Name: Luca Guiga **Last 4 ID:** 4136 **CSE 100**

HW 3 **Due: Wednesday, January 28th, at 11:59 PM in Santa Cruz.** **Winter 2026**

Reading: Sections 3.3-3.4 and 4.1-4.7.

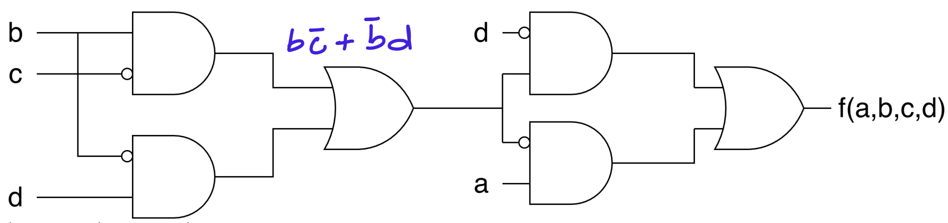
Instructions: Complete the problems below **in your own handwriting**. Box your answers where necessary. The grader will award 1 point for your name and the last four digits of your student ID, and 1 point for neatness.

Note: Σ –> Sum (capital sigma), Π –> Product (capital pi)

Problem 1

For the circuit below (6 points):

- **Use a truth table and a K-map to obtain a minimal Sum-of-Products expression** for the boolean function implemented by the logic network below.
- Compare the gate count and number of gate inputs (the count of all gates and the number of inputs for each gate) in your minimal SoP expression with those for the network below.



a	b	c	d		f
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	0
0	0	1	1	3	1
0	1	0	0	4	0
0	1	0	1	5	1
0	1	1	0	6	0
0	1	1	1	7	0
1	0	0	0	8	1
1	0	0	1	9	1
1	0	1	0	10	1
1	0	1	1	11	1
1	1	0	0	12	0
1	1	0	1	13	1
1	1	1	0	14	1
1	1	1	1	15	1

$$\bar{d}(b\bar{c} + \bar{b}d) + a\overline{(b\bar{c} + \bar{b}d)} = f()$$

cd \ ab	00	01	11	10
00	0: 0	4: 0	12: 0	8: 1
01	1: 1	5: 1	13: 0	9: 1
11	3: 1	7: 0	15: 1	11: 1
10	2: 0	6: 0	14: 1	10: 1

$a(\bar{b}+c)(b+\bar{d})$

a	b	c	d		f
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	0
0	0	1	1	3	1
0	1	0	0	4	0
0	1	0	1	5	1
0	1	1	0	6	0
0	1	1	1	7	0
1	0	0	0	8	1
1	0	0	1	9	1
1	0	1	0	10	1
1	0	1	1	11	1
1	1	0	0	12	0
1	1	0	1	13	1
1	1	1	0	14	1
1	1	1	1	15	1

minimal SOP

$\sum m_i(1,3,5,8,9,10,11,13,14,15)$

1 $\bar a \bar b \bar c d$ +
2 $\bar a \bar b c d$ +
3 $\bar a b \bar c d$ +
4 $a \bar b \bar c \bar d$ +
5 $a \bar b \bar c d$ +
6 $a \bar b c \bar d$ +
7 $a \bar b c d$ +
8 $a b \bar c d$ +
9 $a b c \bar d$ +
10 $a b c d$

→ 10 $\not{0}$ gates w\ ~~9 or gates~~ 1 or gate
w\ 4 inputs
40 pins — 10 pins
4 inverters

total pinout = 40+10+4 = 54

total gates = 10+1+4 = 15

$d(b\bar c + \bar b d) + a\overline{(b\bar c + \bar b d)} = f()$ 0000

0: $0(01+10) = 0(0+0) = 0 \not{+} 0$

$a(\overline{b\bar c}\,\overline{\bar b d}) \rightarrow a(\bar b + \bar{\bar c})(\bar{\bar b} + \bar d) \rightarrow a(\bar b + c)(b + \bar d)$

So $0(1+0)(0+1)$ $0 \not{+} 1 \not{+} 1 = 0$ ← If A=0 rightside fails

1: 0001 abcd → $d(b\bar c + \bar b d)$ $a(\bar b + c)(b + \bar d)$
$1(01 + 11)$ $0(1+0)(0+0)$
$1(1) = 1 + 0 = 1$ $0(1)(0)$ $0 \not{+} 1 \not{+} 0 = 0$

