Pairs Trading Mean Reversion Strategy

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1 INTRODUCTION

Pairs trading is a well-established speculative investment strategy in the financial markets, gaining prominence in the 1980s. Today, it is commonly utilized by hedge funds and institutional investors as a long/short equity strategy. Recent studies have expanded the original analysis of pairs trading to include more contemporary data, demonstrating both economically and statistically significant profits using a straightforward pairs trading approach.

Typically regarded as a form of technical analysis, pairs trading aims to identify the relative overvaluation and undervaluation between two closely related stocks that share a long-term relationship. Relative mispricing occurs when the spread between the two stocks deviates from its equilibrium, and excess returns can be generated if the pair is mean-reverting—meaning any deviations are temporary and will return to equilibrium after a period of adjustment. In such cases, the strategy involves simultaneously shorting the relatively overvalued stock and going long on the relatively undervalued stock.[1]. By working with intraday prices, we fit the most cointegrated pair to an Ornstein-Uhlenbeck (OU) process using maximum likelihood estimation (MLE). Subsequently, we employ an optimized exit rule to adjust positions, aiming to enhance mean reversion.

Asset Pairs and data

We decided to analyze the asset data found in the paper "On the Efficacy of Optimized Exit Rules for Mean Reversion Trading." We were particularly curious to examine the types of correlations described in the study.

- **USDJPY / NK**: This trading strategy relies on the positive correlation between the USD/JPY exchange rate and the Nikkei 225 index. Japan's economy is heavily reliant on exports, so when the Japanese yen strengthens against the US dollar (i.e., USDJPY falls), the Nikkei 225 index is likely to decline due to reduced export competitiveness. Conversely, a weakening Nikkei, often seen as a risk sentiment indicator, typically leads to a stronger yen, causing USDJPY to drop.

[1]Source: Rong Qi Liew, Yuan Wu, *Pairs trading: A copula approach*, 2022.

- **USDCAD / CL**: Canada is one of the world's top oil producers. There is a positive correlation between the price of crude oil and the value of the Canadian dollar. When oil prices rise (or fall), the Canadian dollar tends to appreciate (or depreciate), leading to a decrease (or increase) in the USD/CAD exchange rate.
- **CL / USO**: The United States Oil Fund (USO) is a major ETF that holds futures contracts on West Texas Intermediate (WTI) crude oil. As it tracks the price movements of these futures, USO and crude oil futures (CL) prices generally exhibit a strong positive correlation.
- **GC / SI**: This pair involves trading between gold (GC) and silver (SI) spot prices. Both metals often move in tandem due to their common roles as safe-haven assets and inflation hedges, making this pair a classic case for analyzing relative metal pricing.
- **AUDJPY / SPX**: This strategy involves trading between the AUD/JPY exchange rate, which is often considered a risk-on currency pair, and the S&P 500 (SPX) futures, a benchmark for US equity markets. The S&P 500 is widely used as an indicator of general investor sentiment towards risk.
- **USDCHF / GC**: The Swiss National Bank holds a significant portion of its reserves in gold, implying a correlation between the Swiss franc and gold prices. When gold prices rise, the Swiss franc tends to strengthen (resulting in a decrease in USD/CHF), and vice versa.
- **C / GS**: Citigroup (C) and Goldman Sachs (GS) are two major financial institutions in the banking sector. These large-cap stocks are often subject to similar market forces, which can create opportunities for pairs trading based on their relative performance.
- **AAPL / FB**: Apple (AAPL) and Meta Platforms (FB, formerly known as Facebook) are two leading large-cap technology companies. Their stocks often exhibit related price movements due to their roles in the tech sector, providing a basis for analyzing their relative valuations.

Assets taken from[2].

Data Connection to Yahoo Finance

Data were retrieved from Yahoo Finance using the `yfinance` library. The script fetches historical price data for the specified assets over the past five months, from 01.01.2024 to 31.05.2024, with hourly intervals. This data is used to assess the cointegration between assets.

