

Experimentalphysik II (SS 2023/2024)

Übung 12

Tutorium: 2

Abgabe: 07.07.2023

**Aufgabe 1: Energie des Magnetfeld**

(a)

$$\begin{aligned}L &= N \cdot U + l \\N &= \frac{L - l}{2\pi \cdot r} \\&\approx \frac{1500 \text{ m} - 1.8 \text{ m}}{2\pi \cdot 2.5 \text{ cm}} \\&\approx 9537.84\end{aligned}$$

(b)

$$\begin{aligned}B &= \mu_0 \frac{NI}{l} \\&\approx 1.26 \cdot 10^{-6} \frac{\text{N}}{\text{A}^2} \cdot \frac{9537.84 \cdot 350 \text{ A}}{1.8 \text{ m}} \\&\approx 2.34 \text{ T}\end{aligned}$$

$$\begin{aligned}w &= \frac{W}{V} \\&= \frac{1}{2} \frac{\mu_0 N^2 F}{l} I^2 \cdot \frac{1}{Fl} \\&= \frac{1}{2\mu_0} \left( \mu_0 \frac{NI}{l} \right)^2 \\&= \frac{B^2}{2\mu_0} \\&\approx \frac{2.34^2 \text{ T}^2}{2 \cdot 1.26 \cdot 10^{-6} \frac{\text{N}}{\text{A}^2}} \\&\approx 2.17 \cdot 10^6 \frac{\text{J}}{\text{m}^3}\end{aligned}$$

$$\begin{aligned}E &= \frac{1}{2} LI^2 \\L &= \frac{\mu_0 N^2 F}{l} \\&= \frac{\mu_0 F (L - l)^2}{4l\pi^2 r^2} \\E &= \frac{\mu_0 F (L - l)^2}{8l\pi^2 r^2} I^2\end{aligned}$$

(c)

$$\begin{aligned}W_{total} &= w \cdot V \\&= w \cdot \frac{1}{2} \pi r^2 \cdot l \\&\approx 2.17 \cdot 10^6 \frac{\text{J}}{\text{m}^3} \cdot \frac{1}{2} \pi \cdot 2.5^2 \text{ cm}^2 \cdot 1.8 \text{ m} \\&\approx 3.84 \text{ kJ}\end{aligned}$$

## Aufgabe 2: L-C-Tief- und -Hochpass

(a)

Impedanz des Kondensators:

$$\begin{aligned}U &= \frac{Q}{C} \\ \underline{\dot{u}} &= \frac{\underline{\dot{i}}}{C} \\ j\omega \cdot \underline{u} &= \frac{\underline{\dot{i}}}{C} \\ \underline{Z} &= \frac{\underline{u}}{\underline{\dot{i}}} = \frac{1}{j\omega C}\end{aligned}$$

Impedanz der Spule:

$$\begin{aligned}U &= L\dot{I} \\ \underline{u} &= L \cdot j\omega \cdot \underline{i} \\ \underline{Z} &= \frac{\underline{u}}{\underline{i}} = j\omega L\end{aligned}$$

Frequenzgang:

$$\begin{aligned}\frac{\underline{u_a}}{\underline{u_e}} &= \frac{\underline{Z_a}}{\underline{Z_e}} \\&= \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L} \\&= \frac{\frac{1}{\omega C}}{\frac{1}{\omega C} - \omega L} \\&= \frac{1}{1 - \omega^2 LC} \\&= \frac{1}{1 - \frac{\omega^2}{\omega_0^2}}\end{aligned}$$

(b)

$$\frac{\underline{u_a}}{\underline{u_e}} = \frac{\underline{Z_a}}{\underline{Z_e}}$$

$$\begin{aligned}
&= \frac{j\omega L}{\frac{1}{j\omega C} + j\omega L} \\
&= \frac{1}{1 - \frac{1}{\omega^2 L C}} \\
&= \frac{1}{1 - \frac{\omega_0^2}{\omega^2}}
\end{aligned}$$

Die Ausgangsspannung kann, ohne die Energieerhaltung zu verletzen, größer werden als die Eingangsspannung, da in dem Kondensator und in der Spule Energie temporär in den Feldern gespeichert wird und anschließend auch wieder in den Stromkreis gespeist werden kann wodurch ein kurzer (prinzipiell beliebig groß) peak in der Spannung/Strom entstehen kann. Die Ausgangsspannung kann auch permanent höher sein als die Eingangsspannung, jedoch ist dann der fließende Strom geringer.

