

Winding number of a Dirac cone

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1 Calculation of winding number of a Dirac Cone

Let us define a Dirac Cone as a 2D system governed by the following hamiltonian:

$$H = k_x \otimes \hat{\sigma}_x + k_y \otimes \hat{\sigma}_y \quad (1.1)$$

we may then define the complex number

$$z = \rho e^{i\phi} ; \quad \rho = \sqrt{k_x^2 + k_y^2} ; \quad \phi = \cos^{-1}(k_x)$$

such that the hamiltonian has now the simple form

$$H = \rho \begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix}$$

which is the same 2×2 structure of a 1D system with 2 degrees of freedom per unit cell. We may then -in analogy to what we know about those systems- define a winding number, as:

$$\nu = \frac{1}{2\pi i} \int_{-\pi}^{\pi} \frac{d}{d\phi} \log(e^{i\phi}) d\phi \quad (1.2)$$

which is easy to check equals 1.