[2] Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

2 PRELIMINARY TEST

Before implementing the strategy, it is crucial to select assets that exhibit cointegration with one another. To achieve this, we utilize two models: the Augmented Dickey-Fuller (ADF) test and the Cointegrated Augmented Dickey-Fuller (CADF) test, as detailed below:

Augmented Dickey-Fuller (ADF) test

The Augmented Dickey-Fuller (ADF) test is a widely utilized statistical method to assess whether a time series is stationary. A stationary time series maintains consistent properties over time, including mean, variance, and autocorrelation structure. Stationarity is critical in financial time series analysis for modeling and forecasting, especially for identifying mean-reverting behaviors. We are applying the ADF test to each asset to determine its stationarity, which is essential for subsequent analyses, such as cointegration testing and the formulation of robust trading strategies. This section focuses on verifying the stationarity of each asset individually, as the concept of cointegration is applicable only to non-stationary time series.

Mathematical Foundation

The ADF test is based on the following regression model:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \epsilon_t$$

where:

- y_t is the time series.
- Δy_t is the first difference of y_t, i.e., $y_t - y_{t-1}$.
- α is a constant term.
- βt represents a deterministic trend.
- γ is the coefficient of the lagged level of the series.
- p is the number of lagged difference terms.
- δ_i are coefficients of the lagged differences.
- ϵ_t is the white noise error term.

The null hypothesis (H_0) of the ADF test is that the time series has a unit root, implying it is non-stationary. The alternative hypothesis (H_1) is that the time series is stationary[3].

[3]Source: Rizwan Mushtaq, *Testing Time Series Data For Stationarity*, 2011.

Test Statistic and Interpretation

The key metric in the ADF test is the test statistic, which is compared against critical values at various significance levels (1%, 5%, and 10%). The p-value is also computed to assess the significance of the test. If the p-value is below a certain threshold (commonly 0.05), the null hypothesis is rejected, indicating that the series is stationary[4].

Results

Running the code provides ADF test results for each time series. If the p-value is below 0.05, the series is considered stationary; otherwise, it is non-stationary: The analysis reveals that most of the financial time series examined are non-

Series	**p-value**	**Stationarity**
SPX	0.292718	Non-stationary
USDJPY	0.268162	Non-stationary
AUDJPY	0.996628	Non-stationary
SI	0.887131	Non-stationary
USO	0.233589	Non-stationary
NK	0.043956	Stationary
USDCAD	0.231536	Non-stationary
USDCHF	0.209774	Non-stationary
C	0.706181	Non-stationary
GS	0.956621	Non-stationary
AAPL	0.678282	Non-stationary
CL	0.194642	Non-stationary
GC	0.875268	Non-stationary
META	0.142244	Non-stationary

Table 1: Augmented Dickey-Fuller (ADF) test results for the series.

stationary, as evidenced by p-values exceeding 0.05 for the majority of assets. The sole exception is the Nikkei 225, which is stationary. Consequently, it will not be included in the CADF test, which is designed exclusively for non-stationary variables.

[4]Source: Rizwan Mushtaq, *Testing Time Series Data For Stationarity*, 2011.

Cointegrated Augmented Dickey-Fuller (CADF) Test

In this section, we apply the Cointegrated Augmented Dickey-Fuller (CADF) test to each pair of assets to assess their cointegration. Cointegration analysis is critical for understanding long-term equilibrium relationships between non-stationary time series, which is essential for strategies such as pairs trading.

2.1 Mathematical Foundation

The CADF test extends the Augmented Dickey-Fuller (ADF) test by evaluating whether two or more non-stationary time series are cointegrated, implying they have a stable, long-term relationship despite individual stochastic trends. The mathematical foundation of the CADF test is built on the concept of cointegration.

Given two time series X_t and Y_t, they are considered cointegrated if there exists a linear combination $Z_t = X_t - \beta Y_t$ that is stationary. The CADF test involves estimating the cointegration equation:

$$Z_t = X_t - \beta Y_t + \epsilon_t$$

where ϵ_t is a stationary error term. We then test the null hypothesis that there is no cointegration (i.e., Z_t is non-stationary) against the alternative hypothesis of cointegration (i.e., Z_t is stationary).

