

### **EEIGM**

## École Européenne d'Ingénieurs en Génie des Matériaux

 $2^{'eme}$  Année,  $1^{er}$  Semestre

### MÉCANIQUE DU SOLIDE DÉFORMABLE

### TRAVAUX DIRIGÉS

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List of Acronyms

# **List of Acronyms**

List of Acronyms

List of Sy.	mbols
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# **List of Symbols**

List of Symbols

Abstract

## **Abstract**

### 1. Systèmes de coordonnées curvilignes

### 1.1. Énoncé

#### 1.1.1. Problème A

#### 1.1.2. Problème B

### 1.2. Corrigé

### 1.2.1. Problème A

$$\begin{cases} x = x(\xi, \eta, \zeta) \\ y = x(\xi, \eta, \zeta) \\ z = x(\xi, \eta, \zeta) \end{cases} \longleftrightarrow \begin{cases} \xi = \xi(x, y, z) \\ \eta = \eta(x, y, z) \\ \zeta = \zeta(x, y, z) \end{cases}$$
(1)

$$f(x, y, z) = f(x(\xi, \eta, \zeta), y(\xi, \eta, \zeta), z(\xi, \eta, \zeta))$$
(2)

$$f(\xi, \eta, \zeta) = f(\xi(x, y, z), \eta(x, y, z), \zeta(x, y, z))$$

$$(3)$$

$$\nabla_{xyz}^{2} f(x, y, z) = \frac{\partial^{2} f}{\partial x^{2}} + \frac{\partial^{2} f}{\partial y^{2}} + \frac{\partial^{2} f}{\partial z^{2}}$$

$$\tag{4}$$

$$\begin{cases}
\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial x} \\
\frac{\partial f}{\partial y} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial y} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial y} \\
\frac{\partial f}{\partial z} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial z} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial z}
\end{cases} (5)$$

$$\begin{cases}
\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial x} \right) \\
\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial y} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial y} \right) \\
\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial z} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial z} \right)
\end{cases} (6)$$

$$\begin{cases}
\frac{\partial^2 f}{\partial x^2} = \left(\frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial x} \frac{\partial}{\partial \zeta}\right) \left(\frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial x}\right) \\
\frac{\partial^2 f}{\partial y^2} = \left(\frac{\partial \xi}{\partial y} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial z} \frac{\partial}{\partial \zeta}\right) \left(\frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial y} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial y}\right) \\
\frac{\partial^2 f}{\partial z^2} = \left(\frac{\partial \xi}{\partial z} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial z} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial z} \frac{\partial}{\partial \zeta}\right) \left(\frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial z} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial z}\right)
\end{cases} (7)$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial x^{2}} = \left(\frac{\partial \xi}{\partial x}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial x}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial x}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + \\
+ 2 \left(\frac{\partial \xi}{\partial x}\right) \left(\frac{\partial \eta}{\partial x}\right) \frac{\partial^{2} f}{\partial \xi \partial \eta} + 2 \left(\frac{\partial \xi}{\partial x}\right) \left(\frac{\partial \zeta}{\partial x}\right) \frac{\partial^{2} f}{\partial \xi \partial \zeta} + 2 \left(\frac{\partial \eta}{\partial x}\right) \left(\frac{\partial \zeta}{\partial x}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta} \\
\frac{\partial^{2} f}{\partial y^{2}} = \left(\frac{\partial \xi}{\partial y}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial y}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial y}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + \\
+ 2 \left(\frac{\partial \xi}{\partial y}\right) \left(\frac{\partial \eta}{\partial y}\right) \frac{\partial^{2} f}{\partial \xi \partial \eta} + 2 \left(\frac{\partial \xi}{\partial y}\right) \left(\frac{\partial \zeta}{\partial y}\right) \frac{\partial^{2} f}{\partial \xi \partial \zeta} + 2 \left(\frac{\partial \eta}{\partial y}\right) \left(\frac{\partial \zeta}{\partial y}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta} \\
\frac{\partial^{2} f}{\partial z^{2}} = \left(\frac{\partial \xi}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + \\
+ 2 \left(\frac{\partial \xi}{\partial z}\right) \left(\frac{\partial \eta}{\partial z}\right) \frac{\partial^{2} f}{\partial \xi \partial \eta} + 2 \left(\frac{\partial \xi}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \xi \partial \zeta} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$(8)$$

