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Experimentalphysik II (SS 2023/2024)

Übung 12

Tutorium: 2 Abgabe: 07.07.2023

Aufgabe 1: Energie des Magentfeld

(a)

$$\begin{aligned} L &= N \cdot U + l \\ N &= \frac{L - l}{2\pi \cdot r} \\ &\approx \frac{1500\,\mathrm{m} - 1.8\,\mathrm{m}}{2\pi \cdot 2.5\,\mathrm{cm}} \\ &\approx 9537.84 \end{aligned}$$

(b)

$$\begin{aligned} B &= \mu_0 \frac{NI}{l} \\ &\approx 1.26 \cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2} \cdot \frac{9537.84 \cdot 350\,\mathrm{A}}{1.8\,\mathrm{m}} \\ &\approx 2.34\,\mathrm{T} \end{aligned}$$

$$\begin{aligned} w &= \frac{W}{V} \\ &= \frac{1}{2}\frac{\mu_0 N^2 F}{l} I^2 \cdot \frac{1}{Fl} \\ &= \frac{1}{2\mu_0}\left(\mu_0 \frac{NI}{l}\right)^2 \\ &= \frac{B^2}{2\mu_0} \\ &\approx \frac{2.34^2\,\mathrm{T}^2}{2 \cdot 1.26 \cdot 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2}} \\ &\approx 2.17 \cdot 10^6\,\frac{\mathrm{J}}{\mathrm{m}^3} \end{aligned}$$

$$\begin{aligned} E &= \frac{1}{2} L I^2 \\ L &= \frac{\mu_0 N^2 F}{l} \\ &= \frac{\mu_0 F (L-l)^2}{4 l \pi^2 r^2} \\ E &= \frac{\mu_0 F (L-l)^2}{8 l \pi^2 r^2} I^2 \end{aligned}$$

(c)

$$\begin{aligned}
W_{total} &= w \cdot V \\
&= w \cdot \frac{1}{2}\pi r^2 \cdot l \\
&\approx 2.17 \cdot 10^6 \,\frac{\mathrm{J}}{\mathrm{m}^3} \cdot \frac{1}{2}\pi \cdot 2.5^2\,\mathrm{cm}^2 \cdot 1.8\,\mathrm{m} \\
&\approx 3.84\,\mathrm{kJ}
\end{aligned}$$

Aufgabe 2: L-C-Tief- und -Hochpass

(a)

Impedanz des Kondensators:

$$\begin{aligned}
U &= \frac{Q}{C} \\
\dot{\underline{u}} &= \frac{\underline{i}}{C} \\
j\omega \cdot \underline{u} &= \frac{\underline{i}}{C} \\
\underline{Z} &= \frac{\underline{u}}{\underline{i}} = \frac{1}{j\omega C}
\end{aligned}$$

Impedanz der Spule:

$$\begin{aligned}
U &= L\dot{I} \\
\underline{u} &= L \cdot j\omega \cdot \underline{i} \\
\underline{Z} &= \frac{\underline{u}}{\underline{i}} = j\omega L
\end{aligned}$$

Frequenzgang:

$$\begin{aligned}
\frac{\underline{u_a}}{\underline{u_e}} &= \frac{\underline{Z_a}}{\underline{Z_e}} \\
&= \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L} \\
&= \frac{\frac{1}{\omega C}}{\frac{1}{\omega C} - \omega L} \\
&= \frac{1}{1 - \omega^2 LC} \\
&= \frac{1}{1 - \frac{\omega^2}{\omega_0^2}}
\end{aligned}$$

(b)

$$\frac{\underline{u_a}}{\underline{u_e}} = \frac{\underline{Z_a}}{\underline{Z_e}}$$

$$
\begin{aligned}
&= \frac{j\omega L}{\frac{1}{j\omega C} + j\omega L} \\
&= \frac{1}{1 - \frac{1}{\omega^2 LbC}} \\
&= \frac{1}{1 - \frac{\omega_0^2}{\omega^2}}
\end{aligned}
$$

Die Ausgangsspannung kann, ohne die Energieerhaltung zu verletzten, größer werden als die Eingangsspannung, da in dem Kondensator und in der Spule Energie temporär in den Feldern gespeichert wird und anschließend auch wieder in den Spromkreis gespeist werden kann wodurch ein kurzer (prinzipiell beliebig großer) peak in der Spannung/Storm entstehen kann. Die Ausgangs Spannung kann auch permanent höher sein als die Eingangsspannung, jedoch ist dann der fließende Strom geringer.

