

### **EEIGM**

## École Européenne d'Ingénieurs en Génie des Matériaux

 $2^{'eme}$  Année,  $1^{er}$  Semestre

### MÉCANIQUE DU SOLIDE DÉFORMABLE

### TRAVAUX DIRIGÉS

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List of Acronyms

# **List of Acronyms**

List of Acronyms

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# **List of Symbols**

List of Symbols

Abstract

## **Abstract**

## 1. Systèmes de coordonnées curvilignes

### 1.1. Énoncé

#### 1.1.1. Problème A

#### 1.1.2. Problème B

### 1.2. Corrigé

#### 1.2.1. Problème A

$$\begin{cases} x = x (\xi, \eta, \zeta) \\ y = x (\xi, \eta, \zeta) \\ z = x (\xi, \eta, \zeta) \end{cases} \longleftrightarrow \begin{cases} \xi = \xi (x, y, z) \\ \eta = \eta (x, y, z) \\ \zeta = \zeta (x, y, z) \end{cases}$$
(1)

$$f(x, y, z) = f(x(\xi, \eta, \zeta), y(\xi, \eta, \zeta), z(\xi, \eta, \zeta))$$
(2)

$$f(\xi, \eta, \zeta) = f(\xi(x, y, z), \eta(x, y, z), \zeta(x, y, z))$$
(3)

$$\nabla_{xyz}^{2} f(x, y, z) = \frac{\partial^{2} f}{\partial x^{2}} + \frac{\partial^{2} f}{\partial y^{2}} + \frac{\partial^{2} f}{\partial z^{2}}$$

$$\tag{4}$$

$$\begin{cases}
\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial x} \\
\frac{\partial f}{\partial y} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial y} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial y} \\
\frac{\partial f}{\partial z} = \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial z} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial z}
\end{cases}$$
(5)

$$\begin{cases} \frac{\partial^{2} f}{\partial x^{2}} &= \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial x} \right) = \\ &= \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \xi} \right) \frac{\partial \xi}{\partial x} + \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \eta} \right) \frac{\partial \eta}{\partial x} + \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \zeta} \right) \frac{\partial \zeta}{\partial x} + \frac{\partial f}{\partial \xi} \frac{\partial^{2} \xi}{\partial x^{2}} + \frac{\partial f}{\partial \eta} \frac{\partial^{2} \eta}{\partial x^{2}} + \frac{\partial f}{\partial \zeta} \frac{\partial^{2} \zeta}{\partial x^{2}} \\ \frac{\partial^{2} f}{\partial y^{2}} &= \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial y} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial y} \right) = \\ &= \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \xi} \right) \frac{\partial \xi}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \eta} \right) \frac{\partial \eta}{\partial y} + \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \zeta} \right) \frac{\partial \zeta}{\partial y} + \frac{\partial f}{\partial \xi} \frac{\partial^{2} \xi}{\partial y^{2}} + \frac{\partial f}{\partial \eta} \frac{\partial^{2} \eta}{\partial y^{2}} + \frac{\partial f}{\partial \zeta} \frac{\partial^{2} \zeta}{\partial y^{2}} \\ \frac{\partial^{2} f}{\partial z^{2}} &= \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial z} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial z} \right) = \\ &= \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \xi} \right) \frac{\partial \xi}{\partial z} + \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \eta} \right) \frac{\partial \eta}{\partial z} + \frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \zeta} \right) \frac{\partial \zeta}{\partial z} + \frac{\partial f}{\partial \xi} \frac{\partial^{2} \xi}{\partial z^{2}} + \frac{\partial f}{\partial \eta} \frac{\partial^{2} \eta}{\partial z^{2}} + \frac{\partial f}{\partial \zeta} \frac{\partial^{2} \zeta}{\partial z^{2}} \end{cases}$$

$$\begin{cases}
\frac{\partial}{\partial x} = \left(\frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial x} \frac{\partial}{\partial \zeta}\right) \\
\frac{\partial}{\partial y} = \left(\frac{\partial \xi}{\partial y} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial y} \frac{\partial}{\partial \zeta}\right) \\
\frac{\partial}{\partial z} = \left(\frac{\partial \xi}{\partial z} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial z} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial z} \frac{\partial}{\partial \zeta}\right)
\end{cases} (7)$$

