COMPSCI 2AC3 Assignment 3

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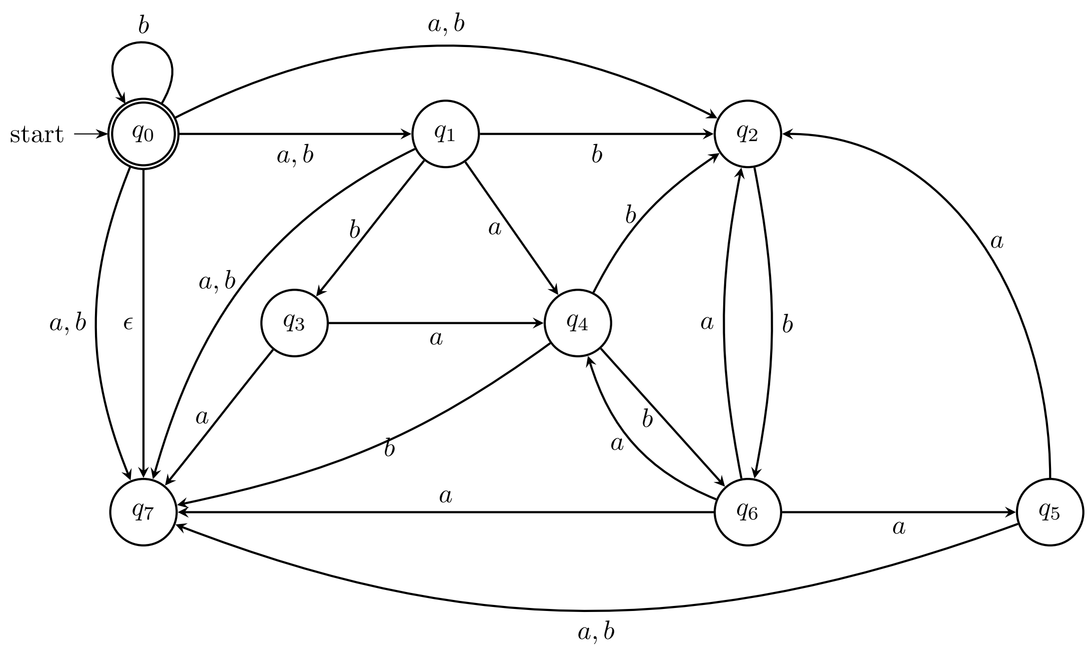
1.a



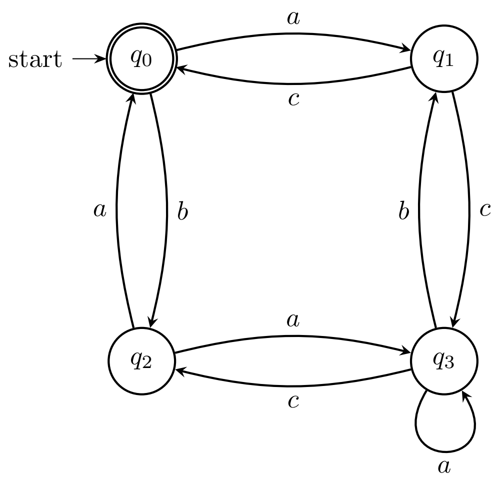
The NFA is correct because it accounts for all possible combinations of a and b that do not have the combinations aaa nor abb with zero or more b's trailing. The NFA accounts for when these combinations may occur in a string, as for example abb may occur at an even or odd index within the string. The NFA also accounts for the fact that abb is only an acceptable string if it is followed by a, as this would invalidate the criteria for $abb(b)^*$. Finally, the NFA also accounts for the fact that there can be at most two a's in a row, and forces the NFA to transition using b, or accepting state is there are two a's in a row.

1.b

$$L(\beta) = b^*(ab)^*(a+\epsilon)(ab+ba)^*(a+\epsilon)$$

This regular expression correctly represents $L(\sim \alpha)$ as it properly accounts for all strings that contain a and b, that do not have the combinations aaa or $abb(b)^*$. This is correct because the string can have any number of b's from the start, because there is no possibility for an a beforehand, allowing for any number of b's to be present without running into $abb(b)^*$. The regular expression also has $(ab+ba)^*$ in order to account for any combination of a and b that does not have aaa or $abb(b)^*$. Finally, the regular expression also accounts for the fact that the string can also start and end with aa, without concatenating with b, or having a third a. This is done by having two $(a+\epsilon)$'s, but also being strategic with where they are placed, as to not encounter instances of $aaab$ or $baaa$.

2



2.a

$$A_{q3,q0}^{\{q1,q2\}} = ca + bc$$

2.b

$$A_{q0,q3}^{\{q1,q2\}} = ba + ac$$

2.c

$$A_{q0,q0}^{\{q1,q2\}} = ba + ac$$

2.d

$$\begin{aligned}
A_{q3,q3}^{\{q1,q2\}} &= ca + bc \\
A_{q0,q0}^{\{q1,q2,q3\}} &= A_{q0,q0}^{\{q1,q2\}} + A_{q0,q3}^{\{q1,q2\}}(A_{q3,q3}^{\{q1,q2\}})^* A_{q3,q0}^{\{q1,q2\}} \\
&= (ba + ac) + (ba + ac)(ca + bc)^*(ca + bc)
\end{aligned}$$

2.e

$$\begin{aligned}
A_{q3,q3}^{\{q1,q2,q3\}} &= a + ca + bc \\
A_{q0,q0}^{\{q1,q2,q3\}} &= A_{q0,q0}^{\{q1,q2\}} + A_{q0,q3}^{\{q1,q2\}}(A_{q3,q3}^{\{q1,q2,q3\}})^* A_{q3,q0}^{\{q1,q2\}} \\
&= (ba + ac) + (ba + ac)(a + ca + bc)^*(ca + bc)
\end{aligned}$$

3

We are given the language:

$$A = \{xx | x \in \{0,1\}^*\}$$

We can use pumping lemma to determine whether or not A is regular. We will assume A is a regular language. We will let P be the pumping length and $S = abc$ be a string in A such that $|S| \geq P$. For A to be regular, it must follow the conditions:

1. $ab^ic \in A$ for all $i \geq 0$
2. $|b| > 0$
3. $|ab| \leq P$

Given the conditions for the languauge A, as x is concatenated with itself, S must always be of the form $x^P x^P$. We will choose string $S = 0^P 0^P$ as example. In this case, $|S| = 2P$, meaning $|S| \geq P$ holds trivially. Now we must split S into components abc:

$$S = abc$$
$$abc = 0^P 0^P$$

Looking at the conditions of the pumping lemma, $|ab| \leq P$, meaning that ab is a substring of 0^P. This means that we can express a and b as:

$$a = 0^{P-k}$$
$$b = 0^k$$

Where $k > 0$ as $|b| > 0$. Now, we need to pump b for some $i \geq 0$. We can use $i = 2$ as an example:

$$\begin{aligned} ab^2c &= 0^{P-k}0^{2k}0^P \\ &= 0^{P+k}0^P \end{aligned}$$

As the length of the first half of the string is not the same as the length of the second half, this means that the string is not expressed in the form xx for some $x \in \{0,1\}^*$. As it contradicts the first condition of the pumping lemma, this means that A is not a regular language.