

Errata-Corrige from Lecture1:

To install packages in anaconda on ECB machines, on the anaconda prompt run

```
conda config --set ssl_verify false
```

then you can run `conda install your-package-name`

Solutions to the assignments:

1.

```
In [3]: def Pascal(n):  
        row = [1]  
        T=[row]  
        for _ in range(n):  
            row=[l+r for l,r in zip(row+[0], [0]+row)]  
            T.append(row)  
        return T  
Pascal(6)
```

```
Out[3]: [[1],  
         [1, 1],  
         [1, 2, 1],  
         [1, 3, 3, 1],  
         [1, 4, 6, 4, 1],  
         [1, 5, 10, 10, 5, 1],  
         [1, 6, 15, 20, 15, 6, 1]]
```

```
In [4]: def bin_exp(x,y,n):  
        return sum([x**(n-k) * y**k *coeff for k,coeff in zip(range(n+1),Pascal(n)[-1  
        ]))]  
def verify_binomial_theorem(x,y,n):  
    return bin_exp(x,y,n) == (x+y)**n  
verify_binomial_theorem(253,28,52)
```

```
Out[4]: True
```

2.

```
In [5]: #sum lines form the bottom to the top and maximise sums
def solution(A):
    A=[ [int(d) for d in str(n)] for n in A] #list becomes list of lists
    while len(A) > 1:
        e1=A[-1]#last level
        e2=A[-2]#penultimate level
        s1=[e1[n] +e2[n] for n in range(len(e2))] #sum below
        s2=[e1[n+1]+e2[n] for n in range(len(e2))] #sum below right
        MS=[max(a,b) for a,b in zip(s1,s2)] #max of possible sums
        A[-2]=MS #Replace penultimate line with MS
        A.pop() #Remove last line
    return A[0][0]
```

```
In [6]: #####Generate long triangle
def gen_T(L):
    from random import randint, seed
    seed(100)
    T=[];
    for n in range(L):
        T.append(randint(10**(n) ,10**(n+1)-1))
    return T
```

```
In [7]: solution([7,38,810,2744,45265])
```

```
Out[7]: 30
```

```
In [8]: solution(gen_T(50))
```

```
Out[8]: 333
```

```

In [9]: #redefine solution to keep track of maxima
def solution(A):
    A=[[int(d) for d in str(n)] for n in A]
    global M
    M=[]
    for l in range(len(A)-1):
        e1=A[-1]#last level
        e2=A[-2]#penultimate level
        s1=[e1[n]+e2[n] for n in range(len(e2))] #sum below
        s2=[e1[n+1]+e2[n] for n in range(len(e2))] #sum below right
        MS=[max(a,b) for a,b in zip(s1,s2)] #max of possible sums
        A[-2]=MS #Replace penultimate line with MS
        A.pop() #Remove last line
        M.append(MS)
    return A[0][0]
def find_path(A):
    global M
    S=solution(A)
    A=[[int(d) for d in str(n)] for n in A]
    ch_el=[M[-1][0]];path=[1]
    for n in range(2,len(M)+1):
        if M[-n][path[-1]-1]>M[-n][path[-1]]:
            path.append(path[-1])
        else:
            path.append(path[-1]+1)
    #add last element of path
    if A[-1][path[-1]-1]>A[-1][path[-1]]:
        path.append(path[-1])
    else:
        path.append(path[-1]+1)
    return path
#function to print triangle out of vector and path
def print_sol_tree(A, start = 0):
    S=solution(A)
    path=find_path(A)
    sm=0;

```

```

    for n in range(len(A)):
        D=[int(d) for d in str(A[n])];
        for d in range(len(D)):
            if d+1==path[n]:
                if n>=start:
                    print("\x1b[31m"+str(D[d])+"\x1b[0m",end="", flush=True)
                    sm+=D[d]
                elif n>=start:
                    print(D[d],end="", flush=True)
            if n>=start:
                print()
        if n>=start:
            print(" "*path[-2]+"\x1b[1;31m"+str(S)+"\x1b[0m")
        if sm==S and n>=start:
            print(sm==S)
        else:
            print(sm==S)
            print("Error!")
    return sm==S
cond=print_sol_tree(gen_T(50),0)

