SAE TECHNICAL PAPER SERIES

2000-01-0837

Optimal Control of Dry Clutch Engagement

Luigi Glielmo
Università di Napoli Federico II

Francesco Vasca
Università del Sannio in Benevento

Reprinted From: Transmission and Driveline Symposium 2000 (SP–1522)

The Engineering Society For Advancing Mobility Land Sea Air and Space®
INTERNATIONAL

SAE 2000 World Congress
Detroit, Michigan
March 6–9, 2000

400 Commonwealth Drive, Warrendale, PA 15096-0001 U.S.A. Tel: (724) 776-4841 Fax: (724) 776-5760

ISSN 0148-7191

Printed in USA

2000-01-0837

Optimal Control of Dry Clutch Engagement

Luigi Glielmo
Università di Napoli Federico II

Francesco Vasca
Università del Sannio in Benevento

ABSTRACT

Based on a state space dynamic model of the powertrain system, a new control technique for the dry clutch engagement process is proposed. The feedback controller is designed following a linear quadratic approach by using the crankshaft speed and the clutch disk speed as state variables. The controller guarantees fast engagement, minimum slipping losses and comfortable lock-up. The critical standing start operating conditions are considered and a comparison with a classical open loop control strategy is presented. The numerical results, carried out by a Simulink/Stateflow simulation scheme and a realistic set of parameters, show the good performance of the closed loop system.

INTRODUCTION

The engagement of dry clutches is a very important process both to ensure small facing wear and good powertrain performance [1]-[3]. The engagement must be controlled in order to satisfy different and sometimes conflicting objectives: small friction losses, minimum time needed for the engagement, preservation of driver comfort. These goals must be reached by applying a suitable normal force to the clutch driven disk. To this aim, several control strategies have been proposed in the literature [4]-[8]. In [4] the problem of avoiding the use of throttle during the engagement in diesel engine vehicles is considered. A fuzzy controller is proposed in [5] where, by using a sixth order state space model of the powertrain, the influence of different friction coefficients and vehicle operating conditions is also analyzed. This influence is also considered in [6] where an H^{∞} controller is designed by using a suitably identified ARMAX discrete time model of the powertrain. The engagement clutch control in parallel hybrid electric vehicles and in heavy duty trucks are considered in [7] and [8], respectively.

In this paper, by using a simple second order dynamic state space model of the clutch engagement system, two different types of control strategies are considered. The first one consists in finding the force profile to be applied to the clutch disk so that the time duration of the engagement process is minimized. As one would expect, analytical results show that the corresponding force must reach as fast as possible its maximum value. This controller is the most common open loop control of the engagement process. On the other hand, this control strategy does not ensure that the friction losses are small and that the difference between the driven disk acceleration and the crankshaft acceleration is small at the engagement, as requested in order to preserve the passengers comfort. To better satisfy these constraints an optimal control state feedback can be designed [9] whose performance index weights the difference between the two speeds and the energy dissipated during the engagement. Moreover, the controller parameters are chosen so that the clutch force does not exceed its maximum value. The control design problem is then formulated as a finite time optimal control problem with initial time constraints (zero initial force) and final time constraints (the two speeds must be equal at the lock-up). The solution consists in the sum of two control signals: a time-varying state feedback component, whose gain depends only on the desired duration of the engagement, and a feedforward component that compensates for the unknown initial conditions and the applied engine torque.

A Simulink/Stateflow simulation scheme is used to check the effectiveness of the proposed controllers [10]. The event driven Stateflow part discriminates between the lock-up and the slipping dynamic models, by enabling two different Simulink schemes. Numerical simulations show the good performance obtained by the optimal control for different load torques.

