The Fiber-Matrix Interface Crack Problem and its Solution by the Finite Element Method

L. Di Stasio[1,2], Z. Ayadi[1], J. Varna[2]

[1]EEIGM & IJL, Université de Lorraine, Nancy, France

[2]Division of Materials Science, Luleå University of Technology, Luleå, Sweden

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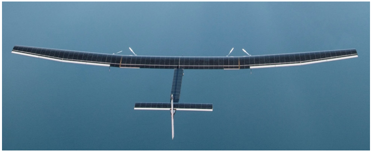
Outline

- Damage Mechanisms in Thin Ply Fiber Reinforced Polymer Laminates
- The Fiber-Matrix Interface Problem in Fiber Reinforced Polymer Laminates
- Analysis of the Infinite Reference Volume Element (RVE)
- Timeline, Conclusions & Outlook
- References

↘ DAMAGE IN THIN PLY FRPC

Spread Tow Technology: Introduction

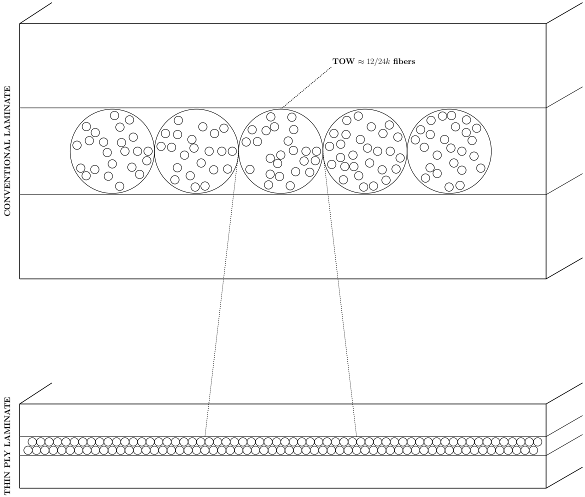
- ➔ Firstly developed for commercial use in Japan between 1995 and 1998 (Kawabe, Tomoda et al. 1997 [1], 2003 [2], 2008 [3], 2009 [4])
- ➔ In the last decade its use has been spreading, from sports' equipments to mission-critical applications as in the *Solar Impulse 2*
- ➔ Only a few producers wolrdwide: NTPT (USA-CH) [5], Oxeon (SE) [6], Chomarat (FR), Hexcel (USA), Technomax (JP)



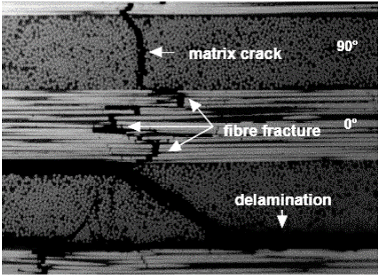
(a) By North Thin Ply Technology.

(b) By TeXtreme.

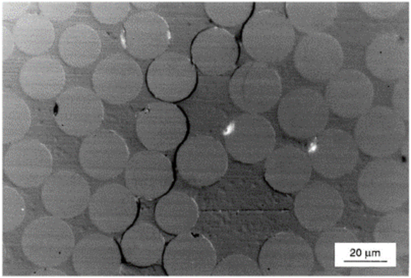
Spread Tow Technology: Foundations



Damage Onset and Propagation



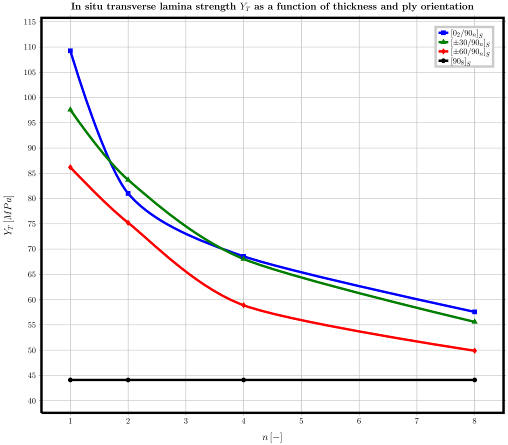
(c) By Dr. R. Olsson, Swerea, SE.



