P.1

1- $\overrightarrow{OI} = \begin{pmatrix} 3,5 \\ -2 \end{pmatrix}_B$ $\overrightarrow{OJ} = \begin{pmatrix} -3,5 \\ -2 \end{pmatrix}_B$

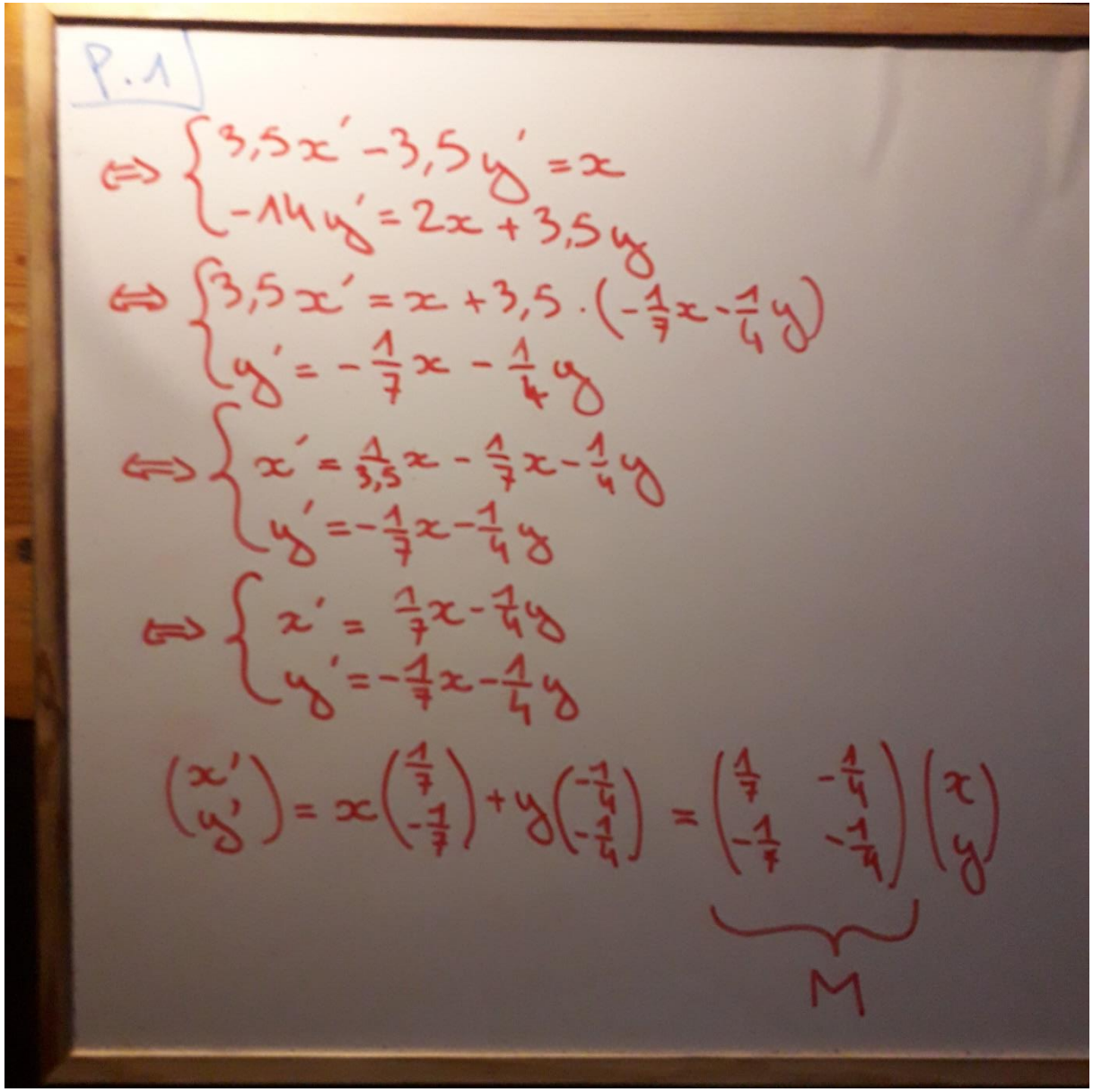
2- $\vec{w} = \begin{pmatrix} x \\ y \end{pmatrix}_B = \begin{pmatrix} x' \\ y' \end{pmatrix}_{B'}$

$= x' \begin{pmatrix} 3,5 \\ -2 \end{pmatrix}_B + y' \begin{pmatrix} -3,5 \\ -2 \end{pmatrix}_B$

$\Leftrightarrow \begin{cases} x = 3,5x' - 3,5y' \\ y = -2x' - 2y' \end{cases}$

$\Leftrightarrow \left(\begin{array}{cc|c} 3,5 & -3,5 & x \\ -2 & -2 & y \end{array}\right) \begin{matrix} l_1 \\ 3,5l_2 + 2l_1 \end{matrix}$

