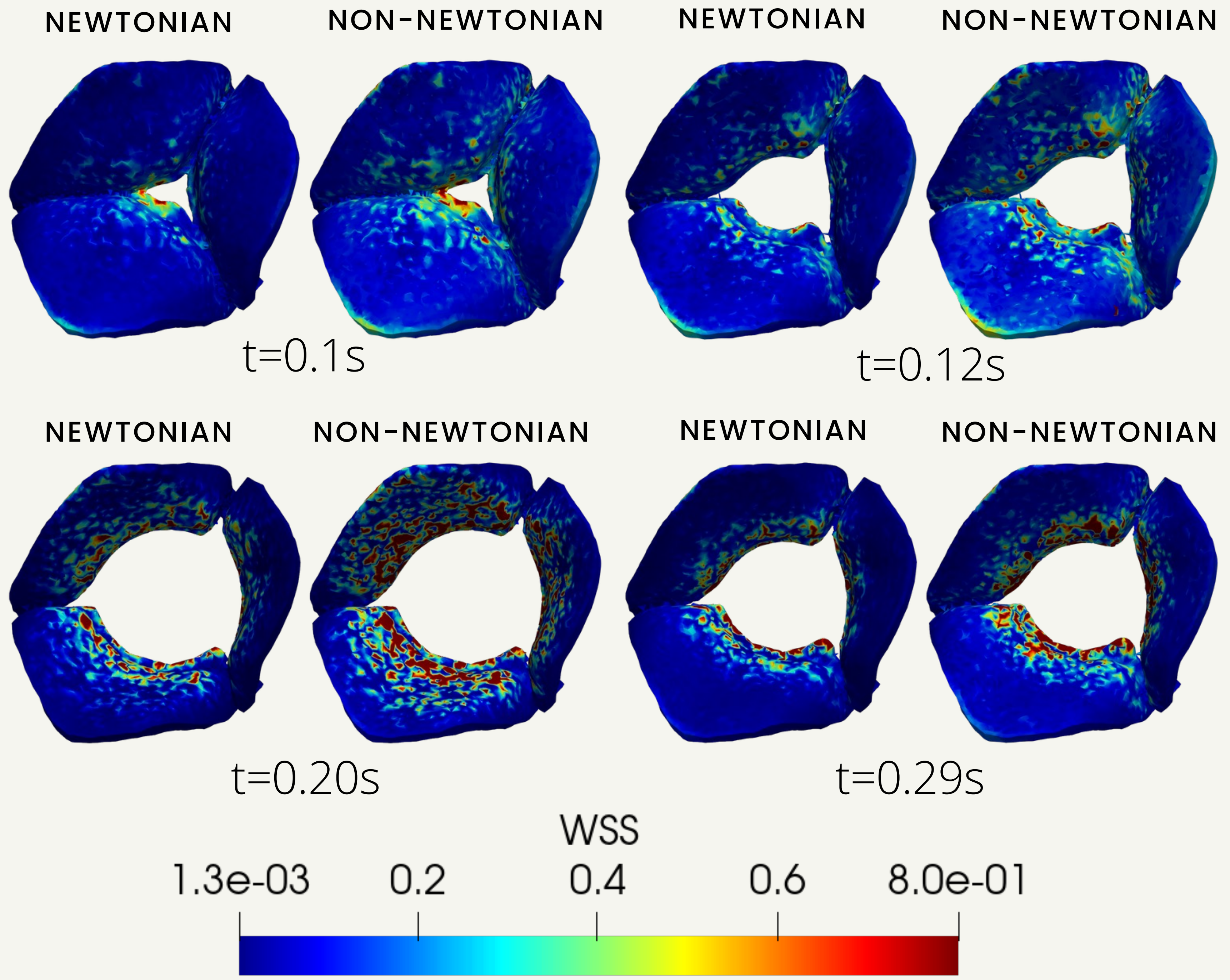
Newtonian vs Non-Newtonian

REGURGITANT CASE