2: 0010 abcd → $d(b\bar c + \bar b d) + \cancel{a(\bar b + c)(b + \bar d)}$
0 0 0

3: 0011 abcd → $d(b\bar c + \bar b d) + \cancel{a(\bar b + c)(b + \bar d)}$
1 0 1 0 1 0
$1(00+11)$ $1(1) = 1 + 0 = 1$

4: 0100 abcd → $\cancel{d(b\bar c + \bar b d)} + \cancel{a(\bar b + c)(b + \bar d)}$
0 0

5: 0101 abcd → $d(b\bar c + \bar b d) + \cancel{a(\bar b + c)(b + \bar d)}$
1 11 0 0

7: 0111 abcd → $d(b\bar c + \bar b d) + \cancel{a(\bar b + c)(b + \bar d)}$
$1(10 + 01) = 0$

8: 1000 abcd → $\cancel{d(b\bar c + \bar b d)} + a(\bar b + c)(b + \bar d)$
0 $1(1+0)(0+1)$
$1(1)(1) = 1$

9: 1001 abcd → $d(b\bar c + \bar b d) + a(\bar b + c)(b + \bar d)$
1 01 11 ~~doesn't matter~~
1(1)

10: 1010 abcd → $\cancel{d(b\bar c + \bar b d)} + a(\bar b + c)(b + \bar d)$
0 $1(1+1)(0+1)$
$1(1)(1) = 1$

11: 1011 abcd → $d(b\bar c + \bar b d) + a(\bar b + c)(b + \bar d)$
1 00 11

12: 1100 abcd → $\cancel{d(b\bar c + \bar b d)} + a(\bar b + c)(b + \bar d)$
1 0 0 1 0

13: 1101 abcd → $d(b\bar c + \bar b d) + a(\bar b + c)(b + \bar d)$
1 11 0

14: 1110 abcd → $\cancel{d(b\bar c + \bar b d)} + a(\bar b + c)(b + \bar d)$
1 $1(0+1)(1+1)$
$1(1)(1) = 1$

15: 1111 abcd → $\cancel{d(b\bar c + \bar b d)} + a(\bar b + c)(b + \bar d)$
1 $1(10+01)$ $1(0+1)(1+0)$
$1(0+0)$ $1(1)(1) = 1$

Minimal SoP: $f(a,b,c,d) = \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d + a\bar{b}\bar{c}\bar{d} + a\bar{b}\bar{c}d + a\bar{b}c\bar{d} + a\bar{b}cd + ab\bar{c}d + abc\bar{d} + abcd$

Original gate count: 6   Original # of gate inputs: 15

Minimal SoP gate count: 15   Minimal SoP # of gate inputs: 54

Problem 2

For each of the functions below:

- Find and **circle all** prime implicants and identify which ones are forced.
- Use Karnaugh Maps to obtain the minimal boolean expression in **both** Sum-of-Products (SoP) and Product-of-Sums (PoS) forms. Draw separate K-maps for the SoP and PoS.
- Finally, give the number of literals for your minimal SOP and POS expressions.

(a) (9 points): $f(x,y,z,w) = \Sigma m(2,3,4,6,7,13,15) + DON'T\_CARE(5,10,11)$

zw \ xy	00	01	11	10
00	0: 0	4: 1	12: 0	8: 0
01	1: 0	5: X	13: 1	9: 0
11	3: 1	7: 1	15: 1	11: X
10	2: 1	6: 1	14: 0	10: X

zw \ xy	00	01	11	10
00	0: 0	4: 1	12: 0	8: 0
01	1: 0	5: X	13: 1	9: 0
11	3: 1	7: 1	15: 1	11: X
10	2: 1	6: 1	14: 0	10: X

PIs: 4|5, 4|6, 5|13|7|15, 3|7|2|6, 3|2|11|10, 15|11,

Forced PIs: 4|6, 5|13|7|15, 11|10|3|2,

SoP: $\bar{x}y\bar{w} + yw + \bar{y}z$

# of literals: 7

PIs: 0|1|8|9, 8|12, 12|14, 14|10, 8|9|11|10, 1|5,

Forced PIs: 0|1|8|9, 12|14, 8|9|11|10, 1|5

PoS: $(\bar{y}+\bar{z})(x+\bar{y})(x+y+\bar{w})(\bar{y}+\bar{z}+w)$

# of literals: 10

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A/ Prime implicants $\boxed{SOP}$