$\Leftrightarrow \left(\begin{array}{cc|c} 3,5 & -3,5 & x \\ 0 & -14 & 2x + 3,5y \end{array}\right)$


P.1
$\Leftrightarrow \begin{cases} 3{,}5x' - 3{,}5y' = x \\ -14y' = 2x + 3{,}5y \end{cases}$
$\Leftrightarrow \begin{cases} 3{,}5x' = x + 3{,}5 \cdot \left(-\frac{1}{7}x - \frac{1}{4}y\right) \\ y' = -\frac{1}{7}x - \frac{1}{4}y \end{cases}$
$\Leftrightarrow \begin{cases} x' = \frac{1}{3{,}5}x - \frac{1}{7}x - \frac{1}{4}y \\ y' = -\frac{1}{7}x - \frac{1}{4}y \end{cases}$
$\Leftrightarrow \begin{cases} x' = \frac{1}{7}x - \frac{1}{4}y \\ y' = -\frac{1}{7}x - \frac{1}{4}y \end{cases}$
$\begin{pmatrix} x' \\ y' \end{pmatrix} = x\begin{pmatrix} \frac{1}{7} \\ -\frac{1}{7} \end{pmatrix} + y\begin{pmatrix} -\frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{7} & -\frac{1}{4} \\ -\frac{1}{7} & -\frac{1}{4} \end{pmatrix}}_{M} \begin{pmatrix} x \\ y \end{pmatrix}$

P.1

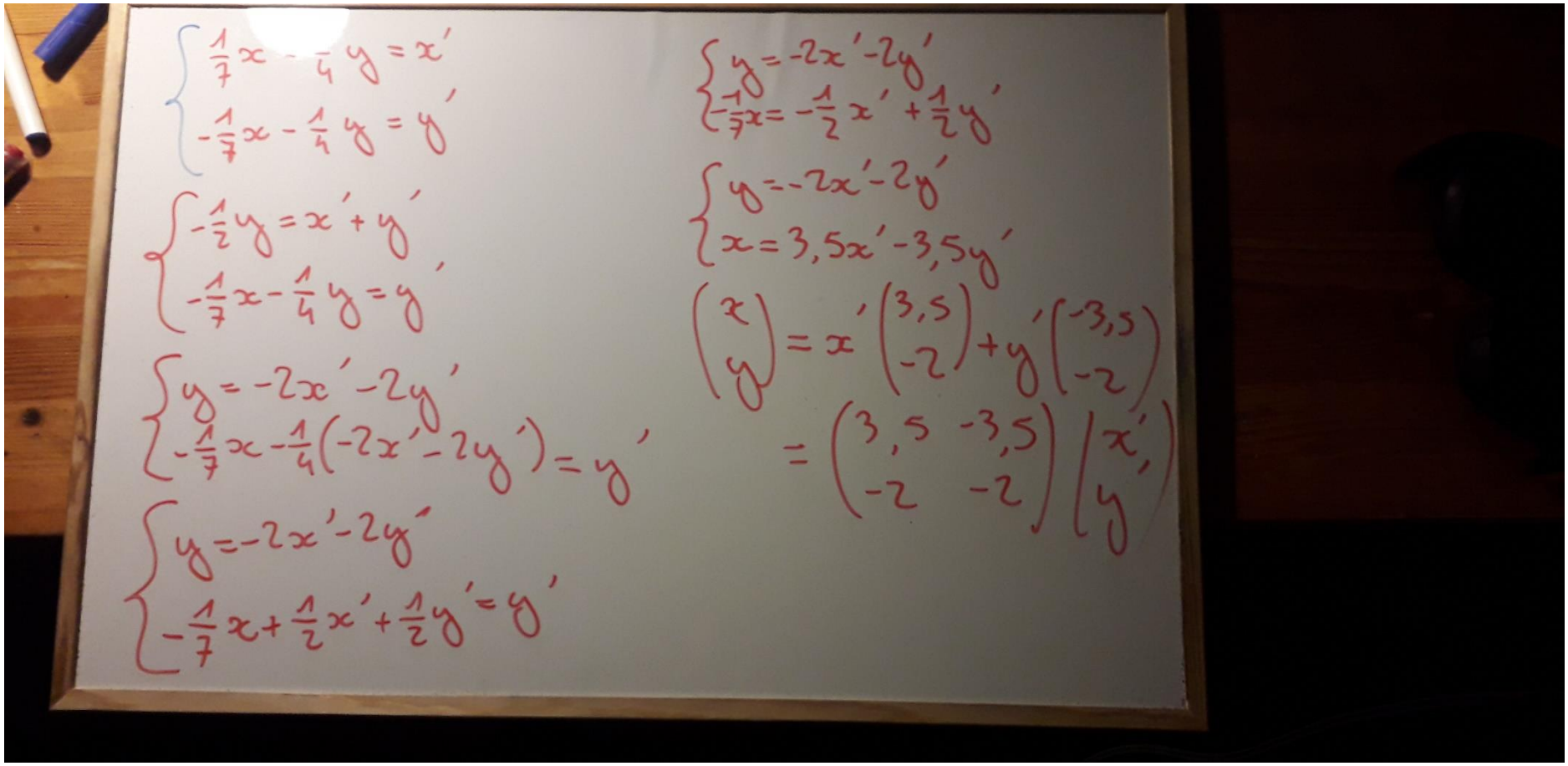
3- $A = M \cdot \begin{pmatrix} -3,5 \\ 2 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} & -\frac{1}{4} \\ -\frac{1}{7} & -\frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} -3,5 \\ 2 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} \cdot (-3,5) - \frac{1}{4} \cdot 2 \\ -\frac{1}{7} \cdot (-3,5) - \frac{1}{4} \cdot 2 \end{pmatrix}_{B'} = \begin{pmatrix} -\frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \end{pmatrix}_{B'} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}_{B'}$

