On the Efficacy of Optimized Exit Rule for Mean Reversion Trading

Donovan Lee* Tim Leung†

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Abstract

We investigate the effect of using an optimized exit rule on pairs trading. For every asset pair, we optimize the positions so that resulting intraday portfolio value is best fitted to an Ornstein-Uhlenbeck (OU) process through maximum likelihood estimation. Using eight asset pairs, we examine the risks and returns of pairs trading strategies with and without an optimize exit rule. We provide empirical evidence that using an optimized exit rule improves the profitability of the trades and reduces turnovers.

*Mathematics Department, Duke University, Durham, NC 27708. Email: donny.lee.wm@gmail.com

†Applied Mathematics Department, University of Washington, Seattle WA 98195. Email: timleung@uw.edu. Corresponding author.

1 Introduction

Pairs trading is among the most popular trading strategies in many markets, ranging from equities and ETFs to currencies and futures markets. It involves taking simultaneous positions in two correlated assets. The premise is that while it is difficult to model the time-series of a single asset, a pairs position may exhibit mean reversion that can be better modeled. In other words, pairs trading is a market-neutral strategy that seeks to profit from the price convergence between the two assets.

How to find mean reverting assets

The pair of assets must be adequately correlated in order for the pair to be profitably traded. Selection of assets can be based on market observations and intuitions, such as risk-on assets trading in the same direction, or stocks in the same industry. Some practical examples of mean-reverting prices can be found in a number of empirical studies, including pairs of stocks and/or ETFs (Gatev et al., 2006; Avellaneda and Lee, 2010; Montana and Triantafyllopoulos, 2011; Kang and Leung, 2017; Leung and Li, 2016), divergence between futures and its spot (Brennan and Schwartz, 1990; Dai et al., 2011), and spreads between physical commodity and commodity stocks/ETFs (Kanamura et al., 2010; Triantafyllopoulos and Montana, 2011; Dunis et al., 2013). There are also automated approaches for identifying mean-reverting portfolios (d'Aspremont, 2011; Leung et al., 2020).

Pairs trading is an intuitive strategy that is easy for traders to understand. However, a successful pairs trading strategy requires intelligent position sizing, good timing for entry and exit, and efficient trade execution. While observing the prevailing market prices, a trader can choose to establish a pairs trading position immediately or wait. To start the trade, the trader needs to determine the sizes of the two positions. After starting a pairs trade, the trader will need to decide when to close the positions. This motivates us to design a procedure for starting and closing a pairs trade with optimized portfolio and exit rule.

In this paper, we present a framework to examine the profitability of pairs trading. In particular, we illustrate how to adjust positions to enhance mean reversion and investigate the effect of using an optimized exit rule on the risks and returns of different pairs trading strategies. Assets or securities considered in this study include not only stocks and ETFs, but also currencies, index futures and commodity futures. Working with intraday prices, we fit any given portfolio value to an Ornstein-Uhlenbeck (OU) process through maximum likelihood estimation (MLE). Using the rule of entry and exit at a multiple of the standard deviation of the portfolio price, we investigate whether using an optimal exit rule improves the profitability of the trades. We start with a canonical mean reversion entry and exit at a standard deviation of its price, study its short comings and thereafter, assess the difference in performance when we liquidate at an optimal exit price given by analytical solutions. No attempts have been made to optimize for performance other than the pair ratio and parameter selection as part of the OU model fitting. Using the rule of entry and exit at a multiple of the standard deviation of the portfolio price, we investigate whether using an optimal exit rule will improve profitability and lead to fewer trades and thus lower trading costs.

In the literature, Nath (2003) examines the risk and rewards of the high frequency pairs trading of U.S. treasury securities for hedge funds. Gatev et al. (2006) provides an empirical study on the performance of pairs trading in the equity market. Do and Faff (2012) investigates the impact of trading costs on pairs trading profitability in the U.S. equity market. We also refer to the survey by Krauss (2017) that reviews the literature on pairs trading frameworks.

The rest of the paper is structured as follows. In Section 2, we describe the asset pairs included in our study and explain our data collection and synchronization. In Section 3, we discuss the procedure for portfolio construction along with exit rules. Then in Section 4, we present the implementation of the trading strategies. We examine the performance of strategies with and without optimized exit rule in Section 5. Conclusions are are provided in Section 6.

2 Traded Assets and Price Data

In this section, we discuss the assets used for our pairs trading strategies and provide a summary of the price data.

2.1 Asset Pairs and Data

Based on our market observations and intuitions, we focus our study to a collection of asset pairs on which we'll test the trading models. The descriptions of the ticker symbols and associated exchange trading hours are shown in Table 1.[1]

USDJPY / NK : The trading premise is based on the positive correlation between these two assets. Given that Japan is export driven, when JPY strengths against USD (i.e. USDJPY weakens), Nikkei is expected to decline because of lower consumption. Conversely, when Nikkei, which is considered a risk sentiment indicator, weakens, JPY strengthens and so USDJPY weakens.

USDCAD / CL : Canada is the 7th largest oil producer in the world.[2] This sugguests a positive correlation between the price of oil and CAD. When crude futures price increases (resp. decreases), Canadian dollar appreciates (resp. depreciates) and so USDCAD decreases (resp. increases).

CL / USO : USO is the largest crude oil ETF that consists of a mixture of West Texas Intermediate (WTI) futures of different maturities. As such, USO and crude oil futures prices are expected to have consistent positive correlation.

GC / SI : Pairs trading between gold spot and silver spot, while staying neutral to metals exposure.

AUDJPY / SPX : This is pairs trading between risk-on currency AUDJPY and equity S&P500 futures. The later is commonly viewed as a barometer of general risk-on appetite of investors.

USDCHF / GC : Swiss National Bank backs up a portion of their Swiss franc holdings with gold thus suggesting a correlation between the two assets. When gold increases in its value, CHF is expected to strengthen (i.e. USDCHF weakens), and vice versa.

C / GS : Citigroup and Goldman are two large-cap stocks are in the banking industry.

AAPL / FB : Apple and Facebook are two large-cap stocks in the technology sector.

Our data set consists of minute-stamped traded prices over the time period from Jan 2012 to Dec 2019.[3] When developing our trading strategies, we choose hourly sampling of the market prices for our tests and analyses. This is at a high enough frequency to differentiate from daily strategies while maintaining a good signal to noise ratio.