DYNAMIC MODEL

A possible dynamic model of the clutch system during slipping conditions consists of the following two differential equations:

$$I_e \dot{\omega}_e = T_{in} - b_e \omega_e - T_{cl} \quad (1)$$

$$I_v \dot{\omega}_v = T_{cl} - b_v \omega_v - T_l \quad (2)$$

where I_e is the engine inertia, ω_e the crankshaft rotor speed, T_{in} the engine torque, b_e the crankshaft friction coefficient, T_{cl} the torque transmitted by the clutch, I_v the equivalent vehicle moment of inertia (it takes into account the presence of the clutch, the mainshaft, the powertrain and the vehicle) - which will also depend on the gear ratio, ω_v the clutch disk rotor speed, b_v the corresponding friction coefficient and T_l the equivalent load torque. Equation (1) models the rotation of the crankshaft, whereas (2) models the rotation of the so called clutch disk. The remaining part of the powertrain transmission system is simply modeled through the equivalent vehicle inertia I_v and the load torque T_l. Though equations (1)-(2) do not model in detail the whole powertrain, this model captures the main dynamics of the system under investigation and is simple enough to design a controller through analytical procedures. The clutch torque T_{cl} can be expressed as a function of the normal force F_n applied to the clutch disk as follows:

$$T_{cl} = kF_n sign(\omega_e - \omega_v) \quad (3)$$

where $k = 4R\mu_d / 3$, R is the equivalent disk ratio and μ_d is the dynamic friction coefficient (see [10] for further details).

When the clutch is engaged, by adding (1) and (2), the dynamic model can be written as

$$(I_e + I_v)\dot{\omega} = T_{in} - (b_e + b_v)\omega - T_l \quad (4)$$

where $\omega = \omega_e = \omega_v$. The switch from the slipping model (1)-(2) to the engaged model (4) is determined by the equality condition $\omega_e = \omega_v$ with the constraint that the clutch torque is smaller than the static friction torque, so that further slipping is avoided.

The control strategy will be based on the model (1)-(2). In particular, the following equivalent state space model will be considered:

$$\dot{x}_1 = -\frac{b_e}{I_e} x_1 - \frac{k}{I_e} F_n + \frac{T_{in}}{I_e} \quad (5)$$

$$\dot{x}_2 = \left(-\frac{b_e}{I_e} + \frac{b_v}{I_v} \right) x_1 - \frac{b_v}{I_v} x_2 - \left(\frac{k}{I_e} + \frac{k}{I_v} \right) F_n + \frac{T_{in}}{I_e} + \frac{T_l}{I_v} \quad (6)$$

where $x_1 = \omega_e$ and $x_2 = \omega_e - \omega_v$ are the state variables, F_n is the control variable and we assume $\omega_e > \omega_v$ which is typical during engagement. The difference between the two rotor speeds has been considered as one of the two state variables since the main goal of the controller will be to ensure a suitable profile to this variable thus guaranteeing a smooth engagement process with small friction losses.

CONTROL STRATEGIES

MINIMUM TIME CONTROLLER – The main objective of the minimum time controller is to minimize the duration of the engagement process. The corresponding control problem can be analytically reformulated as the following optimal control problem: given the state initial conditions $x_1(0)$ and $x_2(0)$, let t^* be the time instant when $x_2(t^*)=0$; find the control signal $F_n(t)$ for $t \in [0, t^*]$ which minimizes t^*.

The control variable F_n is zero at the initial time, i.e. $F_n(0)=0$, it cannot vary discontinuously and cannot exceed a maximum value F_{nmax}. Therefore we introduce as a further state variable $x_3 = F_n$ with the constraints

$$x_3(0) = 0 \quad (7)$$

$$x_3(t) \le F_{nMAX} \quad \forall t \quad (8)$$

and we use as input to the system the time derivative u of the force rather than the force itself. Thus, the dynamic model (1)-(2) can be rewritten as follows:

$$\dot{x}_1 = -\frac{b_e}{I_e} x_1 - \frac{k}{I_e} x_3 + \frac{T_{in}}{I_e} \quad (9)$$

$$\dot{x}_2 = \left(-\frac{b_e}{I_e} + \frac{b_v}{I_v} \right) x_1 - \frac{b_v}{I_v} x_2 - \left(\frac{k}{I_e} + \frac{k}{I_v} \right) x_3 + \frac{T_{in}}{I_e} + \frac{T_l}{I_v} \quad (10)$$

$$\dot{x}_3 = u \quad (11)$$

with the constraint $u \in [u_{MIN}, u_{MAX}]$. Direct computation of the controllability matrix of the system (9)-(11) shows that the system is completely controllable. The input torque and the load torque will be considered as known disturbances.

