

Pairs Trading Mean Reversion Strategy

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1 INTRODUCTION

Pairs trading is a well-established speculative investment strategy in the financial markets, gaining prominence in the 1980s. Today, it is commonly utilized by hedge funds and institutional investors as a long/short equity strategy. Recent studies have expanded the original analysis of pairs trading to include more contemporary data, demonstrating both economically and statistically significant profits using a straightforward pairs trading approach.

Typically regarded as a form of technical analysis, pairs trading aims to identify the relative overvaluation and undervaluation between two closely related stocks that share a long-term relationship. Relative mispricing occurs when the spread between the two stocks deviates from its equilibrium, and excess returns can be generated if the pair is mean-reverting—meaning any deviations are temporary and will return to equilibrium after a period of adjustment. In such cases, the strategy involves simultaneously shorting the relatively overvalued stock and going long on the relatively undervalued stock.¹ By working with intraday prices, we fit the most cointegrated pair to an Ornstein-Uhlenbeck (OU) process using maximum likelihood estimation (MLE). Subsequently, we employ an optimized exit rule to adjust positions, aiming to enhance mean reversion.

Asset Pairs and data

Cointegration is a statistical method that identifies long-term equilibrium relationships between non-stationary time series, such as the prices of different assets. It suggests that certain pairs of assets move together over time, and any deviations from this relationship are temporary imbalances that can present trading opportunities. The cointegration approach involves three key steps: selecting potential asset pairs, analyzing their tradability, and setting criteria for entering and exiting trades based on the observed equilibrium relationship. This method helps traders exploit predictable patterns for profit.² Assets can be cointegrated because they are influenced by common factors such as economic

¹Source: Rong Qi Liew, Yuan Wu, *Pairs trading: A copula approach*, 2022.

²Source: Tetiana Lemishko, Alexandre Landi, Juliana Caicedo-Llano, *Cointegration-Based Strategies in Forex Pair Trading*, 2024.

conditions, market dynamics, trade relationships, or industry-specific forces. These shared influences cause their prices to move together over the long term, reflecting a stable and predictable relationship.

The following are examples of asset pairs that may exhibit cointegration:

- **USDCAD / CL**: Canada is one of the world's top oil producers. There is a positive correlation between the price of crude oil and the value of the Canadian dollar. When oil prices rise (or fall), the Canadian dollar tends to appreciate (or depreciate), leading to a decrease (or increase) in the USD/CAD exchange rate.
- **CL / USO**: The United States Oil Fund (USO) is a major ETF that holds futures contracts on West Texas Intermediate (WTI) crude oil. As it tracks the price movements of these futures, USO and crude oil futures (CL) prices generally exhibit a strong positive correlation.
- **C / GS**: Citigroup (C) and Goldman Sachs (GS) are two major financial institutions in the banking sector. These large-cap stocks are often subject to similar market forces, which can create opportunities for pairs trading based on their relative performance.³.

As we are currently utilizing demo accounts on IBKR, we are restricted from trading futures and indices. Consequently, we have elected to concentrate exclusively on foreign exchange (FX) pairs. We have commenced an analysis of the stationarity of the following pairs, and for those identified as non-stationary, we are further exploring their potential for cointegration:

GBPCHE (British Pound / Swiss Franc), EURGBP (Euro / British Pound), GBPUSD (British Pound / US Dollar), AUDJPY (Australian Dollar / Japanese Yen), USDCAD (US Dollar / Canadian Dollar), GBPJPY (British Pound / Japanese Yen), EURAUD (Euro / Australian Dollar), USDSEK (US Dollar / Swedish Krona), EURCAD (Euro / Canadian Dollar), USDTRY (US Dollar / Turkish Lira), CHFJPY (Swiss Franc / Japanese Yen), AUDUSD (Australian Dollar / US Dollar), USDCHF (US Dollar / Swiss Franc), EURTRY (Euro / Turkish Lira), USDNOK (US Dollar / Norwegian Krone), EURCHF (Euro / Swiss Franc), USDSGD (US Dollar / Singapore Dollar), USDHKD (US Dollar / Hong Kong Dollar), NZDUSD (New Zealand Dollar / US Dollar), CADJPY (Canadian Dollar / Japanese Yen), EURJPY (Euro / Japanese Yen), EURNZD (Euro / New Zealand Dollar), GBPAUD (British Pound / Australian Dollar), GBPCAD (British Pound / Canadian Dollar), AUDNZD (Australian Dollar / New Zealand Dollar), USDJPY (US Dollar / Japanese Yen), EURUSD (Euro / US Dollar), and NZDJPY (New Zealand Dollar / Japanese Yen).

³Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

Data Connection to Interactive Brokers

Data were retrieved from Interactive Brokers (IBKR) using the `ib_insync` library. The script establishes a connection to the IBKR TWS or Gateway to fetch historical price data for the specified currency pairs over the past five months, from January 1, 2024, to May 31, 2024, with hourly intervals. This data is used to assess the cointegration between assets.

2 PRELIMINARY TEST

Before implementing the strategy, it is crucial to select assets that exhibit cointegration with one another. To achieve this, we utilize two models: the Augmented Dickey-Fuller (ADF) test and the Cointegrated Augmented Dickey-Fuller (CADF) test, as detailed below:

Augmented Dickey-Fuller (ADF) test

The Augmented Dickey-Fuller (ADF) test is a widely utilized statistical method to assess whether a time series is stationary. A stationary time series maintains consistent properties over time, including mean, variance, and autocorrelation structure. Stationarity is critical in financial time series analysis for modeling and forecasting, especially for identifying mean-reverting behaviors. We are applying the ADF test to each asset to determine its stationarity, which is essential for subsequent analyses, such as cointegration testing and the formulation of robust trading strategies. This section focuses on verifying the stationarity of each asset individually, as the concept of cointegration is applicable only to non-stationary time series.

Mathematical Foundation

The ADF test is based on the following regression model:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^p \delta_i \Delta y_{t-i} + \epsilon_t$$

where:

- y_t is the time series.
- Δy_t is the first difference of y_t , i.e., $y_t - y_{t-1}$.
- α is a constant term.
- βt represents a deterministic trend.
- γ is the coefficient of the lagged level of the series.
- p is the number of lagged difference terms.

- δ_i are coefficients of the lagged differences.
- ϵ_t is the white noise error term.

The null hypothesis (H_0) of the ADF test is that the time series has a unit root, implying it is non-stationary. The alternative hypothesis (H_1) is that the time series is stationary⁴.

Test Statistic and Interpretation

The key metric in the ADF test is the test statistic, which is compared against critical values at various significance levels (1%, 5%, and 10%). The p-value is also computed to assess the significance of the test. If the p-value is below a certain threshold (commonly 0.05), the null hypothesis is rejected, indicating that the series is stationary⁵.

Results

Running the code yields ADF test results for each time series. If the p-value is below 0.05, the series is considered stationary; otherwise, it is non-stationary.

The analysis reveals that many of the financial time series examined are non-stationary, as indicated by p-values exceeding the 0.05 threshold. This necessitates further analysis for potential cointegration. Stationary pairs, such as GBPCHE, GBPUSD, and AUDUSD, will be excluded from the CADF test, which is designed for non-stationary variables.

The non-stationary pairs identified for the CADF test to assess potential cointegration include EURGBP, AUDJPY, GBPJPY, CHFJPY, USDSGD, CADJPY, EURJPY, GBPCAD, USDJPY, and NZDJPY. These pairs will be analyzed for possible cointegration relationships to develop robust trading strategies.

⁴Source: Rizwan Mushtaq, *Testing Time Series Data For Stationarity*, 2011.

⁵Source: Rizwan Mushtaq, *Testing Time Series Data For Stationarity*, 2011.