As shown in Table 1, the sampling period for FX and futures is Sun 17:00 to Fri 16:00, which is longer than that for equities and ETFs, which is Mon to Fri 9:30–15:30. When a FX or futures is paired with an equity or ETF, sample times follow the latter. Our models are tested during New York's trading session when all assets aforementioned are trading.

[1]For more details, we refer the reader to the following websites: NK: http://www.jpx.co.jp/english/derivatives/rules/trading-hours/; CL: http://www.cmegroup.com/trading/energy/crude-oil/light-sweet-crude_contract_specifications.html; GC: http://www.cmegroup.com/trading/metals/precious/gold_contract_specifications.html; SI: http://www.cmegroup.com/trading/metals/precious/silver_contract_specifications.html

[2]Source: EIA International Energy Statistics

[3]Futures contracts are rolled over as follows: Define $C_{1,t}$ and $C_{2,t}$ as the near-term and far-term futures contracts prices. At the rollover time t', the cumulative return (from the near-term contract) is defined by $S_{t'} = \frac{C_{1,t'}}{C_{1,t'-1}} \cdot S_{t'-1}$ so that the return on the next day (from the new far-term contract) is given by $S_{t'+1} = \frac{C_{2,t'+1}}{C_{2,t'}} \cdot S_{t'}$.

Symbol	Description	Trading Hours
USDJPY	US Dollar valued against Japanese Yen	Sun 17:00 to Fri 17:00
AUDJPY	AUD Dollar valued against Japanese Yen	Sun 17:00 to Fri 17:00
USDCAD	US Dollar valued against Canadian Dollar	Sun 17:00 to Fri 17:00
USDCHF	US Dollar valued against Swiss Franc	Sun 17:00 to Fri 17:00
NK	Nikkei 225 on TSE	Sun to Fri, 20:00 to 16:30
CL	Crude futures on CME	Sun to Fri, 18:00 to 17:00
USO	United States Oil Fund ETF	Mon to Fri, 9:30 to 16:00
GC	Gold spot valued against US Dollar	Sun to Fri, 18:00 to 17:00
SI	Silver spot valued against US Dollar	Sun to Fri, 18:00 to 17:00
SPX	E-mini S&P 500 Futures traded on CME	Sun to Fri, 17:00 to 16:00
C	Citigroup Inc	Mon to Fri, 9:30 to 16:00
GS	Goldman Sachs Group Inc	Mon to Fri, 9:30 to 16:00
AAPL	Apple Inc	Mon to Fri, 9:30 to 16:00
FB	Facebook Inc	Mon to Fri, 9:30 to 16:00

Table 1: List of assets with their ticker symbols, descriptions, and trading hours. All times are Eastern Standard Times.

As some portfolios comprise of assets in different markets with different trading times, we apply the following rules for synchronization:

i) Use last observed minute traded price if price doesn't exist at sampling time.
ii) Use the intersection of sampling times if exchange trading hours are different.

3 Portfolio Construction and Exit Rules

All portfolios examined in this paper are constructed in the form:

$$X_t = S_t^1 - B\,S_t^2 \tag{1}$$

where S_t^1 and S_t^2 are the prices of the first and second assets respectively. The constant B is the pair ratio, which can be positive or negative. For instance, one can long two negatively correlated assets, or simultaneously long and short two positively correlated assets. Likewise, the portfolio value X_t can be positive or negative at different times.

To model the mean reverting price behavior of the portfolio value X_t, we fit it to an OU process. On the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with the historical probability measure $\mathbb{P}$, we consider the evolution of X_t as

$$dX_t = \mu(\theta - X_t)dt + \sigma dW_t \tag{2}$$

with constants $\mu, \sigma > 0$, $\theta \in \mathbb{R}$. Here, W is a standard Brownian motion under $\mathbb{P}$.

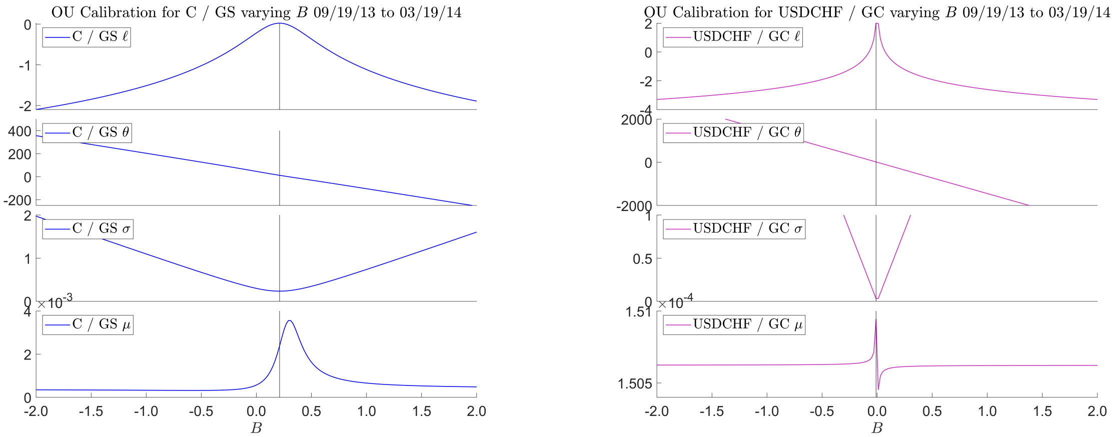
3.1 Optimizing Mean Reversion via MLE

For each choice of B, we observe the corresponding portfolio time series $(X_i^B)_{i=0,1,\ldots,n}$. Using the method of maximum likelihood estimation, we fit the observed portfolio values to an OU process and determine the model parameters (θ, μ, σ). Precisely, we maximize the average log-likelihood

$$\ell(\theta,\mu,\sigma|X_0^B, X_1^B, \ldots, X_n^B) = \frac{1}{n}\sum_{i=1}^{n} \ln f^{OU}\left(X_i^B|X_{i-1}^B; \theta, \mu, \sigma\right) \tag{3}$$

$$= -\frac{1}{2}\ln(2\pi) - \ln(\tilde{\sigma}) - \frac{1}{2n\tilde{\sigma}^2}\sum_{i=1}^{n}\left[X_i^B - X_{i-1}^B e^{-\mu\Delta t} - \theta(1-e^{-\mu\Delta t})\right]^2 \tag{4}$$

We denote this maximum average log-likelihood by $\hat{\ell}(\theta^*, \mu^*\sigma^*)$, where $(\theta^*, \mu^*\sigma^*)$ are the estimated parameters for the OU model. Next, we determine the optimal B that maximizes the



(a) For C / GS, optimal $B = 0.211$

(b) For USDCHF / GC, optimal $B = -0.005$

Figure 1: Estimated log-likelihood $\hat{\ell}(\hat{\theta}, \hat{\sigma}, \hat{\mu})$ for portfolio pairs C / GS and USDCHF / GC, with B varying between -2 and 2.

average log-likelihood. The range of B can be different for different portfolios depending on the scale of S_t^1 and S_t^2.