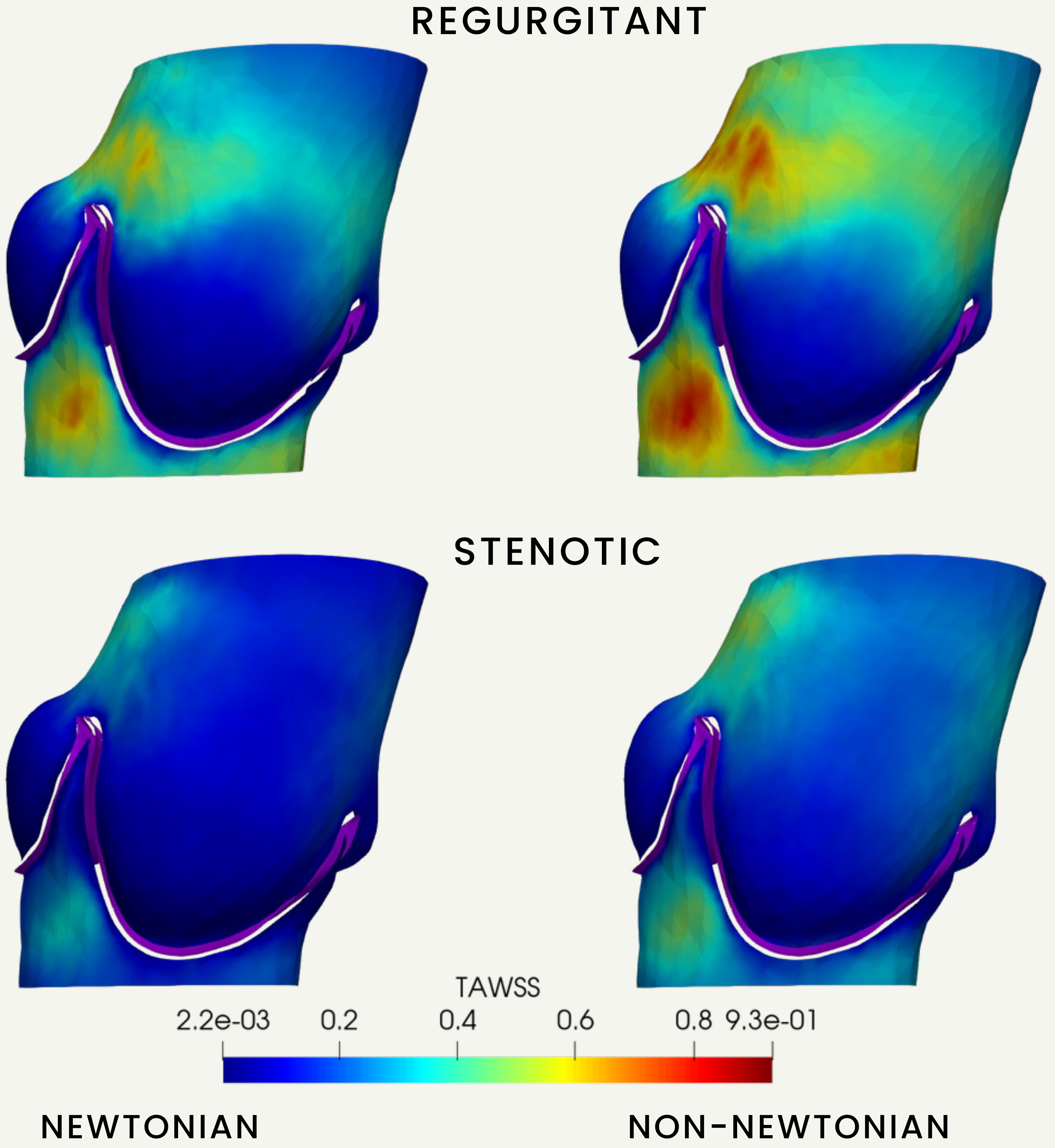


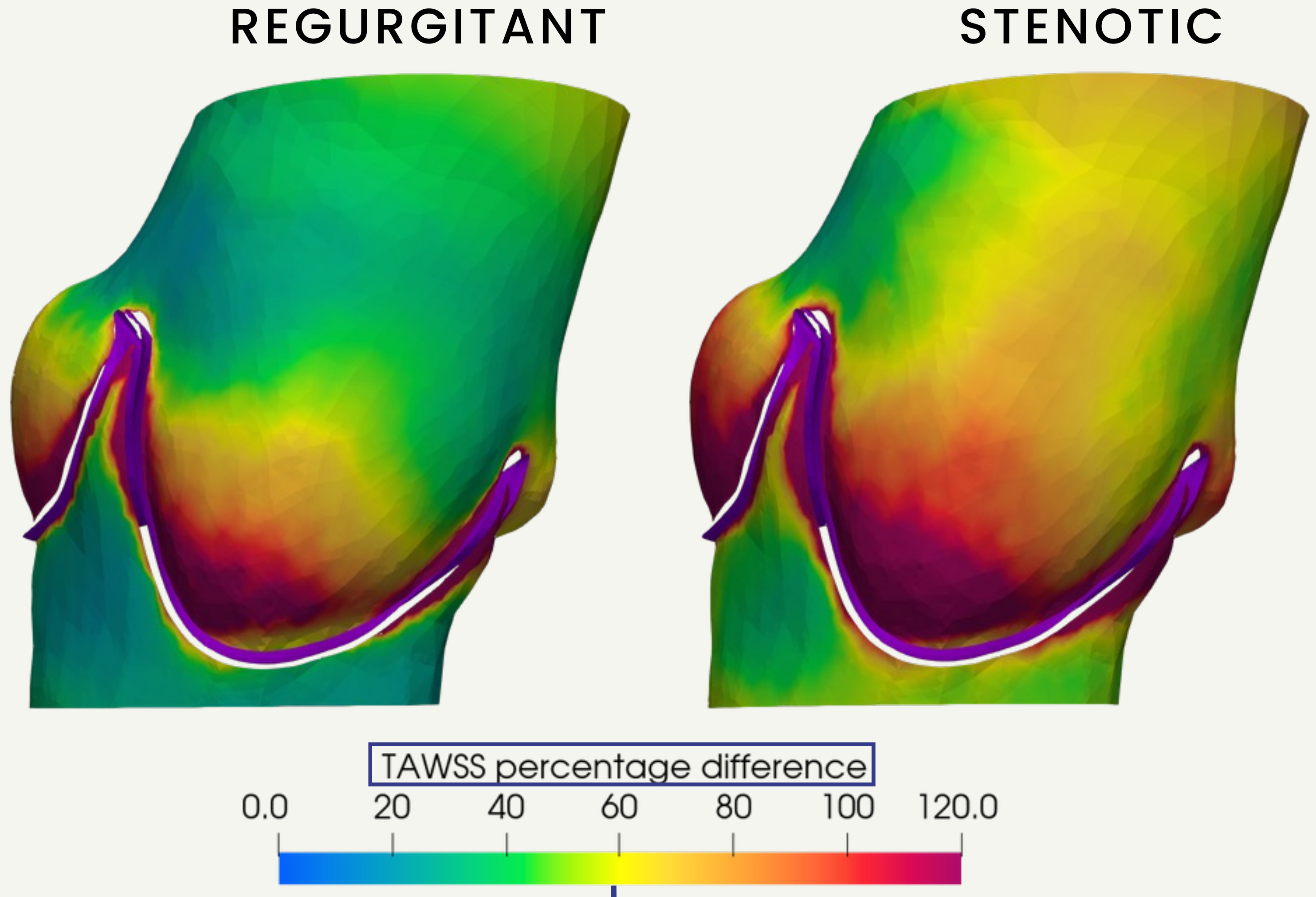