The application of the Maximum Pontryagin Principle [9] on the system (9)-(11) allows one to conclude that, along the optimal trajectory, the control variable can assume only two different values: the maximum value u_{MAX}, corresponding to the maximum time derivative of the normal force, and the minimum value u_{MIN} which is zero since it is assumed that the force does not decrease. In other words the optimal, and somewhat straightforward, solution obtained for this type of controller leads to the force profiles reported in Figure 1 and Figure 2.

The most typical situation is that one reported in Figure 2, since usually lock-up is reached at about 50% of the maximum (also called nominal) value of the force [4].

The minimum time controller does not allow one to control losses and comfort related to the engagement process; to consider both these aspects, in the next section a state feedback controller will be proposed. However the minimum time controller is useful to determine lower bounds for the engagement duration, depending on the maximum allowable derivative of the control force, a typical technological constraint.

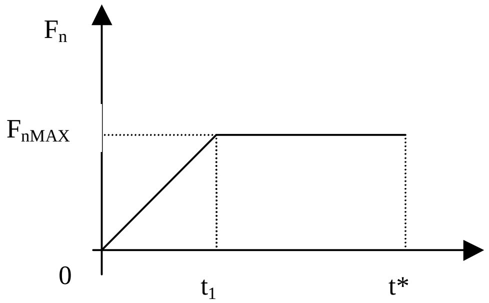

Figure 1. Possible behaviors of the normal force for the minimum engagement time controller when the lock-up is obtained after that the force F_n has reached its maximum value F_{nMAX}.

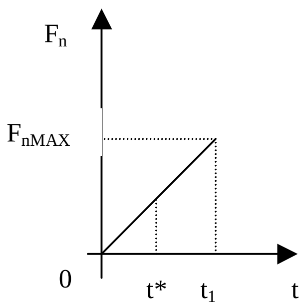

Figure 2. Normal force for the minimum engagement time controller when the lock-up is obtained before that the force F_n reaches its maximum value F_{nMAX}.

LQ CONTROLLER – The controller is obtained by applying the Linear Quadratic control theory [9] to the system (9)-(11). The objective function to be minimized is the following:

$$J = \int_0^{t^*} (q x_2^2(t) + r u^2(t)) dt \tag{12}$$

where t^* is an *a priori* chosen time instant, and q and r are weight constants; the engagement must be forced through the final time constraint $x_2(t^*)=0$.

The control coming out by the application of this technique has the following expression (see Appendix for details):

$$u(t) = \gamma(t)^T x(t) + \rho(t) \tag{13}$$

where $\gamma(t)$ is a vector function obtained by the integration of a differential Riccati equation and $\rho(t)$ is a scalar function depending both on the initial conditions and on the values of the input and load torques. In real applications it is obviously unrealistic to assume the availability of on-line computational power sufficient to solve the Riccati equation (a matrix differential equation which must be solved backwards in time). To overcome this problem it is possible to compute off-line the feedback gains and to find the coefficients of the polynomials which approximate their time evolution. These coefficients can than be stored in a look-up-table whose entries are the input torque, the load torque and the initial conditions, thus allowing actual implementation of the controller.

It is important to stress that u is the time derivative of the force F_n. In order to obtain the force to be applied to the clutch disk an integration is still needed. Therefore, the value of the control u will be zero when the lock-up is reached, i.e. $u(t^*)=0$, so that the force is kept constant to its maximum value for $t>t^*$.

SIMULATIONS

SIMULINK/STATEFLOW SCHEME – The simulation scheme used in this work is an evolution of that one presented in [10]. A general Simulink-like block scheme is reported in Figure 3. The “Scheduler” is a Stateflow block which enables only one of the two Simulink subsystems, i.e. “Slipping” (which integrates the system equations (1)-(2)) and “Locked” (which integrates the system equation (4)), depending on the difference between the crankshaft and the mainshaft rotor speeds.

PARAMETERS – With reference to a medium size car, the following parameters have been chosen:

$$I_e = 0.2 \text{ Kg m}^2,\ I_v = 0.7753 \text{ Kg m}^2,$$

$$b_e = b_v = 0.03 \text{ N m sec},$$

$$\mu_d = 0.4,\ k = 0.098 \text{ m},\ F_{nmax} = 5000 \text{ N}.$$

Moreover, different values of the input torque and load torque have been considered.

