STATISTICAL MODELS

Course 2024-2025

SEMINAR 3. ANOVA-Based Techniques

1 Basic Problems

PROBLEM 1.- In an experiment to assess the quality of earbuds, a total of 22 units from 4 different brands are measured in terms of their sound distortion. The resulting measurements, for brands A, B, C and D, are as follows:

$$\begin{aligned} A: &\quad \{10,\ 15,\ 8\ \ 12\ \ 15\} \\ B: &\quad \{14,\ 18,\ 21,\ 15\} \\ C: &\quad \{17,\ 16,\ 14,\ 15,\ 17,\ 15,\ 18\} \\ D: &\quad \{12,\ 15,\ 17,\ 15,\ 16,\ 15\} \end{aligned}$$

a) Is their a significant difference in the sound quality of the above 4 brands of earbuds? Use significance level $\alpha = 0.05$

b) If you answer YES to (a), find what pairs of brands differ between them in terms of sound quality. Keep the same significance level as above.

c) Compute confidence intervals for the difference between all pairs of groups. Compare your results with those from (b).

PROBLEM 2.- A total of 12 subjects are compared for the effect of 2 drugs. This results in 3 groups (controls, drug A and drug B). The measurements obtained for the subjects in each group are:

$$\begin{aligned} \mathbf{x_{Control}} &= [2, \quad 3, \quad 4, \quad 2] \\ \mathbf{x_{DrugA}} &= [3, \quad 7, \quad 4, \quad 1, \quad 0] \\ \mathbf{x_{DrugB}} &= [8, \quad 10, \quad 7] \end{aligned}$$

a) Compute the total sum of squares of the data (after removing the mean).

b) Break down the total sum of squares into treatment and residuals.

c) Test if you can reject the null hypothesis that the means of the 3 groups are equal at significance level $\alpha = 0.05$

d) If the null hypothesis in (c) is rejected, perform post-hoc comparisons between the different groups to see which one(s) are different.

e) If the null hypothesis in (c) is rejected, compute confidence intervals for the difference between all pairs of groups. Compare your results with those from (d).

PROBLEM 3.- We want to investigate the effect of a certain gym training routine in fitness level. To this end, 6 volunteers are assessed at 3-month intervals (see Table 3), including assessment before starting the routine (0-months). Test whether the routine in question produces a significant effect on fitness level:

a) State the null and alternative hypotheses.

b) Compute the required statistics and clearly indicate your conclusion about the hypotheses in (a), at significance level $\alpha = 0.05$.

Subject	Time on routine		
	0 months	3 months	6 months
1	45	50	55
2	42	42	45
3	37	39	44
4	39	35	40
5	51	55	59
6	44	49	57

Table 1: Measurements of fitness level for Problem 3

PROBLEM 4.- Repeat Problem 3 but under the assumption that the subjects measured at each time point were not the same (i.e. they were subjects that were exposed to the routine by the specified amount of time, but they were not the same subjects). Table 2 shows the corresponding data.

0 months on routine		3 months on routine		6 months on routine	
Subject	Fit. Measure	Subject	Fit. Measure	Subject	Fit. Measure
1	45	7	50	13	55
2	42	8	42	14	45
3	37	9	39	15	44
4	39	10	35	16	40
5	51	11	55	17	59
6	44	12	49	18	57

Table 2: Data for Problem 4

PROBLEM 5.- We want to investigate the effect of a certain diet for weight loss. To this end, Table 3 gathers the measurements from 4 volunteers assessed at 1-month intervals, including assessment before starting the diet (0-months). Test whether the diet produces a significant weight loss:

a) State the null and alternative hypotheses. Include also the mathematical formulation.

b) Indicate what method would you use to test the hypotheses stated in (a). Indicate what assumptions would be needed in order for your method to be valid. For this problem, you do not need to test whether the assumptions hold; we will assume that they do.

c) Compute the required statistics to test the hypothesis in (a).

d) Based on the result from (c), **clearly indicate your conclusion** about the hypotheses in (a), at significance level $\alpha = 0.05$.

