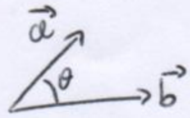
MECCANICA

$\vec{P} = (P-O) = x\hat{i} + y\hat{j} + z\hat{k}$ $\qquad |P| = \sqrt{x^2+y^2+z^2}$



PRODOTTO SCALARE: $S = \vec{a}\cdot\vec{b} = \vec{a}\times\vec{b} = ab\cos(\theta)$

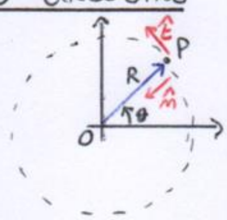
PRODOTTO VETTORIALE: $\vec{c} = \vec{a}\wedge\vec{b} = \vec{a}\times\vec{b} = ab\,\mathrm{sen}(\theta)\left[\hat{a}\wedge\hat{b}\right]$

VERSO CON REGOLA DELLA MANO DX
(PROD. VET. NON È COMMUTATIVO)

TRAIETTORIA: CURVA PERCORSA DAL PUNTO NEL SUO MOTO

SPOSTAMENTO: $\vec{s}$ $\qquad$ VELOCITÀ: $\vec{v} = \dfrac{d\vec{s}}{dt}$ $\qquad$ ACCELERAZIONE: $\vec{a} = \dfrac{d\vec{v}}{dt}$

MOTO CIRCOLARE



$\vec{P} = (P-O) = R\cos(\theta)\hat{i} + R\,\mathrm{sen}(\theta)\hat{j}$

$\vec{v} = R\dot{\theta}\hat{t}$ $\qquad$ se $\dot{\theta} = \omega$: $\vec{v} = R\omega\hat{t}$

$\vec{a} = \underbrace{\dot{\theta}^2 R}_{a_n}\hat{n} + \underbrace{\ddot{\theta}R}_{a_t}\hat{t}$ $\qquad$ se $\ddot{\theta} = \dot{\omega}$: $\vec{a} = \omega^2 R\hat{n} + \dot{\omega}R\hat{t}$

$\left[\vec{\omega} = \omega\hat{k}\right.$ VETTORE VELOCITÀ ANGOLARE $\left.\right]$

RELAZIONI TRIGONOMETRICHE UTILI

$\cos\left(\theta + \frac{\pi}{2}\right) = -\mathrm{sen}\,\theta$ $\qquad$ $\cos(\theta+\pi) = -\cos\theta$

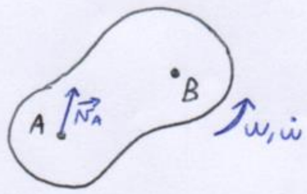
$\mathrm{sen}\left(\theta + \frac{\pi}{2}\right) = \cos\theta$ $\qquad$ $\mathrm{sen}(\theta+\pi) = -\mathrm{sen}\,\theta$

TEOREMA DI RIVALS

$\vec{v}_B = \vec{v}_A + \vec{\omega}\wedge(B-A)$

$\vec{a}_B = \vec{a}_A + \dot{\vec{\omega}}\wedge(B-A) + \vec{\omega}\wedge\left[\vec{\omega}\wedge(B-A)\right]$

SE MOTO PIANO: $\vec{a}_B = \vec{a}_A + \dot{\vec{\omega}}\wedge(B-A) - \omega^2(B-A)$



PURO ROTOLAMENTO

$C \equiv$ C.I.R. CENTRO DI ISTANTANEA ROTAZIONE

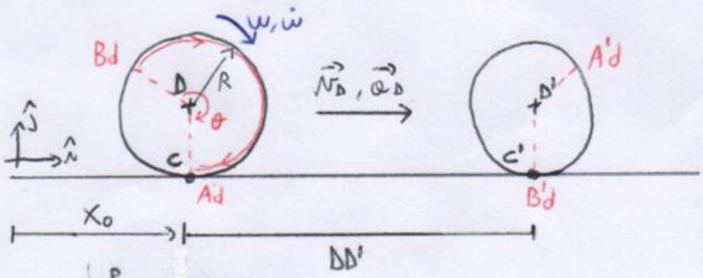
UN SOLO gdl RESIDUO: θ

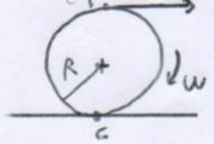
$DD' = x = \theta R$

SPOSTAMENTO: $\begin{cases} x_D = x_0 + \theta R \\ y_D = R \end{cases}$

VELOCITÀ: $\begin{cases} \dot{x}_D = \dot{\theta}R = \omega R \\ \dot{y}_D = 0 \end{cases}$ $\qquad \vec{v}_D = R\omega\hat{i}$

