# COMPSCI 2AC3 Assignment 3

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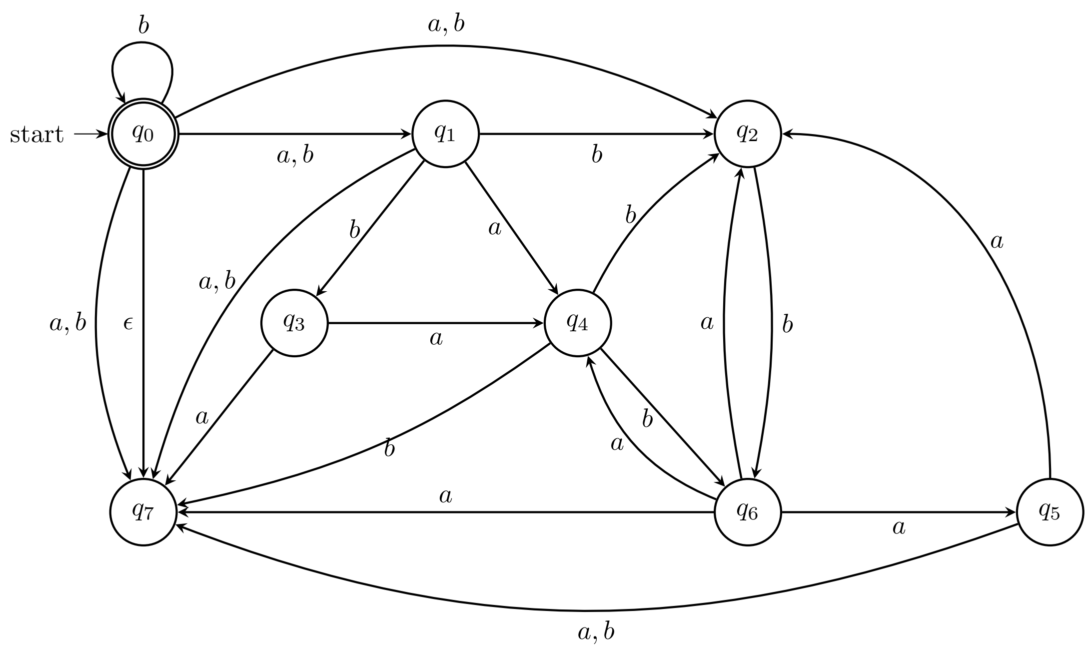
February 25, 2026

## 1.a



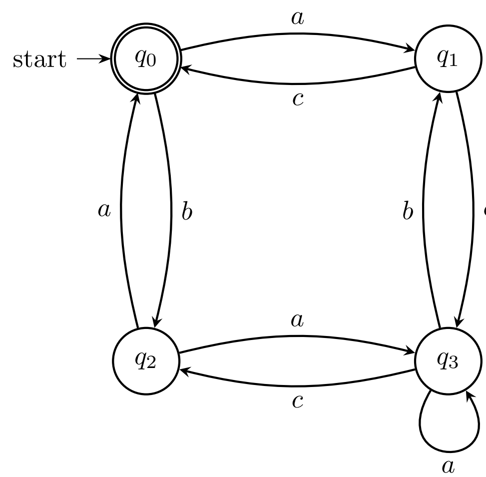

The NFA is correct because it accounts for all possible combinations of $a$ and $b$ that do not have the combinations $aaa$ nor $abb$ with zero or more $b$'s trailing. The NFA accounts for when these combinations may occur in a string, as for example $abb$ may occur at an even or odd index within the string. The NFA also accounts for the fact that $abb$ is only an acceptable string if it is followed by $a$, as this would invalidate the criteria for $abb(b)^*$. Finally, the NFA also accounts for the fact that there can be at most two $a$'s in a row, and forces the NFA to transition using $b$, or accepting state is there are two $a$'s in a row.