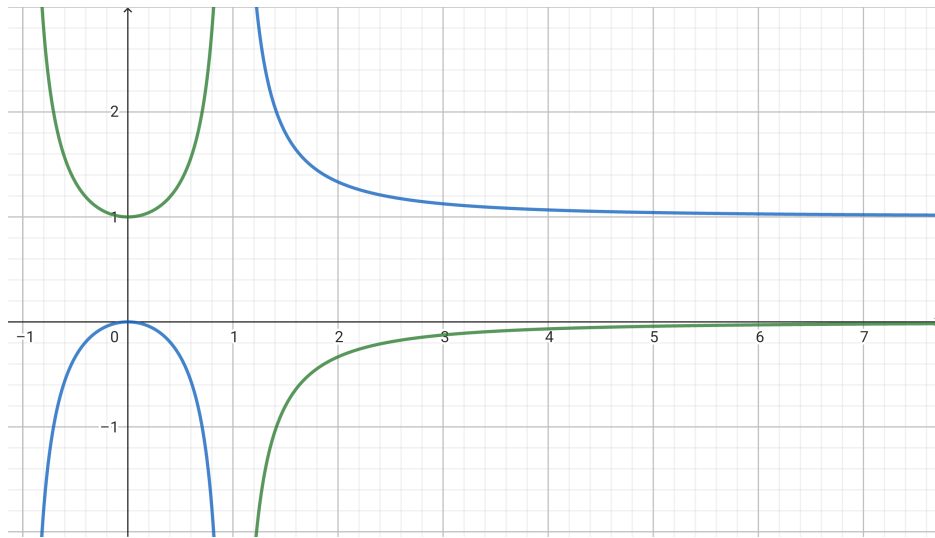


Figure 1: X-Achse:  $\frac{\omega}{\omega_0}$ , Y-Achse:  $\frac{U_a}{U_e}$   
grün: erste Schaltskizze, blau: zweite Schaltskizze

### Aufgabe 3: Transformator

(a)

$$\begin{aligned}
\underline{u_n} &= \sum_m L_m \frac{di_m}{dt} \\
\underline{u_n} &= N \cdot \dot{\Phi} \\
&= N_n (\dot{\Phi}_{nn} + \dot{\Phi}_{mn}) \\
&= N_n (A \dot{B}_n + A \dot{B}_m) \\
&= N_n \left( A \cdot \mu_0 \mu_r \frac{N_n}{l} \dot{i}_n + A \cdot \mu_0 \mu_r \frac{N_m}{l} \dot{i}_m \right) \\
&= \underbrace{\mu_0 \mu_{r,n} \frac{N_n^2 A}{l} \dot{i}_n}_{L_n} + \underbrace{\mu_0 \mu_{r,mn} \frac{N_n N_m A}{l} \dot{i}_m}_{L_{nm}} \\
&= L_n \dot{i}_n + L_{nm} \dot{i}_m
\end{aligned}$$

