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Experimentalphysik II (SS 2023/2024)

Übung 13

Tutorium: 2 | Abgabe: 14.07.2023

Aufgabe 1: Plattenkondensator mit Dielektrikum

(a)

$$\begin{aligned} E_L &= \frac{U_L}{d_L} \\ &= 2\epsilon_r \frac{U_P}{d_P} \\ &= 2\epsilon_r E_P \\ \frac{E_P}{E_L} &= \frac{1}{2\epsilon_r} \\ &\approx 0.278 \end{aligned}$$

Da für isotrope Dielektrika gilt, dass $\vec{E} \propto \vec{D}$, verändert sich das D-Feld um den gleichen Faktor wie das E-Feld.

(b)

$$\begin{aligned} U &= 2U_L + U_P \\ &= 2U_L + \frac{U_L}{\epsilon_r} \\ U_L &= \frac{U}{2 + \frac{1}{\epsilon_r}} \\ E_L &= \frac{U}{d} \frac{1}{2 + \frac{1}{\epsilon_r}} \\ &\approx \frac{600\,\mathrm{V}}{5\,\mathrm{mm}} \frac{1}{2 + \frac{1}{1.8}} \\ &\approx 93.9 \frac{\mathrm{kV}}{\mathrm{m}} \end{aligned}$$

$$\begin{aligned} U_P &= \frac{1}{\epsilon_r} \frac{U}{2 + \frac{1}{\epsilon_r}} \\ E_P &= \frac{1}{\epsilon_r d_P} \frac{U}{2 + \frac{1}{\epsilon_r}} \\ &\approx \frac{1}{1.8 \cdot 5\,\mathrm{mm}} \frac{600\,\mathrm{V}}{2 + \frac{1}{1.8}} \\ &\approx 26.1 \frac{\mathrm{kV}}{\mathrm{m}} \end{aligned}$$

(c)

$$
\begin{aligned}
D_L &= \varepsilon_0 \varepsilon_r E_L \\
&\approx 8.85 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}} \cdot 1 \cdot 93.9 \frac{\mathrm{kV}}{\mathrm{m}} \\
&\approx 8.31 \cdot 10^{-8} \frac{\mathrm{C}}{\mathrm{m}^2}
\end{aligned}
$$

$$
\begin{aligned}
D_P &= \varepsilon_0 \varepsilon_r E_P \\
&\approx 8.85 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}} \cdot 1.8 \cdot 26.1 \frac{\mathrm{kV}}{\mathrm{m}} \\
&\approx 4.16 \cdot 10^{-7} \frac{\mathrm{C}}{\mathrm{m}^2}
\end{aligned}
$$

(d)

$$
\begin{aligned}
\frac{1}{C} &= \frac{2}{C_L} + \frac{1}{C_P} \\
&= \frac{2d_L}{\epsilon_0 A} + \frac{d_P}{\epsilon_0 \epsilon_r A} \\
&= \frac{2d_L}{\epsilon_0 A} \left(1 + \frac{1}{\epsilon_r}\right) \\
C &= \frac{\epsilon_0 A}{2d_L} \frac{1}{1 + \frac{1}{\epsilon_r}} \\
&\approx \frac{8.85 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}} \cdot 200\,\mathrm{cm}^2}{2 \cdot 2.5\,\mathrm{mm}} \frac{1}{1 + \frac{1}{1.8}} \\
&\approx 22.8\,\mathrm{pF}
\end{aligned}
$$

$$
\begin{aligned}
C &= \frac{Q}{U} \\
Q &= CU \\
&= \frac{\epsilon_0 A U}{2d} \frac{1}{1 + \frac{1}{\epsilon_r}} \\
&= \frac{8.85 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}} \cdot 200\,\mathrm{cm}^2 \cdot 600\,\mathrm{V}}{2 \cdot 5\,\mathrm{mm}} \frac{1}{1 + \frac{1}{1.8}} \\
&\approx 6.84\,\mathrm{nC}
\end{aligned}
$$

(e)

$$
\frac{W_P}{W} = \frac{C_P U_P^2}{C U^2}
$$

$$
\begin{aligned}
\frac{C_P}{C} &= \frac{2C_P}{C_L} + 1 \quad \text{, siehe (b)} \\
&= \epsilon_r + 1
\end{aligned}
$$

$$
U_P = \frac{1}{\epsilon_r} \frac{U}{2 + \frac{1}{\epsilon_r}} \quad \text{, siehe (b)}
$$

$$\frac{U_P}{U} = \frac{1}{2\epsilon_r + 1}$$

$$\begin{aligned}
\frac{W_P}{W} &= \frac{C_P U_P^2}{C U^2} \\
&= \frac{\epsilon_r + 1}{(2\epsilon_r + 1)^2} \\
&\approx \frac{1.8 + 1}{(2 \cdot 1.8 + 1)^2} \\
&\approx 13.2\%
\end{aligned}$$

