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HW 3 **Due: October 20, 2025 at 11:59 PM in Santa Cruz.** CSE 100 Fall 2025

Reading: Sections 3.3-3.4 and 4.1-4.7.

Instructions: Complete the problems below **in your own handwriting.** Box your answers where necessary. The grader will award 1 point for your name and the last four digits of your student ID, and 1 point for neatness.

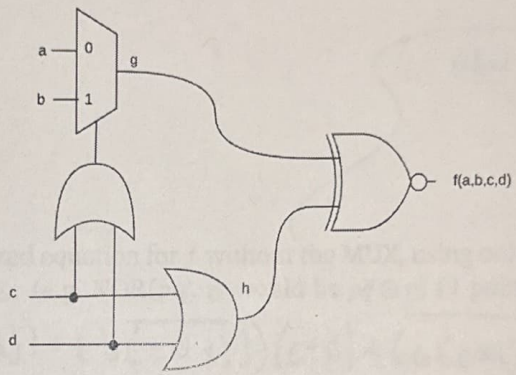
Note:

Σ –> Sum (capital sigma)

Π –> Product (capital pi)

Problem 1

For the following schematic, answer the questions below. (8 points):

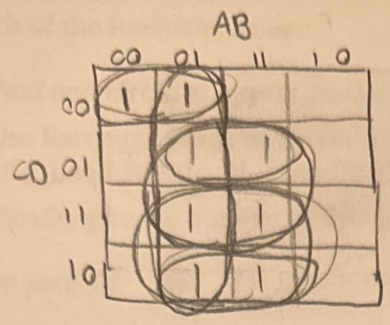


a) Fill out the truth table for the circuit showing both of g and f, draw the resulting k-map for f, circling all prime implicants and listing all the prime implicants as product terms. (5 points).

a	b	c	d	g	f
0	0	0	0	0	1
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1

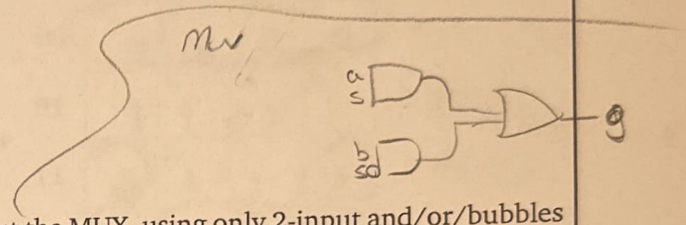
a	b	c	d	g	f
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	1	0
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

Draw K-map and list PIs as product terms:



PIs: $\bar{a}\bar{c}\bar{d}$, bd, bc, $\bar{a}b$, ~~cdb, $acdb$, $\bar{c}\bar{d}b$~~

essential



b) Write the unoptimized equation for f without the MUX, using only 2-input and/or/bubbles and the NOR gate $\odot$: (e.g. NOR(pq', r) would be $p\bar{q} \odot r$) (1 point).

$f = ((a(c \odot d)) + (b\overline{(c \odot d)}))(c+d) + \overline{(a(c \odot d)) + (b\overline{(c \odot d)})}\,\overline{(c+d)}$

c) Write the minimal Sum of Products expression for f (1 point).

$f = bc + bd + \bar{a}\bar{c}\bar{d}$

d) How many literals are there in the unoptimized f and in the optimized SoP for f (1 point)?

unoptimized: 32

optimized: 7

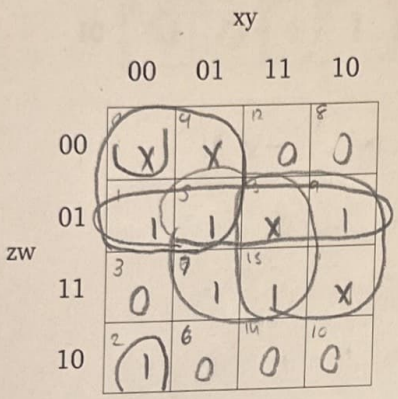
Problem 2

For each of the functions below:

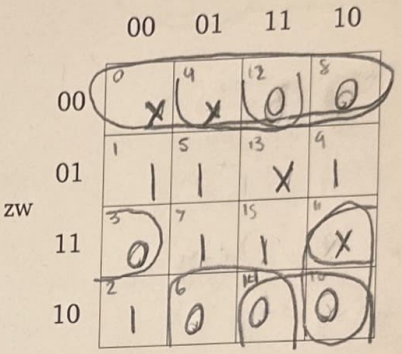
- Find and **circle all** prime implicants and identify which ones are forced.
- Use Karnaugh Maps to obtain the minimal boolean expression in **both** Sum-of-Products (SoP) and Product-of-Sums (PoS) forms. Draw separate K-maps for the SoP and PoS.
- Finally, give the number of literals for your minimal SOP and POS expressions.