$$\begin{aligned}
&= j\omega L_n i_n + j\omega L_{nm} i_m \\
&\rightarrow \begin{cases} u_1 = j\omega L_1 i_1 + j\omega L_{12} i_2 \\ u_2 = Z i_2 = j\omega L_2 i_2 + j\omega L_{12} i_1 \end{cases} \\
&\rightarrow \begin{cases} u_1 = j\omega L_1 i_1 + j\omega L_{12} i_2 \\ i_2 = \frac{j\omega L_{12}}{Z - j\omega L_2} i_1 \end{cases} \\
&\rightarrow \begin{cases} u_1 = j\omega \left( L_1 + \frac{j\omega L_{12}^2}{Z - j\omega L_2} \right) i_1 \\ \quad = \frac{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)}{Z - j\omega L_2} i_1 \end{cases} \\
&\rightarrow \begin{cases} u_1 = \frac{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)}{Z - j\omega L_2} i_1 \\ u_2 = -\frac{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)}{j\omega L_{12}} i_2 \end{cases} \\
&\frac{i_2}{i_1} = -\frac{j\omega L_{12}}{Z + j\omega L_2} \\
&\frac{u_2}{i_1 Z} = -\frac{j\omega L_{12}}{Z + j\omega L_2} \\
&\frac{u_2}{u_1} = -\frac{j\omega L_{12} Z}{j\omega L_1 Z + \omega^2 (L_{12}^2 - L_1 L_2)} \\
&\approx
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{a + bi}{c + di} &= \frac{(a + bi)(c + id)}{c^2 + d^2} \\
&= \frac{(ac - bd) + i(ad + bc)}{c^2 + d^2} \\
\left| \frac{a + bi}{c + di} \right| &= \frac{ac - bd}{c^2 + d^2} \\
\left| \frac{a + bi}{c + di} \right| / \left| \frac{e + fi}{c + di} \right| &= \frac{ac - bd}{c^2 + d^2} \frac{c^2 + d^2}{ec - fd} \\
&= \frac{ac - bd}{ec - fd} \\
i_1 &= \frac{Z - j\omega L_2}{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)} u_1 \\
i_2 &= \frac{-j\omega L_{12}}{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)} u_1 \\
\frac{|i_2|}{|i_1|} &= \frac{\omega^2 Z (L_1 L_2 - L_{12}^2) + \omega^2 Z L_1 L_2}{\omega^2 Z L_1 L_{12}} \\
&= \frac{(L_1 L_2 - L_{12}^2) + L_1 L_2}{L_1 L_{12}} \\
&= \frac{\mu_0 \mu_r \frac{N_1^2 A}{l} \cdot \mu_0 \mu_r \frac{N_2^2 A}{l} - \mu_0^2 \mu_r^2 \frac{N_1^2 N_2^2 A^2}{l^2} + \mu_0 \mu_r \frac{N_1^2 A}{l} \cdot \mu_0 \mu_r \frac{N_2^2 A}{l}}{\mu_0 \mu_r \frac{N_1^2 A}{l} \cdot \mu_0 \mu_r \frac{N_1 N_2 A}{l}} \\
&= \frac{N_1^2 \cdot N_2^2}{N_1^2 \cdot N_1 N_2} \\
&= \frac{N_2}{N_1}
\end{aligned}$$

(c)

$$\begin{aligned}\frac{u_2}{u_1} &= -\frac{j\omega L_{12}Z}{j\omega L_1Z + \omega^2(L_{12}^2 - L_1L_2)} \\ &= -\frac{j\omega L_{12}\frac{1}{j\omega C}}{j\omega L_1\frac{1}{j\omega C} + \omega^2(L_{12}^2 - L_1L_2)} \\ &= -\frac{L_{12}/C}{L_1/C + \omega^2(L_{12}^2 - L_1L_2)}\end{aligned}$$

#### Aufgabe 4: Schwingkreise

(a)

Aus dem Aufbau folgt mit der Knotenregel direkt:

$$\rightarrow \begin{cases} 0 = I_R + I_C + I_L \\ U_R = U_C = U_L \hat{=} U \end{cases}$$

$$\begin{aligned}0 &= I_R + I_C + I_L \\ 0 &\stackrel{(1)}{=} \frac{U}{R} + \dot{Q} + \frac{1}{L} \int U \, dt \\ 0 &= \frac{Q}{RC} + \dot{Q} + \frac{1}{LC} \int Q \, dt \\ 0 &= \ddot{Q} + \frac{\dot{Q}}{RC} + \frac{Q}{LC}\end{aligned}$$

$$(1) : \text{ denn } \begin{cases} U_L = L\dot{I}_L \\ I_L = \frac{1}{L} \int U_L \, dt \end{cases}$$

(b)

$$\begin{aligned}\underline{Y}_{ges} &= \underline{Y}_R + \underline{Y}_C + \underline{Y}_L \\ &= \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \\ \underline{Z}_{ges} &= \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} \\ &= \frac{\frac{1}{R} - j\left(\omega C - \frac{1}{\omega L}\right)}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} \\ Z &= |\underline{Z}_{ges}| = \sqrt{\left(\frac{\frac{1}{R}}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\right)^2 + \left(\frac{\left(\omega C - \frac{1}{\omega L}\right)}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\right)^2} \\ Y_C &= \\ U &= \frac{Q}{C}\end{aligned}$$

$$\begin{aligned}
0 &= \ddot{Q} + \frac{\dot{Q}}{RC} + \frac{Q}{LC} \\
0 &= \ddot{I} + \frac{\dot{I}}{RC} + \frac{I}{LC} \\
0 &\stackrel{(1)}{=} -\omega^2 \cdot \underline{i} + j\omega \cdot \frac{\underline{i}}{RC} + \frac{\underline{i}}{LC} \\
Z &= \frac{U}{I}
\end{aligned}$$