(f)

$$\begin{aligned}
W &= \frac{1}{2} C U^2 \\
&\approx \frac{1}{2} \cdot 6.84\,\mathrm{nC} \cdot 600^2\,\mathrm{V}^2 \\
&\approx 4.11\,\mu\mathrm{J}
\end{aligned}$$

Aufgabe 2: Poynting-Vektor

(a)

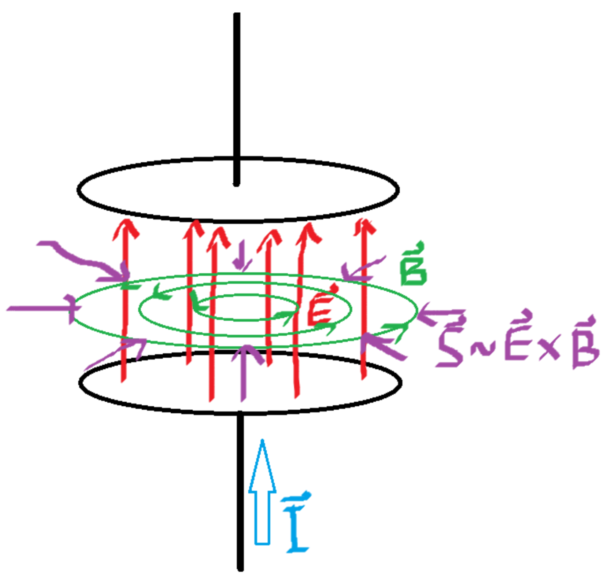

Figure 1: Poynting Vektoren bei Aufladen eines kreisförmigen Plattenkondensators.

(b)

Der Poynting-Vektors zeigt, wie man in der Skizze gut nachvollziehen kann, radial nach innen.

$$\begin{aligned}
0 &= U + U_R + U_C \\
&= U + RI + \frac{Q}{C} \\
&= U + R\dot{Q} + \frac{Q}{C}
\end{aligned}$$

$$
\begin{aligned}
\dot{Q} &= -\frac{Q}{RC} - \frac{U}{R} \\
Q(t) &= c \cdot e^{-\frac{t}{RC}} - UC \quad , \ c \in \mathbb{R} \\
Q(0) &= 0 \implies Q(t) = UC\left(e^{-\frac{t}{RC}} - 1\right) \\
\dot{Q}(t) &= -\frac{U}{R} e^{-\frac{t}{RC}}
\end{aligned}
$$

$$
\begin{aligned}
E &= \frac{U}{d} \\
&= \frac{Q}{dC} \\
&= \frac{U}{d}\left(e^{-\frac{t}{RC}} - 1\right) \\
&= E_{max}\left(e^{-\frac{t}{RC}} - 1\right)
\end{aligned}
$$

9

$$
\begin{aligned}
\vec{\nabla} \times \vec{B} &= \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \\
\oint_{\partial A} \vec{B} &= \varepsilon_0 \mu_0 \oint_S \frac{\partial \vec{E}}{\partial t} \, \mathrm{d}\vec{S} \\
&= \varepsilon_0 \mu_0 \int_0^r \, \mathrm{d}r \int_0^{2\pi} \, \mathrm{d}\phi \, \frac{\partial E}{\partial t} r \\
\dot{E} &= \frac{\dot{U}}{d} = \frac{\dot{Q}}{dC} = -\frac{U_0}{dRC} e^{-\frac{t}{RC}} \\
\oint_{\partial A} \vec{B} &= -\varepsilon_0 \mu_0 \frac{2\pi U_0}{dRC} e^{-\frac{t}{RC}} \int_0^r r \, \mathrm{d}r \\
&= -\varepsilon_0 \mu_0 \frac{\pi U_0 r^2}{dRC} e^{-\frac{t}{RC}} \\
&= \varepsilon_0 \mu_0 \pi r^2 \dot{E}
\end{aligned}
$$

$$
\begin{aligned}
|\vec{S}|(t) &= \varepsilon_0 c^2 |\vec{E}| \cdot |\vec{B}| \\
&= \varepsilon_0 c^2 E \cdot \varepsilon_0 \mu_0 \pi r^2 \dot{E} \\
&= \varepsilon_0^2 \mu_0 \pi c^2 r^2 \dot{E} E \\
&= \varepsilon_0^2 \mu_0 \pi \frac{c^2 r^2}{d^2 c^2} \dot{Q} Q \\
&= \varepsilon_0 c^2 \cdot \varepsilon_0 \mu_0 \frac{\pi U_0 r^2}{dRC} e^{-\frac{t}{RC}}.
\end{aligned}
$$