Newtonian vs Non-Newtonian





Newtonian vs Non-Newtonian



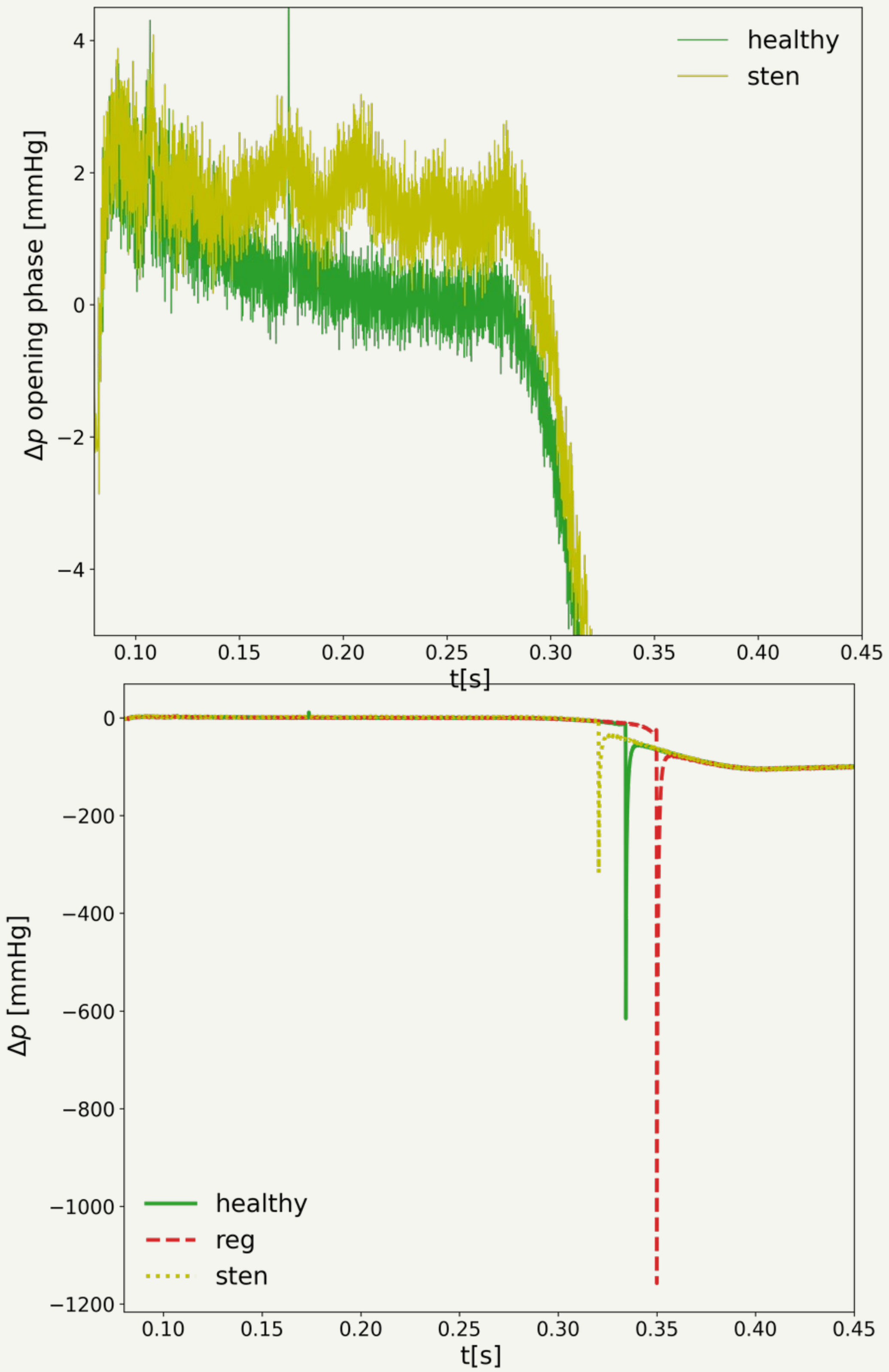