(d) By Prof. Dr. E. K. Gamstedt, KTH, SE.

For a visual definition of intralaminar transverse cracking.

The Thin Ply Effect



Measurements of in-situ transverse strength from D. L. Flaggs & M. H. Kural, 1982 [7].

Characterization of the Fracture Process

➔ Energy Release Rate

$$G_m = G_m\left(p_1, \ldots, p_i, \ldots, p_n\right) \quad \text{where} \quad G = \frac{\partial W}{\partial A} - \left(\frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A}\right)$$

➔ Stress Intensity Factor

$$K_m = K_m\left(p_1, \ldots, p_i, \ldots, p_n\right) \quad \text{where} \quad \sigma_m \sim K_m \frac{\alpha}{(x-a)^{\beta}} \quad \alpha, \beta > 0$$

➔ J-Integral

$$J = J\left(p_1, \ldots, p_i, \ldots, p_n\right) \quad \text{where} \quad J = \lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon} \left(W(\Gamma)\, n_i - n_j \sigma_{jk} \frac{\partial u_k\left(\Gamma, x_i\right)}{\partial x_i}\right) d\Gamma = G$$

➔ Crack Opening & Shear Displacement

$$COD = COD\left(p_1, \ldots, p_i, \ldots, p_n\right) \quad \text{and} \quad CSD = CSD\left(p_1, \ldots, p_i, \ldots, p_n\right)$$

$$p_i \in \{\text{geometry}, \text{materials}, \text{boundary conditions}, \text{loading mode}, \text{scale}\}$$

$$m \in \{I, II, III, I/II, I/III, II/III\}$$

Evaluation of Fracture Parameters

➔ Analytical

✓ Closed form

✓ Every material scale can be studied

✗ Available only for particular configurations, often infinite in size

✗ Complex geometries cannot be studied

➔ Experimental

✓ Complex geometries can be studied

✗ Not every material scale is accessible

➔ Numerical

✓ Complex geometries can be studied

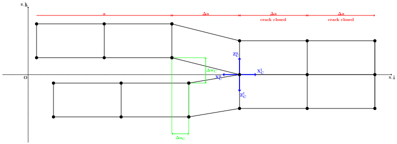
✓ Every material scale can be studied

✗ Discretization

✗ Finite domains

Numerical Estimation of Energy Release Rates

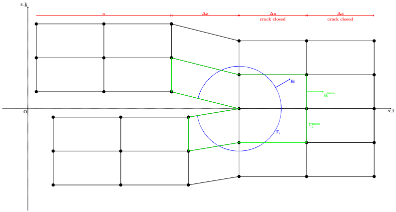
➔ Virtual Crack Closure Technique (VCCT)



$$G_I = \frac{Z_C \Delta w_C}{2B\Delta a} \qquad G_{II} = \frac{X_C \Delta u_C}{2B\Delta a}$$

Krueger, 2004

➔ J-Integral

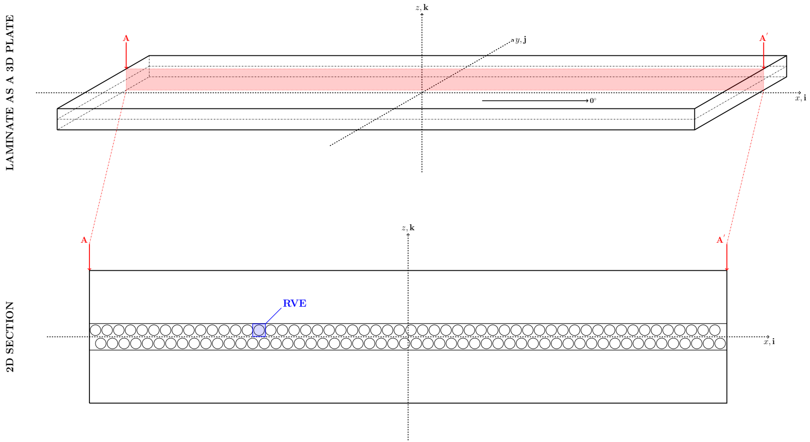


$$J_i = \lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon} \left(W(\Gamma)\, n_i - n_j \sigma_{jk} \frac{\partial u_k(\Gamma, x_i)}{\partial x_i} \right) d\Gamma$$

