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Experimental physics Vb (particle and astro-physics)

Exercise 02

Task 1 *Mandelstam variables* (5 points)

Consider the elastic scattering of identical particles $A + A \to A + A$. Show that in this case:

$$s = 4(\vec{p}^2 + m^2)$$
$$t = -2\vec{p}^2(1 - \cos\theta)$$
$$u = -2\vec{p}^2(1 + \cos\theta),$$

with $\vec{p}$ the momentum of the incoming particle in the center-of-mass frame, m the mass of the particle, and θ the angle of deflection.

Task 2 *Linear accelerator and cyclotron* (4+4=8 points)

You want to accelerate protons to a kinetic energy of $E_{\mathrm{kin}} = 20\,\mathrm{MeV}$. You have a high-frequency AC voltage of $U(t) = U_0 \sin\omega t$, with $U_0 = 200\,\mathrm{kV}$ and $\omega/2\pi = f = 20\,\mathrm{MHz}$, at your disposal.

a. How many drift tubes do you need for a linear accelerator? How large do the tube lengths L_k of the linear accelerator have to be? How long is the accelerator in total?

b. How many turns do the protons need in a cyclotron? How strong does the magenetic field B have to be? Which diameter does the cyclotron have?

Notes: The acceleration is supposed to happen at each maximum of the absolute value of the AC voltage. Perform the calculation in the non-relativistic limit ($E_{\mathrm{kin}} \ll m_p$!).

Task 3 *Luminosity* (1+1+1=3 points)

a. Calculate the instantaneous luminosity of the LHC in units of $1/(\mathrm{cm}^2\,\mathrm{s})$ and $1/(\mathrm{nb\,s})$ (circumference of the LHC: 26.695 km). Use the following parameters:

$$n_B = 2808, \quad N_1 = N_2 = 115 \times 10^9, \quad \sigma_x = \sigma_y = 15\,\mu\mathrm{m}.$$

b. Calculate the integrated luminosity in units of 1/fb over an operation time of six months, with an average of 30 days each. Assume an average efficiency of 25 % for the accelerator.

c. The production cross section for the Higgs boson amounts to $\sigma(pp \to H + X) \simeq 20\,\text{pb}$ at the LHC. How many Higgs bosons are produced at one of the interaction points of the LHC in six months?

Task 4 *Ionisation losses* (5+2+2=9 points)

a. Plot (e.g., using `matplotlib`) the ionisation loss (in units of MeV) of muons, pions, kaons, protons, and α-particles in 1 cm thick polystyrene szintillator ($[C_6H_5CHCH_2]_n$), as a function of momentum. Choose an appropriate way of plotting the results!
The specific energy loss is given by

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = K\rho z^2 \frac{Z}{A}\frac{1}{\beta^2}\left(\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2 W_{\max}}{I^2} - \beta^2\right)$$

to first order and for relatively low momenta, with $K = 4\pi N_A r_e^2 m_e c^2 = 0.307\,\text{MeV}\,\text{g}^{-1}\,\text{cm}^2$, and

$$W_{\max} = \frac{2m_ec^2\beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

is the maximum energy transfer of a particle with mass M to an electron. Polystyrene has a density of $\rho = 1.06\,\text{g/cm}^3$ and $\langle Z/A\rangle = 0.537\,68$, its mean excitation energy is $I = 68.7\,\text{eV}$.

b. At which momentum are the mean energy losses of pions and kaons equal?

c. Which energy does a minimum ionizing particle deposit in 1 cm of szintillator on average?