Ex Vb - sheet 5

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[1]:
```
from sympy import *
import numpy as np
import scipy.constants as c
from IPython.display import display as print
init_printing(use_latex="mathjax")
```

1 Nr. 1 Electron-muon-scattering

The number of particles per second is $W = L\sigma$, where σ is the differential cross section integrated over the central region $\sigma = \int_{45^\circ}^{135^\circ} \mathrm{d}\theta \, 2\pi \sin(\theta) \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$.

[2]:
```
s,L,theta,alpha = symbols("s L theta alpha")
deg = 2*pi/360
gev_to_cm2 = (c.c * c.hbar / (c.e * 1e9))**2 * (1e2)**2 # GeV^-2 to cm^2

subs = {s: 34**2, # GeV^2
        L: 5e30, # cm^-2 s^-1
        alpha: c.alpha}
diff_cross_section = alpha**2/(4*s) * (1 + cos(theta)**2)

cross_section = Integral((2*pi*sin(theta)*diff_cross_section),(theta, 45*deg,
 ↪135*deg))
cross_section
```

[2]:

$$\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi\alpha^2 \left(\cos^2(\theta) + 1\right) \sin(\theta)}{2s} \, d\theta$$

[3]:
```
W = L * cross_section.doit()
W
```

[3]:

$$\frac{7\sqrt{2}\pi L\alpha^2}{12s}$$

[4]:
```
float(W.subs(subs) * gev_to_cm2)
```

[4]:

0.000232432810936274

There are 0.000232 electron-muon-scatterings per second.

2 Nr. 2 Electric dipole moment of the neutron

2.1 Parity

The electric dipole moment, like most vectors, changes signs under a parity transformation,

$$PD = P\int \mathrm{d}r\rho = \int(-\mathrm{d}r)\rho = -D$$

where as the spin transforms like angular momentum, i.e. it does not change its sign:

$$PL = P\left(mr \times \frac{\mathrm{d}r}{\mathrm{d}t}\right) = m(-r) \times \left(\frac{-\mathrm{d}r}{\mathrm{d}t}\right) = L$$

The neutron therefore violates the P symmetry.

2.2 Time

Once again the spin transforms like angular momentum

$$TL = T\left(mr \times v\right) = mr \times \left(\frac{\mathrm{d}r}{-\mathrm{d}t}\right) = -L$$

changing sign under time reversial, on the other hand the dipole moment

$$TD = T\int \mathrm{d}r\rho = \int \mathrm{d}r\rho = D$$

stays the same. It follows that the T symmetry is violated as well.

3 Nr. 3 CKM matrix

3.1 (a)

Let A_i be the three matrices that V_{CKM} is composed of (see problem sheet). Note that A_1 and A_3 are rotation matrices and therefore unitary. A_2 resembles a rotation matrix, except for the fact that it has a complex phase in the entries with the sines. It can be shown to be unitary as well:

$$\begin{aligned}
I &\overset{!}{=} \begin{pmatrix} c & se^{-i\delta} \\ -se^{i\delta} & c \end{pmatrix}^{\dagger} \begin{pmatrix} c & se^{-i\delta} \\ -se^{i\delta} & c \end{pmatrix} \\
&= \begin{pmatrix} c & -se^{-i\delta} \\ se^{i\delta} & c \end{pmatrix} \begin{pmatrix} c & se^{-i\delta} \\ -se^{i\delta} & c \end{pmatrix} \\
&= \begin{pmatrix} c^2 + s^2 & cse^{-i\delta} - cse^{-i\delta} \\ cse^{i\delta} - cse^{i\delta} & c^2 + s^2 \end{pmatrix} \\
&= I
\end{aligned}$$

Using the fact that A_i are unitary, it trivially follows that V_{CKM} is unitary as well:

$$
\begin{aligned}
I &\stackrel{!}{=} V_{CKM}^\dagger V_{CKM} \\
&= (A_1 A_2 A_3)^\dagger A_1 A_2 A_3 \\
&= A_3^\dagger A_2^\dagger A_1^\dagger A_1 A_2 A_3 \\
&= A_3^\dagger A_2^\dagger A_2 A_3 \\
&= A_3^\dagger A_3 \\
&= I
\end{aligned}
$$

3.2 (b)

[5]:
```
c_23, s_23, c_13, s_13, c_12, s_12, delta = symbols("c_23 s_23 c_13 s_13 c_12␣
↪s_12 delta",real=True)
M1 = Matrix([[1,0,0],
             [0,c_23,s_23],
             [0,-s_23,c_23]])
M2 = Matrix([[c_13,0,s_13 * exp(-I*delta)],
             [0,1,0],
             [-s_13 * exp(I*delta),0,c_13]])
M3 = Matrix([[c_12,s_12,0],
             [-s_12,c_12,0],
             [0,0,1]])
# theta_23, theta_13, theta_12, delta = symbols("theta_23 theta_13 theta12␣
↪delta")
# M1 = Matrix([[1,0,0],
#              [0,cos(theta_23),sin(theta_23)],
#              [0,-sin(theta_23),cos(theta_23)]])
# M2 = Matrix([[cos(theta_13),0,sin(theta_13) * exp(-I*delta)],
#              [0,1,0],
#              [-sin(theta_13) * exp(I*delta),0,cos(theta_13)]])
# M3 = Matrix([[cos(theta_12),sin(theta_12),0],
#              [-sin(theta_12),cos(theta_12),0],
#              [0,0,1]])
print(M1, M2, M3)
```