(1) : da  $\underline{i} = \hat{i}e^{j(\omega t + \phi)} \rightarrow \dot{\underline{i}} = j\omega \cdot \hat{i}e^{j(\omega t + \phi)}$

(c)

Ansatz  $\rightarrow$  Exponentialfunktion :

$$\begin{aligned}
Q &= ce^{\lambda t} \quad , \quad c \in \mathbb{C} \\
0 &= \ddot{Q} + \frac{\dot{Q}}{RC} + \frac{Q}{LC} \\
0 &= \lambda^2 + \frac{\lambda}{RC} + \frac{1}{LC} \\
\lambda &= -\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}} \\
Q(t) &= e^{-\frac{t}{2RC}} \left( c_1 e^{t\sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}} + c_2 e^{-t\sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}} \right)
\end{aligned}$$

$$\begin{aligned}
A &= \frac{1}{2RC} \\
B &= \sqrt{A^2 - \frac{1}{LC}} \\
Q(t) &= e^{-At} \left( c_1 e^{Bt} + c_2 e^{-Bt} \right)
\end{aligned}$$

$$Q_{B \in \mathbb{R}}(t) \equiv Q_{\mathbb{R}}(t) = e^{-At} \left( c_1 e^{Bt} + c_2 e^{-Bt} \right)$$

$$Q_{\mathbb{R}}(0) = Q_0$$

$$\dot{Q}_{\mathbb{R}}(0) = 0$$

$$Q_0 = c_1 + c_2$$

$$0 = B(c_1 - c_2) - A(c_1 + c_2)$$

$$0 = B(2c_1 - Q_0) - AQ_0$$

$$c_1 = \frac{Q_0}{2} \left( \frac{A}{B} + 1 \right)$$

$$\begin{aligned}
c_2 &= Q_0 - \frac{Q_0}{2} \left( \frac{A}{B} + 1 \right) \\
&= \frac{Q_0}{2} \left( 1 - \frac{A}{B} \right)
\end{aligned}$$

$$Q_{B \in \mathbb{C}}(t) = e^{-At} \left( c_1 (\cos(|B|t) + i \sin(|B|t)) + c_2 (\cos(|B|t) - i \sin(|B|t)) \right)$$

