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Experimental physics Vb (particle and astro-physics)

Exercise 06

Task 1 *Local gauge invariance of the Schrödinger equation* (7 points)

For the one-dimensional case, prove that the Schrödinger equation written with covariant derivatives is invariant under local gauge transformations: Let $\Psi(t,x)$ be a wave function satisfying the new Schrödinger equation,

$$\frac{1}{2m}(\mathrm{i}D_x)^2\Psi = \mathrm{i}D_0\Psi.$$

Show that then also the locally rotated wave function

$$\Psi' = \mathrm{e}^{\mathrm{i}Q\alpha(t,x)}\Psi$$

satisfies the new Schrödinger equation. The covariant derivatives are given by

$$D_x = -\frac{\partial}{\partial x} + \mathrm{i}QA \quad \text{and} \quad D_0 = \frac{\partial}{\partial t} + \mathrm{i}QV,$$

where A and V are two fields transforming according to

$$A' = A + \partial\alpha/\partial x \quad \text{and} \quad V' = V - \partial\alpha/\partial t.$$

Explain your solution!

You may either do the proof by hand or use the computer algebra system `sympy`. In this case, please hand in your (commented) jupyter notebook both as an `ipynb`-file and exported to `pdf` format as the solution to this task in Moodle.

Hints: In `sympy`, you can define abstract functions of several variables with `Function`. You can use `Derivative` to define an abstract (partial) derivative of a function. To force the evaluation of the product rule in the intermediate terms, use `doit`. To calculate the covariant derivatives, define ordinary `python` functions and apply them to the relevant `sympy` terms.

Task 2 *Gauge invariance and photon mass* (2+1=3 points)

From Maxwell's equations, the wave equation for the photon field A^μ can be derived in covariant notation as

$$\partial_\nu \partial^\nu A^\mu - \partial^\mu \partial^\nu A_\nu = j^\mu,$$

with the current density j^μ. Here we use the summation convention. In classical physics, the magnetic and electric fields are obtained from A^μ as $\vec{B} = \vec{\nabla} \times \vec{A}$ and $\vec{E} = -\vec{\nabla} A^0 - \partial \vec{A}/\partial t$.

a. Prove that this wave equation is invariant under the following local gauge transformation:

$$A^\mu \to A^\mu - \partial^\mu \alpha.$$

b. Prove that the wave equation of a massive vector field,

$$(\partial_\nu \partial^\nu + m^2) A^\mu - \partial^\mu \partial^\nu A_\nu = j^\mu,$$

is however **not** gauge invariant.

So for massive vector particles, there is no local gauge invariance.

Task 3 *Higgs factory* (1+1+4+3=9 points)

To measure the properties of the Higgs boson precisely, an e^+e^- collider is to be built.

a. Which advantages does an e^+e^- collider have compared to a $p\bar{p}$ collider?

b. The dominant production process of the Higgs boson is the so-called Higgs radiation, where a Z boson radiates a Higgs boson. Which center-of-mass energy does the e^+e^- collider need for a Z and a Higgs particle to be produced non-virtually?

c. The Higgs boson most frequently decays to the heaviest fermions which can be produced non-virtually. Draw the complete Feynman graph of the production and this decay. Describe the experimental signature in a particle detector like CMS.

d. Express the four-momentum p_H of the Higgs boson by the four-momenta of the e^+, e^-, and Z. Derive an equation for the Higgs mass m_H as a function of the e^+e^- center-of-mass energy $\sqrt{s}$, the Z mass m_Z, and the energy E_Z of the Z boson. Describe a method to measure the Higgs mass without reconstructing the decay products of the Higgs boson.

Task 4 *Neutrino oscillations* (4+2=6 points)

Like in the lecture, consider the eigenstates $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$ of the weak interaction, related to the mass eigenstates $|\nu_2\rangle$ and $|\nu_3\rangle$ by a mixing matrix:

$$\begin{pmatrix} |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}.$$

a. Prove explicitly that the probability for a muon neutrino produced at time $t = 0$ to be detected again as a muon neutrino after time t is given by

$$P_{\nu_\mu \to \nu_\mu}(t) = 1 - \sin^2(2\theta)\, \sin^2\left(\frac{E_3 - E_2}{2} t\right).$$

b. Plot $P_{\nu_\mu \to \nu_\mu}(L)$ and $P_{\nu_\mu \to \nu_\tau}(L)$ together, for $\Delta m_{23}^2 = 0.0025\,\text{eV}^2$, $E = 1\,\text{GeV}$, and $\theta = 45°$, in the range up to $L = 10\,000\,\text{km}$, using a logarithmic L-axis.