4,5: 0100
0101
$xyzw$
$\bar{x}y\bar{z}$

4: 0100
6: 0110
$xyzw$
$\bar{x}y\bar{w}$

5 0101
13 1101
7 0111
15 1111
$xyzw$
yw

3: 0011
7: 0111
2: 0010
6: 0110
$xyzw$
$\bar{x}z$

3: 0011
2: 0010
11: 1011
10: 1010
$xyzw$
$\bar{y}z$

15: 1111
11: 1011
xzw

Prime implicants $\boxed{POS}$

0: 0000
1: 0001
8: 1000
9: 1001
$xyzw$
$\bar{y}\bar{z}$

8: 1000
9: 1001
11: 1011
10: 1010
$x\bar{y}$

8: 1000
12: 1100
$x\bar{z}\bar{w}$

12: 1100
14: 1110
$xy\bar{w}$

14: 1110
10: 1010
$xz\bar{w}$

1: 0001
5: 1001
$\bar{y}\bar{z}w$

B/ Prime implicants $\boxed{SOP}$

0: 0000
4: 0100
12: 1100
8: 1000
$xyzw$
$\bar{z}\bar{w}$

3: 0011
7: 0111
15: 1111
11: 1011
$xyzw$
zw

0: 0000
8: 1000
2: 0010
10: 1010
$xyzw$
$\bar{y}\bar{w}$

(b) (9 points): $f(x,y,z,w) = \Pi M(1,5,6,9,14) + DON'T\_CARE(7,15)$

F

P

zw \ xy	00	01	11	10
00	0: 1	4: 1	12: 1	8: 1
01	1: 0	5: 0	13: 1	9: 0
11	3: 1	7: X	15: X	11: 1
10	2: 1	6: 0	14: 0	10: 1

zw \ xy	00	01	11	10
00	0: 1	4: 1	12: 1	8: 1
01	1: 0	5: 0	13: 1	9: 0
11	3: 1	7: X	15: X	11: 1
10	2: 1	6: 0	14: 0	10: 1

PIs: 0|4|12|8, 3|2|11|10, 3|7|15|11, 0|2, 8|10, 12|13

Forced PIs: 0|4|12|8, 3|7|15|11, 0|8|2|10,

SoP: $\bar{z}w + zw + \bar{y}\bar{w}$

\# of literals: 6

PIs: 1|5, 1|9, 7|15, 6|14

Forced PIs: 1|5, 1|9, 7|15, 6|14

PoS: $(y+z)(\bar{x}+\bar{z})(\bar{y}+\bar{z}+w)$

\# of literals: 8

7 : 0111
15 : 1111
6 : 0110
14 : 1110
yz

1 : 0001
5 : 0101
$\bar{x}\bar{z}w$

1 : 0001
9 : 1001
$\bar{y}\bar{z}w$

(c) (9 points): $f(x, y, z, w) = \Sigma m(0, 2, 6, 7, 13, 14, 15) + DON'T\_CARE(8, 10, 12)$

zw \ xy	00	01	11	10
00	0: 1	4: 0	12: X	8: X
01	1: 0	5: 0	13: 1	9: 0
11	3: 0	7: 1	15: 1	11: 0
10	2: 1	6: 1	14: 1	10: X

zw \ xy	00	01	11	10
00	0: 1	4: 0	12: X	8: X
01	1: 0	5: 0	13: 1	9: 0
11	3: 0	7: 1	15: 1	11: 0
10	2: 1	6: 1	14: 1	10: X

PIs: 0|8|2|10, 12|8, 12|13|15|14, 7|15|6|14, 2|6, 14|10

PIs: 1|3|9|11, 4|5, 9|11

Forced PIs: 0|8|2|10, 12|13, 7|15|6|14

Forced PIs: $(y+\bar{w})(x+\bar{y}+z)$

SoP: $\bar{y}\bar{w} + xy\bar{z} + yz$

PoS: $(y+\bar{w})(x+\bar{y}+z)$

# of literals: 7

# of literals: 5

0 : 0000
8 : 1000
2 : 0010
10 : 1010
x $\bar{y}$ x $\bar{w}$

12 : 1100
13 : 1101
x
$xy\bar{z}$

7 : 0111
15 : 1111
6 : 0110
14 : 1110
x x
yz

Problem 3

An arcade game ticket counter is implemented with a 4-bit counter.