By varying B, the time series $X_t = S_t^1 - BS_t^2$ may fit better or worse to the OU process in terms of maximum liklihood. Figure 1 shows how the measure of fit ℓ varies with B and how we attain $\text{argmax}_B\{\ell\}$. Consider a 6-month window, derive B that gives the maximum average log-likelihood ℓ. We illustrate this for the pairs C / GS and USDCHF/ GC in Figure 1. We also plot the portfolio series $X_t = S_t^1 - BS_t^2$ over the same 6-month window in Figure 2. Notice how each path resembles an OU process that reverts back to its respective mean θ.

We adopt the practice of using a single lookback of observed portfolio prices for all our pairs. This lookback is decided a priori to any PnL calculations so as to prevent data snooping. On the other hand, as time progresses, re-estimation may be necessary. For any lookback period of N months, we examine model fitting through the following steps:

1. At every NY open, using N-month historical prices of S^1 and S^2, calculate at the current time t the optimal pair ratio, given by

$$B_t = \text{argmax}_B\{\ell\} \tag{5}$$

   where $B \in [-2, 2]$.

2. To test whether these estimated OU parameters reflect indeed a good fit to the model, simulate 100 (or more) OU price paths using the estimated parameters with the same time discretization and time window.

3. Calculate the average log-likelihood, denoted by $\hat{\ell}_{sim}$, from the simulated paths and compare it with the maximum average log-likelihood $\hat{\ell}$ from the empirical prices. Specifically, we compute $|\hat{\ell} - \hat{\ell}_{sim}|$.

We conduct the MLE along with pair ratio optimization with 3-month, 6-month, and 12-month lookback periods. The results are summarized in Table 2. As we can see, 3-month lookback windows give the best fits for 5 out of 8 pairs. We note that the difference in the empirical and simulated likelihoods, measured by $|\hat{\ell} - \hat{\ell}_{\text{sim}}|$, tends to decrease as the lookback period lengthens. In the interest of capturing the most recent price history while maintaining good fits to the model, we choose to use 3-month lookbacks the default setting for OU model fitting.

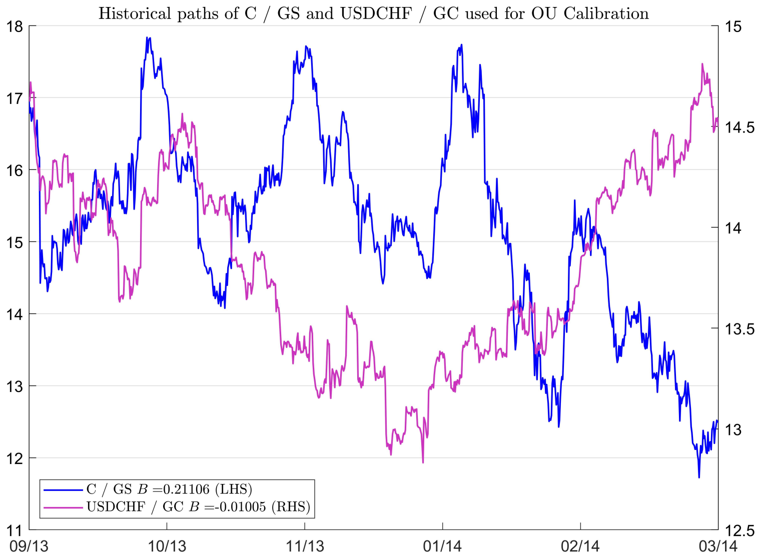


Figure 2: Historical price path of portfolio $X_t = S_t^1 - B \cdot S_t^2$ for the (S_t^1, S_t^2) pairs C / GS and USDCHF / GC, where B is the respective B that gives $\hat{\ell}$ (Figure 1) for that portfolio.

Asset pairs	3-month lookback $\hat{\ell}$	3-month lookback $\lvert\hat{\ell} - \hat{\ell}_{\text{sim}}\rvert$	6 month lookback $\hat{\ell}$	6 month lookback $\lvert\hat{\ell} - \hat{\ell}_{\text{sim}}\rvert$	12 month lookback $\hat{\ell}$	12 month lookback $\lvert\hat{\ell} - \hat{\ell}_{\text{sim}}\rvert$
USDJPY / NK	**0.917**	0.103	0.850	0.067	0.737	0.045
USDCAD / CL	**5.382**	0.097	5.344	0.062	5.095	0.044
CL / USO	0.553	0.161	0.616	0.092	**0.630**	0.058
GC / SI	-2.151	0.096	**-2.137**	0.065	-2.177	0.044
AUDJPY / SPX	**0.744**	0.099	0.686	0.069	0.580	0.047
USDCHF / GC	**5.587**	0.096	5.544	0.063	5.478	0.044
C / GS	**0.422**	0.168	0.384	0.103	0.304	0.067
AAPL / FB	-0.617	0.135	-0.600	0.087	**-0.533**	0.059

Table 2: Average log-likelihoods for all 8 asset pairs over different lookback periods (3, 6, and 12 months). For each pair (row), the highest average log-likelihood is highlighted in bold. For the majority of the pairs portfolios, 3 months gives us the best fit by highest average $\hat{\ell}$.

3.2 Pairs portfolio series construction

The relationship between S^1 and S^2 can vary over time, so we periodically re-estimate the model parameters and re-optimize the value of B. Specifically, we consider a time-dependent version of B, denoted by B_t, that is updated at New York open. The changes in B_t over time will alter the portfolio time series X_t. Therefore, to facilitate the derivation of a trading rule on X_t, we also apply a simple mechanism to smooth out X_t.