## 1.b

$$L(\beta) = b^*(ab)^*(a+\epsilon)(ab+ba)^*(a+\epsilon)$$

This regular expression correctly represents $L(\sim \alpha)$ as it properly accounts for all strings that contain $a$ and $b$, that do not have the combinations $aaa$ or $abb(b)^*$. This is correct because the string can have any number of $b$'s from the start, because there is no possibility for an $a$ beforehand, allowing for any number of $b$'s to be present without running into $abb(b)^*$. The regular expression also has $(ab+ba)^*$ in order to account for any combination of $a$ and $b$ that does not have $aaa$ or $abb(b)^*$. Finally, the regular expression also accounts for the fact that the string can also start and end with $aa$, without concatenating with $b$, or having a third $a$. This is done by having two $(a+\epsilon)$'s, but also being strategic with where they are placed, as to not encounter instances of $aaab$ or $baaa$.

## 2




### 2.a

$$A^{\{q1,q2\}}_{q3,q0} = ca + bc$$

### 2.b

$$A^{\{q1,q2\}}_{q0,q3} = ba + ac$$

### 2.c

$$A^{\{q1,q2\}}_{q0,q0} = ba + ac$$

### 2.d

$$\begin{aligned}
A^{\{q1,q2\}}_{q3,q3} &= ca + bc \\
A^{\{q1,q2,q3\}}_{q0,q0} &= A^{\{q1,q2\}}_{q0,q0} + A^{\{q1,q2\}}_{q0,q3}(A^{\{q1,q2\}}_{q3,q3})^* A^{\{q1,q2\}}_{q3,q0} \\
&= (ba + ac) + (ba + ac)(ca + bc)^*(ca + bc)
\end{aligned}$$

### 2.e

$$\begin{aligned}
A^{\{q1,q2,q3\}}_{q3,q3} &= a + ca + bc \\
A^{\{q1,q2,q3\}}_{q0,q0} &= A^{\{q1,q2\}}_{q0,q0} + A^{\{q1,q2\}}_{q0,q3}(A^{\{q1,q2,q3\}}_{q3,q3})^* A^{\{q1,q2\}}_{q3,q0} \\
&= (ba + ac) + (ba + ac)(a + ca + bc)^*(ca + bc)
\end{aligned}$$

# 3

We are given the language:

$$A = \{xx | x \in \{0,1\}^*\}$$

We can use pumping lemma to determine whether or not A is regular. We will assume A is a regular language. We will let $P$ be the pumping length and $S = abc$ be a string in $A$ such that $|S| \geq P$. For $A$ to be regular, it must follow the conditions:

1. $ab^ic \in A$ for all $i \geq 0$
2. $|b| > 0$
3. $|ab| \leq P$

Given the conditions for the languauge $A$, as $x$ is concatenated with itself, $S$ must always be of the form $x^P x^P$. We will choose string $S = 0^P 0^P$ as example. In this case, $|S| = 2P$, meaning $|S| \geq P$ holds trivially. Now we must split $S$ into components $abc$:

$$S = abc$$
$$abc = 0^P 0^P$$

Looking at the conditions of the pumping lemma, $|ab| \leq P$, meaning that $ab$ is a substring of $0^P$. We can then express the components $a$ and $b$ using $k$:

$$a = 0^{P-k}$$
$$b = 0^k$$

Where $k > 0$ as $|b| > 0$. Now, we need to pump $b$ for some $i \geq 0$. We can use $i = 2$ as an example:

$$\begin{aligned} ab^2c &= 0^{P-k}(0^k)^2 0^P \\ &= 0^{P-k} 0^{2k} 0^P \\ &= 0^{P+k} 0^P \end{aligned}$$

As the length of the first half of the string is not the same as the length of the second half of the string when $i = 2$, this shows that the string is not expressed in the form $xx$ for some $x \in \{0,1\}^*$. As this contradicts the first condition of the pumping lemma, this means that $A$ is not a regular language.