Subject	Time on diet		
	0 months	1 month	2 months
1	79	79	79
2	78	75	75
3	75	76	74
4	80	78	76

Table 3: Measurements of body weight (kg) for Problem 5

PROBLEM 6.- Consider the following bi-variate observations from 3 groups:

$$\mathbf{X_1}: \begin{bmatrix}6\\7\end{bmatrix} \quad \begin{bmatrix}5\\9\end{bmatrix} \quad \begin{bmatrix}8\\6\end{bmatrix} \quad \begin{bmatrix}4\\9\end{bmatrix} \quad \begin{bmatrix}7\\9\end{bmatrix}$$

$$\mathbf{X_2}: \begin{bmatrix}3\\3\end{bmatrix} \quad \begin{bmatrix}1\\6\end{bmatrix} \quad \begin{bmatrix}2\\3\end{bmatrix}$$

$$\mathbf{X_3}: \begin{bmatrix}2\\3\end{bmatrix} \quad \begin{bmatrix}5\\1\end{bmatrix} \quad \begin{bmatrix}3\\1\end{bmatrix} \quad \begin{bmatrix}2\\3\end{bmatrix}$$

a) Break up the observations into mean, treatment and residual components.

b) Compute the Between and Within sums of squares and cross-products.

c) Estimate Wilks' lambda and indicate whether the null hypothesis for MANOVA can be rejected at confidence level $\alpha = 0.01$.

d) If you reject the null hypothesis in (c), determine what groups differ by means of multi-variate post-hoc tests. Hint: do not forget to correct for multiple comparisons.

e) Compute and plot a 99% confidence region for the all pair-wise differences between the population means of the 3 groups. Compare your results to those obtained in (d).

f) Compute 99% confidence intervals for all pair-wise differences between population means of the 3 groups along each dimension.

PROBLEM 7.- Table 4 shows data from a psychological study aimed at improving the social skills of college females and reducing their anxiety. There were 3 groups of subjects in this study: $i)$ control group; $ii)$ behavioral rehearsal and $iii)$ behavioral rehearsal with cognitive restructuring. The table shows 4 of the variables in the original study: anxiety, social skills, appropriateness and assertiveness, all measured in a discrete scale.

a) Is there a significant difference in the measurements of these 3 groups? (test at a significance level $\alpha = 0.05$).

b) In case there is a significant difference between the groups, use multi-variate paired comparisons to determine what groups differ.

c) Compute simultaneous confidence intervals for all group differences in each of the measured variables.

d) Analyze the relation between the results obtained in (a), (b) and (c).

Group 1 - Control											
Variables	Subjects										
	1	2	3	4	5	6	7	8	9	10	11
Anxiety	5	5	4	4	3	4	4	4	5	5	4
Social skills	3	4	5	5	5	5	5	4	4	4	4
Appropriateness	3	4	4	5	5	4	5	4	4	4	4
Assertiveness	3	3	4	4	5	4	5	4	3	3	4

Group 2 - Behavioral rehearsal											
Variables	Subjects										
	12	13	14	15	16	17	18	19	20	21	22
Anxiety	6	6	5	6	4	7	5	5	5	5	6
Social skills	2	2	2	2	4	1	4	2	3	4	2
Appropriateness	1	2	3	2	4	1	3	3	3	3	3
Assertiveness	1	2	3	2	4	1	3	3	3	3	3

Group 3 - Behavioral rehearsal + cognitive restructuring											
Variables	Subjects										
	23	24	25	15	27	28	29	30	31	32	33
Anxiety	4	4	4	4	4	4	4	4	4	5	4
Social skills	4	3	4	5	5	4	5	6	4	3	4
Appropriateness	4	4	4	5	5	4	4	6	4	3	4
Assertiveness	4	3	4	5	5	4	4	5	4	3	4

Table 4: Data for Problem 7

PROBLEM 8.- Consider an example[1] of a cognitive neuroscience study of patient groups, in which 45 subjects (15 amnesiac patients, 15 Huntington patients, and a control group of 15 individuals with no known neurological disorder) were measured in terms of their performance on 3 different tests: (1) artificial grammar task, which consists of classifying letter sequences as either following or not following grammatical rules; (2) classification learning task, which consists of classifying hypothetical patients as either having or not having a certain disease based on symptoms probabilistically related to the disease; and (3) recognition memory task, which consists of recognizing particular stimuli as stimuli that have previously been presented during the task. Table 5 presents the data. Keep in mind that each person has been randomly assigned to one of the three tasks, so there are five observations per cell of the design.

a) Test whether there is a significant interaction between (Diagnosis) and (Task). Clearly state: $i)$ the corresponding null and alternative hypotheses, $ii)$ your conclusion and $iii)$ the relevant numerical details.

b) Based on your results from (a), indicate whether further analysis of these data should be based on assessing *main effects* or *simple effects*. Justify adequately, including any additional analysis that you believe necessary to provide an adequate answer.

c) Proceed to analyze main or simple effects, according to your answer in (b). In all cases provide your conclusions, clearly indicating what they imply stated in plain language.