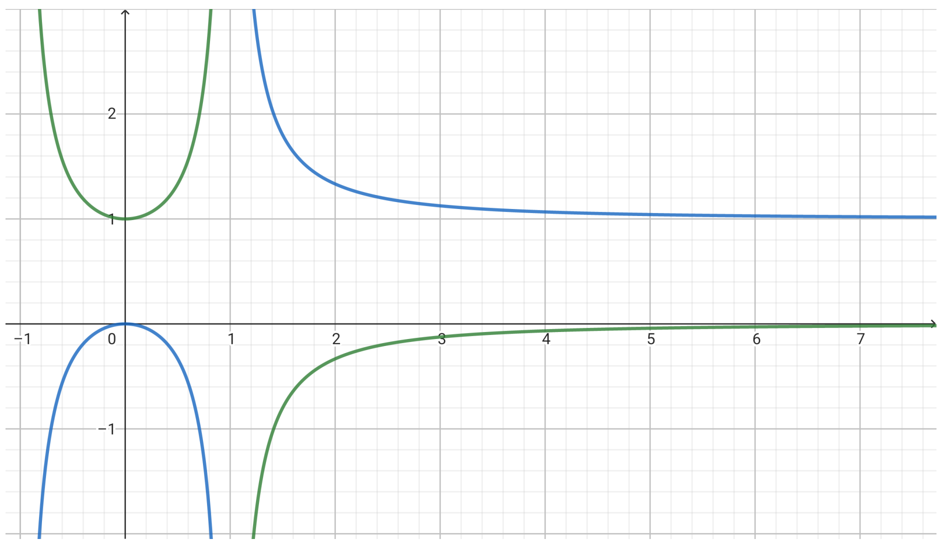

Figure 1: X-Achse: $\frac{\omega}{\omega_0}$, Y-Achse: $\frac{U_a}{U_e}$
grün: erste Schaltskizze, blau: zweite Schaltskizze

Aufgabe 3: Transformator

(a)

$$
\begin{aligned}
\underline{u_n} &= \sum_m L_m \frac{\mathrm{d}\underline{i_m}}{\mathrm{d}t} \\
\underline{u_n} &= N \cdot \dot{\Phi} \\
&= N_n(\dot{\Phi}_{nn} + \dot{\Phi}_{mn}) \\
&= N_n \left(A\dot{B}_n + A\dot{B}_m \right) \\
&= N_n \left(A \cdot \mu_0 \mu_r \frac{N_n}{l} \dot{i}_n + A \cdot \mu_0 \mu_r \frac{N_m}{l} \dot{i}_m \right) \\
&= \underbrace{\mu_0 \mu_{r,n} \frac{N_n^2 A}{l}}_{L_n} \dot{i}_n + \underbrace{\mu_0 \mu_{r,mn} \frac{N_n N_m A}{l}}_{L_{nm}} \dot{i}_m \\
&= L_n \dot{i}_n + L_{nm} \dot{i}_m
\end{aligned}
$$