- The counter holds the number of tickets won and can count up to 15.
- The machine automatically converts tickets into prizes.
- The exchange rate is: **5 Tickets = 1 Prize**.

We need logic to convert the 4 bits of the ticket counter, $\mathbf{C} = (\mathbf{c_3}, \mathbf{c_2}, \mathbf{c_1}, \mathbf{c_0})$, to a format that displays the number of Prizes (P) and remaining Leftover Tickets (L): $\mathbf{P} = (\mathbf{p_1}, \mathbf{p_0})$ and $\mathbf{L} = (\mathbf{l_2}, \mathbf{l_1}, \mathbf{l_0})$.

For example:

- If **C = 9** (9 tickets), we get 1 Prize and 4 Leftovers ($\mathbf{P = 1, L = 4}$).
- If **C = 12** (12 tickets), we get 2 Prizes and 2 Leftovers ($\mathbf{P = 2, L = 2}$).
- If **C = 5** (5 tickets), we get 1 Prize and 0 Leftovers ($\mathbf{P = 1, L = 0}$).

(a) (2 points): Fill in a truth table for the outputs $(\mathbf{p_1}, \mathbf{p_0}, \mathbf{l_2}, \mathbf{l_1}, \mathbf{l_0})$ as a function of the inputs $(\mathbf{c_3}, \mathbf{c_2}, \mathbf{c_1}, \mathbf{c_0})$ for all 16 possible input values.

c_3	c_2	c_1	c_0		p_1	p_0	l_2	l_1	l_0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	1
0	0	1	0	2	0	0	0	1	0
0	0	1	1	3	0	0	0	1	1
0	1	0	0	4	0	0	1	0	0
0	1	0	1	5	0	1	0	0	0
0	1	1	0	6	0	1	0	0	1
0	1	1	1	7	0	1	0	1	0
1	0	0	0	8	0	1	0	1	1
1	0	0	1	9	0	1	1	0	0
1	0	1	0	10	1	0	0	0	0
1	0	1	1	11	1	0	0	0	1
1	1	0	0	12	1	0	0	1	0
1	1	0	1	13	1	0	0	1	1
1	1	1	0	14	1	0	1	0	0
1	1	1	1	15	1	1	0	0	0

(b) (9 points): Using Karnaugh maps, obtain minimal SOP expressions for each of the 5 outputs. Write the expressions on the lines below the K-maps.

P_1

c3 c2 \ c1 c0	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	0	0	1	1

$c3c1 + c3c2$

P_0

c3 c2 \ c1 c0	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	0	0	1	0
10	1	1	0	0

$c0c1c2 + c0c2\overline{c3} + c1c2\overline{c3} + c3\overline{c1}\overline{c2}$

l_2

c3 c2 \ c1 c0	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	0	0	0	1
10	0	0	1	0

$c_1c_2c_3\overline{c_0} + c_0c_3\overline{c_1}\overline{c_2} + c_2\overline{c_0}\overline{c_1}\overline{c_3}$

l_1

c3 c2 \ c1 c0	00	01	11	10
00	0	0	1	1
01	0	0	1	0
11	1	1	0	0
10	1	0	0	0

$c0c1\overline{c3} + c_2c_3\overline{c_1} + c_1\overline{c_2}\overline{c_3} + c_3\overline{c_0}\overline{c_1}$