ACCELERAZIONE: $\begin{cases} \ddot{x}_D = \ddot{\theta}R = \dot{\omega}R \\ \ddot{y}_D = 0 \end{cases}$ $\qquad \vec{a}_D = R\dot{\omega}\hat{i}$





$v_P = 2\omega R$

TEOREMA DEI MOTI RELATIVI

$(P-O) = (P-O_1) + (O_1-O)$

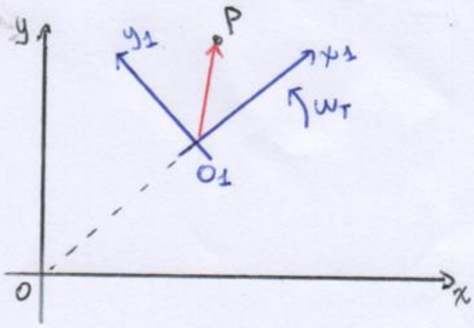
$\vec{v}_P = \vec{v}_{Tr,P} + \vec{v}_{Rel,P}$

DOVE: $\vec{v}_{Tr,P} = \vec{v}_{O1} + \vec{\omega}_T\wedge(P-O_1)$

$\vec{a}_P = \vec{a}_{Tr,P} + \vec{a}_{Rel,P} + \vec{a}_{Cor,P}$

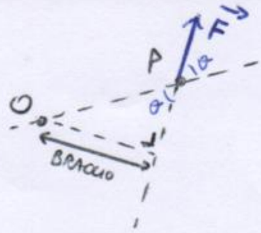
DOVE: $\vec{a}_{Tr,P} = \vec{a}_{O1} + \dot{\vec{\omega}}_T\wedge(P-O_1) - \omega_T^2(P-O_1)$

$\vec{a}_{Cor,P} = 2\vec{\omega}_T\wedge\vec{v}_{Rel,P}$

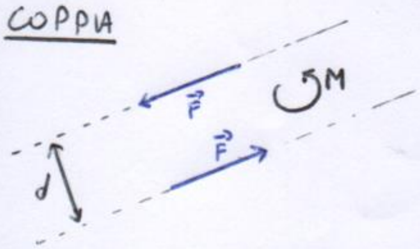


MOMENTO DI UNA FORZA: $M_o = (P-O) \wedge \vec{F}$

$$\begin{cases} |M_o| = \overline{PO}\,|F|\,sen(\theta) \\ \text{DIR. E VERSO CON REGOLA MANO DX} \end{cases}$$



COPPIA



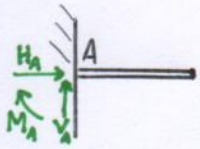
MOMENTO RISULTANTE DALLA COPPIA:

$$M = F \cdot d$$

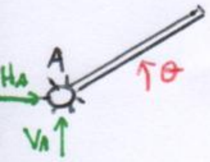
EQUILIBRIO DI UN CORPO RIGIDO NEL PIANO

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_o = 0 \end{cases}$$

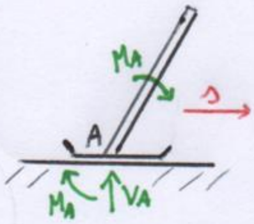
VINCOLI



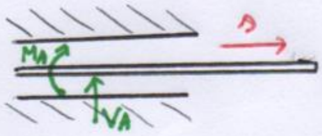
INCASTRO
$\emptyset$ gdl



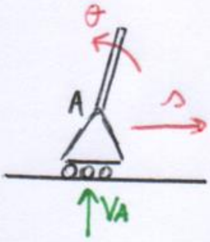
CERNIERA
1 gdl: θ



PATTINO
1 gdl: s



MANICOTTO
1 gdl: s



CARRELLO
2 gdl: s, θ

REGOLA DI GRÜBLER:

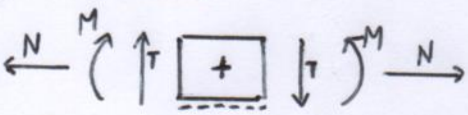
$$\#gdl = \#gdl_{\text{SENZA VINCOLI}} - \#gdv_{\text{DATI DAI VINCOLI}}$$

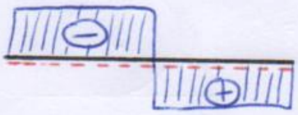
$$\#gdl = 3 \cdot n_{\text{CORPI RIGIDI}} - (1 \cdot n_{\text{CARRELLI}} + 2 \cdot n_{\text{CERNIERE PATTINI MANICOTTI}} + 3 \cdot n_{\text{INCASTRI}})$$

STRUTTURE ISOSTATICHE: $\#gdl = \#gdv$ (IPERSTATICHE: $\#gdl < \#gdv$)

MECCANISMI: $\#gdl > \#gdv$

AZIONI INTERNE



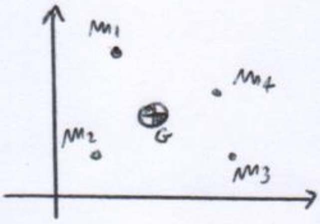


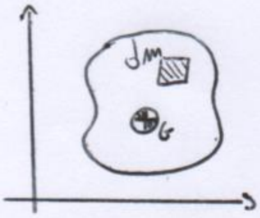
BARICENTRO

$$\begin{cases} X_G = \dfrac{\Sigma_i x_i m_i}{m_{TOT}} \\ y_G = \dfrac{\Sigma_i y_i m_i}{m_{TOT}} \end{cases} \qquad \begin{cases} X_G = \dfrac{\rho h}{m_{TOT}} \displaystyle\int_A x\, dA \\ y_G = \dfrac{\rho h}{m_{TOT}} \displaystyle\int_A y\, dA \end{cases}$$

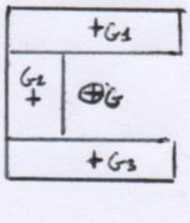
- SE IL CORPO HA UN ASSE DI SIMMETRIA O PIÙ ASSI, IL BARICENTRO STA SU DI ESSI
- IL BARICENTRO DI UN CORPO COMPLESSO PUÒ ESSERE RICAVATO DAI BARICENTRI DEI CORPI IN CUI SI SCOMPONE

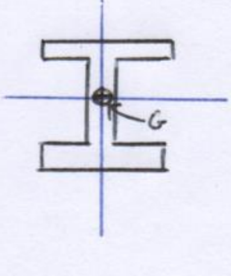
$$X_G = \frac{\Sigma_i X_{Gi} m_i}{m_{TOT}} \qquad y_G = \frac{\Sigma_i y_{Gi} m_i}{m_{TOT}}$$





(OMOGENEO) $dV = h\,dA$

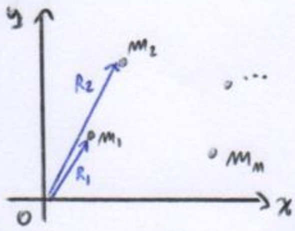




SE NON OMOGENEO:

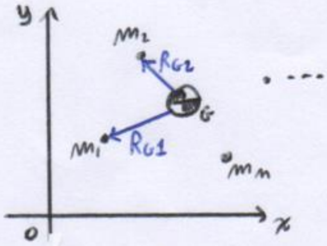
$$\begin{cases} X_G = \dfrac{1}{m} \displaystyle\int_V \rho(x,y,z)\, x\, dV \\ y_G = \dfrac{1}{m} \displaystyle\int_V \rho(x,y,z)\, y\, dV \end{cases}$$

MOMENTO D'INERZIA



$J_O = \sum^{m} m_i R_i^2$ MOMENTO D'INERZIA RISPETTO A O

$J_O = \int_V \rho(x,y,z) R^2 dV \underset{\text{OMOGENEO}}{=} \rho h \int_A R^2 dA$

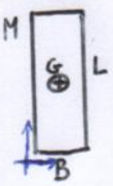


$J_G = \sum^{m} m_i R_{Gi}^2$ MOMENTO D'INERZIA BARICENTRICO

$J_G \underset{\text{OMOGENEO}}{=} \rho h \int_A R_G^2 dA = \rho h \int (x^2+y^2) dA$