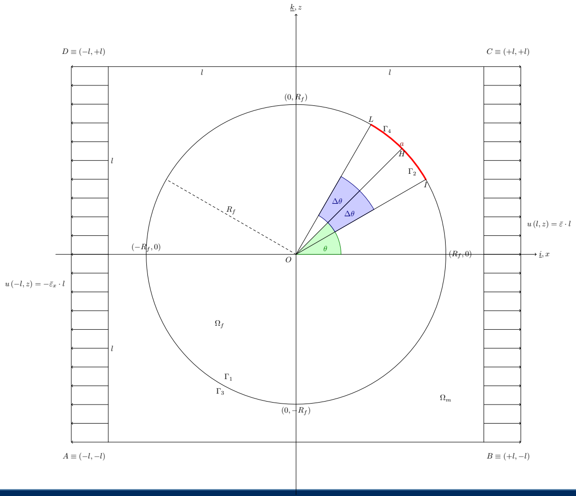
Rice, 1968

The Fiber-Matrix Interface Problem in FRPC

Multi-scale Decomposition of Fiber Reinforced Polymer Laminates



The Fiber-Matrix Interface Crack Problem: Statement



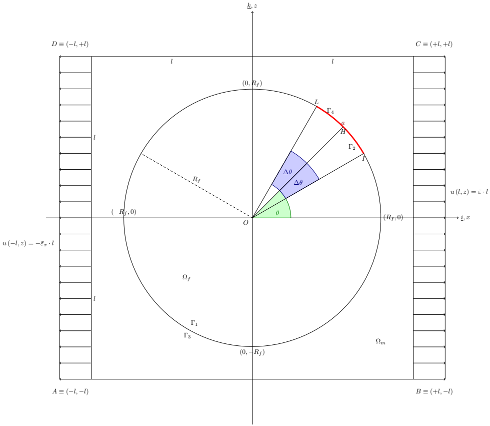
- ➔ 2D space
- ➔ Linear elastic homogeneous isotropic materials
- ➔ Mismatching elastic properties
- ➔ Plane state (strain or stress)
- ➔ Dirichlet-type BC
- ➔ Linear Fracture Mechanics
- ➔ Contact interaction
- ➔ Applied uniaxial traction
- ? SIF, ERR, mode ratio, stress and displacement distribution at the interface

The Fiber-Matrix Interface Crack Problem: Solution

Method	**Domain**	**Natural Variable**	**Conjugate Variable**	**Dirichlet BC**
Analytical (complex) functions	2D, continuous, infinite	Airy stress potential & stress	Displacement & strain	In stress
	M. Toya (1975), A Crack Along the Interface of a Circular Inclusion Embedded in an Infinite Solid [10].			
Boundary Element Method (BEM)	1D, discrete, finite	Stress, by using Green's potentials or Betti's influence functions	Displacement & strain	In stress
	F. París et al. (1996), The fiber-matrix interface crack - A numerical analysis using Boundary Elements [11].			
Finite Element Method (FEM)	2D, discrete, finite	Displacement	Stress	In displacement