Series	p-value	Stationarity
GBPCHE	0.048948	Stationary
EURGBP	0.085131	Non-stationary
GBPUSD	0.003858	Stationary
AUDJPY	0.602053	Non-stationary
USDCAD	0.025954	Stationary
GBPJPY	0.725736	Non-stationary
EURAUD	0.000402	Stationary
USDSEK	0.000006	Stationary
EURCAD	0.000088	Stationary
USDTRY	0.008232	Stationary
CHFJPY	0.926791	Non-stationary
AUDUSD	0.002425	Stationary
USDCHF	0.007920	Stationary
EURTRY	0.015150	Stationary
USDNOK	0.001562	Stationary
EURCHF	0.041691	Stationary
USDSGD	0.107929	Non-stationary
USDHKD	0.040253	Stationary
NZDUSD	0.000014	Stationary
CADJPY	0.743123	Non-stationary
EURJPY	0.714824	Non-stationary
EURNZD	0.000086	Stationary
GBPAUD	0.001020	Stationary
GBPCAD	0.059400	Non-stationary
AUDNZD	0.000009	Stationary
USDJPY	0.498531	Non-stationary
EURUSD	0.001800	Stationary
NZDJPY	0.940668	Non-stationary

Table 1: Augmented Dickey-Fuller (ADF) test results for the currency pairs.

Cointegrated Augmented Dickey-Fuller (CADF) Test

In this section, we apply the Cointegrated Augmented Dickey-Fuller (CADF) test to each pair of assets to assess their cointegration. Cointegration analysis is critical for understanding long-term equilibrium relationships between non-stationary time series, which is essential for strategies such as pairs trading.

2.1 Mathematical Foundation

The CADF test extends the Augmented Dickey-Fuller (ADF) test by evaluating whether two or more non-stationary time series are cointegrated, implying they have a stable, long-term relationship despite individual stochastic trends. The mathematical foundation of the CADF test is built on the concept of cointegration.

Given two time series X_t and Y_t , they are considered cointegrated if there exists a linear combination $Z_t = X_t - \beta Y_t$ that is stationary. The CADF test involves estimating the cointegration equation:

$$Z_t = X_t - \beta Y_t + \epsilon_t$$

where ϵ_t is a stationary error term. We then test the null hypothesis that there is no cointegration (i.e., Z_t is non-stationary) against the alternative hypothesis of cointegration (i.e., Z_t is stationary) ⁶.

2.2 Test Statistic Interpretation

The CADF test generates a test statistic τ for the residuals ϵ_t of the cointegration regression. The critical values for the test statistic depend on the sample size and the number of lags included in the model. If the test statistic τ is less than the critical value at a chosen significance level (e.g., 5%), the null hypothesis of no cointegration is rejected, indicating that the series are cointegrated.

$$\tau = \frac{\hat{\beta}}{SE(\hat{\beta})}$$

Here, $\hat{\beta}$ is the estimated coefficient from the regression of X_t on Y_t , and $SE(\hat{\beta})$ is its standard error ⁷.

Results

The majority of the pairs tested do not exhibit significant evidence of cointegration, as indicated by p-values exceeding the conventional threshold of 0.05.

⁶Source: Bent E. Sørensen, *ECONOMICS* 266, Spring, 1997, 2019.

⁷<https://www.quantstart.com/articles/Basics-of-Statistical-Mean-Reversion-Testing/>
Basics of Statistical Mean Reversion Testing

However, several pairs demonstrated a statistically significant cointegration relationship, suggesting a stable long-term equilibrium between them. This finding is crucial for developing trading strategies that leverage cointegration, as only cointegrated pairs present opportunities to exploit such stable relationships. For the pairs that are not cointegrated, alternative trading strategies may be more appropriate.