$B = M \cdot \begin{pmatrix} 0 \\ -4 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} & -\frac{1}{4} \\ -\frac{1}{7} & -\frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -4 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} \cdot 0 - \frac{1}{4} \cdot -4 \\ -\frac{1}{7} \cdot 0 - \frac{1}{4} \cdot -4 \end{pmatrix}_{B'} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}_{B'}$

$C = M \cdot \begin{pmatrix} 6 \\ 2 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} & -\frac{1}{4} \\ -\frac{1}{7} & -\frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} \cdot 6 - \frac{1}{4} \cdot 2 \\ -\frac{1}{7} \cdot 6 - \frac{1}{4} \cdot 2 \end{pmatrix}_{B'} = \begin{pmatrix} \frac{6}{7} - \frac{1}{2} \\ -\frac{6}{7} - \frac{1}{2} \end{pmatrix}_{B'} = \begin{pmatrix} \frac{5}{14} \\ -\frac{19}{14} \end{pmatrix}_{B'}$

$D = M \cdot \begin{pmatrix} -3,5 \\ -4 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} & -\frac{1}{4} \\ -\frac{1}{7} & -\frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} -3,5 \\ -4 \end{pmatrix}_B = \begin{pmatrix} \frac{1}{7} \cdot (-3,5) - \frac{1}{4} \cdot (-4) \\ -\frac{1}{7} \cdot (-3,5) - \frac{1}{4} \cdot (-4) \end{pmatrix}_{B'} = \begin{pmatrix} -\frac{1}{2} + 1 \\ \frac{1}{2} + 1 \end{pmatrix}_{B'} = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \end{pmatrix}_{B'}$

P1 – Q4 :


$\begin{cases} \frac{1}{7}x - \frac{1}{4}y = x' \\ -\frac{1}{7}x - \frac{1}{4}y = y' \end{cases}$
$\begin{cases} -\frac{1}{2}y = x' + y' \\ -\frac{1}{7}x - \frac{1}{4}y = y' \end{cases}$
$\begin{cases} y = -2x' - 2y' \\ -\frac{1}{7}x - \frac{1}{4}(-2x' - 2y') = y' \end{cases}$
$\begin{cases} y = -2x' - 2y' \\ -\frac{1}{7}x + \frac{1}{2}x' + \frac{1}{2}y' = y' \end{cases}$
$\begin{cases} y = -2x' - 2y' \\ -\frac{1}{7}x = -\frac{1}{2}x' + \frac{1}{2}y' \end{cases}$
$\begin{cases} y = -2x' - 2y' \\ x = 3{,}5x' - 3{,}5y' \end{cases}$
$\begin{pmatrix} x \\ y \end{pmatrix} = x' \begin{pmatrix} 3{,}5 \\ -2 \end{pmatrix} + y' \begin{pmatrix} -3{,}5 \\ -2 \end{pmatrix}$
$= \begin{pmatrix} 3{,}5 & -3{,}5 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$

$$f : \mathbb{R} \to \mathbb{R}$$

$$x, y \longmapsto \frac{1}{7}x - \frac{1}{4}y,\ -\frac{1}{7}x - \frac{1}{4}y$$

$$f^{-1} : \mathbb{R} \to \mathbb{R}$$

$$x, y \longmapsto \frac{7}{2}x - 2y,\ -\frac{7}{2}x - 2y$$

$$\begin{pmatrix} 3{,}5 & -3{,}5 \\ \cdot & -7 \end{pmatrix}$$