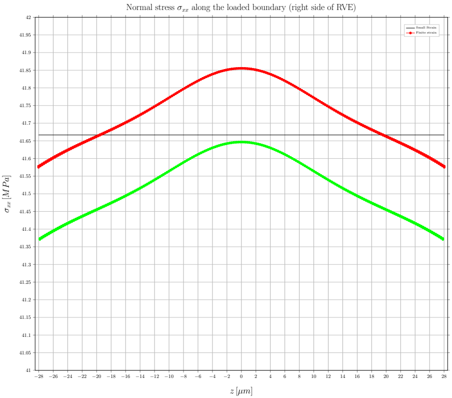
ANALYSIS OF THE INFINITE RVE

The Finite Element Model



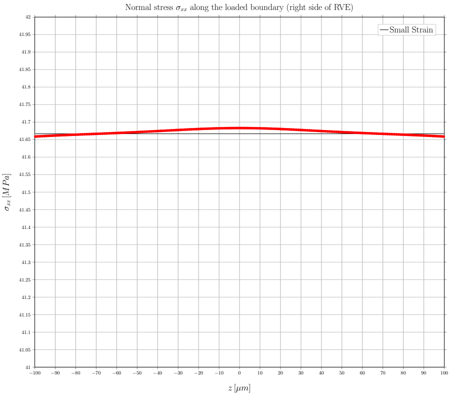
- $\theta\ [^{\circ}] = 0$, angular position of debond's center
- $2\Delta\theta\ [^{\circ}]$, debond's angular size
- $\delta\ [^{\circ}]$, angle subtended by an element at the fiber/matrix interface
- $VF_f\ [-]$, fiber volume fraction
- $2L\ [\mu m]$, RVE's side length
- $R_F\ [\mu m]$, fiber radius
- $\frac{L}{R_f} = \frac{1}{2}\sqrt{\frac{\pi}{VF_f}}\ [-]$, RVE's aspect ratio
- $\sigma_0\ [MPa] = \frac{E_m}{1-\nu_m^2}\varepsilon_{xx}$, reaction stress of undamaged infinite RVE
- $G_0\ \left[\frac{J}{m^2}\right] = \pi R_f \sigma_R^2 \frac{1+(3-4num_m)}{8G_m}$, normalization G following Toya [10] and París [11]
- Small displacement formulation

Reaction Stress at the Boundary



$VF_f = 0.001$, $\frac{L}{R_f} \sim 28$, $\delta = 0.4^{\circ}$,

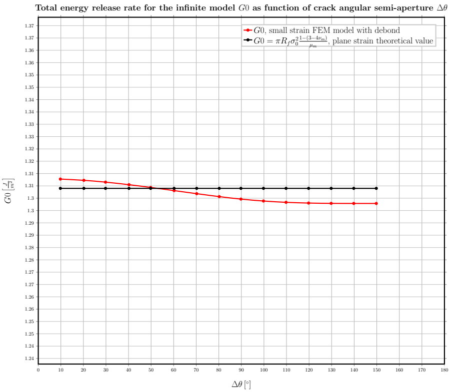
$\frac{\sigma_{max} - \sigma_{mean}}{\sigma_{mean}} = 0.34\%$