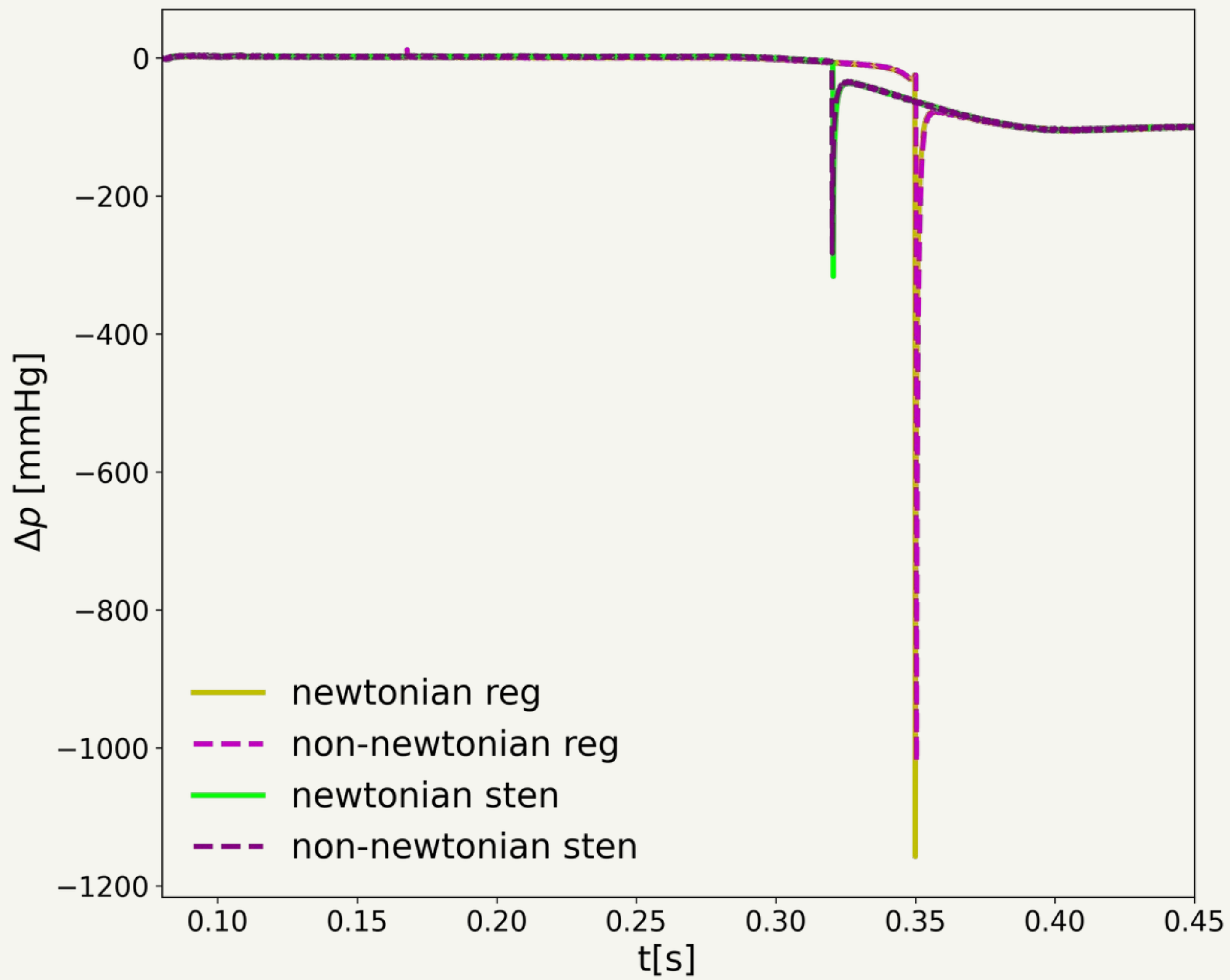
$$100\frac{\mathrm{TAWSS}_{\mathrm{non-Newtonian}} - \mathrm{TAWSS}_{\mathrm{Newtonian}}}{\mathrm{TAWSS}_{\mathrm{Newtonian}}}$$

