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Experimentalphysik Vb (WS 2023/2024)

Exercise 3

Tutorial: 1 Deadline: 08.11.2024

Aufgabe 1: Time-of-flight system

(a)

I feel like the task isn't explained well enough. There is only a well defined „maximum momentum“ if both particles carry the same momentum. So are we to assume that is the case? The task doesn't say. Also isn't there an underlying assumption that we can associate the signals of the scintillator to the correct particle? 1.1

1.2

The time-of-flight of a single particle is

$$t = \frac{L}{v} = \frac{LE}{pc^2} = \frac{LE}{p}$$

The time difference of two particles is then given by

$$\begin{aligned} \Delta t = |t_2 - t_1| &= \left| \frac{LE_2}{p_2} - \frac{LE_1}{p_1} \right| \\ &= Lc \left| \sqrt{m_2^2 c^2/p_2^2 + 1} - \sqrt{m_1^2 c^2/p_1^2 + 1} \right| \\ &= L \left| \sqrt{m_2^2/p_2^2 + 1} - \sqrt{m_1^2/p_1^2 + 1} \right| \\ \implies \sigma_{\Delta t} = \sigma_{t_2} \oplus \sigma_{t_2} &= \sqrt{2}\sigma_t \end{aligned}$$

The time difference must be at least $4\sigma_{\Delta t}$:

$$\begin{aligned} 4\sigma_{\Delta t} &\leq \Delta t \\ 4\sqrt{2}\sigma_t &= L \left| \sqrt{m_2^2/p_{\max}^2 + 1} - \sqrt{m_1^2/p_{\max}^2 + 1} \right| \\ &\overset{*}{\approx} L \left| \frac{1}{2} m_2^2/p_{\max}^2 - \frac{1}{2} m_1^2/p_{\max}^2 \right| \\ p_{\max} &= \sqrt{\frac{L}{8\sqrt{2}\sigma_t} \left| m_2^2 - m_1^2 \right|} \end{aligned}$$

Note that at the following taylor expansion was used in the step with a star:

$$f(x) = \sqrt{1+x^2}, \qquad f'(x) = \frac{x}{f(x)}$$

$$f''(x) = \frac{1}{f(x)} - \frac{x^2}{f^3(x)}, \qquad f'''(x) = -\frac{x}{f^3(x)} - \frac{2x}{f^3(x)} + \frac{3x^3}{f^5(x)}$$

$$f(x) = \sum_{n=0}^{3} \frac{f^{(n)}(0)}{n!} x^n + \mathcal{O}\left(x^4\right) = 1 + \frac{1}{2}x^2 + \mathcal{O}\left(x^4\right)$$

(b)

With the values

$$L = 1m = 5067730 \frac{1}{\mathrm{eV}}, \qquad \sigma_t = 100\,\mathrm{ps} = 151926 \frac{1}{\mathrm{eV}}$$

$$m(K^+) = 494\,\mathrm{MeV}, \qquad m(\pi^+) = 140\,\mathrm{MeV}$$

2.1

the derived formula yields:

$$p_{\max}(K^+, \pi^+) = 0.813\,\mathrm{GeV}$$

Aufgabe 2: Cherenkov detector

(a)

The cherenkov angle can be expressed as:

$$\cos(\theta_C) = \frac{1}{n\beta} = \frac{1}{n}\frac{E}{p} = \frac{1}{n}\frac{\sqrt{m^2+p^2}}{p} = \frac{1}{n}\sqrt{m^2/p^2+1}$$

Using gaussian error propagation the error σ_m is then given by $\sigma_m = \left|\frac{\mathrm{d}m}{\mathrm{d}\theta_C}\right| \cdot \sigma_{\theta_C}$, with

$$\begin{aligned}
-\sin(\theta_C)\frac{\mathrm{d}\theta_C}{\mathrm{d}m} &= \frac{1}{n}\frac{m}{p^2\sqrt{m^2/p^2+1}} \\
\left|\frac{\mathrm{d}m}{\mathrm{d}\theta_C}\right| &= np\sin(\theta_C)\sqrt{1+p^2/m^2} \\
&= np\sqrt{1-\frac{1}{n^2\beta^2}}\sqrt{1+p^2/m^2} \\
&= \frac{p}{\beta}\sqrt{n^2\beta^2-1}\sqrt{1+p^2/m^2}
\end{aligned}$$

2.2

(b)

How can the solution depend on β when there are two particles, each with different velocities and therefore distinct values β_1 and β_2? Additionally, shouldn't the significance of the mass separation be measured by $\frac{\Delta m}{\sigma_{\Delta m}}$ instead of $\frac{\Delta m}{\sigma_m}$? There seem to be several assumptions needed to even approximate the desired solution, so it feels like these should be clarified in the task itself. 3.1

The closest I've been able to get is using the following assumptions: $m = \frac{m_1+m_2}{2}$, $\sigma_{\Delta m} \approx \sigma_m$, $\beta \equiv \beta_m \approx \beta_1 \approx \beta_2$ and $p/m \gg 1$.