[1]From Maxwell et al. (2018) Design of Experiments and Analyzing Data

d) For any significant effect identified in (c), provide the corresponding confidence interval (keep in mind that, even though you are testing just a few selected intervals, you must correct the significance level for all possible multiple comparisons).

	Grammar	Classification	Recognition
Amnesiac	44	72	70
	63	66	51
	76	55	82
	72	82	66
	45	75	56
Huntington	24	53	107
	30	59	80
	51	33	98
	55	37	82
	40	43	108
Control	76	92	107
	98	65	80
	71	86	101
	70	67	82
	85	90	105

Table 5: Data for Problem 8

PROBLEM 9.- A certain study aims to compare the effects of two different diets (Diet 1 and Diet 2) in male and female subjects. A total of 88 subjects were measured for overall health in the study using a scale from 1 to 10. Subjects were distributed in groups as follows:

- Diet 1: 22 male and 22 female
- Diet 2: 22 male and 22 female

The average $\pm$ standard deviations for each Diet $\times$ Sex group are as follows:

- Diet 1, Male: 7.09 ± 1.0
- Diet 1, Female: 7.14 ± 1.0
- Diet 2, Male: 6.18 ± 1.05
- Diet 2, Female: 5.27 ± 1.16

In order to perform a 2-way ANOVA, the following sums of squares were obtained:

- SS Diet = 42.28
- SS Sex = 4.01
- SS Interaction 5.12
- SS Residuals 96.03

a) Test whether there is any significant interaction effect (in all cases use $\alpha = 0.05$).

b) If you find that there exist interaction effects, then use Bonferroni correction to perform pair-wise comparisons to determine if the type of interaction allows interpreting the main effects.

c) If either (a) or (b) suggest that main effects can be meaningfully interpreted, answer the following questions:

c1) When we ignore sex, is the difference between diet large enough to be considered significant at $\alpha = 0.05$?

c2) When we ignore diet, is the difference due to sex large enough to be considered significant at $\alpha = 0.05$?

PROBLEM 10.- We wish to assess the effect of two new drugs, which we will call drug A and drug B. To this end, a total of 60 patients are recruited and divided into 3 groups of 20 patients each: the first group receives drug A, the second one receives drug B, and the third one receives a placebo (which we will refer to as drug C). Further, in each group there are the same number of male and female subjects (10), so that possible effects due to sex can also be assessed.

After a pre-established treatment period of 6 months, each patient is assessed by clinical experts who assign to each of them an overall health score (between 0 and 10 points). We wish to answer the following research questions, at significance level $\alpha = 0.05$: on average, are there differences in the health score of individuals depending on the drug that they take and/or their gender?

To facilitate your task, the results of the following calculations are provided:

Sum of squares for *Drug*	19.0
Sum of squares for *Gender*	7.5
Sum of squares for interaction *Drug* $\times$ *Gender*	15.0
Sum of squares residuals	140.0
Sample mean (over all)	7.5
Sample mean per gender (M; F)	$(7.2;\ 7.8)$
Sample mean per drug (A; B; C)	$(8.0;\ 8.5;\ 6.0)$
Sample mean MALE per drug (A; B; C)	$(7.8;\ 8.1;\ 5.7)$
Sample mean FEMALE per drug (A; B; C)	$(8.2;\ 8.9;\ 6.3)$
Some critical values of interest	$\mathrm{F}_{1,54}(0.95) \simeq 4.0$
	$\mathrm{F}_{2,54}(0.95) \simeq 3.2$
	$\mathrm{t}_{54}(0.05/2) \simeq -2.0$
	$\mathrm{t}_{54}(0.05/6) \simeq -2.5$
	$\mathrm{t}_{54}(0.05/30) \simeq -3.0$

a) Indicate what method would you use to answer the research questions stated above. Indicate the assumptions that should be fulfilled (make sure to specify them clearly and adjusted to the data to be used; e.g. statements like "independence" without further clarification will not be accepted). For this problem, you do not need to test whether the assumptions hold; we will assume that they do.