$$\begin{cases}
\frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \xi} \right) \frac{\partial \xi}{\partial x} &= \left( \frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial x} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \xi} \right) \frac{\partial \xi}{\partial x} = \\
&= \left( \frac{\partial \xi}{\partial x} \right)^2 \frac{\partial^2 f}{\partial \xi^2} + \left( \frac{\partial \eta}{\partial x} \right) \left( \frac{\partial \xi}{\partial x} \right) \frac{\partial^2 f}{\partial \eta \partial \xi} + \left( \frac{\partial \zeta}{\partial x} \right) \left( \frac{\partial \xi}{\partial x} \right) \frac{\partial^2 f}{\partial \zeta \partial \xi} \\
\frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \eta} \right) \frac{\partial \eta}{\partial x} &= \left( \frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial x} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \eta} \right) \frac{\partial \eta}{\partial x} = \\
&= \left( \frac{\partial \eta}{\partial x} \right)^2 \frac{\partial^2 f}{\partial \eta^2} \\
\frac{\partial}{\partial x} \left( \frac{\partial f}{\partial \zeta} \right) \frac{\partial \zeta}{\partial x} &= \left( \frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial x} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \zeta} \right) \frac{\partial \zeta}{\partial x} = \\
&= \left( \frac{\partial \zeta}{\partial x} \right)^2 \frac{\partial^2 f}{\partial \zeta^2}
\end{cases} \tag{8}$$

$$\begin{cases}
\frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \xi} \right) &= \left( \frac{\partial \xi}{\partial y} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial y} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \xi} \right) = \\
&= \\
\frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \eta} \right) &= \left( \frac{\partial \xi}{\partial y} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial y} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \eta} \right) = \\
&= \\
\frac{\partial}{\partial y} \left( \frac{\partial f}{\partial \zeta} \right) &= \left( \frac{\partial \xi}{\partial y} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial y} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \zeta} \right) = \\
&= \\
= \\
\end{cases} \tag{9}$$

$$\begin{cases}
\frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \xi} \right) &= \left( \frac{\partial \xi}{\partial z} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial z} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial z} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \xi} \right) = \\
&= \\
\frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \eta} \right) &= \left( \frac{\partial \xi}{\partial z} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial z} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial z} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \eta} \right) = \\
&= \\
\frac{\partial}{\partial z} \left( \frac{\partial f}{\partial \zeta} \right) &= \left( \frac{\partial \xi}{\partial z} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial z} \frac{\partial}{\partial \eta} + \frac{\partial \zeta}{\partial z} \frac{\partial}{\partial \zeta} \right) \left( \frac{\partial f}{\partial \zeta} \right) = \\
&= \\
&= \\
\end{cases} (10)$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial x^{2}} = \left(\frac{\partial \xi}{\partial x}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial x}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial x}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + \\
+ 2 \left(\frac{\partial \xi}{\partial x}\right) \left(\frac{\partial \eta}{\partial x}\right) \frac{\partial^{2} f}{\partial \xi \partial \eta} + 2 \left(\frac{\partial \xi}{\partial x}\right) \left(\frac{\partial \zeta}{\partial x}\right) \frac{\partial^{2} f}{\partial \xi \partial \zeta} + 2 \left(\frac{\partial \eta}{\partial x}\right) \left(\frac{\partial \zeta}{\partial x}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial y^{2}} = \left(\frac{\partial \xi}{\partial y}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial y}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial y}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + \\
+ 2 \left(\frac{\partial \xi}{\partial y}\right) \left(\frac{\partial \eta}{\partial y}\right) \frac{\partial^{2} f}{\partial \xi \partial \eta} + 2 \left(\frac{\partial \xi}{\partial y}\right) \left(\frac{\partial \zeta}{\partial y}\right) \frac{\partial^{2} f}{\partial \xi \partial \zeta} + 2 \left(\frac{\partial \eta}{\partial y}\right) \left(\frac{\partial \zeta}{\partial y}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial y^{2}} = \left(\frac{\partial \xi}{\partial y}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial y^{2}} = \left(\frac{\partial \xi}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial z^{2}} = \left(\frac{\partial \xi}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial z^{2}} = \left(\frac{\partial \xi}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \zeta^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial z^{2}} = \left(\frac{\partial \xi}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \xi^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \eta \partial \zeta}
\end{cases}$$

$$\begin{cases}
\frac{\partial^{2} f}{\partial z^{2}} = \left(\frac{\partial \xi}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \xi^{2}} + \left(\frac{\partial \eta}{\partial z}\right)^{2} \frac{\partial^{2} f}{\partial \eta^{2}} + \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \xi^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \eta^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial \xi^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}\right) \frac{\partial^{2} f}{\partial z^{2}} + 2 \left(\frac{\partial \eta}{\partial z}\right) \left(\frac{\partial \zeta}{\partial z}$$

