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HW 1 Due: Monday, October 6 at 11:59 PM in Santa Cruz.

CSE 100
Fall 2025

Reading: Chapter 1, and sections 2.1-2.8.

Instructions: Complete the problems below **in your own handwriting.** Box your answers where necessary. The grader will award 1 point for your name and the last four digits of your student ID, and 1 point for neatness.

Boolean algebra laws and identities that you may use on this homework are provided below. You may also use Demorgan's Law:

- Identity Laws: $0 + b = b$, $1 * b = b$
- Complementation Laws: $a+\overline{a} = 1$, $b*\overline{b} = 0$
- Commutative Laws: $a+b = b+a$, $a*b = b*a$
- Distributive Laws: $a * (b+c) = a*b+a*c$, $a + (b * c) = (a + b) * (a + c)$
- Associative Laws: $a + (b+c) = (a+b)+c$, $a * (b * c) = (a * b) * c$
- Idempotency: $a + a = a$, $a * a = a$
- Absorption: $a + a * b = a$, $a * (a + b) = a$
- Involution: $\overline{\overline{a}} = a$
- Domination: $1 + b = 1$, $0 * b = 0$
- Simplification: $a * (\overline{a} + b) = a * b$, $a + (\overline{a} * b) = a + b$
- Uniqueness of complements: if $a + b = 1$ and $a * b = 0$ then $b = \overline{a}$

Problem 1

Verify the identity below using two Truth Tables (2 points):

$$b(a + \overline{c}) + a(\overline{a} + c) = a(\overline{a} + b + c) + b\overline{c}$$

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Problem 2

Verify each of the identities below by transforming one side of the equation using Boolean algebraic laws and identities. **Only use laws and identities demonstrated in lecture.** For full points, show your work by labelling each step with the identity or law that was applied on the right hand side of the vertical bar.

(a) (2 points): $xz \cdot (x + \overline{y} + \overline{z}) == x \cdot z$

$xzx + xz\overline{y} + xz\overline{z}$	Distributive
$xz + xz\overline{y} + xz\overline{z}$	Idempotency
$xz + xz\overline{y}$	Complementation, Domination, Identity
$\boxed{xz}$	Absorption

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(b) (2 points): $xy + xyw(x + \overline{y}) == xy$

$xy + xywx + xyw\overline{y}$	Distributive
$xy + xyw + xy\overline{y}w$	Idempotency
$xy + xyw + \cancel{xw}0$	Complementation / Dom / Identity
$\boxed{xy}$	Absorption

(c) (3 points): $(\overline{x} + \overline{y} + z + w) \cdot (\overline{x} + \overline{y} + \overline{z} \cdot \overline{w}) == \overline{x \cdot y}$

$(\overline{x} + \overline{y} + z + w)(\overline{x} + \overline{y}) + (\overline{x} + \overline{y} + z + w)\overline{z}\,\overline{w}$	Distributive
$(\overline{x} + \overline{y}) + (\overline{x} + \overline{y} + z + w)\overline{z}\,\overline{w}$	Absorption
$(\overline{x} + \overline{y}) + \overline{x}\overline{z}\overline{w} + \overline{y}\overline{z}\overline{w} + z\overline{z}\overline{w} + w\overline{w}\overline{z}$	Distributive
$(\overline{x} + \overline{y}) + \overline{x}\overline{z}\overline{w} + \overline{y}\overline{z}\overline{w}$	Complementation, Domination, Identity
$\overline{x} + \overline{y}$	Absorption
$\boxed{\overline{xy}}$	De Morgan's Law

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(d) (3 points): $ab + \overline{a}c + bc(a + \overline{a}) == ab + \overline{a}c$

$ab + \bar{a}c + abc + bc\bar{a}$	Distributive
$ab + abc + \bar{a}c + bc\bar{a}$	reordering
$ab + \bar{a}c + bc\bar{a}$	Absorption
$ab + \bar{a}c(1 + b)$	Factoring
$ab + \bar{a}c1$	Domination
$ab + \bar{a}c$	Identity

