COMPUTING INFRASTRUCTURES

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Dependability

$R(t) = P(\text{not failed during } [0,t])$ Reliability

$Q(t) = 1 - R(t)$ Unreliability

$A(t) = P(\text{not failed at time t})$ Availability Availability = Uptime / (Uptime + Downtime)

$1 - A(t)$ Unavailability

$MTTF$ Mean Time To Failure $MTTF = \int_0^\infty R(t)dt$

$MTBF$ Mean Time Between Failures MTBF=total operating time / number of failures

λ Failure Rate $\lambda =$ number of failures / total operating time

$$MTBF = {}^1/_\lambda$$

$$R(t) = e^{-\lambda t}$$

Series: $R_s(t) = R_{C1}(t) * R_{C2}(t)$ $R_s(t) = e^{-\lambda_s t}$ where $\lambda_s = \sum_{i=1}^{n} \lambda_i$

$R_s(t) = \prod_{i=1}^{n} R_i$

Parallel: $R_p(t) = 1 - [1 - R_{C1}(t)][1 - R_{C2}(t)] = R_{C1}(t) + R_{C2}(t) - R_{C1}(t)R_{C2}(t)$

$R_p(t) = 1 - \prod_{i=1}^{n}[1 - R_i(t)]$

Series: $MTTF_s = \frac{1}{\lambda_s} = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} \frac{1}{MMT_i}}$

Parallel: $MTTF_p = MTTF_A + MTTF_B - \frac{1}{\frac{1}{MTTF_A} + \frac{1}{MTTF_B}}$ if without repair, otherwise use Availability definition

$MTTR_p = \frac{1}{\sum_{i=1}^{n} \frac{1}{MMTTR_i}}$; if components all have the same MTTR: $MTTR_p = \frac{MTTR}{\#ComponentsInParallel}$

Series: $A_s = \prod_{i=1}^{n} \frac{MTTF_i}{MTTF_i + MTTR_i}$

Parallel: $A_p(t) = 1 - \prod_{i=1}^{n}[1 - A_i(t)]$

r out of n redundancy (RooN): $R_{System} = R_{Voter} \sum_{i=r}^{n} \left[{R_C}^i (1 - R_c)^{n-i} \binom{n}{i} \right]$

Where n is the total number of components and r is the minimum number of components which must survive.

Disk Time

Service Time: $T_{I/O} = T_{seek} + T_{rotation} + T_{transfer} + T_{overhead}$

Response Time: $T_{queue} + T_{I/O}$

$T_{rotation} = \frac{60*1000}{2*RPM} [ms]$

$T_{transfer} = \frac{BlockSize*1000}{TransferRate} [ms]$ where *BlockSize* and *TransferRate* have the same unit of measurement (multiply *1024)

Service Time considering Data Locality: $T_{I/O} = (1 - DL) * (T_{seek} + T_{rotation}) + T_{transfer} + T_{overhead}$

RAID

N the number of available disks

G the number of disks in a stripe of RAID 0 disks

$MTTR$ the Mean Time To Repair of one single disk

RAID 0: $SC = N\ disks$ $MTTF_{RAID0} = \frac{MTTF_{OneDisk}}{N}$

RAID 1: $SC = 1\ disks$ $MTTF_{RAID1} = \frac{{MTTF_{OneDisk}}^2}{N*MTTR}$

RAID 1+0: $SC = {}^{N}/_{2}\ disks$ $MTTF_{RAID1+0} = \frac{{MTTF_{OneDisk}}^2}{N*MTTR}$

RAID 0+1: $SC = {}^{N}/_{2}\ disks$ $MTTF_{RAID0+1} = \frac{{MTTF_{OneDisk}}^2}{N*G*MTTR}$

$MTTF_{RAID0+1} = \frac{{MTTF_{OneDisk}}^2*2}{N^2*MTTR}$ Form of the formula to use to find a needed N to ensure something

RAID 4: $SC = N - 1\ disks$ $MTTF_{RAID4} = \frac{{MTTF_{OneDisk}}^2}{N*(N-1)*MTTR}$

RAID 5: $SC = N - 1\ disks$ $MTTF_{RAI} = \frac{{MTTF_{OneDisk}}^2}{N*(N-1)*MTTR}$

RAID 6: $SC = N - 2\ disks$ $MTTF_{RAID} = \frac{2*{MTTF_{OneDisk}}^3}{N*(N-1)*(N-2)*MTTR^2}$

Performance

T the length of time we observe the system

A_k the number of request arrivals

C_k the number of request completions

B_k the amount of busy time of the resource ($B < T$)

N_k the average number of jobs in the resource (queueing + being served)

Z the think time

$\lambda_k = {}^{A_k}/_{T}$ the arrival rate

$X_k = {}^{C_k}/_{T}$ the throughput

$U_k = {}^{B_k}/_{T}$ the utilization

$S_k = {}^{B_k}/_{C_k}$ the mean service time per completed job

Utilization Law: $U_k = X_k S_k$

Little's Law: $N = XR$

Response Time Law: $R = {}^{N}/_{X} - Z$

$V_k = {}^{C_k}/_{C}$ the visit count

Forced Flow Law: $X_k = V_k X$

$D_k = S_k V_k = {}^{B_k}/_{C}$ the service demand

Utilization Law: $U_k = D_k X$

R_k the residence time

$\widetilde{R_k}$ the response time

$R_k = V_k \widetilde{R_k}$

General Response Time Law: $R = \sum_k V_k \widetilde{R_k} = \sum_k R_k$

$N = \sum_k N_k$

W_k the total accumulated time in the system

$W = NT$ $W_k = R_k - D_k$ $\widetilde{W_k} = \widetilde{R_k} - S_k$ $Q_k = N_k - U_k$

$D = \sum_k D_k$ $D_{max} = max_k(D_k)$

Open Model:

$\lambda_{sat} = {}^{1}/_{D_{max}}$

$X(\lambda) \leq {}^{1}/_{D_{max}}$ $R(\lambda) \geq D$

Closed Model:

$N^* = \frac{D+Z}{D_{max}}$ Population size determining if the light or the heavy-load optimistic bound is to be applied

Batch: $\frac{1}{D} \leq X(N) \leq \min\left(\frac{N}{D}\ ;\ \frac{1}{D_{max}}\right)$

Terminal: $\frac{N}{ND+Z} \leq X(N) \leq \min\left(\frac{N}{D+Z}\ ;\ \frac{1}{D_{max}}\right)$

Transaction: $X(\lambda) \leq \frac{1}{D_{max}}$

Batch: $\max(D\ ; ND_{max}) \leq R(N) \leq ND$

Terminal: $\max(D\ ; ND_{max} - Z) \leq R(N) \leq ND$

Transaction: $D \leq R(\lambda)$

$R_k = \frac{D_k}{1-U_k} = \frac{D_k}{1-\lambda D_k}$ $R = \frac{D_{CPU}}{1-D_{CPU}} + n \frac{{}^{D_{Disk}}/_{n}}{1-\frac{\lambda D_{Disk}}{n}}$