$$\sigma_m = \frac{p}{\beta}\sqrt{n^2\beta^2 - 1}\sqrt{1 + p^2/m^2} \cdot \sigma_{\theta_C} \approx \frac{p^2}{\beta m}\sqrt{n^2\beta^2 - 1} \cdot \sigma_{\theta_C}$$

$$\begin{aligned} N_\sigma &= \frac{\Delta m}{\sigma_m} = \frac{|m_1 - m_2|}{\sigma_m} \\ &= \frac{|m_1 - m_2|\, \beta m}{p^2 \sigma_{\theta_C}\sqrt{n^2\beta^2 - 1}} \\ &= \frac{|m_1 - m_2|\,(m_1 + m_2)\beta}{2p^2 \sigma_{\theta_C}\sqrt{n^2\beta^2 - 1}} \\ &= \frac{|m_1^2 - m_2^2|\,\beta}{2p^2 \sigma_{\theta_C}\sqrt{n^2\beta^2 - 1}} \end{aligned}$$

3.2

(c)

The velocity of the particles has to be greater then the local speed of light, for the cherenkov effect to take place:

$$\begin{aligned} \frac{c}{n} &< v = c\beta = c\frac{p}{\sqrt{m^2 + p^2}} \\ \implies p_{\text{min}} &= \frac{m}{\sqrt{n - 1}} \end{aligned}$$

Python-Code 1:

```python
import numpy as np
import matplotlib.pyplot as plt
import particle
p = particle.Particle.from_name

n = 1.0017
std = 2e-3 # rad
m_K = p("K+").mass * 1e6 # eV
m_pi = p("pi+").mass * 1e6 # eV
m = (m_K + m_pi)/2 # eV

beta = np.vectorize(lambda p: p / np.sqrt(m**2 + p**2))
N_std = np.vectorize(lambda p: abs(m_K**2 - m_pi**2) * beta(p)**2 / (2 * p**2 * std *
↪ np.sqrt(n**2 * beta(p)**2 - 1)))
```

```
lim = [m/np.sqrt(n-1), 1e11]
P = np.linspace(*lim, 1000)
N = N_std(P)
p_max = P[np.argmin(abs(N-3))]

plt.plot(P, N, label=r"$N_\sigma(p)$")
plt.axhline(3, c="r", linestyle="--", label=r"$N_\sigma=3$")
plt.axvline(p_max, c="g", linestyle="--", label=f"p_min = {lim[0]:.3} eV\n{p_max = :.3} eV")
plt.axvline(lim[0], c="g", linestyle="--")

plt.xlabel("p in eV")
plt.ylabel(r"$N_\sigma$")
plt.yscale("log")
plt.legend()
plt.savefig("3.svg")
```