(a) (9 points): $f(x, y, z, w) = \Sigma m(1, 2, 5, 7, 9, 15) + DON'T\_CARE(0, 4, 11, 13)$

SoP



PoS



PIs: $\bar{z}w, \bar{x}\bar{y}\bar{w}, \bar{x}\bar{z}, yw, xw$

Forced PIs: $\bar{x}\bar{y}\bar{w}, yw$

SoP: $\bar{x}\bar{y}\bar{w} + yw + \bar{z}w$

# of literals: 7

PIs: $(y+\bar{z}+w), (z+w), (\bar{x}+\bar{y}+z), (\bar{x}+y+\bar{z}), (\bar{y}+w), (\bar{x}+w)$

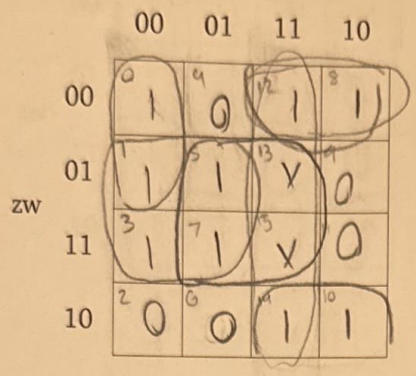
Forced PIs: $(y+\bar{z}+\bar{w}), (\bar{y}+w)$

PoS: $(y+\bar{z}+\bar{w}) \cdot (\bar{y}+w) \cdot (\bar{x}+w)$

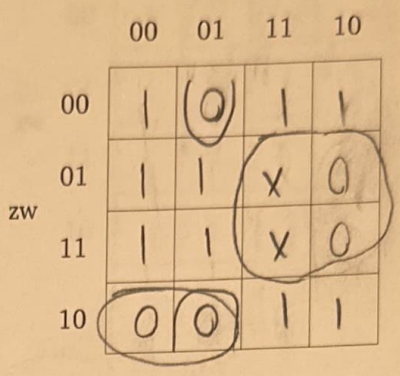
# of literals: 7

(b) (9 points): $f(x, y, z, w) = \Pi M(2, 4, 6, 9, 11) + DON'T\_CARE(13, 15)$

SoP



PoS



PIs: $\bar{x}\bar{y}\bar{z}, \bar{x}w, x\bar{w}, xy, yw$

Forced PIs: $\bar{x}\bar{y}\bar{z}, \bar{x}w, x\bar{w}$

SoP: $\bar{x}\bar{y}\bar{z} + \bar{x}w + x\bar{w}$

# of literals: 7

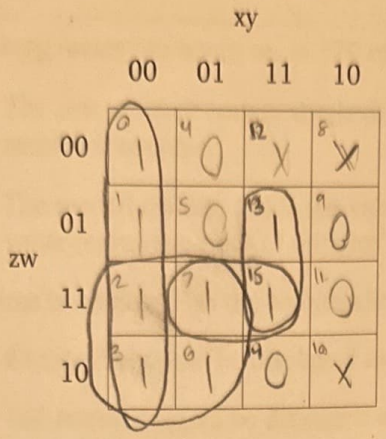
PIs: $(\bar{x}+z+\bar{w}), (x+w), (\bar{x}+y+\bar{w})$

Forced PIs: $(\bar{x}+z+\bar{w}), (x+w), (\bar{x}+y+\bar{w})$

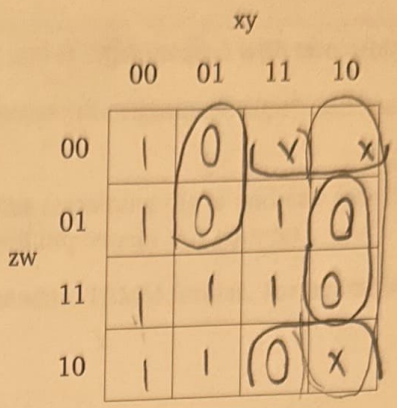
PoS: $(\bar{x}+z+\bar{w})(x+w)(\bar{x}+y+\bar{w})$