b) Test whether analysis of these data should be based on assessing *main effects* or *simple effects*. Clearly state: $i)$ the corresponding null and alternative hypotheses, $ii)$ the numbers that you need to look at and any further calculations that are necessary, $iii)$ your conclusion.

c) Based on your results from (b), proceed to analyze main[2] or simple effects, In all cases provide your conclusions, clearly indicating what they imply for the two research questions proposed in this Problem.

[2]In case of analyzing main effects, make sure to provide a complete analysis, e.g. including any post-hoc tests that might be necessary to determine what groups differ, if any.

PROBLEM 11.- In a hospital, a gait analysis of osteoarthritic subjects was performed to measure the average time needed for each stride (step) and verify if subjects requiring a total knee replacement (TKR - surgery) were significantly slower than the others (conservative group). Subjects were also recruited based on their gender (male, female) and age (60-67, 68-75).

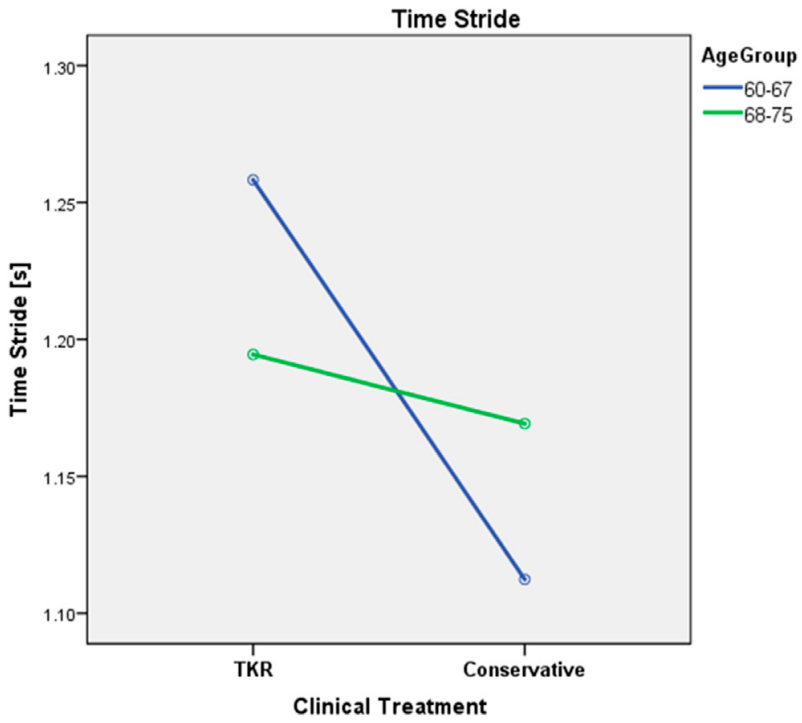
a) What statistical method would you use for this study and why?

b) Which are the dependent and independent variables?

c) What are the hypotheses to test?

d) What are the required assumptions?

e) Suppose now the analysis identified a significant difference between the subject requiring or not TKR, but also a significant interaction between the need for surgery and the age of the patient, as shown in the figure below. Can we accept as significant the difference between the two clinical treatments? Why?



2 Lab Practice Problems

PROBLEM 12.- Data in Table 6 (also in file `SM22_Seminar_3_Fish.mat`) corresponds to an experiment carried out to compare 3 different methods to cook fish. For that purpose, 36 fish samples were randomly split in 3 groups of 12 samples. Samples in each group were cooked according to one of the methods, which we will call A, B, and C.

To compare the 3 methods, judges were asked to taste the cooked f fish samples and rate them in terms of 4 variables: Aroma, Flavor, Texture and Moisture. Table 6 provides these rates. [3]

a) State the null and alternative hypotheses required for the comparison indicated above.

b) Indicate what method would you use to test the hypotheses stated in (a). Indicate what assumptions would be needed in order for your method to be valid. For this problem, you do not need to test whether the assumptions hold; we will assume that they do.

c) Test the hypotheses stated in (a) at significance level $\alpha = 0.05$. Provide all the relevant numerical details and clearly state your conclusion (both in terms of the null and in *plain language*).

d) Compute 95% confidence regions for the average rating differences between the 3 cooking methods. These regions must hold at the specified confidence level simultaneously (e.g. do not forget to correct the significance level for multiple comparisons).