Among the tested pairs, the three that show the strongest evidence of cointegration are as follows:

1. **AUDJPY and CADJPY**: With a test statistic of -5.537 and a p-value of 0.000015, this pair demonstrates the most significant cointegration, indicating a robust long-term relationship.
2. **AUDJPY and GBPJPY**: This pair shows strong cointegration with a test statistic of -4.495 and a p-value of 0.001248, suggesting a significant equilibrium connection.
3. **AUDJPY and EURJPY**: Exhibiting a test statistic of -4.378 and a p-value of 0.001931, this pair is the third most cointegrated, indicating a notable long-term equilibrium.

These results highlight the importance of selecting pairs with proven cointegration for strategies aiming to exploit stable relationships over time. The complete results are presented on the following page.

For our trading strategy, we will proceed with the following pair: AUDJPY and CADJPY.

Pairs	Test Statistic	p-value	Interpretation
EURGBP / AUDJPY	-3.095	0.089215	Not cointegrated
EURGBP / GBPJPY	-2.984	0.113947	Not cointegrated
EURGBP / CHFJPY	-3.695	0.018600	Cointegrated
EURGBP / USDSGD	-3.036	0.101963	Not cointegrated
EURGBP / CADJPY	-3.025	0.104441	Not cointegrated
EURGBP / EURJPY	-3.169	0.075293	Not cointegrated
EURGBP / GBPCAD	-3.807	0.013284	Cointegrated
EURGBP / USDJPY	-3.090	0.090239	Not cointegrated
EURGBP / NZDJPY	-2.701	0.199289	Not cointegrated
AUDJPY / GBPJPY	-4.495	0.001248	Cointegrated
AUDJPY / CHFJPY	-4.084	0.005429	Cointegrated
AUDJPY / USDSGD	-2.076	0.488836	Not cointegrated
AUDJPY / CADJPY	-5.537	0.000015	Cointegrated
AUDJPY / EURJPY	-4.378	0.001931	Cointegrated
AUDJPY / GBPCAD	-1.956	0.551453	Not cointegrated
AUDJPY / USDJPY	-3.183	0.072782	Not cointegrated
AUDJPY / NZDJPY	-1.753	0.652324	Not cointegrated
GBPJPY / CHFJPY	-4.172	0.004028	Cointegrated
GBPJPY / USDSGD	-1.746	0.655673	Not cointegrated
GBPJPY / CADJPY	-3.057	0.097355	Not cointegrated
GBPJPY / EURJPY	-3.033	0.102483	Not cointegrated
GBPJPY / GBPCAD	-1.652	0.698319	Not cointegrated
GBPJPY / USDJPY	-2.475	0.290202	Not cointegrated
GBPJPY / NZDJPY	-1.469	0.772781	Not cointegrated
CHFJPY / USDSGD	-0.832	0.930339	Not cointegrated
CHFJPY / CADJPY	-2.226	0.410821	Not cointegrated
CHFJPY / EURJPY	-2.044	0.505404	Not cointegrated
CHFJPY / GBPCAD	-1.012	0.900594	Not cointegrated
CHFJPY / USDJPY	-1.600	0.720875	Not cointegrated
CHFJPY / NZDJPY	-0.603	0.956164	Not cointegrated
USDSGD / CADJPY	-2.592	0.239886	Not cointegrated
USDSGD / EURJPY	-3.383	0.044304	Cointegrated
USDSGD / GBPCAD	-3.410	0.041283	Cointegrated
USDSGD / USDJPY	-3.133	0.081953	Not cointegrated
USDSGD / NZDJPY	-2.662	0.213578	Not cointegrated
CADJPY / EURJPY	-3.582	0.025796	Cointegrated
CADJPY / GBPCAD	-1.736	0.660049	Not cointegrated
CADJPY / USDJPY	-1.443	0.782238	Not cointegrated
CADJPY / NZDJPY	-1.364	0.809589	Not cointegrated
EURJPY / GBPCAD	-1.736	0.660365	Not cointegrated
EURJPY / USDJPY	-2.349	0.349237	Not cointegrated
EURJPY / NZDJPY	-1.502	0.760275	Not cointegrated
GBPCAD / USDJPY	-3.470	0.035134	Cointegrated
GBPCAD / NZDJPY	-3.115	0.085293	Not cointegrated
USDJPY / NZDJPY	-1.919	0.570294	Not cointegrated

