CT Image Reconstruction

3rd of November 2025

1 Goal

The aim of this project is to gain practical experience with Image Processing. The students are expected to implement some of the techniques applied in the class. This project assignment allows the students to better comprehend the theory and is part of the practical exam.

2 Introduction

A Computed Tomography scan or CT scan is a technology which is often used to observe the inside of an object (such as a human body), without the need to cut open the object. A CT scan uses a large set of X-ray images to compute one or several slices of the object.

An X-ray image is captured by putting the object between an X-ray source and detector. Part of the X-rays that are sent out by the source are absorbed by the object. The amount of absorbed X-rays is dependent on the density of the material through which they pass. The detector captures the remaining X-rays that went through the object. Since dense materials absorb more X-rays, less X-rays will hit the detector than if they would have passed a less dense material. An example of an X-ray image can be seen in Figure 1.

CT Image Reconstruction is done by rotating the source and detector around the object and capturing images at different angles. This way, a so called sinogram can be built for each CT slice (Figure 2). A sinogram contains the X-ray image for that slice for each orientation of the source and detector around the object. The sinogram is used to create the visual representation of the CT slice.

Another way to think of a sinogram is to see it as the result of a radon transform. The radon transform is an integral transform that projects an image on a line defined by a rotation θ.

3 Assignment

You will implement two different ways to calculate a CT reconstruction from a sinogram. The first method is directly based on the Fourier Transform and uses a property, called the Fourier Slice Theorem. The other method is called Filtered Backprojection (FBP). Both solutions only work in the case of parallel projection.

Additionally, you will implement a rebinning algorithm, which undistorts the sinogram captured with a fan-beam scanner, such that it can be used as input for above algorithms.

Additional information can be found in section 5.11 “Image Reconstruction from Projections” in “Digital Image Processing (Gonzalez & Woods)”.

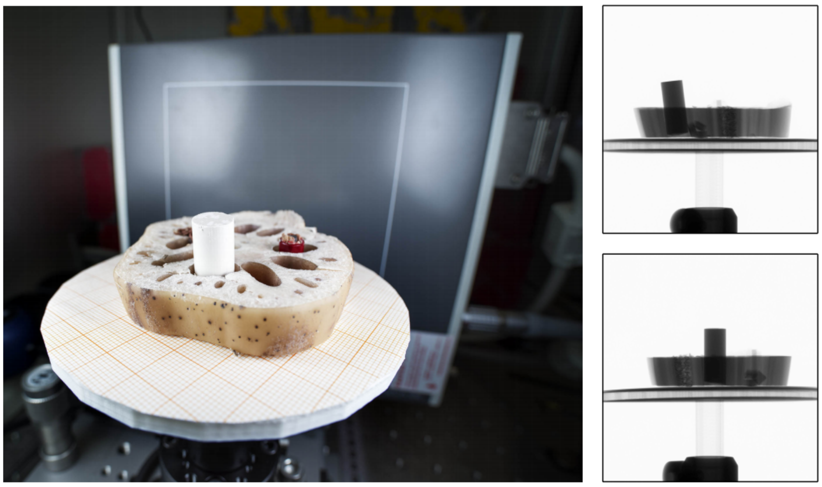

Figure 1: Left: an X-ray imaging setup. Right: X-ray images of the setup. Darker regions represent parts of the object with a dense material (From [1]).

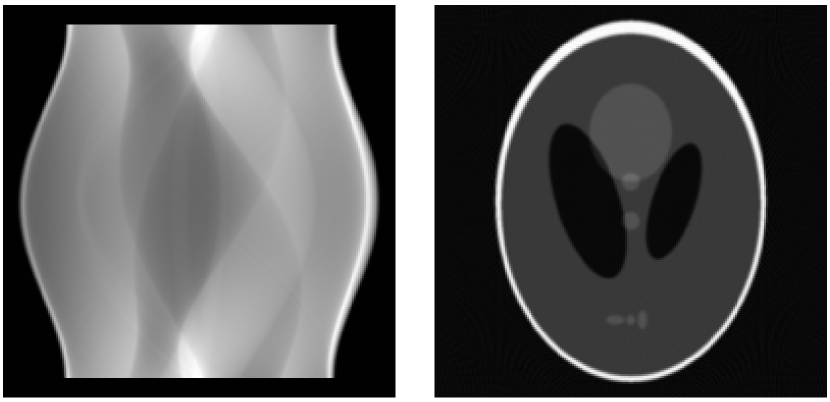

Figure 2: Left: Sinogram/radon transform of the Shepp-Logan Phantom. Right: Shepp-Logan Phantom (slice).