Cooking Method "A"												
Aroma	5.4	5.2	6.1	4.8	5.0	5.7	6.0	4.0	5.7	5.6	5.8	5.3
Flavor	6.0	6.2	5.9	5.0	5.7	6.1	6.0	5.0	5.4	5.2	6.1	5.9
Texture	6.3	6.0	6.0	4.9	5.0	6.0	5.8	4.0	4.9	5.4	5.2	5.8
Moisture	6.7	5.8	7.0	5.0	6.5	6.6	6.0	5.0	5.0	5.8	6.4	6.0
Cooking Method "B"												
Aroma	5.0	4.8	3.9	4.0	5.6	6.0	5.2	5.3	5.9	6.1	6.2	5.1
Flavor	5.3	4.9	4.0	5.1	5.4	5.5	4.8	5.1	6.1	6.0	5.7	4.9
Texture	5.3	4.2	4.4	4.8	5.1	5.7	5.4	5.8	5.7	6.1	5.9	5.3
Moisture	6.5	5.6	5.0	5.8	6.2	6.0	6.0	6.4	6.0	6.2	6.0	4.8
Cooking Method "C"												
Aroma	4.8	5.4	4.9	5.7	4.2	6.0	5.1	4.8	5.3	4.6	4.5	4.4
Flavor	5.0	5.0	5.1	5.2	4.6	5.3	5.2	4.6	5.4	4.4	4.0	4.2
Texture	6.5	6.0	5.9	6.4	5.3	5.8	6.2	5.7	6.8	5.7	5.0	5.6
Moisture	7.0	6.4	6.5	6.4	6.3	6.4	6.5	5.7	6.6	5.6	5.9	5.5

Table 6: Data for Problem 11

[3] Actually, the values indicated in the table correspond to the average rates provided by several judges tasting each sample, but the purpose of our analysis this does not matter.

PROBLEM 13.- File `SM22_Seminar_3_Diet` contains data from a study aimed to compare the effect of 3 different diets from a sample of 76 individuals. Each row of the file corresponds to one individual, and provides the gender, the type of diet (from 1 to 3) and two measurements of weight: one before starting the diet and another one after 6 weeks of diet. For the purpose of this problem we will assume that the data is suitable for analysis under a 2-way factorial ANOVA (i.e. the data fulfills all the assumptions required for such model), and we will use significance level $\alpha = 0.05$:

a) Test whether there is a significant interaction between (type of diet) and (gender). Clearly state: $i)$ the corresponding null and alternative hypotheses, $ii)$ your conclusion and $iii)$ the relevant numerical details.

b) Based on your results from (a), indicate whether further analysis of these data should be based on assessing *main effects* or *simple effects*. Justify adequately, including any additional analysis that you believe necessary to provide an adequate answer.

c) Proceed to analyze main or simple effects, according to your answer in (b). In all cases provide your conclusions, clearly indicating what they imply stated in plain language.

d) For any significant effect identified in (c), provide the corresponding confidence interval (keep in mind that, even though you are testing just a few selected intervals, you must correct the significance level for all possible multiple comparisons).

PROBLEM 14.- In one experiment involving remote sensing, the spectral reflectance of three species of 1-year-old seedlings was measured at various wavelengths during the growing season. The species of seedlings used were sitka spruce (SS), Japanese larch (JL), and lodgepole pine (LP).
Files `SM22_Seminar_3_Remote_??.xslx` provides two of the variables that were measured:

$X_1 =$ percent of spectral reflectance at wavelengt 560 nm (green)

$X_2 =$ percent of spectral reflectance at wavelength 720 nm (near infrared)

where `??` are replaced by the abbreviations provided above for the 3 species of seedlings (SS, JL and LP).

a) Test whether the mean reflectance (bi-dimensional, in terms of the two provided variables) is different between the 3 species of seedlings (at significance level $\alpha = 0.05$). For the purpose of this question, consider that any required assumptions are met (no need to assert them).

b$_1$) In case you find a significant difference in (a), test what groups differ.

b$_2$) In case you do not find a significant difference in (a), construct confidence intervals for each pair of groups along each dimension. The intervals should simultaneously guarantee a FWER (family-wise error rate) of $\alpha = 0.05$

c) Assess whether the data fulfills the required assumptions for the test methodology that you applied in (a). Based on this, indicate whether you believe your results can be considered valid or not (justify).