SIMULATION RESULTS – The engagement process during standing starts has been considered. Figure 4 shows the crankshaft and vehicle rotor speeds under the LQ feedback control. Each group refers to a constant input torque step (respectively 100 Nm and 150 Nm) applied at t=0. Within each group, the different behaviors are determined by different values of the load torque (from 5 Nm to 30 Nm approximately).

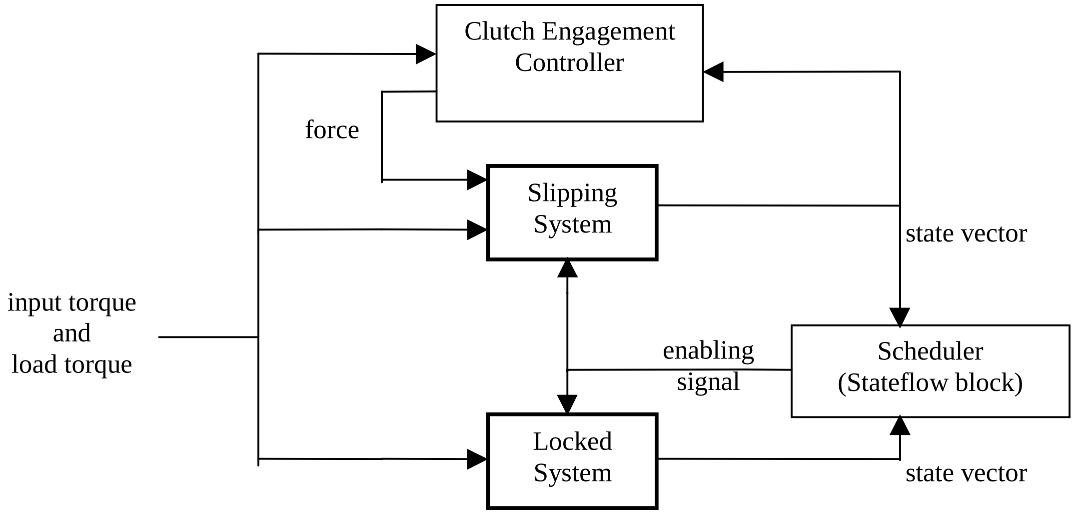

Figure 3. Simulink-like block scheme of the simulator: the enabling signal selects between the locked system and the slipping system depending on the clutch operating conditions.

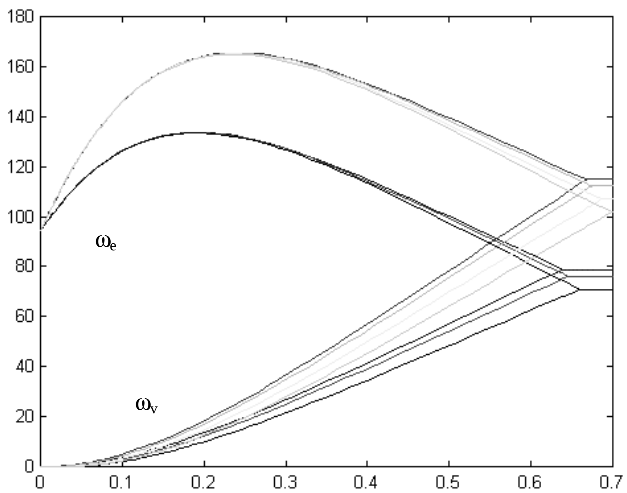

Figure 4. Crankshaft and vehicle rotor speeds (measured in rad/sec) as a function of time (measured in sec) during the LQ controlled engagement process at standing starts.

Figure 5 shows the normal force F_n time history (obtained as the integral of the control variable u) during one of the previous simulations.

