Lenia - DAI project

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February 21, 2022

What are Lenia?

Lenia is a particular kind of cellular automata, which is continuous in time, space and state. They differ from other cellular automata patterns in being geometric, fuzzy, resilient, adaptive, and rule-generic. They were first presented in the paper *Lenia — Biology of Artificial Life* by *Bert Wang-Chak Chan*.

Cellular automata

A cellular automata (CA) is a mathematical system where a grid of cells is evolved according to a set of local rules. By using simple rules, interesting patterns start to emerge and lead to complex phenomena. One of the most famous examples of CA is the Game of Life (GoL) developed by Conway in 1970.

Game of Life

In GoL we have a grid of cells, each of which can be either alive or dead. The rules of the game are simple and consider only a 3x3 neighborhood:

- Any live cell with fewer than two live neighbours dies, as if by underpopulation.
- Any live cell with two or three live neighbours lives on to the next generation.
- Any live cell with more than three live neighbours dies, as if by overpopulation.
- Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction.

Game of Life

In other words, GoL can be seen as a CA with:

- a finite number of states (dead or alive)
- a finite neighborhood
- a finite time step, every cells immediately evolves to the next state

Can we generalize these finite properties to be continuous?

Definition of CA

Before we can generalize the rules of GoL, we need to define a CA mathematically. A CA is a tuple $\mathcal{A} = (\mathcal{L}, \mathcal{T}, \mathcal{S}, \mathcal{N}, \phi)$ where $\mathcal{L}$ is the d-dimensional lattice of grid, $\mathcal{T}$ is the timeline, $\mathcal{S}$ is the state, $\mathcal{N} \subset \mathcal{L}$ is the neighborhood, $\phi : \mathcal{S}^{\mathcal{N}} \rightarrow \mathcal{S}$ is the update rule.
With $\mathcal{A}^t : \mathcal{L} \rightarrow \mathcal{S}$ we define the collection of states over the whole grid at timestep t, while $\mathcal{A}^t(x)$ is the state of the cell $x \in \mathcal{L}$

Implementation of GoL

To implement GoL we follow a different path than usual, that will help us to generalize the CA in the future. Given a 2D grid of cells $\mathcal{L}$, we define a kernel $\mathcal{K}$ that will be convolved over each cell of $\mathcal{L}$.
For GoL we define the following kernel

$$\mathcal{K} = \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{matrix} \tag{1}$$

The first thing to compute is the so called *potential distribution* U^t for each cell $x \in \mathcal{L}$:

$$U^t(x) = \mathcal{K} * \mathcal{A}^t(x) \tag{2}$$

Implementation of GoL

The next step is to feed the potential distribution into a growth mapping function $G(x) : \mathbb{R} \rightarrow [-1, +1]$, this gives as the *growth distribution* G^t, which for GoL is defined as:

$$G^t(x) = \begin{cases} 0 & \text{if } U^t(x) == 2 \\ +1 & \text{if } U^t(x) == 3 \\ -1 & \text{otherwise} \end{cases} \tag{3}$$

Now we can finally update the grid of cells by adding the growth distribution and clipping everything to the two states $0, 1$:

$$\mathcal{A}^{t+1}(x) = [\mathcal{A}^t(x) + G^t(x)]_0^1 \tag{4}$$

Implementation of GoL

Basically the growth distribution G^t tells us how each cells change in the next step, 0 mean that it remains in the same state, $+1$ mean the cell is born, -1 mean that the cell died.

From GoL to Lenia

From now on, we will start from the Game of Life, and try to achieve continuous states, continuous time, and continuous space, one by one.
The first thing that can be done is to update every cell just by a fraction Δt of G^t, when $\Delta t \to 0$ we get continuous time.
We change equation 4 to:

$$\mathcal{A}^{t+1}(x) = [\mathcal{A}^t(x) + G^t(x)\Delta t]_0^1 \tag{5}$$

From GoL to Lenia

This make sense only if the state can be continuous too instead of only $0, 1$. Then let's change $\mathcal{S}$ to be continuous:

$$\mathcal{S} = [0, 1] \tag{6}$$

To make the space continuous, the convolution kernel is enlarged to radius R and defined to have a ring shape. As R approaches infinity, the space become continuous.

From GoL to Lenia

To make the ring shape and the growth function *smooth* we can define them using the gaussian function:

$$\mathcal{K}(\mu, \sigma, x) = exp(-\frac{(x-\mu)^2}{2\sigma^2}) \tag{7}$$

$$G^t(\mu, \sigma, x) = 2exp(-\frac{(x-\mu)^2}{2\sigma^2}) - 1 \tag{8}$$

Lenia

We now have achieved continuos time, space and state. We can now start to look for creatures by setting random G_μ, G_σ, K_μ, K_σ and also initializing the grid with random values between 0 and 1.

The first creature found by Bert Wang-Chak Chan was named *Orbium*. By setting the parameters to $G_\mu = 0.15$, $G_\sigma = 0.015$, $K_\mu = 0.5$, $K_\sigma = 0.15$, we can get a creature that is able to move and survive in the grid.

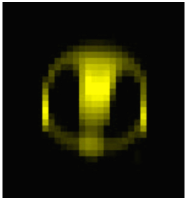
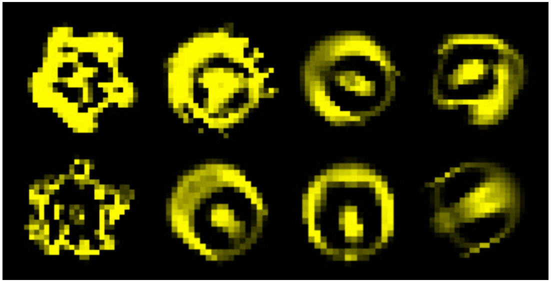


Figure: Orbium

Lenia

Different creature have been found just by initializing the parameters at random and playing with the grid:



Extending Lenia

How can we push this further? We can actually use more that a single kernel, each one with a different growth function. Then we can weight the different growth distribution by a factor h_i. Then the formula became:

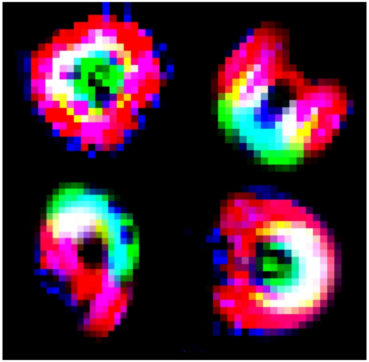
$$\mathcal{A}^{t+1}(x) = [\mathcal{A}^t(x) + \sum_{i=1}^{N} h_i G_i^t(U_i^t(x))\Delta t]_0^1 \tag{9}$$

Extending Lenia

We can also add more grid and make them interact with each other by assigning to a kernel a source and a target. The source grid is the one that is used to compute U^t and G^t while the target grid is the one that will be updated.
By using 3 grid, we can have RGB life-forms.

Extending Lenia

This obliviously gives us more parameters to tune and thus is harder to explore the whole parameters space and to find living creatures, but life-forms can exibith more complex behaviors such as self-replication, which was never been found in the "standard" Lenia.



Extending Lenia

Until now, every cell relied on a global synchronous clock. We can now introduce a local clock to each cell and make the whole CA asynchronous. We can update a cell state with a probability of 50%. Will the whole system still manage to self-organize and able to generate a living creature?

Extending Lenia

The answer is yes, the system is able to self-organize and some of the life-forms found in the synchronous Lenia can live in the asynchronous too. A drawback is that these life-forms are way more sensitive to noise.