(e) (3 points): $(x + \overline{y} + z) \cdot (x + y + z) == (x + z)$

$xx + xy + xz + x\bar{y} + \bar{y}y + \bar{y}z + zx + zy + zz$	Distributive
$x + xy + xz + x\bar{y} + 0 + \bar{y}z + zx + zy + z$	Idempotent / Complementation
$x + x\bar{y} + \bar{y}z + zx + zy + z$	Absorption x 2
$x + \bar{y}z + zx + zy + z$	Absorption
$x + z\bar{y} + zy + z$	Absorption
$x + z + z(\bar{y} + y)$	Factoring
$x + z + z1$	Complementation
$x + z + z$	Identity
$x + z$	Idempotency

Problem 3

For each expression below:

1. Write the complement of each of the expressions (e.g. the complement of (abc) is $\overline{(abc)}$)
2. Then simplify the resulting expression using Boolean algebraic laws and identities. You will need to use DeMorgan's Law at least once. **Your resulting expression must have the minimal number of literals possible for full points.**

For full points, show your work by labelling each step with the identity or law that was applied on the right hand side of the vertical bar.

(a) (2 points): $(x \cdot y \cdot z + \overline{y})$ = $\overline{(xyz+\overline{y})}$

$\overline{(xyz+\overline{y})} = \overline{(xyz)} \cdot \overline{\overline{y}}$	De Morgan's
$\overline{(xyz)} \cdot y$	Involution
$(\overline{x}+\overline{y}+\overline{z})y$	De Morgan's
$\overline{x}y+y\overline{y}+\overline{z}y$	Distribute
$\overline{x}y+\overline{z}y$	Complementation / Dom / Identity
$y(\overline{x}+\overline{z})$	Factoring

(b) (2 points): $(x \cdot z) + (\overline{x} + y)$ = $\overline{(xz+(\overline{x}+y))}$

$\overline{(xz)} + \overline{(\overline{x}+y)}$	De Morgan's
$(\overline{x}+\overline{z})(\overline{\overline{x}}\,\overline{y})$	De Morgan's
$(\overline{x}+\overline{z})(x\overline{y})$	Involution
$\overline{x}x\overline{y}+x\overline{y}\overline{z}$	Distribute
$x\overline{y}\overline{z}$	Complementation / Dom / Identity

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(c) (3 points): $x \cdot \overline{z} + (x + \overline{y})\overline{(x \cdot y + \overline{z})}$ $\overline{x\bar{z} + (x+\bar{y})\overline{(xy+\bar{z})}}$

$\overline{x\bar{z}}\,\overline{(x+\bar{y})(xy+\bar{z})}$	
$(\bar{x}+\bar{\bar{z}})(\bar{x}\,\bar{\bar{y}})+(xy+\bar{z})$	De Morgan's
$(\bar{x}+z)[(\bar{x}y)+(xy+\bar{z})]$	Involution
$(\bar{x}+z)[(\bar{x}y+xy)+\bar{z}]$	Associative
$(\bar{x}+z)[y(\bar{x}+x)+\bar{z}]$	Factor
$(\bar{x}+z)(y+\bar{z})$	Complementation/Identity

Problem 4

Suppose $f(a,b,c) = \overline{a} \cdot (\overline{b} + c) + (\overline{a} + b) \cdot c$.

(a) (1 point): Make a truth table for $f(a,b,c)$.

a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(b) (1 point): Make a Karnaugh Map for $f(a,b,c)$ and circle all of the prime implicants.

c \ ab	00	01	11	10
0	1	0	0	0
1	1	1	1	0

(c) (1 point): Write $f(a, b, c)$ in **canonical** Sum-of-Products form, i.e. $\sum m(...)$.