In order to compare the improvements achievable with the LQ controller with respect to those of the minimum time controller, two parameters have been considered: the dissipated energy and the comfort. The former is computed through the expression

$$E_d = \int_0^{t^*} k x_2(t) x_3(t) dt \tag{14}$$

where we used the notation with the above defined state variables. The second comfort-related parameter is the difference between the angular accelerations of the two disks at lock-up. Table 1 summarizes the simulation results obtained with the minimum time controller, and Table 2 those achieved with the LQ controller for q=1000 and r=1. In both cases the initial crankshaft speed was $x_1(0)$=95 rad/sec and the maximum time derivative of the normal force has been chosen equal to u_{MAX}=8000 N/sec.

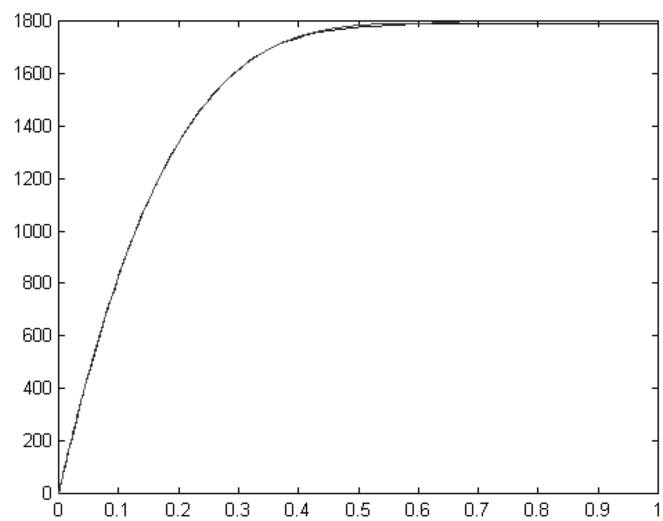

Figure 5. Normal force (measured in N) during one of the engagement simulations reported in Figure 3.

Note that for the minimum time controller a ramp signal has been added to the nominal value of the engine torque in order to maintain the crankshaft speed larger than its minimum value (approximately 70 rad/sec). It is important to stress that this fuel consuming action it is not needed in the case of the LQ controller.

Table 1. *Simulation results obtained with the minimum time controller.*

T_{in} [Nm]	T_l [Nm]	t* [sec]	$F_n(t^*)$ [N]	E_d [J]	$\dot{x}_2(t^*)$ [rad/sec^2]
100 + 250 t	4.8	0.39	3168	4890	960
100 + 250 t	10	0.40	3190	4998	963
100 + 250 t	20	0.40	3233	5215	970
150 + 150 t	4.8	0.45	3616	8216	1145
150 + 150 t	10	0.45	3637	8378	1156
150 + 150 t	20	0.46	3678	8698	1176
150 + 150 t	30	0.46	3718	9029	1197

Table 2. *Simulation results obtained with the LQ controller.*

T_{in} [Nm]	T_l [Nm]	t* [sec]	$F_n(t^*)$ [N]	E_d [J]	$\dot{x}_2(t^*)$ [rad/sec^2]
100	4.8	0.64	1304	4757	308
100	10	0.65	1312	4885	307
100	20	0.66	1330	5136	304
150	4.8	0.67	1789	8270	363
150	10	0.67	1798	8455	361
150	20	0.69	1817	8817	359
150	30	0.70	1835	9186	357

The two tables allow to compare the performance of the two controllers. Let us consider, for instance, the first rows of the two tables. It is apparent that the minimum time controller allows one to reach the lock-up in a shorter time and the dissipated energies of the two controlled systems are almost the same. However, two important advantages are obtained with the LQ controller: both $F_n(t^*)$ and $\dot{x}_2(t^*)$ are much lower than those obtained with the minimum time controller. Since the mechanical powertrain torsional oscillations generated after lock-up will depend on $\dot{x}_2(t^*)$ [4]-[5], it is possible to conclude that the LQ controller ensures a better driver comfort during and soon after the engagement process.

CONCLUSION

A Linear Quadratic state feedback controller for dry clutch engagement has been proposed. The direct use of the state variables (engine speed and clutch disk speed) allows one to design the controller parameters so that a fast engagement process with small dissipated energy and good comfort can be achieved (see Table 1 and Table 2 for a quantitative analysis).

The controller implementation seems possible due to its simple structure, consisting of a state feedback component and a feedforward compensation. In particular, the feedback gains are scheduled in order to consider different load torque and engine torque.