2.2 Test Statistic Interpretation

The CADF test generates a test statistic τ for the residuals ϵ_t of the cointegration regression. The critical values for the test statistic depend on the sample size and the number of lags included in the model. If the test statistic τ is less than the critical value at a chosen significance level (e.g., 5%), the null hypothesis of no cointegration is rejected, indicating that the series are cointegrated.

$$\tau = \frac{\hat{\beta}}{\mathrm{SE}(\hat{\beta})}$$

Here, $\hat{\beta}$ is the estimated coefficient from the regression of X_t on Y_t, and $\mathrm{SE}(\hat{\beta})$ is its standard error.

2.3 Summary

The CADF test is a powerful tool for identifying long-term relationships between non-stationary time series. By analyzing the cointegration of pairs of assets, we can gain insights into their underlying economic relationships and develop trading strategies that exploit these connections. For further details on the CADF test, refer to engle1987cointegration, which provides a comprehensive introduction to cointegration and the methodology used in the CADF test.

Source: Engle, R. F., & Granger, C. W. J. (1987). Cointegration and Error Correction: Representation, Estimation, and Testing. Econometrica, 55(2), 251-276. Retrieved from `https://www.jstor.org/stable/1913236`

Results

Running the code these are the results:

Pairs	**Test Statistic**	**p-value**	**Interpretation**
USDJPY / NK	-1.019	0.899169	Not cointegrated
USDCAD / CL	-0.650	0.951786	Not cointegrated
CL / USO	-0.208	0.931990	Not cointegrated
GC / SI	-1.244	0.845952	Not cointegrated
AUDJPY / SPX	-0.795	0.935377	Not cointegrated
USDCHF / GC	-3.959	0.008211	Cointegrated
C / GS	-1.941	0.559243	Not cointegrated
AAPL / META	-1.671	0.690029	Not cointegrated

Table 2: ADF Test Results for Various Asset Pairs

Most of the pairs tested do not show significant evidence of cointegration, with the exception of the USD/CHF and Gold Futures pair, which demonstrates a strong long-term equilibrium relationship. This result is important for the development of trading strategies based on cointegration, as only cointegrated pairs offer opportunities to exploit such stable relationships. Therefore, for most of the pairs analyzed, other trading methodologies may be more appropriate.

3 PARAMETERS ESTIMATION

Mean reversion is a financial theory suggesting that asset prices and historical returns eventually revert to their long-term average levels. This phenomenon is frequently observed in various financial markets where prices tend to fluctuate around a long-term mean or equilibrium level. The key assumption is that deviations from the mean are temporary and will eventually correct, allowing for potential trading opportunities. Mathematically, mean reversion can be represented as:

$$X_t = \mu + \phi(X_{t-1} - \mu) + \epsilon_t \tag{1}$$

where X_t is the price at time t, μ is the long-term mean, ϕ is the mean-reversion speed, and ϵ_t is a white noise error term. SOURCE!!!

The mean-reversion trading strategy implemented in the provided script leverages the Ornstein-Uhlenbeck (OU) process to model the statistical behavior of financial time series, such as stock prices. This approach captures the tendency of asset prices to revert to their long-term mean, allowing the strategy to exploit temporary deviations. By identifying and trading these deviations, the strategy aims to capitalize on the expected price corrections over time. We take the past data of the last ten days to estimate the parameters.

Data Connect to IBKR

Data were taken from Interactive Brokers Trader Workstation (IBKR TWS) using the `ib_insync` library. The script fetches historical price data for USD/CHF and GC (Gold Futures) over the past 10 days, with minute intervals. This data is used for parameter estimation.

Definition of portfolio

We start constructing a portfolio by holding a risky asset S^1 and shorting B shares of another risky asset S^2. This resulting in

$$X_t = S_t^1 - BS_t^2 \tag{2}$$

In which B is the pair ratio (which can be positive or negative), that determines how many units of the second asset s^2 are needed to offset or hedge one unit of the first asset S^{1}[5].