Consider two New York open times (t_1, t_2) with $t_1 < t_2$. We determine the corresponding optimal pair ratios B_{t_1} and B_{t_2}, then $B_t = B_{t_1}$ for $t_1 \leq t < t_2$. Now consider the initial series $Y_t = S_t^1 - B_t S_t^2$. From here we calculate its log-returns and set it to 0 for when $\Delta B_t \neq 0$, $\Delta B_t = B_t - B_{t-1}$, namely at New York open. This gives the log-returns for X_t,

$$\log\left(\frac{X_t}{X_{t-1}}\right) = \begin{cases} \log\left(\frac{Y_t}{Y_{t-1}}\right) & \text{if } \Delta B = 0 \\ 0 & \text{if } \Delta B \neq 0 \end{cases}$$

3.3 Pairs Ratio and Optimal Exit level series

Given an existing pairs trading position, the trader seeks the best timing to close the position to capture the value X_τ at exit time τ. This leads to an optimal stopping problem. Specifically, the trader seeks to maximize

$$V(x) = \sup_{\tau \in T} \mathrm{E}\left\{e^{-r\tau}\left(X_\tau - c\right)\right\} \tag{6}$$

By solving this optimal liquidation problem, the trader determines the optimal time to exit a position. Leung and Li (2015) provide the analytic solution to this problem while the finite-horizon version of this problem requires numerical solutions (Kitapbayev and Leung (2017)).

The solution to the optimal liquidation problem (6) is given by

$$V(x) = \begin{cases} (b^* - c)\dfrac{F(x)}{F(b^*)} & \text{if } x \in (\infty, b^*) \\ x - c & \text{otherwise} \end{cases} \tag{7}$$

The corresponding optimal liquidation strategy is described by the critical price level b^* at which the trader should exit. It is determined from the equation:

$$F(b) = (b - c)F'(b) \tag{8}$$

where

$$F(x) := \int_0^\infty u^{\frac{r}{\mu}-1} e^{\sqrt{\frac{2\mu}{\sigma^2}}(\theta - x)u - \frac{u^2}{2}}\, du\,, \tag{9}$$

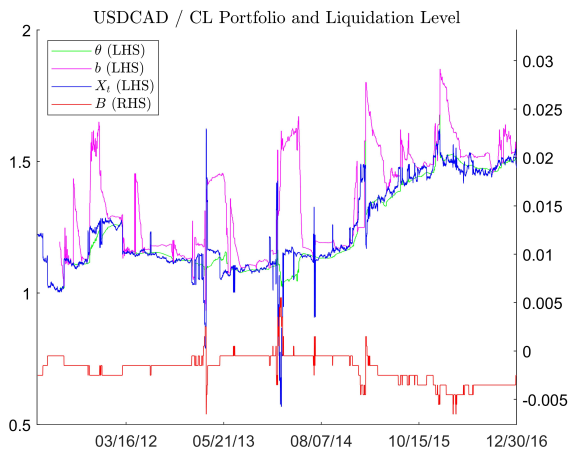
which is strictly increasing, strictly positive and convex.

Let us now examine how the pair ratio B_t, optimal exit level b_t, and long-run mean θ_t interact with the portfolio value X_t over time. In Figure 3, we present their time series for four different pairs, namely, USDCAD/CL, AUDJPY/SPX, GC/SI, and AAPL/FB. As we can see in all four examples, the portfolio value tends to fluctuates around the periodically estimated long-run mean θ_t over time, as is expected from the OU model. Prior to exit, the exit level b_t hovers above X_t, as seen in all the plots. Furthermore, when b_t and X_t converge, that means the trader exits the pairs trade temporarily. Often b_t starts above X_t and moves lower when X_t increases to a price that represents the critical liquidation price. In Figure 3a, the optimal exit level b resets to be higher from X_t then converges to X_t multiple times, implying immediate exits.

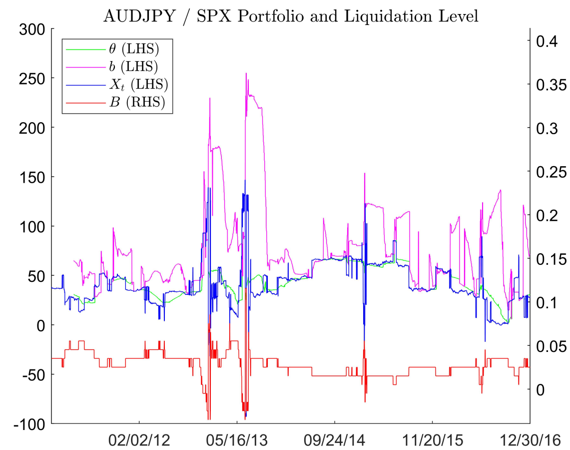
It's worth mentioning that the different nature in pair ratios B. Figure 3b shows that B_t for AUDJPY / SPX changes around every quarter. This implies that over the last decade or so, risk-on currency prices tend to follow risk-on sentiment, a common phenomenon followed by most trading desks. Conversely, Figure 3d shows more volatile B_t while keeping a mean of around 0.4. We infer that AAPL and FB are correlated but their correlation may break during earnings.

We also notice in Figure 3c that the optimal pair ratio is constant at $B_t = 1$ for the pair GC / SI. This suggests that gold and silver spot prices exhibit high level of co-movement. Compared to other pairs, a relatively stable pair ratio is advantageous since it reduces the need to adjust the positions over time and reduces transaction costs.

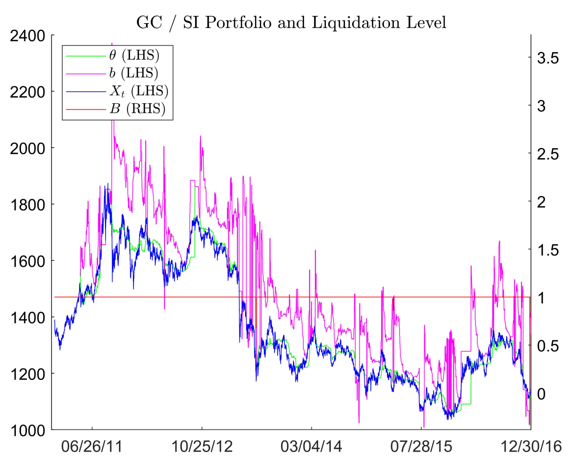
For each pair, we can now appreciate how we create a portfolio position based on B_t. Recall that our portfolio is $X_t = S_t^1 - B_t S_t^2$. A long position in X_t involves going long in S^1 and short in S^2 if $B > 0$. If $B < 0$, this means that one should long S^1 and long $|B|S^2$. For AUDJPY / SPX, $B_t > 0$, long X_t means long AUDJPY and short SPX. On the other hand, $B > 0$ implies that AUDJPY and SPX move in the same direction together, so we need to be long short in order for the portfolio to stay market neutral to the two assets. Conversely for USDCAD / CL, $B_t < 0$, long X_t means long USDCAD and long CL. USDCAD and CL move in opposite directions so we need to long both to stay market neutral.