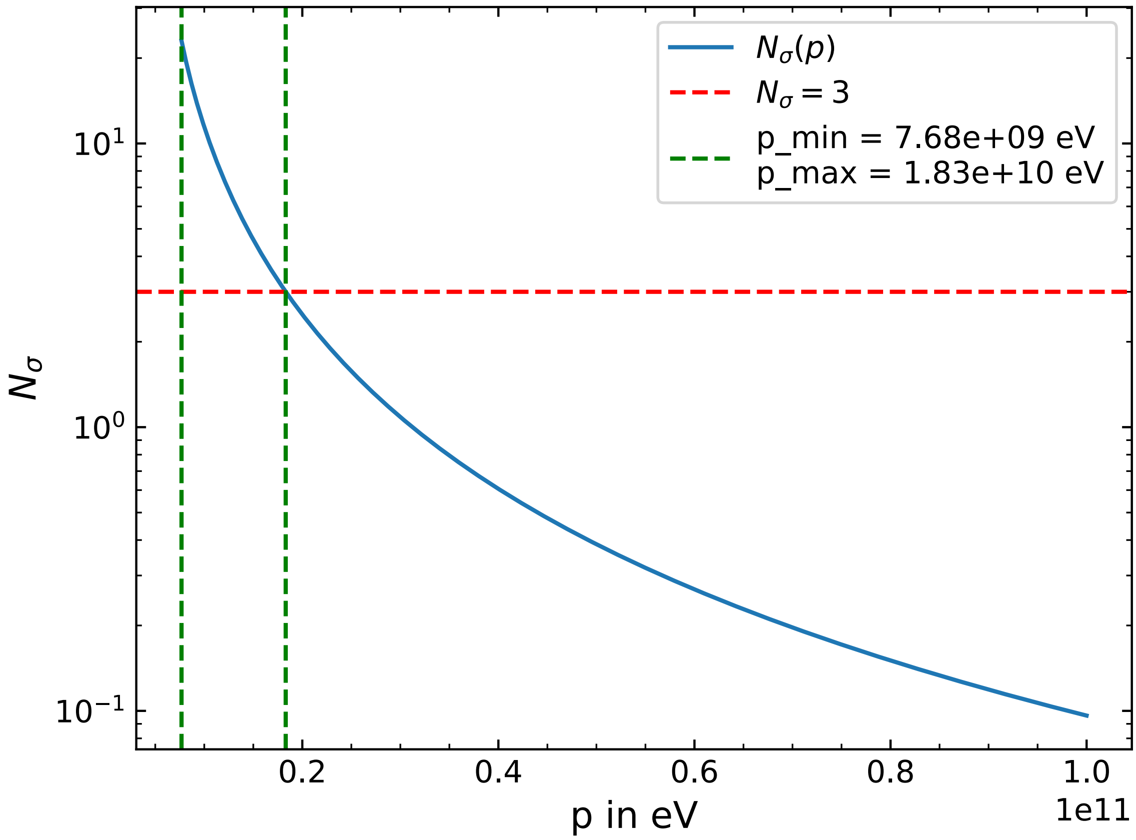

Figure 1: resulting plot

In this model the range of momenta in which the detector can differentiate the two particles with 3σ significance is given by $p \in (7.68, 18.3)\,\mathrm{GeV}$. 4.1

Aufgabe 3: Decay to four photons

$$P = \sum_i p_i = (1.412, -8 \cdot 10^{-5}, 0)\,\text{GeV}$$
$$E = \sum_i |p_i| = 1.497\,\text{GeV}$$

$$E_0 = \sqrt{E^2 - |P|^2} = 499\,\text{MeV}$$
$$E_{\text{kin}} = E - E_0 = 998\,\text{MeV}$$

In conclusion the particle must have the following proporties:

1. chargeless
2. no baryon
3. whole number spin
4. rest mass $\approx 499\,\text{MeV}$

Based on these observations a K^0 with restmass $E_0(K^0) = 498\,\text{MeV}$ seems the fit the bill, since it can decay into an intermediate state of $\pi^0\pi^0$ which then decays into four photons.
If this is correct, then it should be possible to pair the photons, such that the invariant mass of each pair matches the $E_0(\pi^0) = 135\,\text{MeV}$ mass. This is indeed the case when grouping p_0 with p_2, and p_1 with p_3:

$$E_{0,(0,2)} = \sqrt{(|p_0| + |p_2|) - (p_0 + p_2)^2} = 135\,\text{MeV}$$
$$E_{0,(1,3)} = \sqrt{(|p_1| + |p_3|) - (p_1 + p_3)^2} = 135\,\text{MeV}$$

The observed particle is therefore a K^0. 5.1

Aufgabe 4: Particle decays

Task	Reaction	Possible?	Conservation laws
(a)	$\mu^- \to e^-\bar{\nu}_e\nu_\mu$	✓	
(b)	$\mu^- \to e^-\gamma\gamma$	✗	lepton number
(c)	$\mu^+ \to e^+\bar{\nu}_c\nu_\mu$	✗	lepton number
(d)	$K^+ \to \pi^+\pi^0\pi^-$	✗	charge
(e)	$K^+ \to \pi^+\pi^+\pi^+\pi^-\pi^-$	✗	energy
(f)	$\Lambda \to \pi^+\pi^-$	✗	baryon number
(g)	$\Lambda \to p\pi^-\gamma$	✓	
(h)	$\Lambda \to p\mu^-\bar{\nu}_\mu$	✓	
(i)	$\Omega^- \to \Sigma^-\pi^+\pi^-$	✗	strangeness $\Delta S = \pm 1, 0$
(j)	$\Sigma^+ \to p$	✗	energy/momentum

5.2

Index der Kommentare

1.1 It may help to think in terms of observables. The particle detector gives you a track of a given momentum. Now the question is, with the time-of-flight system described in the question, under what conditions (i.e. for tracks of what momentum) can I ascertain the mass with the required confidence.

1.2 That is indeed an inherent assumption. Naturally this is a spherical cow model of a ToF system.

2.1 You can in principle keep the length and time in metric units, so long as you keep track of the c terms (the mass would be MeV/c^2 and momentum MeV/c)

2.2 This reads as pE/mbeta^2 sqrt(n^2beta^2-1), but in reality it's p^3/mbeta^2 [sqrt(...)]. I suspect a mistake when flipping the derivative

3.1 Again it's useful to think with detector observables. Given a track that's measured to have momentum p and produce Cherenkov radiation at angle theta_c, what is its mass? The resolution leads to an uncertainty on m, which depends only on the detector quantities (in this case theta_c). In order to differentiate between two particles of masses m1 and m2 one has to do a statistical test such as the one in this exercise.

3.2 Close, and the assumptions are correct, but it should be beta^2 in the numerator

4.1 You will still be able to distinguish between pions and kaons between 2.4 and 7.68 GeV. The former produce rings and the other do not.

5.1 missing kinetic energy

5.2 1: 4+2

2: 0 points due to not presenting

3: 2+2+0

4: 5