For the USDCHF/GC pair, we define:

- S_t^1: USDCHF (exchange rate)
- S_t^2: GC (price of Gold)

[5]Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck (OU) process is a continuous-time stochastic process used to model mean-reverting behavior in financial markets. It is particularly suitable for describing the dynamics of asset prices that exhibit tendencies to revert to a mean level. The OU process is defined by the following stochastic differential equation (SDE):

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t \tag{3}$$

Here, θ represents the rate of mean reversion, μ is the long-term mean, σ is the volatility (the degree of variation or dispersion of the price), and W_t is a standard Wiener process (Brownian motion). The term $\theta(\mu - X_t)$ drives the process towards the mean μ, while σdW_t adds randomness to the movement of X_t.[6].

Parameters Estimation

Estimating the parameters θ, μ, and σ is crucial for accurately modeling the OU process. We estimate the parameters through maximum likelihood estimation, tracking the time series data for each portfolio. By maximizing the average log-likelihood $\ell(\theta, \mu, \sigma \mid X_{B_0}, X_{B_1}, \ldots, X_{B_n})$, we determine the parameters θ^*, μ^*, and σ^*, which in turn help us identify optimal entry and exit points for trades.

$$\begin{aligned} \ell(\theta, \mu, \sigma \mid X_0^B, X_1^B, \ldots, X_n^B) &= \frac{1}{n} \sum_{i=1}^{n} \ln f^{OU}(X_i^B \mid X_{i-1}^B; \theta, \mu, \sigma) \\ &= -\frac{1}{2} \ln(2\pi) - \ln(\tilde{\sigma}) - \frac{1}{2\tilde{\sigma}^2} \sum_{i=1}^{n} \left[X_i^B - X_{i-1}^B - \theta(1 - e^{-\mu \Delta t})\right]^2 \end{aligned} \tag{4}$$

where Δt is the time step between observations. The parameters are estimated by maximizing this likelihood function over the historical price data.[7].

The OU parameters estimated based on the historical data are the following:

- $\mu = 0.01602$ (mean reversion speed)
- $\theta = -590.8839$ (long-term mean price)
- $\sigma = 0.3190$ (volatility)

[6]Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

[7]Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

Optimization of B

The optimization of the parameter B is integral to constructing a mean-reverting portfolio. The portfolio value X_t is defined as a linear combination of two assets, S_1 and S_2, with a ratio B:

$$X_t = S_1(t) - BS_2(t) \tag{5}$$

The optimal B is determined by maximizing the log-likelihood of fitting the portfolio values to the OU process. This involves finding the value of B that best aligns the portfolio with the mean-reverting dynamics described by the OU model. The optimization is typically done using a lookback window, as shown in the formula:

$$\hat{B} = \arg\max_{B} \left\{ \frac{1}{n} \sum_{i=1}^{n} \ln \mathcal{L} \left(X_{t_i} | X_{t_{i-1}}, \theta, \mu, \sigma \right) \right\} \tag{6}$$

[8].

For the USDCHF/GC pair, the optimal value of B is calculated as follows:

- **USDCHF/GC:** $B = 0.2553$

This value of B is used to construct a mean-reverting portfolio between the USD/CHF exchange rate and the price of Gold.

IMPORTANT: The portfolio series need to be updated periodically using the optimized pair ratio.

4 ENTRY AND OPTIMIZED EXIT

In this section, we will detail the methodology for opening and closing positions within our trading strategy.

Entry Condition

At every hour, the strategy evaluates whether to generate an entry signal. The entry signal, denoted as Sig_{S1}, is triggered if the portfolio value X_t falls below the moving average minus one standard deviation. Mathematically, this condition is expressed as:

$$\text{Sig}_{S1} = \begin{cases} 1, & \text{if } X_t < \text{MA}(X_t) - \sigma(X_t) \\ 0, & \text{otherwise} \end{cases}$$

This criterion indicates that the portfolio value is substantially lower than its historical average, suggesting a potential opportunity for mean reversion.

[8]Source: Donovan Lee, Tim Leung, *On the Efficacy of Optimized Exit Rule for Mean Reversion Trading*, 2020.