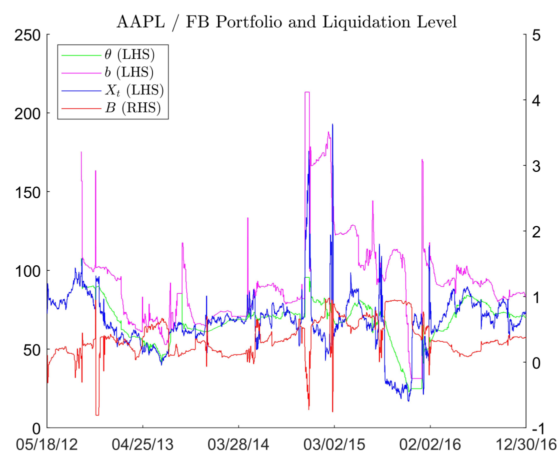
(a) b resets to be higher from X_t then drops down to meet X_t when exit occurs



(b) b adjusts over time as B_t changes rapidly



(c) $B = 1$ throughout the whole sample suggests a stable relationship between these two assets



(d) B is fast varying and b follows accordingly

Figure 3: Time series of daily updated OU parameters and b (magenta) for selected portfolio pairs. $X_t < b$ implies b is the optimal liquidation price level for a given long X_t position. $X_t > b$ implies market is trending and we trade using σ bands to exit long X_t.

4 Trade Generation

We proceed to describe our Baseline and Optimal Exit Price models. In calibrating θ, σ and μ for our OU process X_t, we decide, based on Table 2, on a 3 month lookback period.

4.1 Rebalancing and transaction costs assumptions

The slow varying nature of the pair ratio B means that the amounts of S^1 and S^2 that need to be adjusted every day is likely to be less than a unit share. As such, we assume fractional shares. Also, the rebalancing between S^1 and S^2 is assumed to costless. Nevertheless, we account for the costs of trade execution. Specifically, we assume losing 2bps, or 0.02%, of returns for a crossing the bid-ask spread at 100% of our trading asset under management (AUM). For example, suppose our AUM is US$10M. Buying US$10M of C shares from a cash position costs 0.01% of AUM, and buying US$10M of C shares from a short position of US$10M costs 0.02% of AUM.

To study the performance of the optimal liquidation level, consider the following trading rules. Define the followings:

– Sig S^1 is the percentage of our AUM which we assign to asset S^1
– Liq L is the indicator to tell us to use optimal exit rule
– $MA(S^1_t)$ is the 30 hour simple moving average of S^1_t
– $\sigma(S^1_t)$ is the 30 hour standard deviation of S^1_t
– liquadation level $b\,(\theta,\mu,\sigma)$ is the solution to $F(b) = (b-c)F'(b)$. See Theorem 4.2.

And recall that our portfolio is $X_t = S^1_t - B_t S^2_t$.

4.2 Baseline

At every New York open[4], update $(B_t, \theta_t, \mu_t, \sigma_t)$ and b_t.

At every hour t:

– If Sig $S^1_t = 0$, check for entry

$$\text{Sig } S^1_t = \begin{cases} 1, & \text{if } X_t < \text{MA}(X_t) - \sigma(X_t) \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

$$\text{Sig } S^2_t = -B_t \cdot \text{Sig } S^1_t \tag{11}$$

– If Sig $S^1_t = 1$, check for liquidation

$$\text{Sig } S^1_t = \begin{cases} 0, & \text{if } X_t > \text{MA}(X_t) + \sigma(X_t) \\ \text{Sig } S^1_{t-1}, & \text{otherwise} \end{cases} \tag{12}$$

$$\text{Sig } S^2_t = \begin{cases} 0, & \text{if } X_t > \text{MA}(X_t) + \sigma(X_t) \\ \text{Sig } S^2_{t-1}, & \text{otherwise} \end{cases} \tag{13}$$

[4] If the portfolio involves FX and futures, New York open means 9am EST. For equities and ETFs, New York open means 9:30am EST.

4.3 Optimal Exit Level

At every New York open, update model parameters $(\theta_t, \mu_t, \sigma_t)$ and pair ratio B_t, and then compute b_t via (8) using the data up to the current time t.

At every hour t:

– If Sig $S_t^1 = 0$, check for entry

$$\text{Sig } S_t^1 = \begin{cases} 1, & \text{if } X_t < \text{MA}(X_t) - \sigma(X_t) \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

$$\text{Liq } L_t = \begin{cases} 1, & \text{if } b_t > X_t \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

$$\text{Sig } S_t^2 = -B_t \cdot \text{Sig } S_t^1 \tag{16}$$

– If Sig $S_t^1 = 1$, check for liquidation

$$\text{Liq } L_t = \text{Liq } L_{t-1} \tag{17}$$

$$\text{Sig } S_t^1 = \begin{cases} 0, & X_t > \text{MA}(X_t) + \sigma(X_t) \text{ and } L_t = 0 \\ 0, & \text{if } X_t > b_t \text{ and } L_t = 1 \\ \text{Sig } S_{t-1}^1, & \text{otherwise} \end{cases} \tag{18}$$

$$\text{Sig } S_t^2 = \begin{cases} 0, & X_t > \text{MA}(X_t) + \sigma(X_t) \text{ and } L_t = 0 \\ 0, & \text{if } X_t > b_t \text{ and } L_t = 1 \\ \text{Sig } S_{t-1}^2, & \text{otherwise} \end{cases} \tag{19}$$

An explanation of signal generation for the optimal exit model is in order. Consider the GC / SI portfolio X_t, its $\sigma(X_t)$ bands and b in Figure 4. Lower Band is $\text{MA}(X_t) - \sigma(X_t)$ and Upper Band is $\text{MA}(X_t) + \sigma(X_t)$. Recall that b is our optimal liquidation level and we shall see that how we exit the position differs depending on whether $X_t > b$ or $X_t < b$ at time of entry.