The performance of the engagement process is also influenced by the throttle command (in this paper modeled by the engine torque). The LQ controller, in order to properly control the engagement process, does not needs any further throttle command during the engagement process, thus ensuring a reduction of fuel consumption with respect to open loop control strategies.

Future work will consider the elastic transmission in the powertrain model and the effects of uncertainties of the system parameters on the feedback controller.

ACKNOWLEDGMENTS

Authors would like to thank Dr. Francesco Cimmino and Dr. Fabrizio Amisano from Magneti Marelli (Venaria, Italy) for the interesting discussions and the clutch parameters data, and Dr. Valeria Chiusolo for her help in producing the simulation results.

REFERENCES

1. B. K. Powell, K. E. Bailey, S. R. Cikanek, “Dynamic Modeling and Control of Hybrid Electric Vehicle Powertrain Systems”, *IEEE Control Systems Magazine*, October 1998, p. 17-33.
2. C. Bader, “Powertrain Dynamics – Progress on the Use and Development of the Computer Aided Gearshift Systems”, *Proc. of Transportation Electronics*, 1990, p. 375-385 (paper #901160).
3. M. Schwab, “Electronically-Controlled Transmission Systems – Current Position and future Developments”, *Proc. of Transportation Electronics*, 1990, p. 335-342 (paper #901156).
4. A. Szadkowski, R. B. Morford, “Clutch Engagement Simulation: Engagement Without Throttle”, *SAE Technical Paper Series 920766*, p. 103-117, 1992.
5. G. Ercole, G. Mattiazzo, S. Mauro, M. Velardocchia, F. Amisano, R. Flora, “Co-operating Clutch and Engine Control for Servoactuated Shifting through Fuzzy Supervisor”, *SAE Technical Paper Series 1999-01-0746*, p. 101-109, 1999.
6. R. Hibino, M. Osawa, M. Yamada, K. Kono, M. Tanaka, “H• Control Design for Torque-Converter-Clutch Slip System”, *Proc. of 35th IEEE Conference on Decision and Control*, Kobe, Japan, 1996, p. 1797-1802.
7. H. D. Lee, S. K. Sul, H. S. Cho, J. M. Lee, “Advanced Gear Shifting and Clutching Strategy for Parallel Hybrid Vehicle with Automated Manual Transmission”, *Proc. of 33rd IEEE Industry Applications Conference*, 1998, p. 1709-1714.
8. J. Slicker, R. N. K. Loh, “Design of Robust Vehicle Launch Control System”, *IEEE Trans. on Control System Technology*, vol. 4, no. 4, 1996, p. 326-335.

9. A. E. Bryson, Y. C. Ho, *Applied Optimal Control*, Hemisphere Publ. Corporation.
10. “Using Simulink and Stateflow in Automotive Applications”, The Mathworks Inc., 1998.

NOMENCLATURE

I_e: engine inertia
ω_e: crankshaft rotor speed
T_{in}: engine torque
b_e: crankshaft friction
T_{cl}: clutch torque
I_v: vehicle moment of inertia
ω_v: clutch disk rotor speed
b_v: clutch disk friction
T_l: equivalent load torque
F_n: clutch normal force
x_1: ω_e
x_2: ω_e-ω_v
x_3: F_n
u: time derivative of F_n
R: equivalent disk ratio
μ_d: dynamic friction coefficient
k: clutch torque constant
J: objective function of the LQ controller
q* and *r: weight constants of the LQ controller
$\gamma(t)$: feedback controller gain vector
$\rho(t)$: feedforward controller gains
E_d: energy dissipated during engagement