PROBLEM 15.- In this problem we will analyze continuous evaluation data that corresponds to Statistical Models from 2019 to 2021. To this end, file `SM_SamplesMarks_2019_2021.xlsx` contains data from 78 students. For each of them (corresponding to a row in the referenced file), we have collected:

1. The year in which the student took the course.
2. The marks obtained in each of the 5 labs with deliverables.
3. The resulting Continuous Evaluation mark (which is simply the average of the lab marks).
4. The result of the student in the final exam, as a binary variable ($1 =$ pass).

Students included in the file were selected randomly according to the following criteria:

- For each year, 13 students who passed the exam and 13 students who did not pass de exam were selected.
- Students with a Continuous Evaluation mark below 5.0 were not eligible, since they could not take the final exam.

We wish to assess whether, on average, there is a difference in the Continuous Evaluation mark of students depending on:

- Whether they successfully passed the final exam or not.
- The year in which they took the course.[4]

Because we are using the same dataset for the 3 problems of this lab, we will set our significance level for each problem to $\alpha = 0.017$, which is approximately $0.05/3$.

a) Indicate what method would you use to answer the research questions of this Problem. Indicate the assumptions that should be fulfilled (make sure to specify them clearly and adjusted to the data to be used; e.g. statements like "independence" without further clarification will not be accepted). For this problem, you do not need to test whether the assumptions hold; we will assume that they do.

b) Test whether there is a significant interaction effect. Clearly state: $i)$ the corresponding null and alternative hypotheses, $ii)$ your conclusion and $iii)$ the relevant numerical details.

c) Based on your results from (b), indicate whether further analysis of these data should be based on assessing *main effects* or *simple effects*. Justify adequately, including any additional analysis that you believe necessary to provide an adequate answer.

d) Proceed to analyze main[5] or simple effects, according to your answer in (c). In all cases provide your conclusions, clearly indicating what they imply for the two research questions proposed in this Problem.

[4] This question is important because, in our case, the subject has both been updated every year and has changed some of its teachers.

[5] In case of analyzing main effects, make sure to provide a complete analysis, e.g. including any post-hoc tests that might be necessary to determine what groups differ, if any.

PROBLEM 16.- Using data from Problem 15, we wish to assess whether, on average, there is a difference considering all the 5 Lab Marks of students (not simply their mean), depending on:

- Whether they passed the final exam or not.
- The year in which they took the course.

Because we are using the same dataset for the 3 problems of this lab, we will set our significance level for each problem to $\alpha = 0.017$, which is approximately $0.05/3$.

a) Indicate what method would you use to answer the research questions of this Problem. Indicate the assumptions that should be fulfilled (make sure to specify them clearly and adjusted to the data to be used; e.g. statements like "independence" without further clarification will not be accepted). For this problem, you do not need to test whether the assumptions hold; we will assume that they do.

b) Test whether there is a significant interaction effect. Clearly state: $i)$ the corresponding null and alternative hypotheses, $ii)$ your conclusion and $iii)$ the relevant numerical details.

c) Based on your results from (a), indicate whether further analysis of these data should be based on assessing *main effects* or *simple effects*. Justify adequately, including any additional analysis that you believe necessary to provide an adequate answer.

d) Proceed to analyze main[6] or simple effects, according to your answer in (b). In all cases provide your conclusions, clearly indicating what they imply for the two research questions proposed in this Problem.

[6]In case of analyzing main effects, make sure to provide a complete analysis, e.g. including any post-hoc tests that might be necessary to determine what groups differ, if any.