Table 2: CADF Test Results for Various Currency Pairs

3 PARAMETERS ESTIMATION

Mean reversion is a financial theory that suggests asset prices and returns will eventually revert to their long-term average levels. This phenomenon is commonly observed in financial markets where prices tend to oscillate around an average value. The key idea is that short-term deviations from this average will eventually correct themselves, presenting potential trading opportunities. The larger the deviation from the mean, the higher the likelihood that the asset's price will move back towards it in the future.⁸

The mean-reversion trading strategy in the provided script uses the Ornstein-Uhlenbeck (OU) process to model the behavior of asset prices like stocks. This approach takes advantage of the tendency of prices to return to their average by identifying and trading on temporary deviations.

Data Connection to Interactive Brokers

Historical price data were obtained from Interactive Brokers Trader Workstation (IBKR TWS) using the `ib_insync` library. The script retrieves data for the currency pairs AUDJPY and CADJPY over a 30-day period, spanning from May 1, 2024, to May 31, 2024, at 15-minute intervals. This data is utilized for parameter estimation in our analysis.

Definition of portfolio

We start constructing a portfolio by holding a risky asset S^1 and shorting B shares of another risky asset S^2 . This resulting in

$$X_t = S_t^1 - BS_t^2 \quad (1)$$

In which B is the pair ratio (which can be positive or negative), that determines how many units of the second asset s^2 are needed to offset or hedge one unit of the first asset S^1 ⁹.

For our trading strategy, we define the portfolio as follows:

- S_t^1 : AUDJPY (the first risky asset)
- S_t^2 : CADJPY (the second risky asset)

Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck (OU) process is a continuous-time stochastic process used to model mean-reverting behavior in financial markets. It is particularly

⁸<https://www.investopedia.com/terms/m/meanreversion.asp>

What Is Mean Reversion, and How Do Investors Use It? by James Chen, reviewed by Gordon Scott.

⁹Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

suitable for describing the dynamics of asset prices that exhibit tendencies to revert to a mean level. The OU process is defined by the following stochastic differential equation (SDE):

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t \quad (2)$$

Here, θ represents the rate of mean reversion, μ is the long-term mean, σ is the volatility (the degree of variation or dispersion of the price), and W_t is a standard Wiener process (Brownian motion). The term $\theta(\mu - X_t)$ drives the process towards the mean μ , while σdW_t adds randomness to the movement of X_t .¹⁰

Parameters Estimation

Estimating the parameters θ , μ , and σ is crucial for accurately modeling the OU process. We estimate the parameters through maximum likelihood estimation, tracking the time series data for each portfolio. By maximizing the average log-likelihood $\ell(\theta, \mu, \sigma \mid X_{B_0}, X_{B_1}, \dots, X_{B_n})$, we determine the parameters θ^* , μ^* , and σ^* , which in turn help us identify optimal entry and exit points for trades.

$$\begin{aligned} \ell(\theta, \mu, \sigma \mid X_0^B, X_1^B, \dots, X_n^B) &= \frac{1}{n} \sum_{i=1}^n \ln f^{OU}(X_i^B \mid X_{i-1}^B; \theta, \mu, \sigma) \\ &= -\frac{1}{2} \ln(2\pi) - \ln(\tilde{\sigma}) - \frac{1}{2\tilde{\sigma}^2} \sum_{i=1}^n [X_i^B - X_{i-1}^B - \theta(1 - e^{-\mu\Delta t})]^2 \end{aligned} \quad (3)$$

where Δt is the time step between observations. The parameters are estimated by maximizing this likelihood function over the historical price data.¹¹