$$
\begin{aligned}
&= j\omega L_n i_n + j\omega L_{nm} i_m \\
&\rightarrow \begin{cases} u_1 = j\omega L_1 i_1 + j\omega L_{12} i_2 \\ u_2 = Z i_2 = j\omega L_2 i_2 + j\omega L_{12} i_1 \end{cases} \\
&\rightarrow \begin{cases} u_1 = j\omega L_1 i_1 + j\omega L_{12} i_2 \\ i_2 = \frac{j\omega L_{12}}{Z - j\omega L_2} i_1 \end{cases} \\
&\rightarrow \begin{cases} u_1 = j\omega \left(L_1 + \frac{j\omega L_{12}^2}{Z - j\omega L_2} \right) i_1 \\ \quad\;\; = \frac{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)}{Z - j\omega L_2} i_1 \end{cases} \\
&\rightarrow \begin{cases} u_1 = \frac{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)}{Z - j\omega L_2} i_1 \\ u_1 = -\frac{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)}{i\omega L_{12}} i_2 \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\frac{i_2}{i_1} &= -\frac{j\omega L_{12}}{Z + j\omega L_2} \\
\frac{u_2}{i_1 Z} &= -\frac{j\omega L_{12}}{Z + j\omega L_2} \\
\frac{u_2}{u_1} &= -\frac{j\omega L_{12} Z}{j\omega L_1 Z + \omega^2 (L_{12}^2 - L_1 L_2)}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{a+bi}{c+di} &= \frac{(a+bi)(c+id)}{c^2+d^2} \\
&= \frac{(ac-bd) + i(ad+bc)}{c^2+d^2} \\
\left| \frac{a+bi}{c+di} \right| &= \frac{ac-bd}{c^2+d^2} \\
\left| \frac{a+bi}{c+di} \right| / \left| \frac{e+fi}{c+di} \right| &= \frac{ac-bd}{c^2+d^2} \frac{c^2+d^2}{ec-fd} \\
&= \frac{ac-bd}{ec-fd} \\
i_1 &= \frac{Z - j\omega L_2}{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)} u_1 \\
i_2 &= \frac{-j\omega L_{12}}{j\omega L_1 Z + \omega^2 (L_1 L_2 - L_{12}^2)} u_1 \\
\frac{|i_2|}{|i_1|} &= \frac{\omega^2 Z (L_1 L_2 - L_{12}^2) + \omega^2 Z L_1 L_2}{\omega^2 Z L_1 L_{12}} \\
&= \frac{(L_1 L_2 - L_{12}^2) + L_1 L_2}{L_1 L_{12}} \\
&= \frac{\mu_0 \mu_r \frac{N_1^2 A}{l} \cdot \mu_0 \mu_r \frac{N_2^2 A}{l} - \mu_0^2 \mu_r^2 \frac{N_1^2 N_2^2 A^2}{l^2} + \mu_0 \mu_r \frac{N_1^2 A}{l} \cdot \mu_0 \mu_r \frac{N_2^2 A}{l}}{\mu_0 \mu_r \frac{N_1^2 A}{l} \cdot \mu_0 \mu_r \frac{N_1 N_2 A}{l}} \\
&= \frac{N_1^2 \cdot N_2^2}{N_1^2 \cdot N_1 N_2} \\
&= \frac{N_2}{N_1}
\end{aligned}
$$

(c)

$$\begin{aligned}
\frac{u_2}{u_1} &= -\frac{j\omega L_{12} Z}{j\omega L_1 Z + \omega^2 (L_{12}^2 - L_1 L_2)} \\
&= -\frac{j\omega L_{12} \frac{1}{j\omega C}}{j\omega L_1 \frac{1}{j\omega C} + \omega^2 (L_{12}^2 - L_1 L_2)} \\
&= -\frac{L_{12}/C}{L_1/C + \omega^2 (L_{12}^2 - L_1 L_2)} \\
&\approx -1.5
\end{aligned}$$

Aufgabe 4: Schwingkreise

(a)

Aus dem Aufbau folgt mit der Knotenregel direkt:

$$\rightarrow \begin{cases} 0 = I_R + I_C + I_L \\ U_R = U_C = U_L \hat{=} U \end{cases}$$

$$\begin{aligned}
0 &= I_R + I_C + I_L \\
0 &\overset{(1)}{=} \frac{U}{R} + \dot{Q} + \frac{1}{L} \int U \,\mathrm{d}t \\
0 &= \frac{Q}{RC} + \dot{Q} + \frac{1}{LC} \int Q \,\mathrm{d}t \\
0 &= \ddot{Q} + \frac{\dot{Q}}{RC} + \frac{Q}{LC}
\end{aligned}$$

$$(1): \text{ denn } \begin{cases} U_L = L\dot{I}_L \\ I_L = \frac{1}{L} \int U_L \,\mathrm{d}t \end{cases}$$

(b)

$$\begin{aligned}
\underline{Y}_{ges} &= \underline{Y}_R + \underline{Y}_C + \underline{Y}_L \\
&= \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \\
\underline{Z}_{ges} &= \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} \\
&= \frac{\frac{1}{R} - j\left(\omega C - \frac{1}{\omega L}\right)}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} \\
Z = |\underline{Z}_{ges}| &= \sqrt{\left(\frac{\frac{1}{R}}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\right)^2 + \left(\frac{\left(\omega C - \frac{1}{\omega L}\right)}{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\right)^2}
\end{aligned}$$

$$\begin{aligned}
Y_C &= \\
U &= \frac{Q}{C}
\end{aligned}$$

$$
\begin{aligned}
0 &= \ddot{Q} + \frac{\dot{Q}}{RC} + \frac{Q}{LC} \\
0 &= \ddot{I} + \frac{\dot{I}}{RC} + \frac{I}{LC} \\
0 &\overset{(1)}{=} -\omega^2 \cdot \underline{i} + j\omega \cdot \frac{\underline{i}}{RC} + \frac{\underline{i}}{LC} \\
Z &= \frac{U}{I}
\end{aligned}
$$