3.1 Direct Fourier Reconstruction

The direct Fourier reconstruction method uses the Fourier Slice Theorem. Simplified, this theorem states that taking the inverse Fourier transform of a 1D-slice, under angle θ, through the origin (DC coefficient) of the frequency domain of a 2D-image, corresponds to a parallel projection of that image under an angle θ in image space.

Since a sinogram represents a set of parallel projections at different angles, one can use the sinogram and the Fourier Slice Theorem to rebuild the frequency domain of the slice. Details on how to accomplish this can be found in [2].

Tips:

- Start by watching the following videos, as they give a clear explanation of the Fourier Slice Theorem: `https://www.youtube.com/watch?v=YIvTpW3IevI` and `https://www.youtube.com/watch?v=DcmL1JGoiPs`
- Define a function 'CTSlice', that takes as input a sinogram and a parameter that defines the angular range of the sinogram (typically 180° or 360°).
- You do not need to implement the NUFFT methods.
- To calculate the FFT of an image in Python, you should actually use

  $$\text{freq} = \text{fftshift}(\text{fft}(\text{ifftshift}(\text{image})))$$

  to make sure that the phase information in the frequency domain is correct. The inverse FFT is calculated as

  $$\text{image} = \text{fftshift}(\text{ifft}(\text{ifftshift}(\text{freq})))$$

  The reason is that fft actually assumes the data to be centered around 0: the center of the image should be at $(0, 0)$. The additional shift that we apply shifts the center of the image to $(0, 0)$, making sure that we obtain the correct phase for each frequency. Often, only the magnitude is important, and this additional shift of the image can be ignored, but in this project phase information is important in this project, since you will move frequency information from one image to another one.
- You may also use some functionality, such as `scipy.ndimage.map_coordinates`, from the scipy package.

3.2 Filtered Backprojection Reconstruction

The backprojection algorithm is an algorithm that simply deprojects each 1D parallel projection in the sinogram on the 2D-target image. This is accomplished by 'smearing out' each 1D projection in the its projection direction and summing them together. Using this naive backprojection method results in a blurry reconstruction. To remove the blur one needs to apply a filter to the sinogram before doing the backprojection. This is explained in [3].

Tips:

- Watch `https://www.youtube.com/watch?v=pZ7JlXagT0w` for a clear explanation of the Filtered Backprojection algorithm.

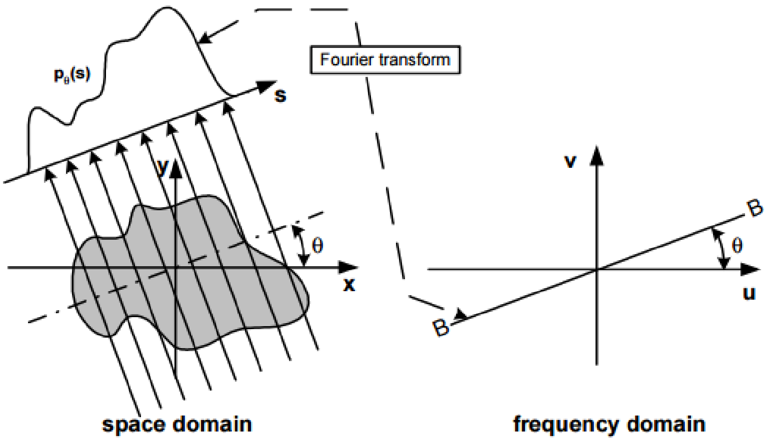

Figure 3: Left: Sinogram/radon transform of the Shepp-Logan Phantom. Right: Shepp-Logan Phantom (slice) [2].

- Define a function 'CTRadon', that takes as input a sinogram and a parameter that defines the angular range of the sinogram (typically 180° or 360°).
- To rotate a 1D parallel deprojection, you may use the `getRotationMatrix2D` and `warpAffine` functions from OpenCV.
- You may utilize skimage's `iradon` to compare your results with those of an existing implementation. Of course, you may NOT utilize `iradon` in your own reconstruction code.