$$
\begin{aligned}
\vec{S}(t) &= \epsilon_0 c^2 (\vec{E}(t) \times \vec{B}(t)) \\
|\vec{S}|(t) &= \epsilon_0 c^2 |\vec{E}| \cdot |\vec{B}| \\
&= \epsilon_0 c E^2 \\
&= \epsilon_0 c \frac{U^2}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \epsilon_0 c \frac{Q^2}{d^2 C^2} \\
&= \epsilon_0 c \frac{U^2 C^2 \left(e^{-\frac{t}{RC}} - 1\right)^2}{d^2 C^2} \\
&= \epsilon_0 c \frac{U^2}{d^2} \left(e^{-\frac{t}{RC}} - 1\right)^2 \\
&= \epsilon_0 c E_{max}^2 \left(e^{-\frac{t}{RC}} - 1\right)^2 \\
&= I_{max} \left(e^{-\frac{t}{RC}} - 1\right)^2 \quad , \; I \equiv \text{Intensität}
\end{aligned}
$$

(c)

$$
\begin{aligned}
P &= \int_S \vec{S} \, \mathrm{d}\vec{A} \\
&= \int_{Mantelflache} \vec{S} \, \mathrm{d}\vec{A} + \underbrace{\int_{KondensatorPlatten} \vec{S} \, \mathrm{d}\vec{A}}_{=0,\text{da } \vec{S} \perp \mathrm{d}\vec{A}} \\
&= \int_0^d \mathrm{d}z \int_0^{2\pi} \mathrm{d}\phi \, |S(t)| \quad , \; \text{da } \vec{S} \parallel \vec{A} \\
&= 2\pi \cdot d \cdot |S(t)| \\
&= 2\pi \cdot d \cdot I_{max} \left(e^{4-\frac{t}{RC}} - 1\right)^2
\end{aligned}
$$

(d)

$$
\begin{aligned}
P &= 2\pi \cdot d \cdot I_{max} \left(e^{-\frac{t}{RC}} - 1\right)^2 \\
&= 2\pi \epsilon_0 c \frac{U^2}{d} \left(e^{-\frac{t}{RC}} - 1\right)^2 \\
W &= \int_0^\infty P \, \mathrm{d}t \\
&= 2\pi \epsilon_0 c \frac{U^2}{d} \int_0^\infty \left(e^{-\frac{t}{RC}} - 1\right)^2 \, \mathrm{d}t
\end{aligned}
$$

Aufgabe 3: Dipolstrahlung
(a)

$$
\begin{aligned}
c &= I(\theta, r) \quad , \; c \in \mathbb{R} \text{ beliebig aber fest} \\
&= \frac{\omega^4 P_0^2}{2(4\pi)^2 \epsilon_0 c^3} \cdot \frac{\sin^2(\theta)}{r^2} \\
c &= \frac{\sin^2(\theta)}{r^2} \\
c &= \frac{\sin(\theta)}{r} \\
r &= c \sin(\theta)
\end{aligned}
$$

Letzteres ist die bekannte Vorschrift für einen den Ursprung tangierenden Kreis in Polarkoordinaten mit Radius $r = \frac{c}{2}$.

(b)

$$
\begin{aligned}
I_{ges} &= \int_{\mathcal{S}} I(\theta, r)\,\mathrm{d}\vec{S} \\
&= \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \mathrm{d}\theta\, I(\theta, r) \cdot r^2 \sin\theta \\
&= 2\pi \cdot \frac{1}{2} \frac{\omega^4 P_0^2}{(4\pi)^2 \epsilon_0 c^3} \int_0^{\pi} \mathrm{d}\theta\, \frac{\sin^2\theta}{r^2} \cdot r^2 \sin\theta \\
&= \underbrace{\frac{\omega^4 P_0^2}{16\pi^2 \epsilon_0 c^3}}_{\eta} \int_0^{\pi} \mathrm{d}\theta\, \sin^3\theta \\
&= \frac{\eta}{2} \int_0^{\pi} \mathrm{d}\theta\, (\sin\theta - \cos(2\theta)\sin\theta) \\
&= \eta - \frac{\eta}{2} \int_0^{\pi} \mathrm{d}\theta\, \cos(2\theta)\sin\theta \\
&= \eta - \frac{\eta}{4} \int_0^{\pi} \mathrm{d}\theta\, (\sin(3\theta) - \sin\theta) \\
&= \eta + \frac{\eta}{2} - \frac{\eta}{4} \int_0^{\pi} \mathrm{d}\theta\, (\sin(3\theta)) \\
&= \eta + \frac{\eta}{2} + \frac{\eta}{12} \cos(3\theta) \Big|_0^{\pi} \\
&= \eta + \frac{\eta}{2} - \frac{\eta}{6} \\
&= \frac{4}{3}\eta \\
&= \frac{\omega^4 P_0^2}{12\pi^2 \epsilon_0 c^3}
\end{aligned}
$$