```

3
 68
 565
 3863
 61515
 867514
 6867610
 68185322
 644242175
 4436061708
 25173696048
 907798109191
 5631933017735
 38780348552268
 358622896049262
 8556180242414099
 30295924558305991

526652620926830762
4778751146409500022
43847364149707529459
652592522033620805229
9509036238695415327786
66683487566751174353559
884039315950094523656025
2747111867082114488982338
71203172009075049610999007
338477573475126558386533925
1258826705890267193204111302
99259652769278215515445130201
176734376896159134659564783099
731585522450074888886893280825
64951890914616622368685328430331
804127876779347044119691539169541
1643313783827797330974154076903364
59829434940177015637734372146571438
418744425182106254762324765268583738
8517027184494201168094853201776174995
19905495404807847318174851463000018942
211190092620649635838104821711183330845
8726513375633751610457942716768734238283
89399357577698226976873855548857615377619
368644597566770658191227891180199031930058
7641876277040658233068710319771596314829353
37027850249890306589849593995595782274793195
780384603136050422247697990613367538901646873
6083671798297876918069677488887949895817468912
89731873933709622126568196609975917250560892768
304579873520784682720034826644804968407591091808
1367712352974592097373652119073990828810352961386
59744921223361202330047087747395949536721275921715

333

True

A Python Lecture Series

Lecture 2

by Luca Mingarelli

Lecture 2: split into 2.1 and 2.2

Content:

- I/O
- Modules
- NumPy and SciPy
- Pandas
- Matplotlib
- Importing data from the web

Input/Output

To write in a file:

```
In [10]: f = open('a_work_file', 'w') # opens the file workfile
         #more specifically it creates a file object
         f.write('This is a test\n')
         for n in range(5):
             f.write(str(n)+'\n')
         f.close()
```

```
In [11]: !ls #notice a new file!
```

ECB Python Lectures - Lecture 2.1.ipynb a_work_file

To read from a file:

```
In [12]: f = open('a_work_file', 'r')
s = f.read()
print(s)
f.close()
```

This is a test

0
1
2
3
4

Iterating over a file

For reading lines from a file, you can loop over the file object. This is memory efficient, fast, and leads to simple code.

```
In [13]: f = open('a_work_file', 'r')
         for line in f:
             print(line, end = ' ')
         f.close()
```

This is a test

0
1
2
3
4

File modes

- Read-only: r
- Write-only: w
 - Note: Create a new file or overwrite existing file.
- Append to a file: a
- Read and Write: r+
- Binary mode: b

It is good practice to use the `with` keyword when dealing with file objects. The advantage is that the file is properly closed after its suite finishes, even if an exception is raised at some point (using `with` is also much shorter than writing equivalent try-finally blocks).

```
In [14]: with open('A_new_test', 'w') as f:
          f.write('This is a NEW test\n\n')
          for n in range(6):
              f.write(f'{n} squared is {n**2}\n') # A formatted string - notice the 'f'
                                                  at the beginning of the string
```

```
In [15]: with open('A_new_test', 'r') as f:
          print(f.read())
```

This is a NEW test

0 squared is 0
1 squared is 1
2 squared is 4
3 squared is 9
4 squared is 16
5 squared is 25

Modules

i.e. how to write reusable code

A module is a file containing Python definitions and statements. The file name is the module name with the suffix `.py` appended.

```
In [16]: %%writefile my_new_module.py

def a_complicated_function():
    print("Working... Done.")
```

Writing my_new_module.py

```
In [17]: import my_new_module as mnm
mnm.a_complicated_function()
```

Working... Done.

```
In [18]: !mkdir MODULES
```

```
In [19]: %%writefile MODULES/module2.py
def function2():
    print("Working... Done.")
```

Writing MODULES/module2.py

We could now call this as `MODULES.module2.function2`. However, to make our life easier we can instead write the following `__init__.py` file (notice the `!`):

```
In [20]: %%writefile MODULES/__init__.py
from .module2 import function2
```

Writing MODULES/__init__.py

```
In [21]: import MODULES as MD
MD.function2()
```

Working... Done.