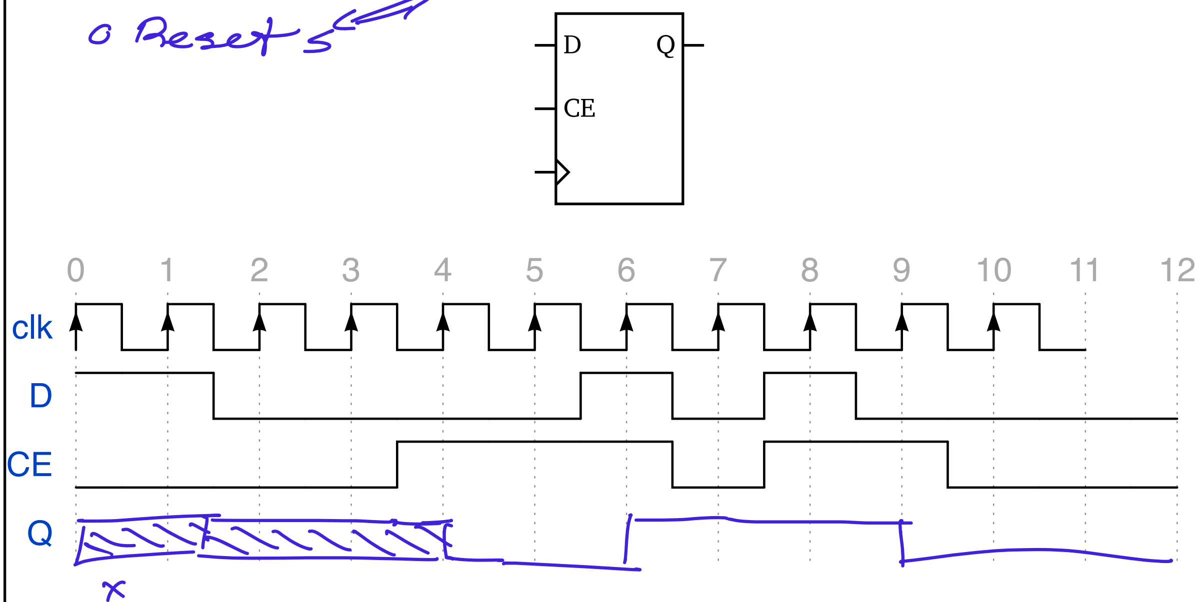
c3 c2 \ c1 c0	00	01	11	10
00	0	1	1	0
01	0	0	0	1
11	0	1	0	0
10	1	0	1	0

$c0c1\overline{c2} + c_0\overline{c_2}\overline{c_3} + c_0c_2c_3\overline{c_1} + c_3\overline{c_0}\overline{c_1}\overline{c_2}$

Problem 4

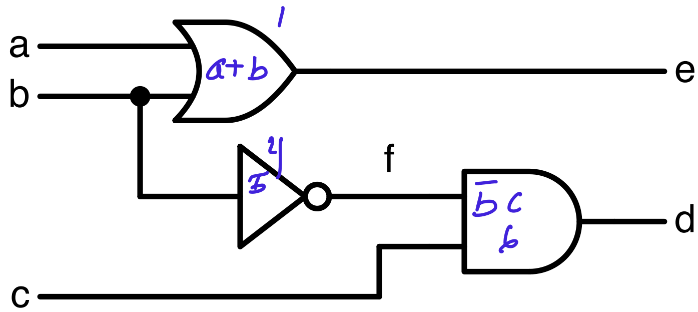
For the sequential circuit below, fill in the waveform for Q in the timing diagram. Assume the Flip-Flops are not initialized. Assume that R is 0 at all times. (5 points)