3.3 Rebinning

Both methods only work on sinograms captured using parallel projection, e.g.: the rays captured by the sensor are assumed to be parallel to each other. In reality, often, a divergent projection is used in which the rays originate from one point. You will need to implement a rebinning algorithm to undistort the sinogram of a divergent projection, so that it becomes the sinogram of a parallel projection. This way the sinograms obtained using a divergent projection can be used as input for both implemented methods.

As mentioned earlier, often a divergent projection is used to capture the sinograms. These sinograms need to be undistorted so that they can be used with above reconstruction methods. This technique is called rebinning as is described in [2].

Tips:

- Define a function 'Rebinning', that takes as input a sinogram and the parameters (FOD, FDD, sensor width and center offset) that define the divergent fan-beam projection. This function outputs the rebinned sinogram.
- Remember that the axes of the coordinate system in images are typically placed such that the positive X-axis points to the right and the positive Y-axis points to the bottom.

3.4 Sinogram Generation

Implement a method to generate sinograms from other images. Describe in your report what you observe when recreating the images from the sinogram. E.g. how does the number of angles impact the reconstruction? Can you obtain an exact reconstruction? Is it possible to apply the sinogram generation and image reconstruction on a photograph?

3.5 Extras

Implementing extra functionality may result in a higher score. Some possible extras include:

- Detect if the sinogram is a 180° or 360° sinogram.
- Detect if the sensor images are stacked horizontally or vertically in the sinogram.
- Additional filtering approaches.

3.6 Input

We provide two kinds of input: sinograms from a parallel projection and sinograms from a divergent projection. The parameters required for the divergent projection are described in the documents that accompany them.

Tip:

- Some of the provided sinograms capture the full 360°(e.g. the sensor-detector pair or the object did a full rotation), while others span only 180°. In both cases you have enough data to reconstruct the object.

4 Modalities

- The project is made in groups of two people. You are free to choose your partner. Subscribe to a group on Blackboard before the 19th of November. Only students that subscribed before that date will be informed of important project dates and updates.
- The project counts for 34% of the exam score for this course. Normally, both partners in a group will be graded the same.
- The project is made independently from the other groups. Using code from other groups, or letting other groups use your code, is only allowed if both parties have beforehand reported this to the teaching assistants, and if the work is compensated by other work (in consultation with the teaching assistants). It is best to not discuss possible solutions related to this project beforehand with other groups. Irregularities will be sanctioned in the same way as cheating during an exam.
- Reporting: Your program code and a brief report need to be submitted to blackboard. The deadline of the project is set on Thursday 22/1/2026 at 23h59. Use a compressed archive (.zip) with the following subdirectories:

  src contains Python source code and input data. Make sure that we can easily run your code.

doc contains the electronic version of the report (preferably in a pdf format).

results several input images and their corresponding output images.

The report gives a short description (approximately 5 pages) about the contents in the *src* directory, how to use the program, and contains all information that can be relevant for the grading of the project, for example: parts of the assignment that you did or did not implement, whether you worked together with other groups and how you compensated that work, extras, ...

- Demo: each group needs to demonstrate their project on the day of the exam. During this demonstration we can ask you to execute the program on an alternative dataset provided by us. We can ask questions about the solutions of subtasks and we may gauge your knowledge in the material. You will also have time to inform us of any extras you have implemented, and you can, of course, inform us of any problems you had during the project. The demonstration will take approximately 15 minutes per group and will be considered in assigning the grade.
- Grade: a minimal, working implementation of the complete exercise will result in a score of 7 out of 10. This score will be increased or decreased based on the quality of your work and extras that you have implemented.

5 Contact

You can always contact us:

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References

[1] Bubba, Tatiana A., et al. "Tomographic X-ray data of a lotus root filled with attenuating objects." arXiv preprint arXiv:1609.07299 (2016).

[2] S. De Francesco, and A. Silva, "Fourier Methods in CT: projection and reconstruction algorithms", BIOENG 2003

[3] Nasser M. Abbasi, "The application of Fourier analysis in solving the Computed Tomography (CT) inverse problem.", July 2010