Most of the useful operations needed for scientific computing are contained within some module (e.g. **NumPy**, **SciPy**, etc.).

This means that in order to access them we will need to import that module as

- `import module,`

or giving it an alias as

- `import module as md.`

Then we will be able to call the function as `module.specific_function()` or as `md.specific_function()`. Alternatively we can import the required tool/function as

- `from module import specific_funtion.`

NumPy and its arrays

NumPy provides an efficient extension package to Python for multidimensional arrays.

```
In [22]: import numpy as np
x = np.array([1,2,3])
# convert list to numpy array object
x
```

```
Out[22]: array([1, 2, 3])
```

```
In [23]: ###--- Notice that
x+x
###--- More on this later.
```

```
Out[23]: array([2, 4, 6])
```

```
In [24]: x = np.linspace(0,10,11) # as in Matlab!
x
```

```
Out[24]: array([ 0.,  1.,  2.,  3.,  4.,  5.,  6.,  7.,  8.,  9., 10.])
```

```
In [25]: x = np.arange(1.5,10,2) # same as Matlab's [1.5:2:10]
x
```

```
Out[25]: array([1.5, 3.5, 5.5, 7.5, 9.5])
```

NumPy's number types and associated risk (overflow)

```
In [26]: x=np.array([0,1])  
print("x =",x,"and has dtype",x.dtype)
```

x = [0 1] and has dtype int64

```
In [27]: x=np.array([0,1],dtype = np.int8)  
x[:] = 2**7-1  
print("x =",x,"and has dtype",x.dtype)
```

x = [127 127] and has dtype int8

```
In [28]: print("x + 1 =",x + 1,"\t (because dtype is"  
          ,x.dtype, "!)")
```

x + 1 = [-128 -128] (because dtype is int8 !)

```
In [29]: x=np.array([2**63-1,2**63-1])  
print("x[0] =",x[0],"and has dtype",x.dtype)  
sum(x)
```

x[0] = 9223372036854775807 and has dtype int64

/anaconda/lib/python3.6/site-packages/ipykernel_launcher.py:3: RuntimeWarning:
overflow encountered in long_scalars

This is separate from the ipykernel package so we can avoid doing imports un
til

Out[29]: -2

Some of NumPy arrays' attributes and methods

```
In [30]: x.ndim
```

```
Out[30]: 1
```

```
In [31]: x.shape
```

```
Out[31]: (2,)
```

```
In [32]: len(x)
```

```
Out[32]: 2
```

```
In [33]: x = np.array([[0, 1, 2], [3, 4, 5]])    # 2 x 3 array
```

```
In [34]: x.shape
```

```
Out[34]: (2, 3)
```

```
In [35]: x.mean() ## an object's method
```

```
Out[35]: 2.5
```

```
In [36]: x.dtype ## data type
```

```
Out[36]: dtype('int64')
```

```
In [37]: print("itemsize:", x.itemsize, "bytes")  
         print("nbytes:", x.nbytes, "bytes")
```

```
itemsize: 8 bytes  
nbytes: 48 bytes
```

Higher dimensional arrays

```
In [38]: np.zeros((2,3))
```

```
Out[38]: array([[0., 0., 0.],  
               [0., 0., 0.]])
```

```
In [39]: np.ones((2,2))
```

```
Out[39]: array([[1., 1.],  
               [1., 1.]])
```

```
In [40]: np.eye(3)
```

```
Out[40]: array([[1., 0., 0.],  
               [0., 1., 0.],  
               [0., 0., 1.]])
```

```
In [41]: np.diag(range(1,5))
```

```
Out[41]: array([[1, 0, 0, 0],  
               [0, 2, 0, 0],  
               [0, 0, 3, 0],  
               [0, 0, 0, 4]])
```

Indexing and Slicing

Recall: `x[start:stop:step]` ; when any is omitted the default values are `start=0` ,
`stop=size` , `step=1`