$$\frac{\partial^2 f}{\partial \eta \partial \xi} = \frac{\partial^2 f}{\partial \xi \partial \eta} \qquad \frac{\partial^2 f}{\partial \eta \partial \zeta} = \frac{\partial^2 f}{\partial \zeta \partial \eta} \qquad \frac{\partial^2 f}{\partial \xi \partial \zeta} = \frac{\partial^2 f}{\partial \zeta \partial \xi} \tag{9}$$

$$\nabla_{\xi\eta\zeta}^{2}f\left(\xi,\eta,\zeta\right) = \left[\left(\frac{\partial\xi}{\partial x}\right)^{2} + \left(\frac{\partial\xi}{\partial y}\right)^{2} + \left(\frac{\partial\xi}{\partial z}\right)^{2}\right] \frac{\partial^{2}f}{\partial\xi^{2}} + \\
+ \left[\left(\frac{\partial\eta}{\partial x}\right)^{2} + \left(\frac{\partial\eta}{\partial y}\right)^{2} + \left(\frac{\partial\eta}{\partial z}\right)^{2}\right] \frac{\partial^{2}f}{\partial\eta^{2}} + \\
+ \left[\left(\frac{\partial\zeta}{\partial x}\right)^{2} + \left(\frac{\partial\zeta}{\partial y}\right)^{2} + \left(\frac{\partial\zeta}{\partial z}\right)^{2}\right] \frac{\partial^{2}f}{\partial\zeta^{2}} + \\
+ 2\left[\left(\frac{\partial\xi}{\partial x}\right)\left(\frac{\partial\eta}{\partial x}\right) + \left(\frac{\partial\xi}{\partial y}\right)\left(\frac{\partial\eta}{\partial y}\right) + \left(\frac{\partial\xi}{\partial z}\right)\left(\frac{\partial\eta}{\partial z}\right)\right] \frac{\partial^{2}f}{\partial\xi\partial\eta} + \\
+ 2\left[\left(\frac{\partial\eta}{\partial x}\right)\left(\frac{\partial\zeta}{\partial x}\right) + \left(\frac{\partial\eta}{\partial y}\right)\left(\frac{\partial\zeta}{\partial y}\right) + \left(\frac{\partial\eta}{\partial z}\right)\left(\frac{\partial\zeta}{\partial z}\right)\right] \frac{\partial^{2}f}{\partial\eta\partial\zeta} + \\
+ 2\left[\left(\frac{\partial\xi}{\partial x}\right)\left(\frac{\partial\zeta}{\partial x}\right) + \left(\frac{\partial\xi}{\partial y}\right)\left(\frac{\partial\zeta}{\partial y}\right) + \left(\frac{\partial\xi}{\partial z}\right)\left(\frac{\partial\zeta}{\partial z}\right)\right] \frac{\partial^{2}f}{\partial\xi\partial\zeta}$$

### 1.2.2. Problème B

$$\begin{cases} x = x(\xi, \eta, \zeta) \\ y = x(\xi, \eta, \zeta) \\ z = x(\xi, \eta, \zeta) \end{cases} \longleftrightarrow \begin{cases} \xi = \xi(x, y, z) \\ \eta = \eta(x, y, z) \\ \zeta = \zeta(x, y, z) \end{cases}$$
(11)

$$f(x, y, z) = f(x(\xi, \eta, \zeta), y(\xi, \eta, \zeta), z(\xi, \eta, \zeta))$$

$$(12)$$

$$f(\xi, \eta, \zeta) = f(\xi(x, y, z), \eta(x, y, z), \zeta(x, y, z))$$

$$(13)$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial x} \\ \frac{\partial f}{\partial \eta} \\ \frac{\partial f}{\partial \zeta} \end{bmatrix}$$
(14)

Introduction

## 2. Second section

Second section

4	7.	4
Apper	ndix	A
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# A. First appendix

4	7.	4
Ap	pendix	A