$(1):$ da $\underline{i} = \hat{\imath}e^{j(\omega t+\phi)} \rightarrow \dot{\underline{i}} = j\omega \cdot \hat{\imath}e^{i(\omega t+\phi)}$

(c)

Ansatz $\rightarrow$ Exponentialfunktion :

$$
\begin{aligned}
Q &= ce^{\lambda t} \quad , \ c \in \mathbb{C} \\
0 &= \ddot{Q} + \frac{\dot{Q}}{RC} + \frac{Q}{LC} \\
0 &= \lambda^2 + \frac{\lambda}{RC} + \frac{1}{LC} \\
\lambda &= -\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}} \\
Q(t) &= e^{-\frac{t}{2RC}} \left(c_1 e^{t\sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}} + c_2 e^{-t\sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}} \right)
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{1}{2RC} \\
B &= \sqrt{A^2 - \frac{1}{LC}} \\
Q(t) &= e^{-At} \left(c_1 e^{Bt} + c_2 e^{-Bt} \right)
\end{aligned}
$$

$$
\begin{aligned}
Q_{B\in\mathbb{R}}(t) \equiv Q_{\mathbb{R}}(t) &= e^{-At} \left(c_1 e^{Bt} + c_2 e^{-Bt} \right) \\
Q_{\mathbb{R}}(0) &= Q_0 \\
\dot{Q}_{\mathbb{R}}(0) &= 0 \\
Q_0 &= c_1 + c_2 \\
0 &= B(c_1 - c_2) - A(c_1 + c_2) \\
0 &= B(2c_1 - Q_0) - AQ_0 \\
c_1 &= \frac{Q_0}{2} \left(\frac{A}{B} + 1 \right) \\
c_2 &= Q_0 - \frac{Q_0}{2} \left(\frac{A}{B} + 1 \right) \\
&= \frac{Q_0}{2} \left(1 - \frac{A}{B} \right)
\end{aligned}
$$

$$
Q_{B\in\mathbb{C}}(t) = e^{-At} \left(c_1(\cos(|B|t) + i\sin(|B|t)) + c_2(\cos(|B|t) - i\sin(|B|t)) \right)
$$