In [42]:

```
x
```

Out[42]: `array([[0, 1, 2],
 [3, 4, 5]])`

In [43]: `print('x[0] = ', x[0])
print('x[1] = ', x[1])`

```
x[0] = [0 1 2]  
x[1] = [3 4 5]
```

In [44]: `x[0][-1]`

Out[44]: `2`

In [45]: `x[0,-1]`

Out[45]: `2`

In [46]: `x[:,::-1]`

Out[46]: `array([[2, 1, 0],
 [5, 4, 3]])`

```
In [47]: x[::-1,:]
```

```
Out[47]: array([[3, 4, 5],  
               [0, 1, 2]])
```

```
In [48]: ## Notice the equivalence x[0] = x[0,:] for multidimensional arrays  
print(x[0,:]) # first row of x  
print(x[0])   # still first row of x
```

```
[0 1 2]  
[0 1 2]
```

IMPORTANT 1: Be carefull about the datatype:

```
In [49]: print('type: ',x.dtype)
x[:,:] = np.pi
x
```

type: int64

```
Out[49]: array([[3, 3, 3],
               [3, 3, 3]])
```

```
In [50]: y = x.astype(bool)
# y = y.astype(float)
print('type: ',y.dtype)
y[:,:] = np.pi
y
```

type: bool

```
Out[50]: array([[ True,  True,  True],
               [ True,  True,  True]])
```

IMPORTANT 2: Slices return views, NOT copies!

```
In [51]: x_slice = x[:2,:2]
x_slice
```

```
Out[51]: array([[3, 3],
               [3, 3]])
```



```
In [52]: x_slice[:] = 2  
x
```

```
Out[52]: array([[2, 2, 3],  
               [2, 2, 3]])
```

This behavior is quite useful: when working with large datasets, we can access and process pieces of these datasets without the need to copy the data.

Copying NumPy arrays

```
In [53]: x[:] = 3  
         x_slice_copy = x[:2, :2].copy()  
         x_slice_copy
```

```
Out[53]: array([[3, 3],  
               [3, 3]])
```

```
In [54]: x_slice_copy[:] = 2  
         x
```

```
Out[54]: array([[3, 3, 3],  
               [3, 3, 3]])
```

Reshaping

```
In [55]: x = np.array([1, 2, 3])  
         # reshape to row vector  
         x.reshape((1, 3))
```

```
Out[55]: array([[1, 2, 3]])
```

```
In [56]: # reshape to row vector  
         x.reshape((3, 1))
```

```
Out[56]: array([[1],  
                [2],  
                [3]])
```

Concatenation

```
In [57]: x = np.array([1, 2, 3])
          y = np.array([4, 5, 6])
          Z = np.concatenate([x, y])
          Z
```

```
Out[57]: array([1, 2, 3, 4, 5, 6])
```

```
In [58]: # for multidimensional arrays as well

          np.concatenate([Z, Z])
```

```
Out[58]: array([1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6])
```

Although `np.vstack` and `np.hstack` might be clearer:

```
In [59]: x = np.array([1, 2, 3])
          Z = np.array([[4, 5, 6],
                        [7, 8, 9]])
          # vertically stack the arrays
          np.vstack([x, Z])
```

```
Out[59]: array([[1, 2, 3],
                [4, 5, 6],
                [7, 8, 9]])
```

```
In [60]: # horizontally stack the arrays
y = np.array([[456],
               [789]])
np.hstack([z, y])
```

```
Out[60]: array([[ 4,  5,  6, 456],
                [ 7,  8,  9, 789]])
```

Use `np.dstack` to stack arrays along higher dimensional axis.