$$\frac{\partial^2 f}{\partial \eta \partial \xi} = \frac{\partial^2 f}{\partial \xi \partial \eta} \qquad \frac{\partial^2 f}{\partial \eta \partial \zeta} = \frac{\partial^2 f}{\partial \zeta \partial \eta} \qquad \frac{\partial^2 f}{\partial \xi \partial \zeta} = \frac{\partial^2 f}{\partial \zeta \partial \xi} \tag{12}$$

$$\nabla_{\xi\eta\zeta}^{2}f\left(\xi,\eta,\zeta\right) = \left[\left(\frac{\partial\xi}{\partial x}\right)^{2} + \left(\frac{\partial\xi}{\partial y}\right)^{2} + \left(\frac{\partial\xi}{\partial z}\right)^{2}\right] \frac{\partial^{2}f}{\partial\xi^{2}} + \\
+ \left[\left(\frac{\partial\eta}{\partial x}\right)^{2} + \left(\frac{\partial\eta}{\partial y}\right)^{2} + \left(\frac{\partial\eta}{\partial z}\right)^{2}\right] \frac{\partial^{2}f}{\partial\eta^{2}} + \\
+ \left[\left(\frac{\partial\zeta}{\partial x}\right)^{2} + \left(\frac{\partial\zeta}{\partial y}\right)^{2} + \left(\frac{\partial\zeta}{\partial z}\right)^{2}\right] \frac{\partial^{2}f}{\partial\zeta^{2}} + \\
+ 2\left[\left(\frac{\partial\xi}{\partial x}\right)\left(\frac{\partial\eta}{\partial x}\right) + \left(\frac{\partial\xi}{\partial y}\right)\left(\frac{\partial\eta}{\partial y}\right) + \left(\frac{\partial\xi}{\partial z}\right)\left(\frac{\partial\eta}{\partial z}\right)\right] \frac{\partial^{2}f}{\partial\xi\partial\eta} + \\
+ 2\left[\left(\frac{\partial\eta}{\partial x}\right)\left(\frac{\partial\zeta}{\partial x}\right) + \left(\frac{\partial\eta}{\partial y}\right)\left(\frac{\partial\zeta}{\partial y}\right) + \left(\frac{\partial\eta}{\partial z}\right)\left(\frac{\partial\zeta}{\partial z}\right)\right] \frac{\partial^{2}f}{\partial\eta\partial\zeta} + \\
+ 2\left[\left(\frac{\partial\xi}{\partial x}\right)\left(\frac{\partial\zeta}{\partial x}\right) + \left(\frac{\partial\xi}{\partial y}\right)\left(\frac{\partial\zeta}{\partial y}\right) + \left(\frac{\partial\xi}{\partial z}\right)\left(\frac{\partial\zeta}{\partial z}\right)\right] \frac{\partial^{2}f}{\partial\xi\partial\zeta}$$

#### Coordonnées cylindriques

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \longleftrightarrow \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1} \left(\frac{y}{x}\right) \\ z = z \end{cases}$$
 (14)

$$\begin{cases}
\frac{\partial r}{\partial x} = \frac{1}{2} \frac{2x}{\sqrt{x^2 + y^2}} = \frac{r' \cos \theta}{r'} \\
\frac{\partial r}{\partial y} = \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2}} = \frac{r' \sin \theta}{r'} \\
\frac{\partial r}{\partial x} = 0
\end{cases}$$
(15)

#### Coordonnées spheriques

#### 1.2.2. Problème B

$$\begin{cases} x = x(\xi, \eta, \zeta) \\ y = x(\xi, \eta, \zeta) \\ z = x(\xi, \eta, \zeta) \end{cases} \longleftrightarrow \begin{cases} \xi = \xi(x, y, z) \\ \eta = \eta(x, y, z) \\ \zeta = \zeta(x, y, z) \end{cases}$$
(16)

$$f(x, y, z) = f(x(\xi, \eta, \zeta), y(\xi, \eta, \zeta), z(\xi, \eta, \zeta))$$

$$(17)$$

$$f(\xi, \eta, \zeta) = f(\xi(x, y, z), \eta(x, y, z), \zeta(x, y, z))$$

$$(18)$$

$$\nabla f_{xyz} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \frac{\partial f}{\partial x} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \frac{\partial f}{\partial y} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \frac{\partial f}{\partial z} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{\partial f}{\partial x} \mathbf{i}_x + \frac{\partial f}{\partial y} \mathbf{j}_y + \frac{\partial f}{\partial z} \mathbf{k}_z$$
 (19)