CONTACT

Luigi Glielmo
glielmo@unina.it

Francesco Vasca
vasca@unina.it

APPENDIX

The system (9)-(11) can be rewritten into the following compact form

$$\dot{x}(t) = Ax(t) + Bu(t) + \Gamma \tag{A.1}$$

where we assumed the input torque and the load torque to be constant (in the vector *G*). We look for a control which minimizes the performance index

$$J = \frac{1}{2}\int_0^{t^*} (x^T(t)Q(t)x(t) + u^T(t)R(t)u(t))\, dt \tag{A.2}$$

Using the Hamiltonian approach and the so called adjoint equation it is simple to show that this corresponds to solve the following set of differential equations [9]

$$\begin{cases} \dot{x}(t) = Ax(t) + BR^{-1}B^T\lambda(t) + \Gamma \\ \dot{\lambda}(t) = Qx(t) - A^T\lambda(t) \end{cases} \tag{A.3}$$

with the set of conditions

$$x(0) = x_0, \quad x_2(t^*) = 0, \quad \lambda_1(t^*) = 0, \quad \lambda_3(t^*) = 0. \tag{A.4}$$

Some of the conditions (A.4) are given at the initial time instant and some at the final one *t**. Moreover $\lambda_2(t^*)$, $x_1(t^*)$ and $x_3(t^*)$ are not given and the controller must also take into account the presence of the vector Γ in (A.1). Both these problems can be solved by imposing

$$\lambda(t) = -P(t)x(t) + M(t)\nu + h(t) \tag{A.5}$$

$$\psi = U(t)x(t) + F(t)\nu + K(t) \tag{A.6}$$

where $\psi = x_2(t^*)$, $\nu = \lambda_2(t^*)$, $P(t)$ is the matrix that solves the Riccati equation, $h(t)$ has been introduced in order to compensate for the presence of the disturbance vector Γ, $M(t)$ is used to solve the problem of the unknown final condition $\lambda_2(t^*)$. By computing (A.5)-(A.6) at t^* one obtains that $P(t^*)$, $h(t^*)$, $F(t^*)$ and $K(t^*)$ must all be zero and

$$M(t^*) = \begin{pmatrix}0 & 1 & 0\end{pmatrix}^T, \qquad U(t^*) = \begin{pmatrix}1 & 0\end{pmatrix} \tag{A.7}$$

By substituting (A.5)-(A.6) in (A.3), after simple algebraic manipulations one obtains the following three matrix differential equations:

$$-\dot{P}(t) - A^TP(t) - P(t)A + P(t)BR^{-1}B^TP(t) - Q = 0 \tag{A.8}$$

$$P(t)BR^{-1}B^TM(t) - \dot{M}(t) - A^TM(t) = 0 \tag{A.9}$$

$$P(t)BR^{-1}B^Th(t) + P(t)\Gamma - \dot{h}(t) - A^Th(t) = 0 \tag{A.10}$$

which can be solved backward in time from the above terminal conditions. Now, by differentiating (A.6) we have

$$\dot{U}(t)x(t) + U(t)\dot{x}(t) + \dot{F}(t)\nu + \dot{K} = 0 \tag{A.11}$$

and, by using (A.3)

$$\begin{aligned} &[\dot{U}(t) + U(t)A - U(t)BR^{-1}B^TP(t)]x(t) = \\ &= [-\dot{F}(t) - U(t)BR^{-1}B^TM(t)]\nu - \dot{K}(t) \\ &\quad - U(t)BR^{-1}B^Th(t) - U(t)\Gamma \end{aligned} \tag{A.12}$$

that holds for any $x(t)$ and ν. Therefore

$$\dot{U}(t) + U(t)A - U(t)BR^{-1}B^TP(t) = 0 \tag{A.13}$$

$$-\dot{F}(t) - U(t)BR^{-1}B^TM(t) = 0 \tag{A.14}$$

$$-\dot{K}(t) - U(t)BR^{-1}B^Th(t) - U(t)\Gamma = 0 \tag{A.15}$$

which allow to determine $U(t)$, $F(t)$ and $K(t)$. From (A.13), (A.9) and the final conditions on $U(t)$ and $M(t)$, one can conclude that $U(t) = M^T(t)$. From (A.6), since n is a constant one obtains

$$\nu = F^{-1}(0)[\psi - U(0)x_0 - K(0)] \tag{A.16}$$

Finally the control variable $u(t)$ has the following expression, resembling (13),

$$u(t) = R^{-1}B^T\lambda(t) = R^{-1}B^T(-P(t)x(t) + M(t)\nu + h(t)) \tag{A.17}$$

where $P(t)$, $M(t)$, n and $h(t)$ can be obtained as described above.