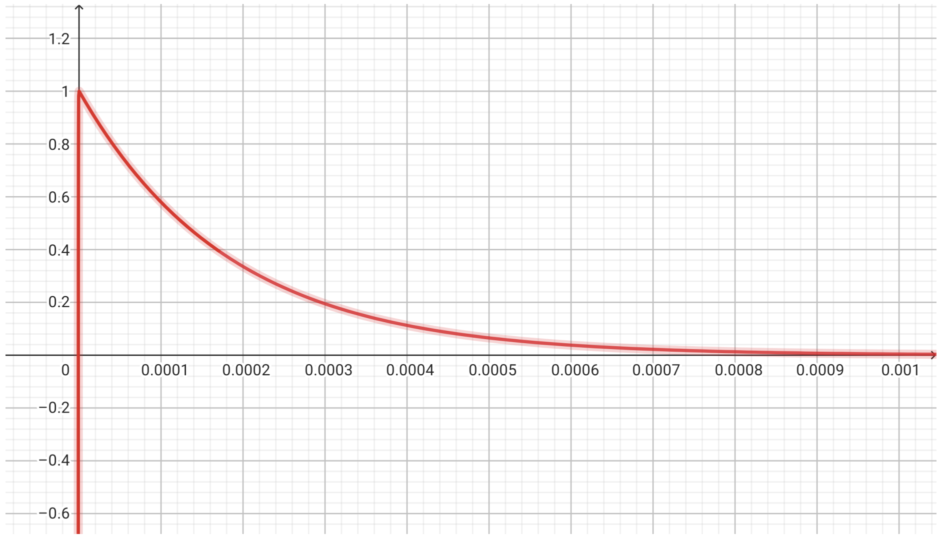

Figure 2: $R = 0.024\,\Omega$

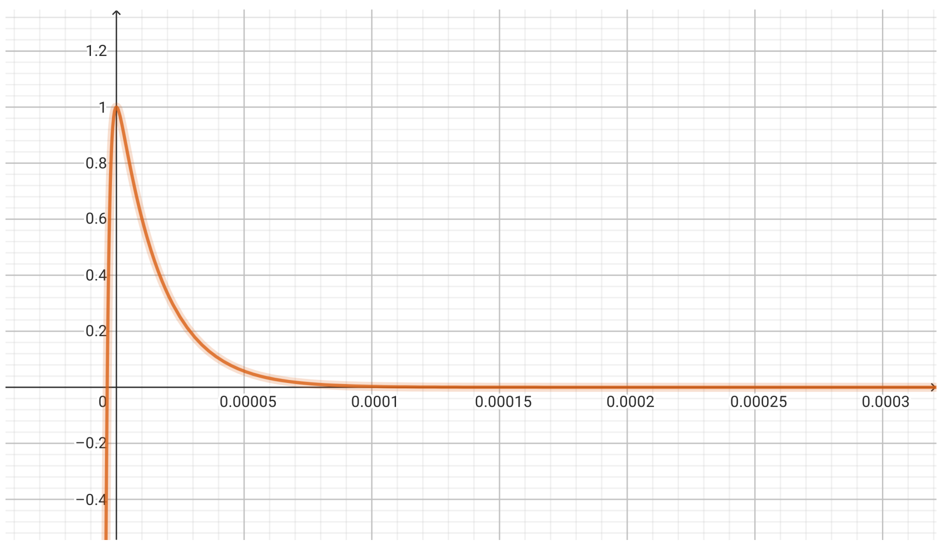

Figure 3: $R = 0.24\,\Omega$

$$
\begin{aligned}
\Re\left(Q_{B\in\mathbb{C}}(t)\right) \equiv Q_{\mathbb{C}}(t) &= e^{-At}\left(c_{11}\cos(|B|t) - c_{12}\sin(|B|t) + c_{21}\cos(|B|t) + c_{22}\sin(|B|t)\right)\\
&= e^{-At}\left((c_{11}+c_{21})\cos(|B|t) + (c_{22}-c_{12})\sin(|B|t)\right)\\
&= e^{-At}\left(c_1\cos(|B|t) + c_2\sin(|B|t)\right)\\
Q_{\mathbb{C}}(0) &= Q_0\\
\dot{Q}_{\mathbb{C}}(0) &= 0\\
Q_0 &= c_1\\
0 &= -Ac_1 + |B|c_2\\
c_2 &= Q_0\frac{A}{|B|}
\end{aligned}
$$

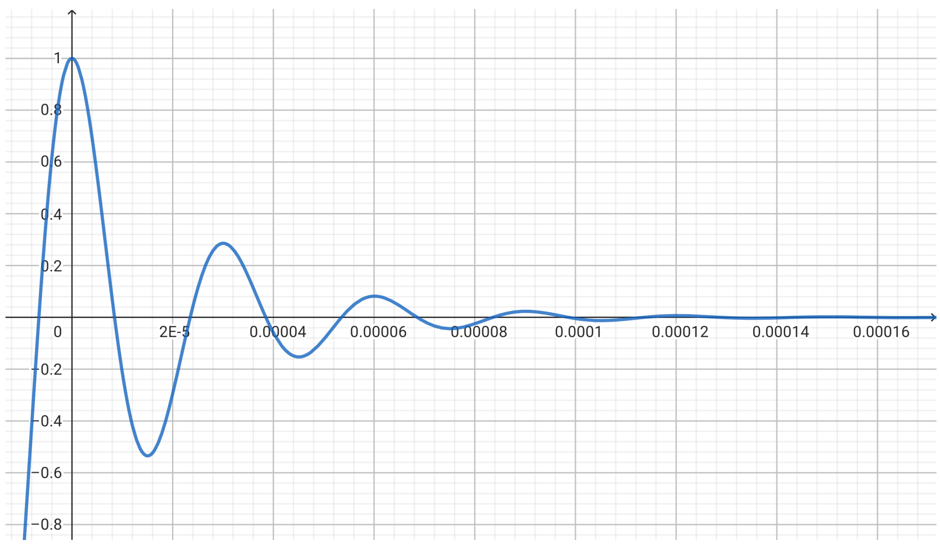

Figure 4: $R = 2.4\,\Omega$