TEOREMA DI HUYGENS: $J_O = J_G + m\,\overline{OG}^2$

ASTA / TRAVE



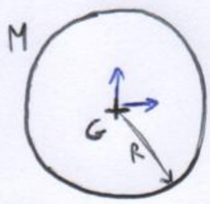
$\begin{cases} x_G = \frac{B}{2} \\ y_G = \frac{L}{2} \end{cases}$

$J_G = \frac{\rho h L B}{12}(B^2+L^2)$ $(M = \rho h L B)$

SE SOTTILE: $(L >> B)$

$J_G = \frac{ML^2}{12}$ (TRAVE "SNELLA")

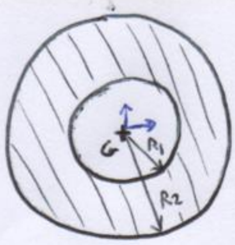
DISCO



$\begin{cases} x_G = 0 \\ y_G = 0 \end{cases}$

$J_G = \frac{1}{2} M R^2$

CORONA CIRCOLARE



$\begin{cases} x_G = 0 \\ y_G = 0 \end{cases}$

$J_G = \frac{1}{2} M (R_1^2 + R_2^2)$

PRINCIPIO DI D'ALEMBERT

FORZA D'INERZIA: $\vec{F}_{IN} = -m\,\vec{a}_G$

COPPIA D'INERZIA: $\vec{C}_{IN} = -J_G\,\dot{\vec{\omega}}$

EQUAZIONI DI EQUILIBRIO DINAMICO O PRINCIPIO DI D'ALEMBERT:

$$\begin{cases} \sum_j \vec{F}_j + \vec{F}_{IN} = 0 \\ \sum_j \left[(P_j - O) \wedge \vec{F}_j\right] + \sum_i \vec{C}_i + (G-O)\wedge \vec{F}_{IN} + \vec{C}_{IN} = 0 \end{cases}$$

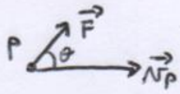
(IN ROSSO I TERMINI INERZIALI)

BILANCIO DI POTENZE

$$\underbrace{\sum \vec{F}_i \cdot \vec{v}_i + \sum \vec{C}_j \cdot \vec{\omega}_j}_{\text{POTENZE DI FORZE E COPPIE ATTIVE}} + \underbrace{\sum \vec{F}_{IN,K} \cdot \vec{v}_{G,K}}_{\text{POTENZA DELLE FORZE D'INERZIA}} + \underbrace{\sum \vec{C}_{IN,K} \cdot \vec{\omega}_K}_{\text{POTENZA DELLE COPPIE D'INERZIA}} = 0$$

POTENZA DI UNA FORZA: $W = \vec{F} \cdot \vec{v}_P$

$W = Fv\cos\theta$



POTENZA DI UNA COPPIA: $W = \vec{C} \cdot \vec{\omega}$

ENERGIA CINETICA DEL PUNTO: $E_c = \frac{1}{2} m \vec{v}_P \cdot \vec{v}_P = \frac{1}{2} m v_P^2$

ENERGIA CINETICA DEL CORPO RIGIDO: $E_c = \frac{1}{2} M \vec{v}_G \cdot \vec{v}_G + \frac{1}{2} J_G \vec{\omega} \cdot \vec{\omega}$
(II Th. DI KÖNING)

$$E_c = \frac{1}{2} M v_G^2 + \frac{1}{2} J_G \omega^2$$

TEOREMA DELL'ENERGIA CINETICA

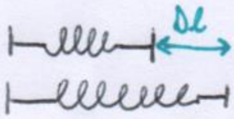
$$\frac{dE_c}{dt} = \sum W_{i,ATTIVE}$$

Hp: VINCOLI FISSI, LISCI E BILATERI

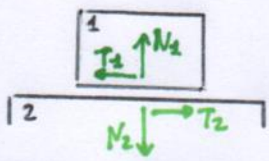
$$\frac{dE_c}{dt} = \underbrace{\sum \vec{F}_i \cdot \vec{v}_i + \sum \vec{C}_j \cdot \vec{\omega}_j}_{\text{POTENZA DI FORZE E COPPIE ATTIVE}}$$