First, let's look at how buys and sells are generated when $X_t > b_t$. Not all the time will liquidation level b_t be above our portfolio series X_t. When $X_t > b_t$ at time of trade entry, as we have intentionally shown here, we deem that b is not a valid liquidation level. In this case, from equation (15), we set $L_t = 0$ when signal $S_t^1 = 1$ and use $X_t > \text{MA}(X_t) + \sigma(X_t)$ as the exit condition for this trade.

The above describes our optimal exit model for the case when $X_t > b_t$ upon trade entry. It's clear that our baseline model is a special case of our optimal exit model when $b_t = -\infty$ for all t. Then entry condition of $X_t <$ Lower Band stays the same and we always exit with $X_t >$ Upper Band.

Now, when $b_t > X_t$ at time our trade entry, we set $L_t = 1$ and use our optimal liquidation level b_t as the trigger to liquidate this position as shown in Figure 5. Notice that the position involves a long GC position and a short SI position until X_t converges to the profitable exit signal when $X_t > b$. We expect to save on unnecessary trading and exit with a profit.

Had we used the baseline model rules in Figure 5, we would have incurred higher turnover due to whipsaw price action triggering exits when X_t is $\sigma(X_t)$ from mean portfolio level $\text{MA}(X_t)$. Optimal exit b dynamically adjusts during volatile periods to a profit taking level after those periods.

Figure 4: Trading the GC / SI Pair. Top panel: Portfolio value X_t stays above the optimal liquidation level b. Baseline model is optimal exit model with $b_t = -\infty$ so $X_t > b_t$ for all t. Middle panel: Time series of GC and SI prices. Bottom panel: Positions in GC and SI (bottom panel).

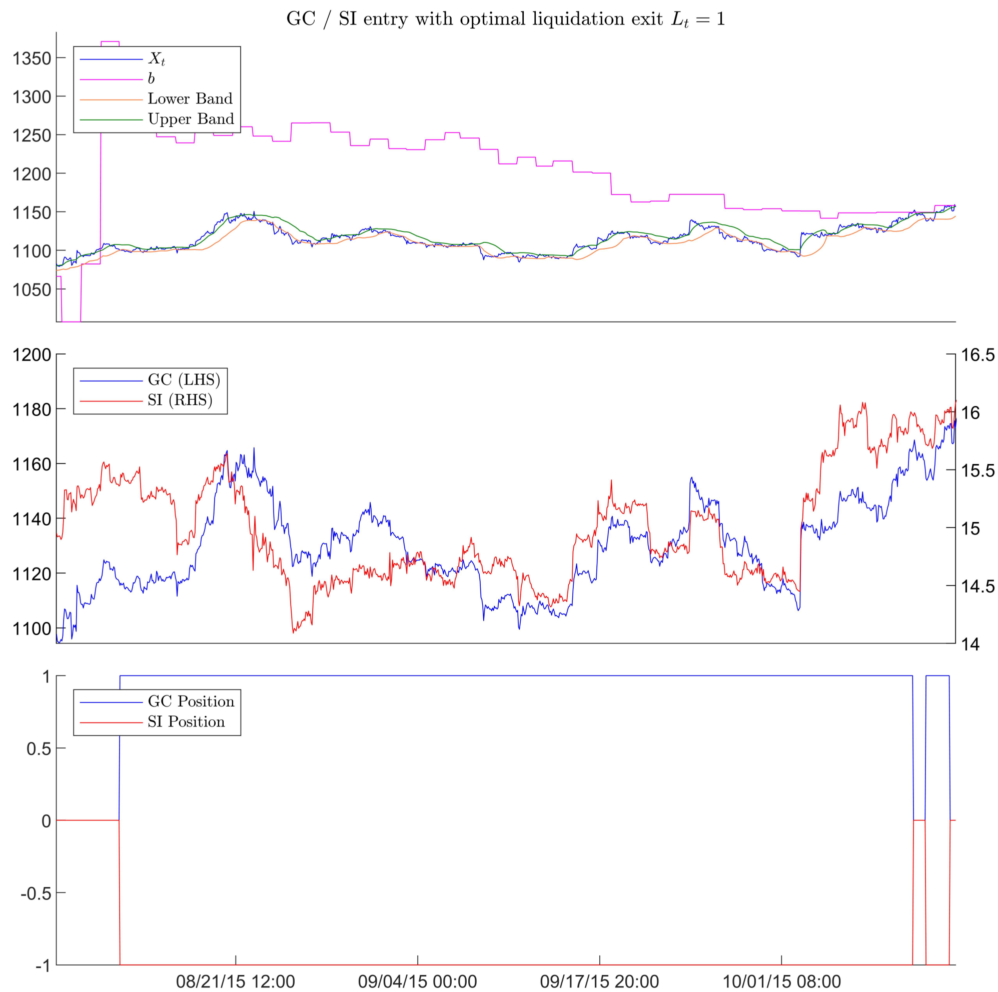


Figure 5: Trading the GC / SI Pair. Top panel: Portfolio value X_t stays mostly below the optimal liquidation level b. Middle panel: Time series of GC and SI prices. Bottom panel: Positions in GC and SI (bottom panel).

5 Performance

We compare the performance of the baseline strategy and the optimal exit strategy for all 8 asset pairs, and then later for a combined portfolio P consisting of equally weighted asset pairs.

We observe from Table 3 that modeling co-moving assets as a portfolio X_t whose dynamics is driven by an OU processes proves to be a promising framework that is opportune for profitable mean reversion trading. The baseline model of long X_t (i.e. long S^1 and short BS^2 if $B > 0$, or long S^1 and long $-BS_t^2$ if $B < 0$) when its price is below $\mathrm{MA}(X_t) - \sigma(X_t)$ and liquidate when price is above $\mathrm{MA}(X_t) + \sigma(X_t)$ shows promise as a positive PnL strategy. Most of the pairs achieve a positive Sharpe.

When using our optimal stopping model which liquidates when price is above an optimal exit level b, we notice two metrics that are consistent with the model's objective: higher returns and lower turnover. For seven of the eight pairs, Sharpe increases by as much as 0.7, and daily turnover decreases by an average of 34%. See Table 4.

