Name: ________________________________ **Last 4 ID:** __________ **CSE 100**

HW 1 **Due: Wednesday, January 14 at 11:59 PM in Santa Cruz.** **Winter 2026**

Reading: Chapter 1, and sections 2.1-2.8.

Instructions: Complete the problems below **in your own handwriting**. Box your answers where necessary. The grader will award 1 point for your name and the last four digits of your student ID, and 1 point for neatness.

Boolean algebra laws and identities that you may use on this homework are provided below. You may also use Demorgan's Law:

- Identity Laws: $0 + b = b$, $1 * b = b$
- Complementation Laws: $a + \overline{a} = 1$, $b * \overline{b} = 0$
- Commutative Laws: $a + b = b + a$, $a * b = b * a$
- Distributive Laws: $a * (b + c) = a * b + a * c$, $a + (b * c) = (a + b) * (a + c)$
- Associative Laws: $a + (b + c) = (a + b) + c$, $a * (b * c) = (a * b) * c$
- Idempotency: $a + a = a$, $a * a = a$
- Absorption: $a + a * b = a$, $a * (a + b) = a$
- Involution: $\overline{\overline{a}} = a$
- Domination: $1 + b = 1$, $0 * b = 0$
- Simplification: $a * (\overline{a} + b) = a * b$, $a + (\overline{a} * b) = a + b$
- Uniqueness of complements: if $a + b = 1$ and $a * b = 0$ then $b = \overline{a}$

Problem 1

Verify the identity below using two Truth Tables (2 points):

$$((x \cdot \overline{z}) + w) \cdot (\overline{z} + \overline{w}) = \overline{(z \cdot \overline{w})} \cdot \overline{(z \cdot w)} \cdot \overline{(\overline{x} \cdot \overline{w})}$$

Problem 2

Verify each of the identities below by transforming one side of the equation using Boolean algebraic laws and identities. **Only use laws and identities demonstrated in lecture.** For full points, show your work by labelling each step with the identity or law that was applied on the right hand side of the vertical bar.

(a) (2 points): $(x + \overline{z} + y) \cdot (x + z + y) == (x + y)$

(b) (2 points): $x + (\overline{x} \cdot \overline{y} \cdot \overline{z}) == (x + \overline{y}) \cdot (x + \overline{z})$

(c) (2 points): $(a+b+c+d)\cdot(a+c+\overline{b}\cdot\overline{d}) == a+c$

(d) (3 points): $(a+b)\cdot(\overline{a}+\overline{c})\cdot(\overline{c}+\overline{b}) == (a+c+b)\cdot\overline{c}$

(e) (3 points): $\overline{x} \cdot \overline{y} + \overline{y} \cdot z + x \cdot z == \overline{x} \cdot \overline{y} + x \cdot z$

Problem 3

For each expression below:

1. Write the complement of each of the expressions (e.g. the complement of (abc) is $\overline{(abc)}$)
2. Then simplify the resulting expression using Boolean algebraic laws and identities. You will need to use DeMorgan's Law at least once. **Your resulting expression must have the minimal number of literals possible for full points.**

For full points, show your work by labelling each step with the identity or law that was applied on the right hand side of the vertical bar.

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(a) (2 points): $(\overline{a} + b \cdot c)$

(b) (2 points): $(\overline{a} + \overline{b}) \cdot (c \cdot \overline{d})$

(c) (3 points): $\overline{(a + \overline{d})} + \overline{(a \cdot b \cdot \ c + d)} \cdot d$

Problem 4

Suppose $f(a,b,c) = \bar{b} \cdot (c + a) + (\bar{c} + \bar{b}) \cdot a$.

(a) (1 point): Make a truth table for $f(a,b,c)$.

(b) (1 point): Make a Karnaugh Map for $f(a,b,c)$ and circle all of the prime implicants.

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(c) (1 point): Write $f(a,b,c)$ in **canonical** Sum-of-Products form, i.e. $\sum m(...)$.

(d) (1 point): Write $f(a,b,c)$ in **canonical** Product-of-Sums form, i.e. $\prod M(...)$.