Entry Signal Execution

Upon the condition $\mathrm{Sig}_{S1} = 1$, the strategy proceeds to enter a position. This entails taking a long position in the first asset $S1$ and a corresponding short position in the second asset $S2$, adjusted by the optimized pair ratio B. The corresponding signal for the **second asset** is given by:

$$\mathrm{Sig}_{S2} = -B \cdot \mathrm{Sig}_{S1}$$

This strategy maintains market neutrality and capitalizes on mean reversion by entering positions when the portfolio is undervalued. It leverages expected price corrections towards the historical mean to maximize returns.

Optimal Exit Level b_t

To find the optimal exit point we have to find before the optimal exit level, b_t, that is essential for maximizing trading strategy profits. The steps to determine and utilize this level are as follows:

To find the optimal exit level, solve an optimal stopping problem. This aims to maximize the expected payoff by exiting at the most favorable time:

$$V(x) = \sup_{\tau} \mathbb{E}\left[e^{-r\tau}(X_\tau - c)\right]$$

where r is the discount rate, and c represents costs related to holding or exiting a position.

The optimal exit level b_t is found by solving:

$$F(b) = (b - c)F'(b)$$

Here:

$$F(x) = \int_0^\infty u^{\frac{2\mu}{\sigma^2}-1} e^{\sqrt{\frac{2\mu}{\sigma^2}}(\theta - x)u - \frac{u^2}{2}} \, du$$

Using numerical methods, approximate $F(x)$ and its derivative to find b. This level b_t is a dynamic threshold adjusting to market conditions, ensuring exits at optimal points.

Exit Condition

The strategy uses the optimal exit level b_t to decide when to exit positions to maximize returns. Hourly, the strategy checks if the portfolio value X_t exceeds the optimal exit level b_t or the moving average plus one standard deviation. The exit signals are defined as follows:

1. **Moving Average Exit**: - Exit if X_t exceeds the moving average plus one standard deviation:

$$\mathrm{Sig}_{S1} = \begin{cases} 0, & \text{if } X_t > \mathrm{MA}(X_t) + \sigma(X_t) \\ \mathrm{Sig}_{S1,t-1}, & \text{otherwise} \end{cases}$$

2. **Optimal Exit Level**: - Exit if X_t exceeds the optimal exit level b_t:

$$\mathrm{Sig}_{S1} = \begin{cases} 0, & \text{if } X_t > b_t \\ \mathrm{Sig}_{S1,t-1}, & \text{otherwise} \end{cases}$$

The optimal exit level b_t helps to:

- Reduce trade frequency and transaction costs.
- Ensure exits at the best possible prices, maximizing returns.

For the **second asset**, adjust the exit signal based on the pair ratio B_t:

$$\mathrm{Sig}_{S2} = \begin{cases} 0, & \text{if } \mathrm{Sig}_{S1} = 0 \\ -\mathrm{Sig}_{S2,t-1}, & \text{otherwise} \end{cases}$$

Exit Signal Execution

Upon meeting exit conditions, the strategy liquidates positions by selling the long in $S1$ and covering the short in $S2$, optimizing the capture of profits.

5 BACKTESTING OF THE STRATEGY

Before proceeding with our trading algorithm, we conducted a thorough backtest. The backtest involved estimating the parameters using minute-level data from the entire month of May. As previously explained, we estimated the parameters as of May 31, 2024. The estimated parameters are:

- $\mu = 0.1789$
- $\theta = -172.8241$
- $\sigma = 0.0819$
- $B = 0.0744$
- $b = -172.4148$

Using these parameters, we tested the strategy with an initial capital of 100,000 CHF, resulting in a final value of 102,000 CHF. The return.....etc.

6 HOW TO USE THE STRATEGY IN A LIVE ACCOUNTS

It is crucial to follow two main steps:

Step 1: Estimate the parameters μ, θ, σ, B, and the optimal exit level (b) for today using minute-level data from the past 30 days.

Step 2: After estimating these parameters, connect the code to Interactive Brokers (IBKR) to access live market data. The algorithm will then autonomously monitor and manage positions, ensuring the strategy is executed in real-time.