3 Answers to Selected Problems

PROBLEM 2.-

a) $SS_{Total} = 104.25$

b) $SS_{Total} = 104.25 = SS_B + SS_W = 66.8333 + 37.4167$

c) $\hat{\theta}_F = 8.038 > F_{critic} = 4.257 \rightarrow$ Reject.

d) Using Bonferroni correction:
Drug A not different from Controls: $\hat{\theta}_t = 0.183 < t_{critic} = 2.933$
Drug B different from Controls: $\hat{\theta}_t = 3.5853 > t_{critic} = 2.933$
Drug B different from Drug A: $\hat{\theta}_t = 3.5817 > t_{critic} = 2.933$

e) Bonferroni-corrected intervals:
Mean of drug A - Mean of Controls within $(-3.76\,;\,4.26)$
Mean of drug B - Mean of Controls within $(1.02\,;\,10.2)$
Mean of drug A - Mean of drug B within $(-9.70\,;\,-0.966)$

PROBLEM 3.-

a) The null hypothesis is that the routine does not produce an effect on fitness level; therefore, the means for each time point are the same, $H_0:\ \mu_0 = \mu_3 = \mu_6$. The alternative is simply the negation of H_0 (i.e. there are at least two time points with a significant difference in average fitness level).

b) $\hat{\theta}_F = 13.45 > F_{critic} = 4.1 \Rightarrow\ H_0$ is rejected and there is a significant effect in fitness level due to the time on the routine.

PROBLEM 4.-

a) The null hypothesis is that the routine does not produce an effect on fitness level; therefore, the means for each time point are the same, $H_0:\ \mu_0 = \mu_3 = \mu_6$. The alternative is simply the negation of H_0 (i.e. there are at least two time points with a significant difference in average fitness level).

b) $\hat{\theta}_F = 1.617 < F_{critic} = 3.68 \Rightarrow\ H_0$ is not rejected and our analysis does not suggest a significant effect in fitness level due to the time on the routine, i.e. we cannot reject that the training might not work.

PROBLEM 6.-

b)

$$\mathbf{B} = \begin{bmatrix} 36 & 48 \\ 48 & 84 \end{bmatrix} \qquad \mathbf{W} = \begin{bmatrix} 18 & -13 \\ -13 & 18 \end{bmatrix}$$

c) $\hat{\theta}_\Lambda = 0.0362$
$s = 2, \quad df_1 = 4, \quad df_2 = 16,$
$\hat{\theta}_F = \frac{df_2}{df_1} \times \frac{1-\sqrt{\theta_\Lambda}}{\sqrt{\theta_\Lambda}} = 17.03 > F_{4,16}(0.01) = 4.77$

PROBLEM 7.-

a) Yes; $\hat{\theta}_\Lambda = 0.3691$ which yields $\hat{\theta}_F = 4.3611$, well above the critical value $F_{8,54}(0.95) = 2.1152$.

b) Controls vs Behavioral: $\hat{\theta}_{T^2} = 34.88$, with $T^2_{critic} = \frac{120}{27} F_{4,27}(0.9833) \simeq 16.25 \Rightarrow$ Significant.
Controls vs Behavioral + Cognitive: $\hat{\theta}_{T^2} = 2.69$ (same T^2_{critic} as above) $\Rightarrow$ Not significant.
Behavioral vs Behavioral + Cognitive: $\hat{\theta}_{T^2} = 33.41$ (same T^2_{critic} as above) $\Rightarrow$ Significant.

PROBLEM 9.-

a) There is interaction: $\hat{\theta}_{interaction} = 4.479 > F_{1,84}(0.05) = 3.955$

b) Confidence intervals using Bonferroni correction:
(Diet 1, Male) vs (Diet 1, Fem.) = $(-0.92, \quad +0.82)$
(Diet 2, Male) vs (Diet 2, Fem.) = $(+0.04, \quad +1.78)$
(Diet 1, Male) vs (Diet 2, Male) = $(+0.04, \quad +1.78)$
(Diet 1, Fem.) vs (Diet 2, Fem.) = $(+1.00, \quad +2.74)$
(Diet 1, Male) vs (Diet 2, Fem.) = $(+0.95, \quad +2.69)$
(Diet 1, Fem.) vs (Diet 2, Male) = $(+0.09, \quad +1.83)$

c1) All comparisons of Diet 1 - Diet 2 gave positive confidence intervals, then we can interpret the main effect of diet: $\hat{\theta}_{Diet} = 36.9834 > F_{critic} = 3.955$ and we reject the null that diet has no effect.

c2) We cannot interpret the main effect of Sex regardless of Diet because neither (a) nor (b) support this.