$\sum m(0,1,3,7) = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}bc + abc$

(d) (1 point): Write $f(a, b, c)$ in **canonical** Product-of-Sums form, i.e. $\prod M(...)$.

$\prod M(2,4,5,6) = (\bar{a}+b+\bar{c})(a+\bar{b}+\bar{c})(a+\bar{b}+c)(a+b+\bar{c})$

Problem 5

For the functions below, use a Karnaugh Map to obtain a minimal Sum-of-Products (SoP) form. **To receive full points, you must:**

1. Fill in the variables on Karnaugh maps provided, and **circle all of the prime implicants.**
2. List all of the **prime implicants** and **forced prime implicants** in SoP form (e.g. $\bar{a}b\bar{c}$, $a\bar{b}$).
3. Give the cost of your minimal SOP form in terms of the number of literals (variables, or their complement). For example, the cost of $ab\bar{c}$ is 3.

(a) (3 points): $f(a, b, c) = \sum m(0, 1, 2, 4, 5, 7)$

a	b	c	f
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

ab

c	00	01	11	10
0	1	1	0	1
1	1	0	1	1

Prime implicants:

$\bar{b}, \bar{a}\bar{c}, bc$

Forced PIs:

$\bar{b}, \bar{a}\bar{c}, bc$

Minimal SoP:

$\bar{b} + \bar{a}\bar{c} + bc$

Cost (# of literals):

5

(b) (3 points): $f(a, b, c) = \prod M(0, 2, 3, 7)$

ab

c	00	01	11	10
0	0	0	1	1
1	1	0	0	1

Prime implicants:

$a\bar{b}, a\bar{c}, \bar{b}c$

Forced PIs:

$a\bar{c}, \bar{b}c$

Minimal SoP:

$a\bar{c} + \bar{b}c$

Cost (# of literals):

4

(c) (4 points): $f(x,y,z,w) = \prod M(1,3,8,9,10,11,13,15)$

x	y	z	w	f
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

zw \ xy	00	01	11	10
00	1	1	1	0
01	0	1	0	0
11	0	1	0	0
10	1	1	1	0

Prime implicants:

$\bar{x}y, \bar{x}\bar{w}, y\bar{w}$

Forced PIs:

$\bar{x}y, \bar{x}\bar{w}, y\bar{w}$

Minimal SoP:

$\bar{x}y + \bar{x}\bar{w} + y\bar{w}$

Cost (# of literals):

6

(d) (4 points): $f(x,y,z,w) = \sum m(1,2,4,5,6,8,12,13,14,15)$

zw \ xy	00	01	11	10
00	0	1	1	1
01	1	1	1	0
11	0	0	1	0
10	1	1	1	0

Prime implicants:

$xy, \bar{x}z\bar{w}, \bar{x}\bar{z}w, x\bar{z}\bar{w}, y\bar{z}$

Forced PIs:

$xy, \bar{x}z\bar{w}, \bar{x}\bar{z}w, x\bar{z}\bar{w}, y\bar{z}$

Minimal SoP:

$xy + \bar{x}z\bar{w} + \bar{x}\bar{z}w + x\bar{z}\bar{w} + y\bar{z}$

Cost (# of literals):

13

(e) (4 points): $f(x,y,z,w) = \sum m(0,1,3,4,5,6,9,10,11,13)$

zw \ xy	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	1	0	0	1
10	0	1	0	1

Prime implicants:

$\bar{z}w, \bar{x}\bar{z}, zw\bar{x}, x\bar{y}z, \bar{x}y\bar{w}$

Forced PIs:

$\bar{z}w, \bar{x}\bar{z}, zw\bar{x}, x\bar{y}z, \bar{x}y\bar{w}$

Minimal SoP:

$\bar{z}w + \bar{x}\bar{z} + zw\bar{x} + x\bar{y}z + \bar{x}y\bar{w}$

Cost (# of literals):

13

Problem 6

For the truth table below, use a Karnaugh Map to obtain a minimal Sum-of-Products (SoP) form. **To receive full points, you must:**

1. Fill in the variables on Karnaugh maps provided, and **circle all of the prime implicants.**
2. List all of the **prime implicants** and **forced prime implicants** in SoP form (e.g. $\bar{a}b\bar{c}$, $a\bar{b}$).
3. Give the cost of your minimal SOP form in terms of the number of literals (variables, or their complement). For example, the cost of $ab\bar{c}$ is 3.