# of literals: 8

(c) (9 points): $f(x, y, z, w) = \Sigma m(0, 1, 2, 3, 6, 7, 13, 15) + DON'T\_CARE(8, 12, 10)$



SoP



PoS

PIs: $\bar{x}\bar{y}, \bar{x}z, xyw, yzw$

Forced PIs: $\bar{x}\bar{y}, \bar{x}z, xyw$

SoP: $\bar{x}\bar{y} + \bar{x}z + xyw$

# of literals: 7

PIs: $(\bar{z}+w), (y+\bar{z}), (x+z+\bar{w}), (x+y+\bar{w})$

Forced PIs: $(\bar{z}+w), (y+\bar{z}), (x+z+\bar{w})$

PoS: $(\bar{z}+w)(y+\bar{z})(x+z+\bar{w})$

# of literals: 7

Problem 3

A parking meter can count up to 179 minutes and is implemented with two 4-bit counters:

- The first counter counts single-digit minutes (the rightmost digit), but we *won't* implement this counter.
- The second counter holds the value for the remaining digits and can count up to 15. Its value represents "tens of minutes". We *will* implement this counter.

The time is displayed on the counters in the standard H:MM format. For example:

- 89 minutes would be displayed as 1:29
- 123 minutes would be displayed as 2:03
- 45 minutes would be displayed as 0:45

Hence we need logic to convert the 4 bits of the second counter, $\mathbf{C} = (c_3, c_2, c_1, c_0)$, to a format that displays hours (H) and tens of minutes (T),
$\mathbf{H} = (h_1, h_0)$ and $\mathbf{T} = (t_2, t_1, t_0)$

So for example, if $\mathbf{C = 9}$, then $\mathbf{H = 1}$ and $\mathbf{T = 3}$,
or if $\mathbf{C = 12}$, then $\mathbf{H = 2}$ and $\mathbf{T = 0}$,
or if $\mathbf{C = 5}$, then $\mathbf{H = 0}$ and $\mathbf{T = 5}$.

(a) (2 points): Fill in a truth table for the outputs $(h_1, h_0, t_2, t_1, t_0)$ as a function of the inputs (c_3, c_2, c_1, c_0) for all 16 possible input values.

c_3	c_2	c_1	c_0	h_1	h_0	t_2	t_1	t_0
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1	0
0	0	1	1	0	0	0	1	1
0	1	0	0	0	0	1	0	0
0	1	0	1	0	0	1	0	1
0	1	1	0	0	1	0	0	0
0	1	1	1	0	1	0	0	1
1	0	0	0	0	1	0	1	0
1	0	0	1	0	1	0	1	1
1	0	1	0	0	1	1	0	0
1	0	1	1	0	1	1	0	1
1	1	0	0	1	0	0	0	0
1	1	0	1	1	0	0	0	1
1	1	1	0	1	0	0	1	0
1	1	1	1	1	0	0	1	1

(b) (9 points): Using Karnaugh maps, obtain minimal SOP expressions for each of the 5 outputs. Write the expressions on the lines below the K-maps.

h_1

$c_1 c_0$ \ $c_3 c_2$	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$h_1 = c_3 c_2$

h_0

$c_1 c_0$ \ $c_3 c_2$	00	01	11	10
00	0	0	0	1
01	0	0	0	1
11	0	1	0	1
10	0	1	0	1

$h_0 = c_3 \bar{c}_2 + \bar{c}_3 c_2 c_1$

t_2

$c_1 c_0$ \ $c_3 c_2$	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	0	0	0	1
10	0	0	0	1

