$[a_{ij}]$ adjacency matrix

$[d_{ij}]$ shortest path matrix

$[\varepsilon_{ij}]$ efficiency matrix $\qquad \varepsilon_{ij} = \frac{1}{d_{ij}}$

CHARACTERISTIC PATH LENGTH $\qquad L = \frac{\sum_{i \neq j} d_{ij}}{N(N-1)}$

NETWORK DIAMETER $\qquad d = \max d_{ij}$

CLUSTERING COEFFICIENT $\qquad C_i = \frac{\#\text{ of edges connecting the neighbours of } i \text{ (between them)}}{\binom{\#\text{ of neighbours}}{2}}$

AVERAGE CLUSTERING COEFF. $\qquad C = \frac{1}{N} \sum_i C_i$

GLOBAL EFFICIENCY $\qquad E[G] = \frac{\sum_{i \neq j} \varepsilon_{ij}}{N(N-1)}$

VULNERABILITY INDEX $\qquad V^* = \frac{E_{global}[G] - E_{global}[G^*]}{E_{global}[G]}$

G^* is the graph after the disconnection of one or more links

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Centrality measures:

$N = \#$ nodes

TOPOLOGICAL DEGREE CENTRALITY $\qquad C_i^D = \frac{\sum a_{ij}}{N-1}$

Note i is fixed!

TOPOLOGICAL CLOSENESS CENTRALITY $\qquad C_i^C = \frac{N-1}{\sum d_{ij}}$

TOPOLOGICAL BETWEENNES CENTRALITY $\qquad C_i^B = \frac{\sum \frac{m_{jk}(i)}{m_{jk}}}{(N-1)(N-2)}$

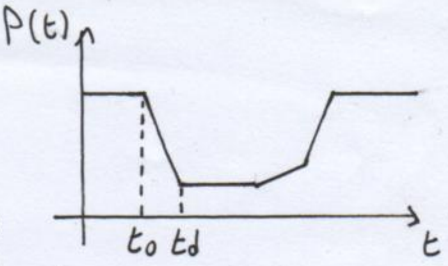
$m_{jk}(i) = \#$ of shortest paths from j to k which pass through i

$m_{jk} = \#$ of shortest paths from j to k

TOPOLOGICAL INFORMATION CENTRALITY $\qquad C_i^I = \frac{E[G] - E[G^*(i)]}{E[G]}$

$G^*(i)$ is the graph after the disconnection of node i

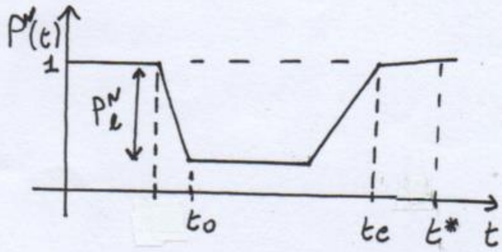
Resilience metrics:



HENRY $$R(t) = \frac{P(t) - P(t_d)}{P(t_0) - P(t_d)} \qquad (t \geq t_d)$$

t_0 = time when the external disruptive event occurs

t_d = time when the performance reaches its lowest level



ZOBEL $$R = \frac{T^* - P_l^N \cdot \frac{T}{2}}{T^*} = 1 - \frac{P_l^N T}{2T^*}$$

P_l^N = Loss of normalized system performance after a disruption

t_0 = instant when $P(t)$ reaches its minimum

t_e = instant when $P(t)$ returns to original level

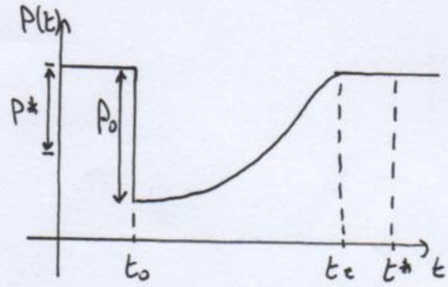
t^* = (given) strict upper bound for t_e

$T = t_e - t_0 \qquad T^* = t^* - t_0$

BRUNEAU $$R = \int_{t_{0E}}^{t_{0E}+T_{RE}} \frac{P^N(t)}{T_{RE}}\, dt$$

t_{0E} = time when $P(t)$ reaches its minimum

T_{RE} = total recovery time



CHANG R = Probability ($P_0 < P^*$ and $t_e < t^*$)

P_0 = initial $P(t)$ loss after disruption

P^* = maximum acceptable loss of $P(t)$

t_e = time when $P(t)$ returns to original level

t^* = maximum acceptable system recovery time