	USDJPY / NK		**USDCAD / CL**		**CL / USO**		**GC / SI**	
	Baseline	OptExit	Baseline	OptExit	Baseline	OptExit	Baseline	OptExit
AnnualRet	-0.2%	1.1%	2.3%	3.8%	5.2%	4.6%	0.1%	9.3%
AnnualStd	5.9%	7.9%	4.8%	6.6%	3.2%	3.2%	10.0%	12.8%
Sharpe	-0.04	0.13	0.49	0.57	1.63	1.44	0.01	0.72
MaxDD	31%	32%	10%	17%	7%	7%	20.5%	20.2%
RetPerTO	0.00%	0.02%	0.01%	0.09%	0.01%	0.01%	0.00%	0.12%
DailyRet	0.00%	0.00%	0.01%	0.02%	0.02%	0.02%	0.00%	0.04%
DailyTO	64%	21%	75%	17%	147%	144%	140%	30%
CumulPnL	-1.8%	8.7%	19.4%	31.3%	43.5%	38.3%	0.5%	77.0%
CumulTC	13.4%	4.3%	15.6%	3.5%	30.7%	29.9%	29.3%	6.3%

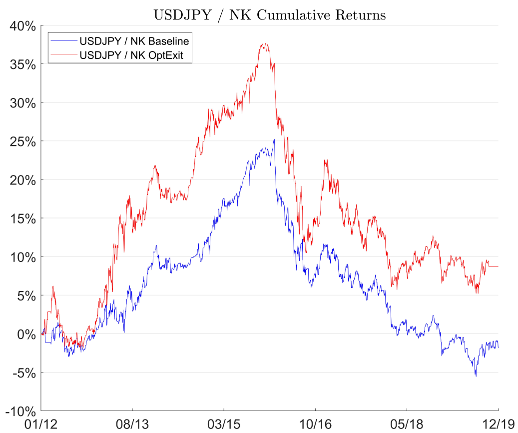
	AUDJPY / SPX		**USDCHF / GC**		**C / GS**		**AAPL / FB**	
	Baseline	OptExit	Baseline	OptExit	Baseline	OptExit	Baseline	OptExit
AnnualRet	-0.8%	0.9%	0.8%	1.8%	5.1%	5.8%	3.3%	10.1%
AnnualStd	7.8%	9.6%	5.6%	5.8%	12.2%	13.3%	14.0%	18.1%
Sharpe	-0.10	0.09	0.15	0.31	0.42	0.44	0.24	0.56
MaxDD	22.3%	21.1%	10%	10%	24%	26%	34.9%	42.8%
RetPerTO	-0.01%	0.01%	0.00%	0.01%	0.06%	0.08%	0.03%	0.23%
DailyRet	0.00%	0.00%	0.00%	0.01%	0.02%	0.02%	0.01%	0.04%
DailyTO	62%	31%	72%	71%	35%	30%	44%	18%
CumulPnL	-6.6%	7.5%	7.0%	14.6%	42.2%	48.4%	26.2%	79.7%
CumulTC	12.9%	6.4%	14.9%	14.8%	7.2%	6.3%	8.8%	3.5%

Table 3: Performance summary for all 8 pairs portfolios with and without the optimal exit. From the top to bottom rows, we report the annualized return, annualized standard deviation, Sharpe ratio, maximum drawdown, return per turnover, daily return, daily turnover, cumulative PnL, and cumulative transaction costs.

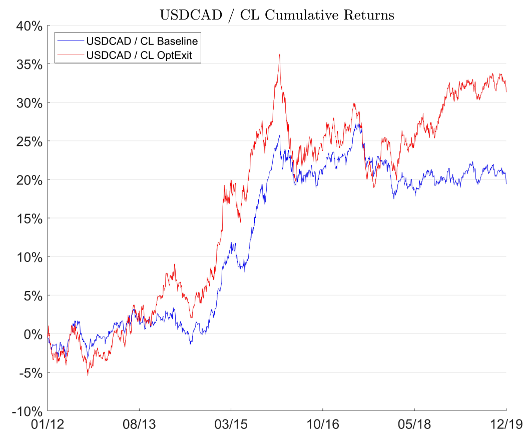
Optimal exit model outperforming baseline model is more apparent at the pairs portfolio level shown in Figure 6. An interesting observation is that both models suffer draw downs together. On the other hand, during the periods of steady positive returns for baseline, optimal exit model generates more returns as evident for pairs USDJPY / NK during 2012 to 2015 in Figure 6a and AUDJPY / SPX during late 2013 to early 2015 in Figure 6d.

Let us highlight the significant outperformance from using the optimal exit for the pair GC / SI in Figure 6c. This can be partly explained by how both precious metals have the same volatility range unlike the other pairs in which S^1 being a currency, has a different volatility range than S^2, being a commodity or equity futures.[5] We theorize that S^1 and S^2 having

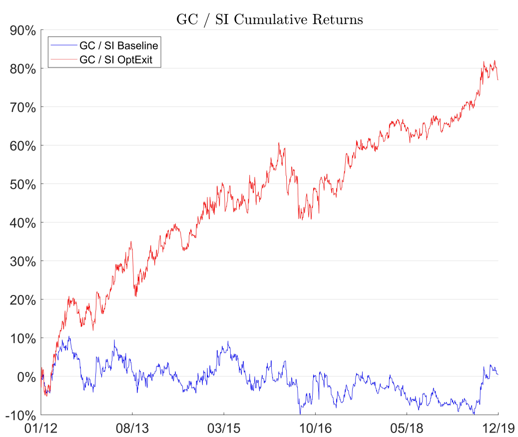
[5]From simple calculations, one can be easily deduced that currencies, on a percentage return metric, are less volatile than commodities and equities



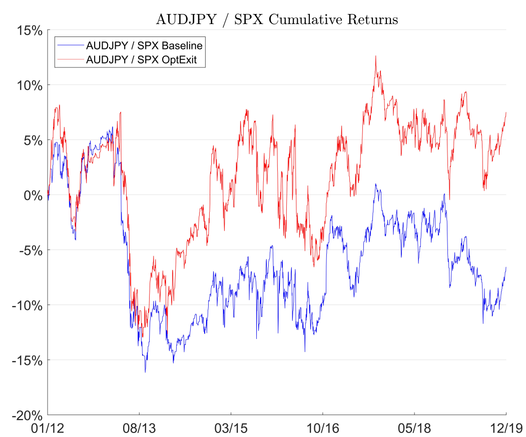
(a) USDJPY/NK



(b) USDCAD/CL



(c) GC/SI



(d) AUDJPY/SPX

Figure 6: In all four examples, trading using the optimal exit rule (red) performs better than the baseline (blue) strategy.