The OU parameters estimated based on the historical data are the following:

- $\mu = 0.0000$ (mean reversion speed)
- $\theta = 94.2406$ (long-term mean price)
- $\sigma = 0.0010$ (volatility)

Optimization of B

The optimization of the parameter B is integral to constructing a mean-reverting portfolio. The portfolio value X_t is defined as a linear combination of two assets, S_1 and S_2 , with a ratio B :

$$X_t = S_1(t) - BS_2(t) \quad (4)$$

¹⁰Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

¹¹Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

The optimal B is determined by maximizing the log-likelihood of fitting the portfolio values to the OU process. This involves finding the value of B that best aligns the portfolio with the mean-reverting dynamics described by the OU model. The optimization is typically done using a lookback window, as shown in the formula:

$$\hat{B} = \arg \max_B \left\{ \frac{1}{n} \sum_{i=1}^n \ln \mathcal{L}(X_{t_i} | X_{t_{i-1}}, \theta, \mu, \sigma) \right\} \quad (5)$$

¹².

For our portfolio pair, the optimal value of B is as follows:

- **AUDJPY/CADJPY:** $B = -0.0000$

This value of B is utilized to construct a mean-reverting portfolio between the exchange rate S_t^1 and the price of S_t^2 .

IMPORTANT: The portfolio series need to be updated periodically using the optimized pair ratio.

4 ENTRY AND OPTIMIZED EXIT

In this section, we will detail the methodology for opening and closing positions within our trading strategy.

Entry Condition

At every hour, the strategy evaluates whether to generate an entry signal. The entry signal, denoted as Sig_{S1} , is triggered if the portfolio value X_t falls below the moving average minus one standard deviation. Mathematically, this condition is expressed as:

$$\text{Sig}_{S1} = \begin{cases} 1, & \text{if } X_t < \text{MA}(X_t) - \sigma(X_t) \\ 0, & \text{otherwise} \end{cases}$$

This criterion indicates that the portfolio value is substantially lower than its historical average, suggesting a potential opportunity for mean reversion¹³.

¹²Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

¹³Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

Entry Signal Execution

Upon the condition $\text{Sig}_{S1} = 1$, the strategy proceeds to enter a position. This entails taking a long position in the first asset $S1$ and a corresponding short position in the second asset $S2$, adjusted by the optimized pair ratio B . The corresponding signal for the **second asset** is given by:

$$\text{Sig}_{S2} = -B \cdot \text{Sig}_{S1}$$

This strategy maintains market neutrality and capitalizes on mean reversion by entering positions when the portfolio is undervalued. It leverages expected price corrections towards the historical mean to maximize returns¹⁴.

Optimal Exit Level b_t

To find the optimal exit point we have to find before the optimal exit level, b_t , that is essential for maximizing trading strategy profits. The steps to determine and utilize this level are as follows:

To find the optimal exit level, solve an optimal stopping problem. This aims to maximize the expected payoff by exiting at the most favorable time:

$$V(x) = \sup_{\tau} \mathbb{E} [e^{-r\tau} (X_{\tau} - c)]$$

where r is the discount rate, and c represents costs related to holding or exiting a position.

The optimal exit level b_t is found by solving:

$$F(b) = (b - c)F'(b)$$

Here:

$$F(x) = \int_0^{\infty} u^{\frac{2\mu}{\sigma^2}-1} e^{\sqrt{\frac{2\mu}{\sigma^2}}(\theta-x)u - \frac{u^2}{2}} du$$

Using numerical methods, approximate $F(x)$ and its derivative to find b . This level b_t is a dynamic threshold adjusting to market conditions, ensuring exits at optimal points¹⁵.