Problem 5

For the functions below, use a Karnaugh Map to obtain a minimal Sum-of-Products (SoP) form. **To receive full points, you must**:

1. Fill in the variables on Karnaugh maps provided, and **circle all of the prime implicants**.
2. List all of the **prime implicants** and **forced prime implicants** in SoP form (e.g. $\overline{a}b\overline{c}$, $a\overline{b}$).
3. Give the cost of your minimal SoP form in terms of the number of literals (variables, or their complement). For example, the cost of $ab\overline{c}$ is 3. **Do not simplify your Karnaugh map SoP**.

(a) (3 points): $f(a,b,c) = \sum m(2,3,4,5,7)$

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Prime implicants:

Forced PIs:

Minimal SoP:

Cost (# of literals):

(b) (3 points): $f(a,b,c) = \prod M(2,3,4,6,7)$

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Prime implicants:

Forced PIs:

Minimal SoP:

Cost (# of literals):

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(c) (4 points): $f(x, y, z, w) = \sum m(0, 1, 5, 7, 9, 10, 13, 15)$

Prime implicants:

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Forced PIs:

Minimal SoP:

Cost (# of literals):

(d) (4 points): $f(x, y, z, w) = \prod M(1, 4, 5, 6, 7, 9, 12, 13, 14)$

Prime implicants:

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Forced PIs:

Minimal SoP:

Cost (# of literals):

(e) (4 points): $f(x, y, z, w) = \sum m(0, 2, 4, 5, 6, 8, 10, 12, 14, 15)$

Prime implicants:

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Forced PIs:

Minimal SoP:

Cost (# of literals):

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Problem 6

For the truth table below, use a Karnaugh Map to obtain a minimal Sum-of-Products (SoP) form. **To receive full points, you must**:

1. Fill in the variables on Karnaugh maps provided, and **circle all of the prime implicants**.
2. List all of the **prime implicants** and **forced prime implicants** in SoP form (e.g. $\overline{a}b\overline{c}$, $a\overline{b}$).
3. Give the cost of your minimal SoP form in terms of the number of literals (variables, or their complement). For example, the cost of $ab\overline{c}$ is 3. **Do not simplify your Karnaugh map SoP**.

(a) (1 point): What are the minterms of the logic function described by the truth table?:

a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Minterms of f (i.e. m_0, m_1, etc.):

(b) (4 points): Use the Karnaugh-Map to obtain a minimal Sum-of-Products (SoP) form.

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Prime implicants:

Forced PIs:

Minimal SoP:

Cost (# of literals):

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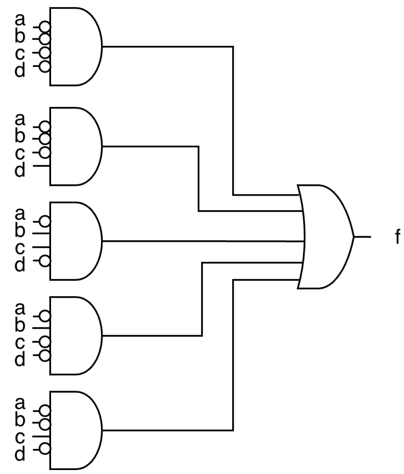
Problem 7

In this problem, you will optimize the initially suboptimal circuit given below by minimizing the logic using a K-map.

- First, fill out the truth table with the output of the circuit for all possible inputs, and calculate the number of literals in the initial circuit.
- Next, use a K-map to find the minimal SoP form of the circuit. **Do not simplify your K-map SoP**.
- Finally, draw the optimized circuit and calculate the number of literals used in the optimized circuit.



(a) (2 points): Fill out the truth table for the initial circuit, and calculate how many literals are in the initial circuit.

a	b	c	d	f

\# of literals in the initial circuit:

(b) (4 points): Use a K-map to find the minimal SoP form of the circuit.

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Prime implicants:

Forced PIs:

Minimal SoP:

(c) (2 points): Draw the optimized circuit, and calculate how many literals are in the optimized circuit.

Cost (# of literals):