$t_2 = \bar{c}_3 c_2 \bar{c}_1 + c_3 \bar{c}_2 c_1$

t_1

$c_1 c_0$ \ $c_3 c_2$	00	01	11	10
00	0	0	0	1
01	0	0	0	1
11	1	0	1	0
10	1	0	1	0

$t_1 = c_3 c_2 c_1 + c_3 \bar{c}_2 \bar{c}_1 + \bar{c}_3 \bar{c}_2 c_1$

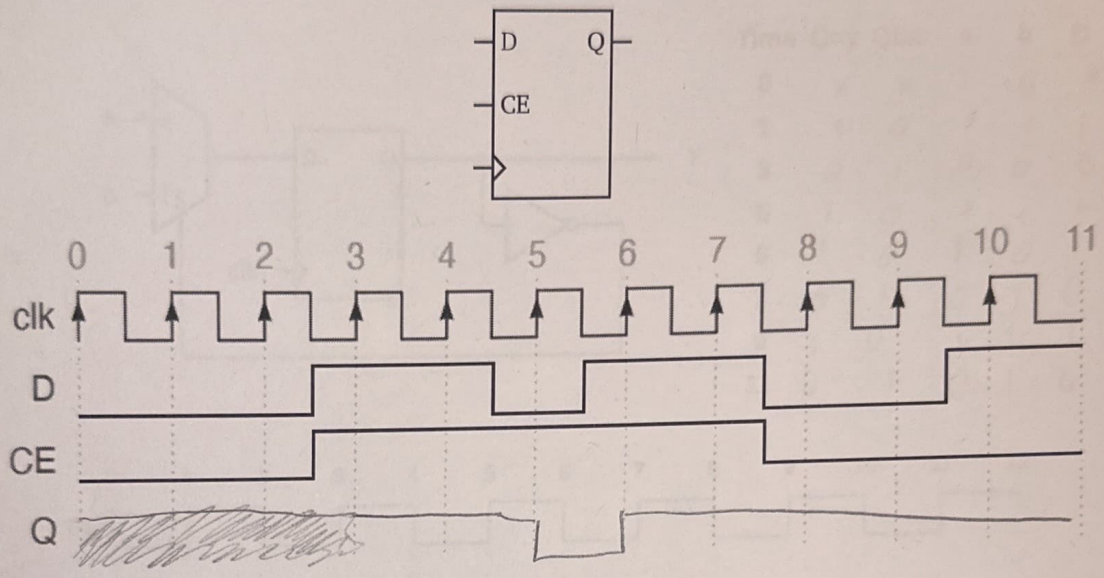
t_0

$c_1 c_0$ \ $c_3 c_2$	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$t_0 = c_0$

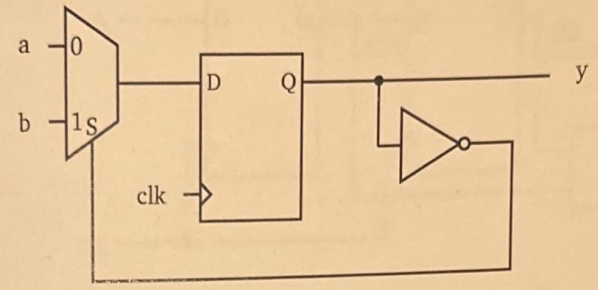
Problem 4

For the sequential circuit below, fill in the waveform for Q in the timing diagram. Assume the Flip-Flops are not initialized. Assume that R is 0 at all times. (5 points)

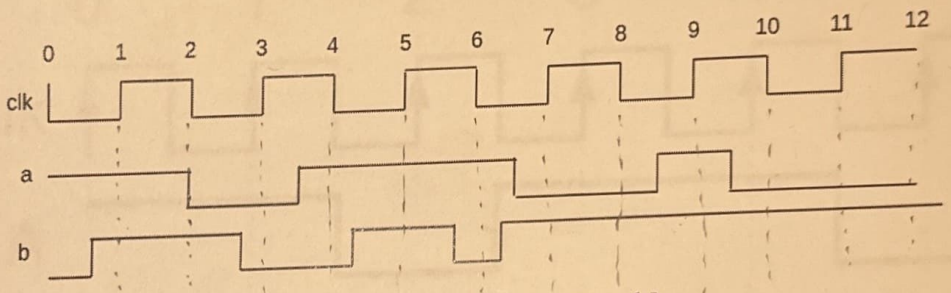


Problem 5

Consider the following circuit diagram and the waveforms shown for inputs a, b and clk. (4 points)



Time	Q=y	Qbar	a	b	D
0	x	x	1	0	x
1	1	0	1	1	1
3	0	1	0	0	0
5	1	0	1	1	1
6	1	0	1	0	1
7	0	1	0	1	0
9	1	0	1	1	1
11	0	1	0	1	0



For each of the following, circle the correct value [1 point each].

The value of Q at time t=1: 0 1 (x) z

The value of D at time t=1: 0 1 (x) z

The value of D at time t=7: (0) 1 x z

The value of y at time t=11: 0 (1) x z

Problem 6

For the sequential circuit below, fill in the waveform for S, Q and Y in the timing diagram. Assume the Flip-Flops are not initialized. (10 points)