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial x} \\ \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial y} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial y} \\ \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial z} + \frac{\partial f}{\partial \zeta} \frac{\partial \zeta}{\partial \zeta} \end{bmatrix} = \frac{\partial f}{\partial \xi} \begin{bmatrix} \frac{\partial \xi}{\partial x} \\ \frac{\partial \xi}{\partial y} \\ \frac{\partial \xi}{\partial z} \end{bmatrix} + \frac{\partial f}{\partial \eta} \begin{bmatrix} \frac{\partial \eta}{\partial x} \\ \frac{\partial \eta}{\partial y} \\ \frac{\partial \eta}{\partial z} \end{bmatrix} + \frac{\partial f}{\partial \zeta} \begin{bmatrix} \frac{\partial \zeta}{\partial x} \\ \frac{\partial \zeta}{\partial y} \\ \frac{\partial \zeta}{\partial z} \end{bmatrix}$$
(20)

$$\mathbf{i}_{\xi} = \frac{1}{\sqrt{\left(\frac{\partial \xi}{\partial x}\right)^{2} + \left(\frac{\partial \xi}{\partial y}\right)^{2} + \left(\frac{\partial \xi}{\partial z}\right)^{2}}} \begin{bmatrix} \frac{\partial \xi}{\partial x} \\ \frac{\partial \xi}{\partial y} \\ \frac{\partial \xi}{\partial z} \end{bmatrix} 
\mathbf{j}_{\eta} = \frac{1}{\sqrt{\left(\frac{\partial \eta}{\partial x}\right)^{2} + \left(\frac{\partial \eta}{\partial y}\right)^{2} + \left(\frac{\partial \eta}{\partial z}\right)^{2}}} \begin{bmatrix} \frac{\partial \eta}{\partial x} \\ \frac{\partial \eta}{\partial y} \\ \frac{\partial \eta}{\partial y} \\ \frac{\partial \eta}{\partial z} \end{bmatrix} 
\mathbf{k}_{\zeta} = \frac{1}{\sqrt{\left(\frac{\partial \zeta}{\partial x}\right)^{2} + \left(\frac{\partial \zeta}{\partial y}\right)^{2} + \left(\frac{\partial \zeta}{\partial z}\right)^{2}}} \begin{bmatrix} \frac{\partial \zeta}{\partial x} \\ \frac{\partial \zeta}{\partial y} \\ \frac{\partial \zeta}{\partial z} \end{bmatrix}$$
(21)

$$\nabla_{\xi\eta\zeta}f = \frac{\partial f}{\partial\xi}\sqrt{\left(\frac{\partial\xi}{\partial x}\right)^{2} + \left(\frac{\partial\xi}{\partial y}\right)^{2} + \left(\frac{\partial\xi}{\partial z}\right)^{2}} \cdot \mathbf{i}_{\xi} + 
+ \frac{\partial f}{\partial\eta}\sqrt{\left(\frac{\partial\eta}{\partial x}\right)^{2} + \left(\frac{\partial\eta}{\partial y}\right)^{2} + \left(\frac{\partial\eta}{\partial z}\right)^{2}} \cdot \mathbf{j}_{\eta} + 
+ \frac{\partial f}{\partial\zeta}\sqrt{\left(\frac{\partial\zeta}{\partial x}\right)^{2} + \left(\frac{\partial\zeta}{\partial y}\right)^{2} + \left(\frac{\partial\zeta}{\partial z}\right)^{2}} \cdot \mathbf{k}_{\zeta} = 
= \begin{bmatrix} \frac{\partial f}{\partial\xi}\sqrt{\left(\frac{\partial\xi}{\partial x}\right)^{2} + \left(\frac{\partial\xi}{\partial y}\right)^{2} + \left(\frac{\partial\xi}{\partial z}\right)^{2}} \\ \frac{\partial f}{\partial\zeta}\sqrt{\left(\frac{\partial\eta}{\partial x}\right)^{2} + \left(\frac{\partial\eta}{\partial y}\right)^{2} + \left(\frac{\partial\eta}{\partial z}\right)^{2}} \\ \frac{\partial f}{\partial\zeta}\sqrt{\left(\frac{\partial\zeta}{\partial x}\right)^{2} + \left(\frac{\partial\zeta}{\partial y}\right)^{2} + \left(\frac{\partial\zeta}{\partial z}\right)^{2}} \end{bmatrix}$$
(22)

#### Coordonnées cylindriques

#### Coordonnées spheriques

## 2. Second section

Second section

| Λ             | • | nond | 1: | Λ |
|---------------|---|------|----|---|
| $\mathcal{A}$ | μ | pena | IΧ | Α |

# A. First appendix

| Appen | dix | $\overline{A}$ |
|-------|-----|----------------|
|       |     |                |