LEGGE DI HOOK

$F_e = k \cdot \Delta l$



ATTRITO STATICO

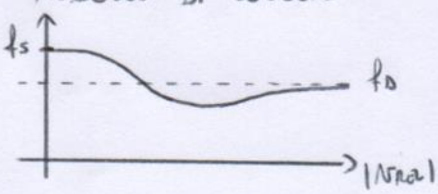


$v_{REL,1-2} = 0$

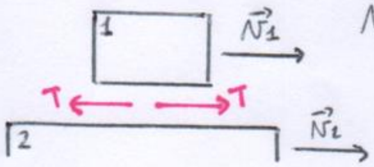
CONDIZIONE DI ADERENZA:

$$|T| \leq f_s |N|$$

MODELLO DI COULOMB:



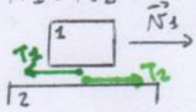
ATTRITO DINAMICO



$v_{REL,1-2} \neq 0$

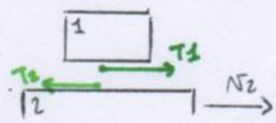
$$\vec{T} = -f_D \vec{N} \frac{\vec{v}_{12}}{|v_{12}|} = f_D \vec{N} \frac{\vec{v}_{21}}{|v_{21}|}$$

- $v_1 > v_2$



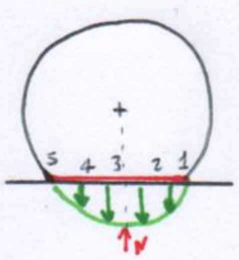
IL CORPO PIÙ LENTO "FRENA" IL PIÙ VELOCE

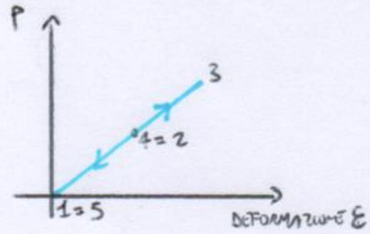
- $v_1 < v_2$



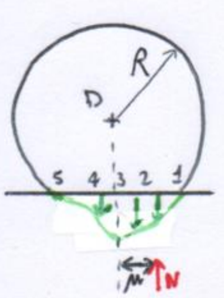
$$W = \vec{T}_1 \cdot \vec{v}_1 + \vec{T}_2 \cdot \vec{v}_2 = -|T| \cdot |v_{12}|$$

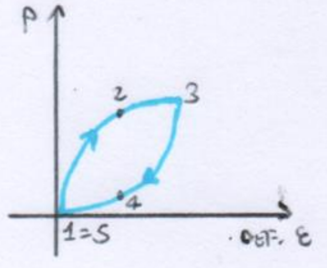
ATTRITO VOLVENTE (RESISTENZA A ROTOLAMENTO)





DISTRIBUZIONE DI PRESSIONI SIMMETRICA

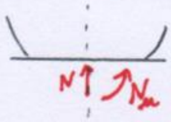




DISTRIBUZIONE DI PRESSIONI NEL CASO REALE (MATERIALE NON PERFETTAMENTE ELASTICO)

AREA TRA LE CURVE È ENERGIA DISSIPATA

SISTEMA EQUIPOLLENTE:



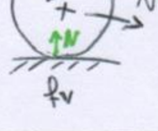
N_μ = MOMENTO DI TRASPORTO

$$N = \int_S P \, ds \qquad \mu = f_v \cdot R$$

$$W = -|N_\mu| \cdot |\omega|$$

$$W = -|N| \cdot |\omega| \cdot f_v \cdot R$$

$$W = -N f_v v_D$$



$\vec{v}_D = \omega R \hat{i}$

MOTORE - TRASMISSIONE - UTILIZZATORE

$$W_M + W_u + W_P = \left.\frac{dE_c}{dt}\right|_{MOTORE} + \left.\frac{dE_c}{dt}\right|_{UTILIZZATORE}$$