For our portfolio pair, the optimal value of b is as follows:

- **AUDJPY/CADJPY:** $b = 94.2354$

¹⁴Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

¹⁵Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

Exit Condition

The strategy uses the optimal exit level b_t to decide when to exit positions to maximize returns. Hourly, the strategy checks if the portfolio value X_t exceeds the optimal exit level b_t or the moving average plus one standard deviation. The exit signals are defined as follows:

1. ****Moving Average Exit****: - Exit if X_t exceeds the moving average plus one standard deviation:

$$\text{Sig}_{S1} = \begin{cases} 0, & \text{if } X_t > \text{MA}(X_t) + \sigma(X_t) \\ \text{Sig}_{S1,t-1}, & \text{otherwise} \end{cases}$$

2. ****Optimal Exit Level****: - Exit if X_t exceeds the optimal exit level b_t :

$$\text{Sig}_{S1} = \begin{cases} 0, & \text{if } X_t > b_t \\ \text{Sig}_{S1,t-1}, & \text{otherwise} \end{cases}$$

The optimal exit level b_t helps to:

- Reduce trade frequency and transaction costs.
- Ensure exits at the best possible prices, maximizing returns.

For the **second asset**, adjust the exit signal based on the pair ratio B_t :

$$\text{Sig}_{S2} = \begin{cases} 0, & \text{if } \text{Sig}_{S1} = 0 \\ -\text{Sig}_{S2,t-1}, & \text{otherwise} \end{cases}$$

Exit Signal Execution

Upon meeting exit conditions, the strategy liquidates positions by selling the long in $S1$ and covering the short in $S2$, optimizing the capture of profits¹⁶.

5 BACKTESTING OF THE STRATEGY

Before proceeding with our trading algorithm, we conducted a thorough backtest. The backtest involved estimating the parameters using 15 minute-level data from the entire month of May. As previously explained, the optimal parameters are:

- $\mu = 0.0000$
- $\theta = 94.2406$

¹⁶Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

- $\sigma = 0.0010$
- $B = -0.0000$
- $b = 94.2354$

Over the course of one month, the portfolio demonstrated a modest yet positive performance. Starting with an initial value of \$100,000 the portfolio's value increased to \$100,905.95 by the end of the month, resulting in a total return of 0.91%. This gain shows a small, steady improvement in the portfolio's value over the short term.

The portfolio's Sharpe ratio for the month was 0.01, indicating a minimal risk-adjusted return. This means the portfolio's returns were only slightly higher than the risk-free rate when accounting for its volatility. While the portfolio made a gain, its performance barely exceeded the expected return for the risk taken.

The portfolio achieved a high win rate of 81.21%, indicating that a significant majority of trades or investment decisions made during the month were profitable. This high success rate points to effective trade selection and strategy execution. However, it is important to evaluate the win rate in conjunction with the average size of gains and losses to fully understand its impact on the overall portfolio performance.

In summary, the portfolio's performance over the one-month period was encouraging, showing a notable increase in value and a high win rate. Nevertheless, the relatively low Sharpe ratio underscores an opportunity to enhance the portfolio's risk-return profile. Future strategies should aim to improve returns relative to risk to achieve more favorable performance metrics, especially for similar short-term trading periods.

6 HOW TO USE THE STRATEGY IN A LIVE ACCOUNTS

It is crucial to follow two main steps:

Step 1: Estimate the parameters μ , θ , σ , B , and the optimal exit level (b) for today using 15 minute-level data from the past 30 days.

Step 2: After estimating these parameters, connect the code to Interactive Brokers (IBKR) to access live market data. The algorithm will then autonomously monitor and manage positions, ensuring the strategy is executed in real-time.

7 Methodology

OpenAI, ChatGPT 4.0, 2024-06-15: Conduct a CADF test using the previously mentioned currency pairs.

OpenAI, ChatGPT 4.0, 2024-06-15: Develop optimal exit signals following the steps in the provided images.

OpenAI, ChatGPT 4.0, 2024-06-16: Optimize parameters and execute trades by following the instructions in the images.