Transvalvular pressure drop

Pathological cases

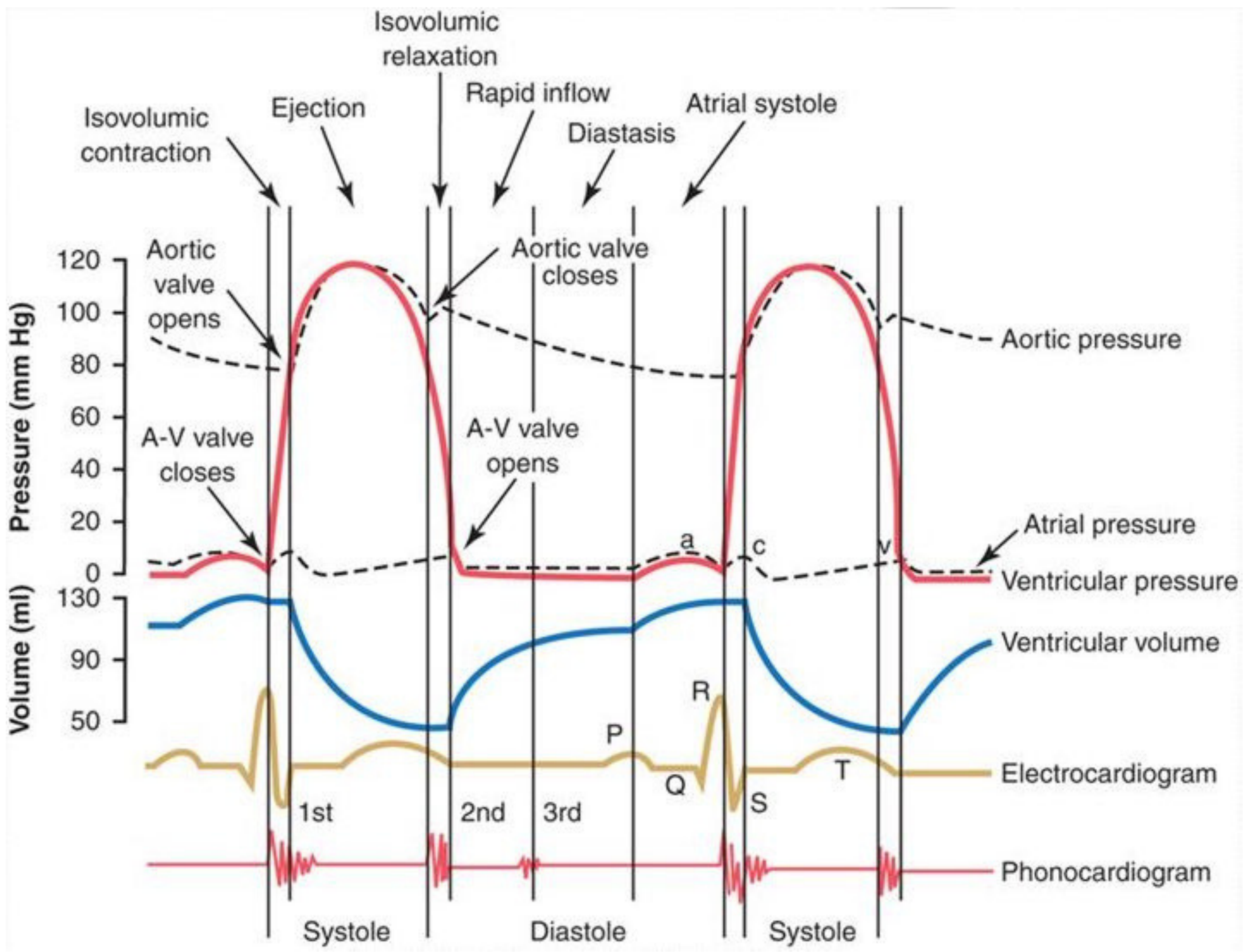
Control volumes



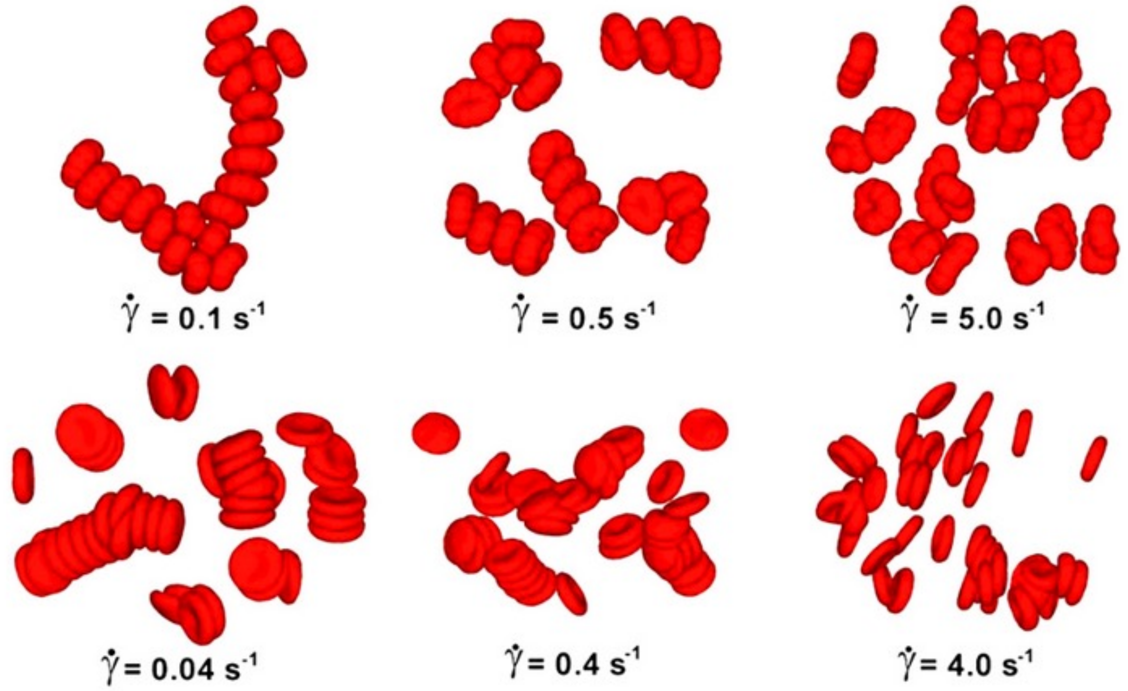


Newtonian vs non-Newtonian