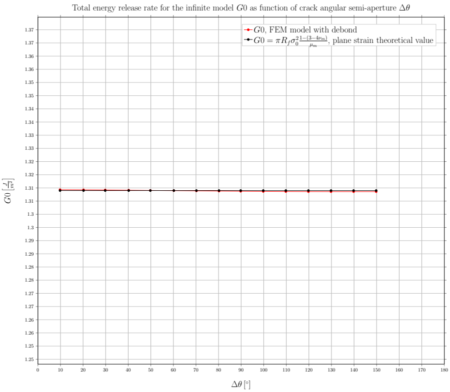


$VF_f = 0.000079$, $\frac{L}{R_f} \sim 100$, $\delta = 0.4^{\circ}$,

$\frac{\sigma_{max} - \sigma_{mean}}{\sigma_{mean}} = 0.03\%$

Normalization G_0 as a function of $\Delta\theta$

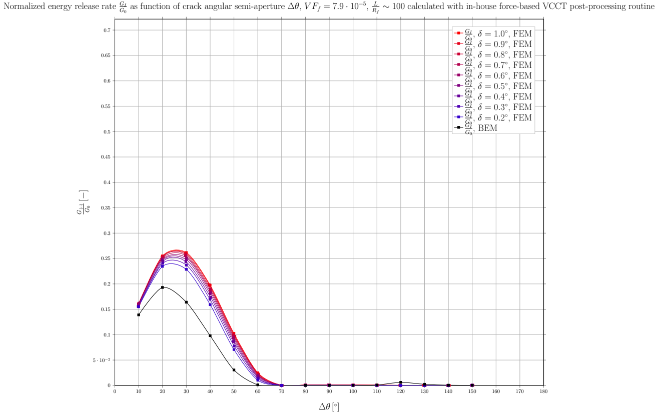




$VF_f = 0.001$, $\frac{L}{R_f} \sim 28$, $\delta = 0.4°$

$VF_f = 0.000079$, $\frac{L}{R_f} \sim 100$, $\delta = 0.4°$

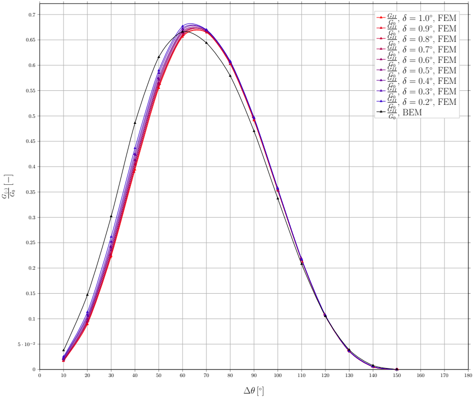
Mode I Energy Release Rate G_I from VCCT



$Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$; fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

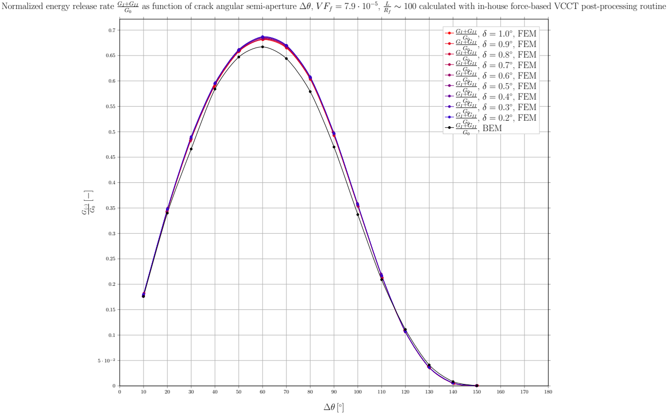
Mode II Energy Release Rate G_{II} from VCCT

Normalized energy release rate $\frac{G_{II}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, $VF_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine



$Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$; fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

Total Energy Release Rate G_{TOT} from VCCT



$Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$; fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

Conclusions

Timeline

- Oct. 2015 - Dec. 2017: research in Nancy
- Jan. 2018 - Dec. 2019: research in Luleå

- May - June 2016: DocMASE Summer School 2016, Luleå; presentation
- June 2016: quarter-term auto-evaluation report, Nancy
- April 2017: International Materials Research Meeting of the Greater Region, Sarrebrücken; presentation
- May 2017: EMMA doctoral school seminar, Nancy; poster
- July 2017: seminar day of the Research Group 304, presentation (in French)
- October 2017: mid-term defence, Nancy
- September 2018: mid-term defence, Luleå
- December 2019: doctoral defence, Nancy - Luleå

Conclusions & Outlook

Conclusions

- ➔ There is a limiting value of $\frac{L}{R_f}$ after which models are effectively infinite
- ➔ For models larger than this value, domain size and mesh refinement at the interface has a similar effect on the energy release rate
- ➔ The discrepancy in modes with the use of linear elements might be linked to the deformed shape of crack faces

Outlook

- ➔ Modeling extreme ply geometries, for example a ply with a single layer of fibers bounded by stiffer plies
- ➔ Investigate the effect of clusters of fibers in thin plies
- ➔ Analyzing the effect of complex stress and deformation states, thermal loads, different sets of boundary conditions

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