	Baseline		Optimal Exit		Difference	
Pair	AnnualRet	DailyTO	AnnualRet	DailyTO	AnnualRet	DailyTO
USDJPY / NKA	-0.2%	64%	1.1%	21%	+1.3%	-43.0%
USDCAD / CL	2.3%	75%	3.8%	17%	+1.5%	-58.0%
CL / USO	5.2%	147%	4.6%	144%	-0.6%	-3.0%
GC / SI	0.1%	140%	9.3%	30%	+9.2%	-110.0%
AUDJPY / SPX	-0.8%	62%	0.9%	31%	+1.7%	-31.0%
USDCHF / GC	0.8%	72%	1.8%	71%	+1.0%	-1.0%
C / GS	5.1%	35%	5.8%	30%	+0.7%	-5.0%
AAPL / FB	3.3%	44%	10.1%	18%	+6.8%	-26.0%

Table 4: Comparison of the baseline portfolio and portfolio with optimal exit in terms of annual returns and daily turnover. Using the optimal exit rule leads to higher annual returns for all pairs except CL / USO. For all pairs, the optimal exit strategy leads to fewer trades than the baseline strategy implies lower transaction costs and thereby improving profitability.

similar volatility levels implies a less volatile X_t and therefore avoids the unwanted whipsaw price action we mentioned earlier. For the other pairs, with S^2 having a higher volatility than S^1, X_t is likely to experience jumps in price action which will trigger more optimal exit levels. Exiting long X_t at these levels is still better than exiting using $\sigma(X_t)$ bands, but exiting more frequently implies paying more transaction cost which hurts returns.

Turning to the equally weighted portfolio P in Table 5, using an optimal exit increases the annualized return from 5.4% to 7.4% (+2.0%). The Sharpe ratio increases from 1.19 to 1.43 (+0.24) while the daily turnover decreases by -35% compared to the baseline strategy. By inspecting the trade signals, we observe that the optimal exit level b adjusts to be wider during high volatility periods, reducing unnecessary turnover, and then later adjusts back to an appropriate level which likely results in profit taking. The return profile in Figure 7 infers this behavior from how the optimal exit model doesn't have drops in PnL seen in the baseline model that comes from paying transaction cost for more frequent liquidations. This observation is most obvious from mid-2017 to mid-2018. This mechanism dovetails with b's purpose, which is the optimal level to liquidate a position for positive expected returns.

	Portfolio P	
	Baseline	OptExit
AnnualRet	5.4%	7.4%
AnnualStd	4.5%	5.2%
Sharpe	1.19	1.43
MaxDD	9%	12%
RetPerTO	0.03%	0.07%
DailyRet	0.02%	0.03%
DailyTO	80%	45%
CumulPnL	44.6%	61.3%
CumulTC	16.1%	9.1%

Table 5: Performance summary for the equally weighted portfolios (consisting of 8 pairs) with and without the optimal exit.

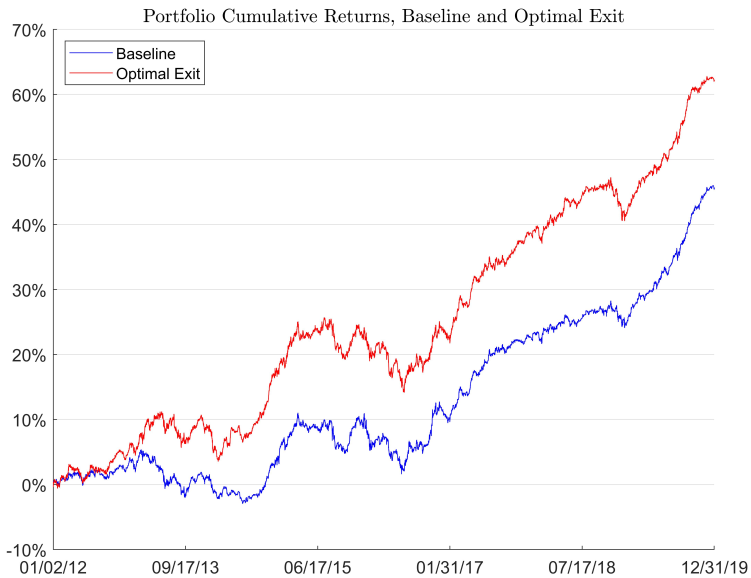


Figure 7: Cumulative returns of the equally weighted portfolios (consisting of 8 pairs) with and without the optimal exit rule. The portfolio with the optimal exit consistently outperforms the baseline portfolio during 1/2/2012 – 12/31/2019.

6 Conclusion

We have presented a framework for optimizing a pairs trading strategy. Each portfolio consisting of two assets is constructed with enhanced mean reversion through maximum likelihood estimation. An optimized exit rule is also incorporated to each pairs trading strategy. A total of eight asset pairs across different markets have been used for pairs trading. We have computed various performance metrics for the trading strategy with an optimized exit and compared to the baseline strategy. Our results show that optimized exit is capable of enhancing returns while reducing daily turnovers.

There are several directions for future research. We have analyzed a general pairs trading framework applicable to a risk-neutral trader. A natural extension is to directly incorporate the trader's risk preferences into the trading problem. The selection of the exit rule is likely to depend on the trader's risk aversion as well as the underlying dynamics.[6] Other types of exit rules can also be considered. In particular, path-dependent exit rules, such as trailing stop, can offer an adaptive rule to close out trades. In the study by Leung and Zhang (2019), the determination of the optimal trailing stop also leads to a non-trivial optimal stopping problem. Lastly, the pairs trading framework herein is also applicable to other asset classes and securities, including bonds and interest rate swaps (Nath (2003)), as well as cryptocurrencies (Leung and Nguyen (2019)).

[6]See Leung and Wang (2019) for an analysis on the optimal risk-averse timing to sell an asset under different dynamics.

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