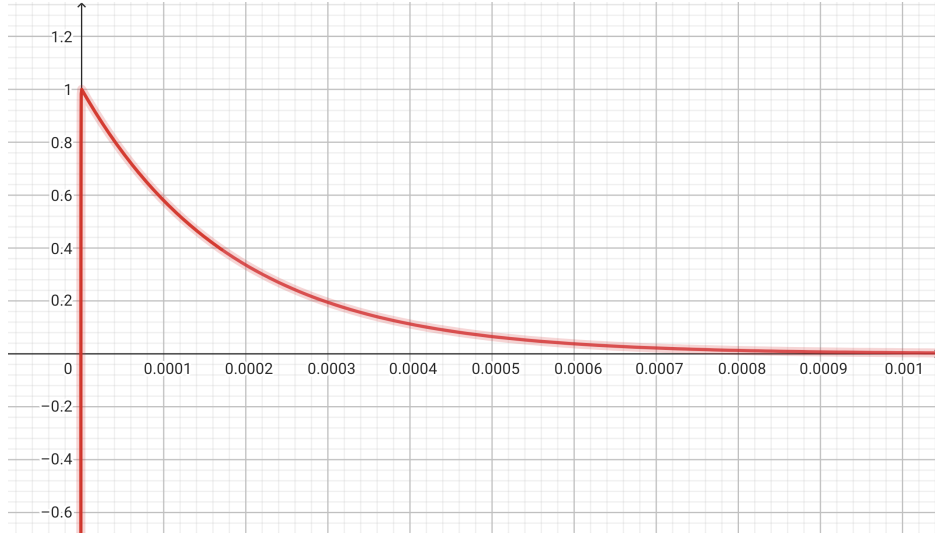


Figure 2:  $R = 0.024 \Omega$

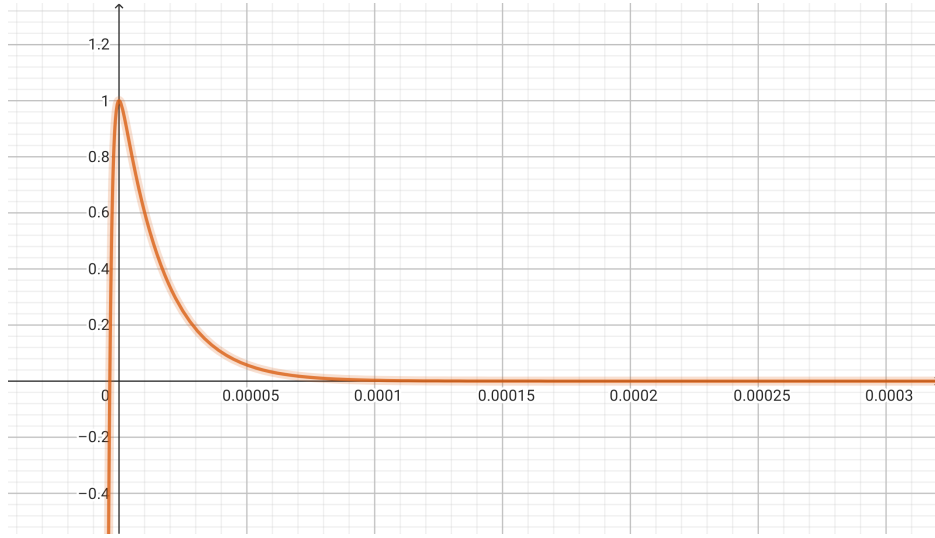


Figure 3:  $R = 0.24 \Omega$

$$\begin{aligned}
 \Re(Q_{B \in \mathbb{C}}(t)) &\equiv Q_{\mathbb{C}}(t) = e^{-At} (c_{11} \cos(|B|t) - c_{12} \sin(|B|t) + c_{21} \cos(|B|t) + c_{22} \sin(|B|t)) \\
 &= e^{-At} ((c_{11} + c_{21}) \cos(|B|t) + (c_{22} - c_{12}) \sin(|B|t)) \\
 &= e^{-At} (c_1 \cos(|B|t) + c_2 \sin(|B|t)) \\
 Q_{\mathbb{C}}(0) &= Q_0 \\
 \dot{Q}_{\mathbb{C}}(0) &= 0 \\
 Q_0 &= c_1 \\
 0 &= -Ac_1 + |B|c_2 \\
 c_2 &= Q_0 \frac{A}{|B|}
 \end{aligned}$$

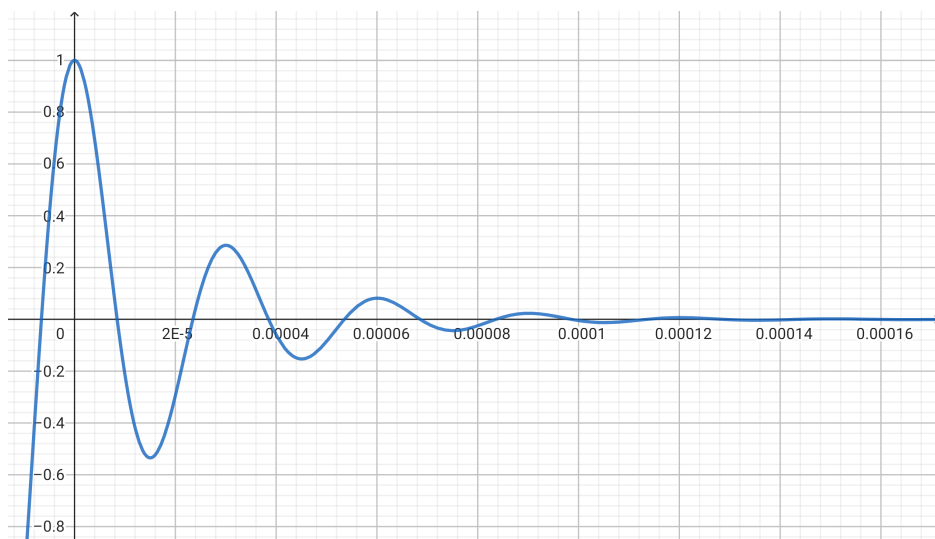


Figure 4:  $R = 2.4 \Omega$