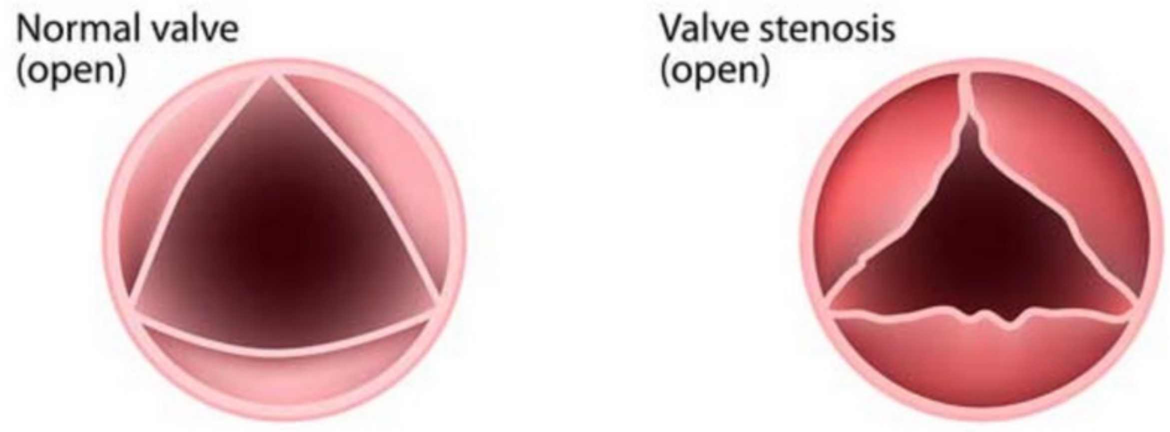
Anatomy and physiology



Cardiac cycle



Rouleaux aggregation



Stenosis

Sensibility of the healthy configuration