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}
$$

$$
\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}
$$

$$
\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

[6]:
```python
V = M1*M2*M3
V
```

[6]:

$$\begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{12}s_{13}s_{23}e^{i\delta} - c_{23}s_{12} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ -c_{12}c_{23}s_{13}e^{i\delta} + s_{12}s_{23} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix}$$

3.3 (c)

[7]:
```python
lamb, A, rho, eta = symbols("lambda A rho eta", real=True)
subs = {c_12: sqrt(1-s_12**2),
        c_23: sqrt(1-s_23**2),
        c_13: sqrt(1-s_13**2),
        s_12: lamb,
        s_23: A*lamb**2,
        s_13: A*lamb**3*(rho-I*eta)*exp(I*delta)}
M2_ = Matrix([[c_13,0,s_13 * exp(-I*delta)],
              [0,1,0],
              [-A*lamb**3*(rho + I*eta),0,c_13]])

V_W = (M1.subs(subs)*M2_.subs(subs)*M3.subs(subs))
V_W
```

[7]:

$$\begin{bmatrix} \sqrt{1-\lambda^2}\sqrt{-A^2\lambda^6\left(-i\eta+\rho\right)^2 e^{2i\delta}+1} & \lambda\sqrt{-A^2\lambda^6\left(-i\eta+\rho\right)^2 e^{2i\delta}+1} & A\lambda^3\left(-i\eta\right. \\ -A^2\lambda^5\sqrt{1-\lambda^2}\left(i\eta+\rho\right) - \lambda\sqrt{-A^2\lambda^4+1} & -A^2\lambda^6\left(i\eta+\rho\right) + \sqrt{1-\lambda^2}\sqrt{-A^2\lambda^4+1} & A\lambda^2\sqrt{-A^2\lambda^6\left(-i\eta\right.} \\ -A\lambda^3\sqrt{1-\lambda^2}\left(i\eta+\rho\right)\sqrt{-A^2\lambda^4+1} + A\lambda^3 & -A\lambda^4\left(i\eta+\rho\right)\sqrt{-A^2\lambda^4+1} - A\lambda^2\sqrt{1-\lambda^2} & \sqrt{-A^2\lambda^4+1}\sqrt{-A^2\lambda^6\left(\right.} \end{bmatrix}$$

[8]:
```python
V_W_taylor = simplify(V_W.applyfunc(lambda x: series(x, lamb, 0, 4).removeO()))
V_W_taylor
```

[8]:

$$\begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(-i\eta+\rho\right) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3\left(-i\eta-\rho+1\right) & -A\lambda^2 & 1 \end{bmatrix}$$

3.4 (d)

1. identity:

[9]:
```python
simplify(abs(V[0,1]) / sqrt(abs(V[0,0])**2 + abs(V[0,1])**2))
```

[9]:

$$\frac{\left|s_{12}\right|}{\sqrt{c_{12}^2+s_{12}^2}}$$

In practice all the angles are small and positive, so $|s_{ij}| = s_{ij}$ and $|c_{ij}| = c_{ij}$ all hold. The denominator then cancels by the pythagorean theorem, leaving the desired identity.

With the Wolfenstein parametrization:

[15]:
```
V_W = V_W_taylor
simplify(abs(V_W[0,1]) / sqrt(abs(V_W[0,0])**2 + abs(V_W[0,1])**2))
```

[15]:

$$\frac{2\left|\lambda\right|}{\sqrt{\lambda^{4}+4}}$$

since λ is small and positive: $\frac{2|\lambda|}{\sqrt{\lambda^4+4}} \approx \lambda$

2. identity:

[11]:
```
simplify(lamb * abs(V[1,2] / V[0,1]))
```

[11]:

$$\lambda\left|\frac{s_{23}}{s_{12}}\right|$$

the λ and $1/|s_{12}| = 1/s_{12}$ cancel, leaving the desired identity.