MOTORE:

$$W_m = \vec{C}_m \cdot \vec{\omega}_m \qquad \text{OPPURE} \qquad W_m = \vec{M}^*_m \cdot \vec{\omega}_m = \sum \vec{F}_i \cdot \vec{v}_{Pi} + \sum \vec{C}_j \cdot \vec{\omega}_j$$

$$E_{c,m} = \frac{1}{2} J^*_m \omega_m^2 = \sum \frac{1}{2} m_i v_{Gi}^2 + \sum \frac{1}{2} J_{Gi} \omega_i^2$$

$$\left.\frac{dE_c}{dt}\right|_m = J^*_m \dot{\omega}_m \omega_m$$

UTILIZZATORE:

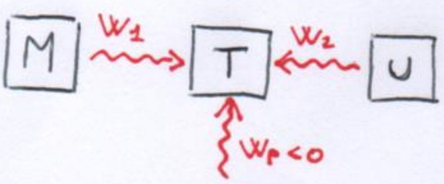
$$W_u = \sum \vec{F}_i \cdot \vec{v}_{Pi} + \sum \vec{C}_j \cdot \vec{\omega}_j = \vec{M}^*_u \cdot \vec{\omega}_u$$

$$E_{c,u} = \sum \frac{1}{2} m_{Gi} v_{Gi}^2 + \sum \frac{1}{2} J_{Gj} \omega_j^2 = \frac{1}{2} J^*_u \omega_u^2$$

$$\left.\frac{dE_c}{dt}\right|_u = J^*_u \dot{\omega}_u \omega_u$$

TRASMISSIONE:

$$\tau = \frac{\omega_u}{\omega_m} \longrightarrow \omega_u = \tau \omega_m \qquad (\tau < 1)$$



$$W_1 + W_2 + W_P = \left.\frac{dE_c}{dt}\right|_T = \emptyset$$

$$W_1 = W_m - \left.\frac{dE_c}{dt}\right|_{MOT.} \qquad W_2 = W_u - \left.\frac{dE_c}{dt}\right|_{UT.} \qquad \eta = \frac{W_{OUT}}{W_{IN}}$$

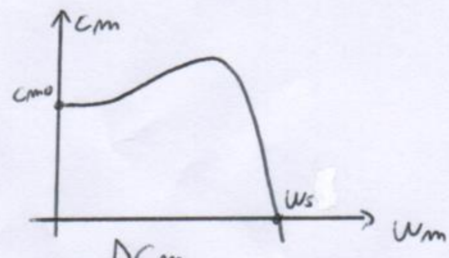
MOTO DIRETTO: $W_1 > 0,\ W_2 < 0 \longrightarrow W_P = -(1 - \eta_D) W_1$

MOTO RETROGRADO: $W_1 < 0,\ W_2 > 0 \longrightarrow W_P = -(1 - \eta_R) W_2$

SPUNTO: $C_m = C_{m0}$; $\left.\frac{dE_c}{dt}\right|_{UT.} > 0$

REGIME: $\dot{\omega}_m = 0$; $\omega_m = \text{cost.}$; $\left.\frac{dE_c}{dt}\right|_{MOT.} = 0$; $\left.\frac{dE_c}{dt}\right|_{UT.} = 0$

MOTO VARIO: $W_m + W_u + W_P = \frac{dE_c}{dt}$



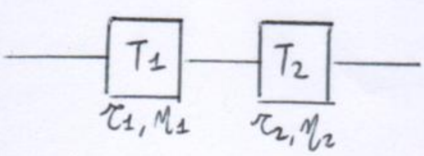
$$s = \frac{\Delta C_m}{\Delta \omega_m}$$

SE SONO VICINO A ω_s:

$$s = \frac{\bar{C}_m - 0}{\bar{\omega}_m - \omega_s}$$

SE NON CONOSCO $\dot{\omega}_m$ E QUINDI NON POSSO STUDIARE IL SEGNO DI W_1 E W_2 PER CAPIRE SE IL MOTO È DIRETTO O RETROGRADO, FACCIO UN'IPOTESI DI TIPO DI MOTO, CALCOLO $\dot{\omega}_m$ E QUINDI VERIFICO L'IPOTESI TROVANDO ANCHE W_1 O W_2

TRASMISSIONI IN SERIE



$$\begin{cases} \tau = \tau_1 \cdot \tau_2 \cdot \ldots \cdot \tau_n \\ \eta = \eta_1 \cdot \eta_2 \cdot \ldots \cdot \eta_n \end{cases}$$