(a) (1 point): What are the minterms of the logic function described by the truth table?:

	a	b	c	f
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0

Minterms of f (i.e. $\sum m(\ldots)$):

$\Sigma m(1, 3, 4, 6)$

(b) (4 points): Use the Karnaugh-Map to obtain a minimal Sum-of-Products (SoP) form.

c \ ab	00	01	11	10
0	0	0	1	1
1	1	1	0	0

Prime implicants:

$a\bar{c}$, $\bar{a}c$

Forced PIs:

$a\bar{c}$, $\bar{a}c$

Minimal SoP:

$a\bar{c} + \bar{a}c$

Cost (# of literals):

4

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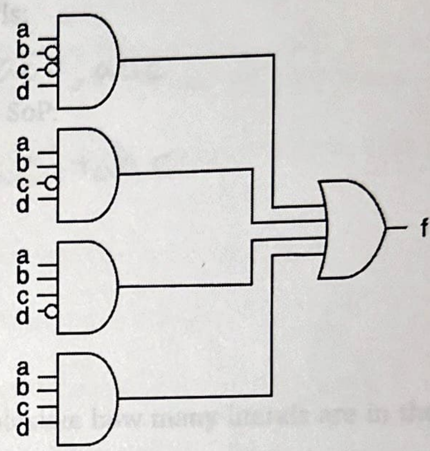
Problem 7

In this problem, you will optimize the initially suboptimal circuit given below by minimizing the logic using a K-map.

- First, fill out the truth table with the output of the circuit for all possible inputs, and calculate the number of literals in the initial circuit.
- Next, use a K-map to find the minimal SOP form of the circuit.
- Finally, draw the optimized circuit and calculate the number of literals used in the optimized circuit.



(a) (2 points): Fill out the truth table for the initial circuit, and calculate how many literals are in the initial circuit.

	a	b	c	d	f
0	0	0	0	0	0
	0	0	0	1	0
	0	0	1	0	0
	0	0	1	1	0
	0	1	0	0	0
	0	1	0	1	0
	0	1	1	0	0
	0	1	1	1	0
	1	0	0	0	0
9	1	0	0	1	1
	1	0	1	0	0
	1	0	1	1	0
	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

\# of literals in the initial circuit:

$A\bar{B}\bar{C}D + AB\bar{C}D + ABC\bar{D} + ABCD$

16 literals

(b) **(4 points):** Use a K-map to find the minimal SoP form of the circuit.

cd \ ab	00	01	11	10
00	0 (0)	0 (4)	0 (12)	0 (8)
01	0 (1)	0 (5)	1 (13)	1 (9)
11	0 (3)	0 (7)	1 (15)	0 (11)
10	0 (2)	0 (6)	1 (14)	0 (10)

Prime implicants:

$a\bar{c}d$, abd, abc

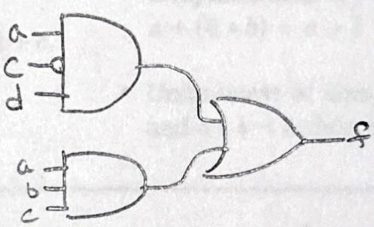
Forced PIs:

$a\bar{c}d$, abc

Minimal SoP:

$a\bar{c}d + abc$

(c) **(2 points):** Draw the optimized circuit, and calculate how many literals are in the optimized circuit.



6 literals total

Cost (# of literals):