With the Wolfenstein parameterization:

[19]:
```
simplify(lamb * abs(V_W[1,2] / V_W[0,1]))
```

[19]:

$$\lambda\left|A\lambda\right|$$

again, λ is small and positive so $\lambda|A\lambda| = A\lambda^2$

3. identity:

[12]:
```
simplify(V[0,2].conjugate())
```

[12]:

$$s_{13}e^{i\delta}$$

and with the Wolfenstein parameterization:

[20]:
```
simplify(V_W[0,2].conjugate())
```

[20]:

$$A\lambda^{3}\left(i\eta+\rho\right)$$

3.5 (e)

The tip of the triangle as defined in the lectures is located at $-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$ in the complex plane:

[34]:
```
-(V_W[0,0] * conjugate(V_W[0,2])) / (V_W[1,0] * conjugate(V_W[1,2]))
```

[34]:

$$\left(1-\frac{\lambda^{2}}{2}\right)\left(i\eta+\rho\right)$$

using the definition of $\bar{\rho} = \rho \cdot (1 - \lambda^2/2 + \dots)$ and equivalently for η this will be equal to

$\bar{\rho} + i\bar{\eta}$

The tip of the other triangle points to $1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}$ which yiels:

[37]:
```
1+(V_W[2,0] * conjugate(V_W[2,2])) / (V_W[1,0] * conjugate(V_W[1,2]))
```

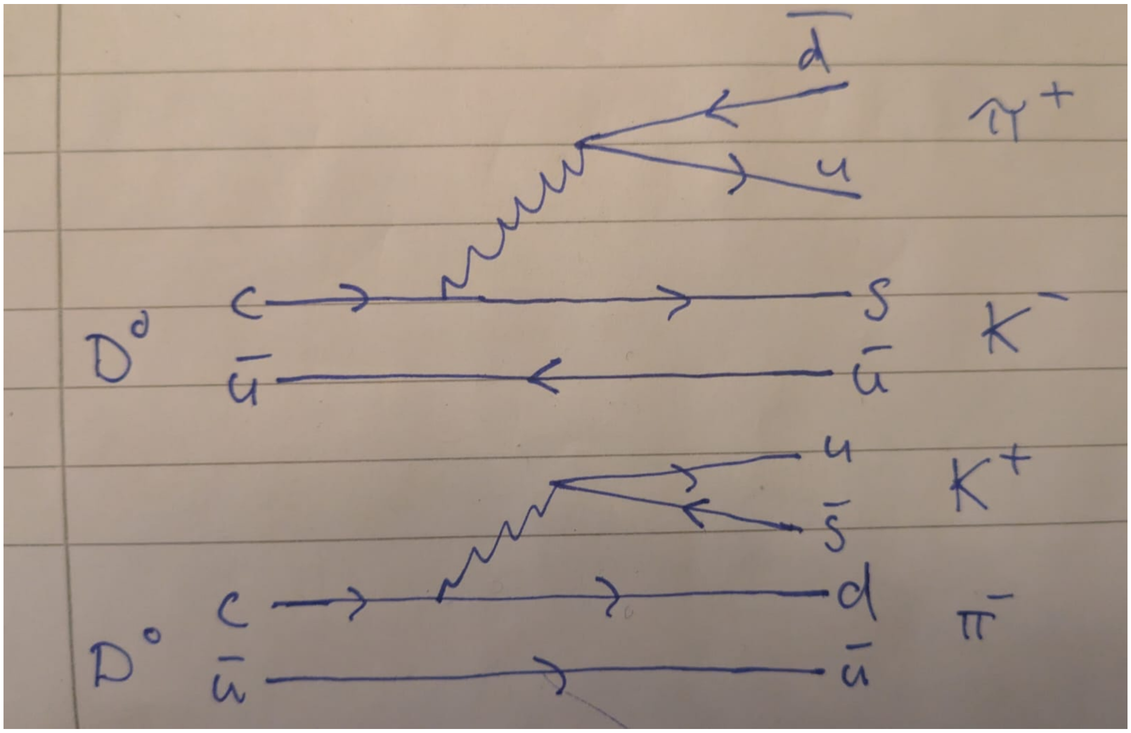
[37]:

$$i\eta+\rho$$

Showcasing that both identities hold

4 Nr. 4 Decay of the D^0 meson

4.1 (a)



The decay $D^0 \to K^-\pi^+$ is significantly more likely, because the transition $c \to W^+ + s$ does not involve a change in the quark family. The relevant amplitude of the CKM matrix is therefore on the diagonal making it the most probable transition. Further more the decay $W^+ \to \bar{d} + u$ is also more likely then $W^+ \to \bar{s} + u$ since its transition amplitude also lies on the diagonal.

Mathematically the ration of the decay widths is

$$\begin{aligned}\frac{P(D^0 \to K^- + \pi^+)}{P(D^0 \to K^+ + \pi^-)} &= \frac{|V_{cs}|^2 |V_{ud}|^2}{|V_{cd}|^2 |V_{us}|^2} \\ &= \left(\frac{0.97349 \cdot 0.97435}{0.22486 \cdot 0.22500}\right)^2 \\ &= 351.482\end{aligned}$$

4.2 (b)

The relative decay widths (source: pdg) are:

$D^0 \to K^-\pi^+ :\ \Gamma_1/\Gamma = (3.947 \pm 0.030) \cdot 10^{-2}$

$D^0 \to K^+\pi^- :\ \Gamma_2/\Gamma = (1.363 \pm 0.025) \cdot 10^{-4}$

$\frac{P(D^0 \to K^- + \pi^+)}{P(D^0 \to K^+ + \pi^-)} = \frac{\Gamma_1}{\Gamma_2} = 289.581$

The rough estimate for the ration of the decay width, aligns nicely with the experimental data.