0 Resets



Problem 5

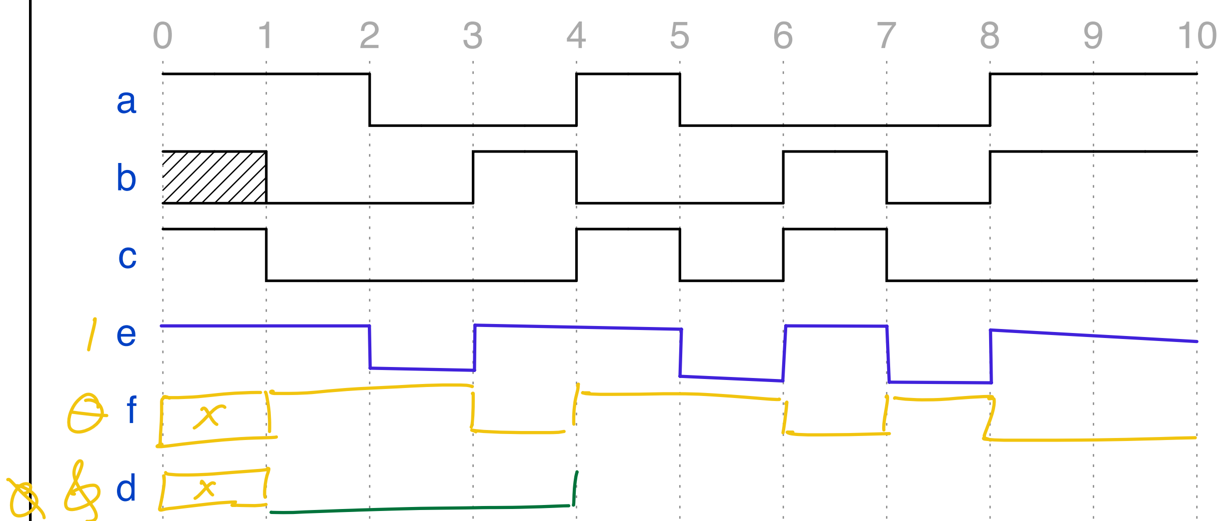
For the circuit shown below:



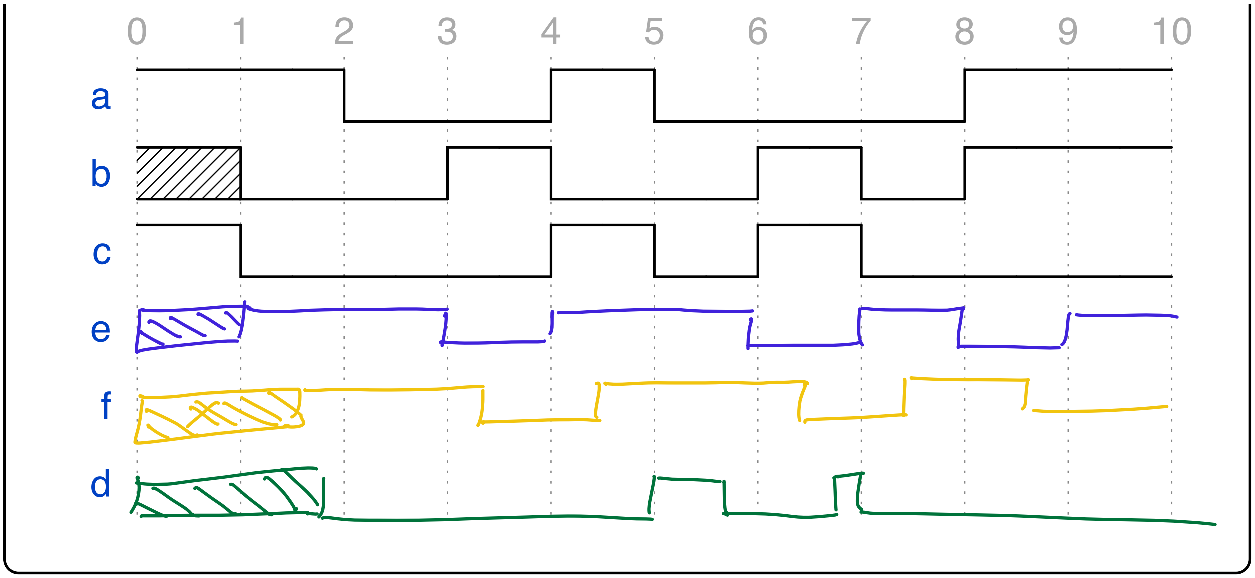
(a) (3 points) Write the boolean expression for the variables d, e, and f in terms of a, b, and c.

$e = a+b$ $f = \bar{b}$ $d = \bar{b}c$

(b) (3 points): Assume the gate delays are 0 time units (idealized). Fill in the waveforms for d, e, and f in the waveform below. (The shaded region is the indeterminate value 'X'.)



(c) (3 points): Assume that the AND delay is .6 time units, the NOT delay is .4 time units, and the OR delay is 1 time unit. Fill in the waveforms for d, e, and f in the waveform below. Each dotted vertical line is .2 units of delay.



Problem 6

For the sequential circuit below, fill in the waveform for S, Q and Y in the timing diagram. Assume the Flip-Flops are not initialized. (10 points)