Splitting

```
In [61]: z1 = np.vstack([x, Z]).reshape((9,))
          print(z1)
          x, y, z = np.split(z1,[3,5])
          print(x,y,z)
```

```
[1 2 3 4 5 6 7 8 9]
[1 2 3] [4 5] [6 7 8 9]
```

```
In [62]: Z = np.arange(16).reshape((4, 4))
          print(Z)
```

```
[[ 0  1  2  3]
 [ 4  5  6  7]
 [ 8  9 10 11]
 [12 13 14 15]]
```

```
In [63]: upper, lower = np.vsplit(Z, [2])
          print('Upper part:\n',upper)
          print('-'*15)
          print('Lower part: \n',lower)
```

```
Upper part:
[[0 1 2 3]
 [4 5 6 7]]
-----
Lower part:
[[ 8  9 10 11]
 [12 13 14 15]]
```

```
In [64]: left, right = np.hsplit(Z, [2])
print('Left part:\n',left)
print('-'*15)
print('Right part:\n',right)
```

Left part:

```
[[ 0  1]
 [ 4  5]
 [ 8  9]
 [12 13]]
```

Right part:

```
[[ 2  3]
 [ 6  7]
 [10 11]
 [14 15]]
```

Operations on NumPy arrays

Whenever possible, avoid looping: it's slow!

Instead it is advisable to make use of **NumPy**'s built in functions. These are highly optimised and are applied elementwise.

```
In [65]: x = np.arange(-5,5)
         np.abs(x)
```

```
Out[65]: array([5, 4, 3, 2, 1, 0, 1, 2, 3, 4])
```

```
In [66]: x = np.linspace(1,2,1000)
         %timeit [1/x[n] for n in range(len(x))]
         %timeit 1/x
```

400 μ s \pm 16.8 μ s per loop (mean \pm std. dev. of 7 runs, 1000 loops each)

4.8 μ s \pm 89.4 ns per loop (mean \pm std. dev. of 7 runs, 100000 loops each)

The most common functions (trigonometric, exponentials, logarithms, etc.) can be found within **NumPy**. More specialised functions on the other hand, can be found in **SciPy**, within the sub-module `scipy.special`:

```
In [67]: from scipy import special
x = np.array([0.5, 1.])
print("Γ(x)=", special.gamma(x))
print("B(x,2)=", special.beta(x, 2))
print("erf(x)=", special.erf(x))
```

```
Γ(x)= [1.77245385 1.          ]
B(x,2)= [1.33333333 0.5        ]
erf(x)= [0.52049988 0.84270079]
```

More *special* mathematical functions can be found [here](https://docs.scipy.org/doc/scipy/reference/special.html#module-scipy.special)
(<https://docs.scipy.org/doc/scipy/reference/special.html#module-scipy.special>).

Even when computing aggregates: use NumPy's functions.

```
In [68]: x = np.arange(1000)
          %timeit sum(x)
          %timeit x.sum()
          %timeit np.sum(x) # the same as above!
```

```
122  $\mu$ s  $\pm$  1.51  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 10000 loops each)
5.38  $\mu$ s  $\pm$  98 ns per loop (mean  $\pm$  std. dev. of 7 runs, 100000 loops each)
7.42  $\mu$ s  $\pm$  153 ns per loop (mean  $\pm$  std. dev. of 7 runs, 100000 loops each)
```

```
In [69]: %timeit max(x)
          %timeit x.max()
          %timeit np.max(x) # the same as above!
```

```
88.3  $\mu$ s  $\pm$  1.69  $\mu$ s per loop (mean  $\pm$  std. dev. of 7 runs, 10000 loops each)
6.15  $\mu$ s  $\pm$  122 ns per loop (mean  $\pm$  std. dev. of 7 runs, 100000 loops each)
7.68  $\mu$ s  $\pm$  78.2 ns per loop (mean  $\pm$  std. dev. of 7 runs, 100000 loops each)
```

Same for max and min.

These operations can also be done along one axis only:

```
In [70]: print(Z)
print("\nSum columns:")
Z.sum(axis = 1 )
```

```
[[ 0  1  2  3]
 [ 4  5  6  7]
 [ 8  9 10 11]
 [12 13 14 15]]
```

Sum columns:

```
Out[70]: array([ 6, 22, 38, 54])
```

```
In [71]: print("Max along columns:")
print(Z.max(axis = 0 ))
print("\nMax along rows:")
print(Z.max(axis = 1 ))
```

Max along columns:

```
[12 13 14 15]
```

Max along rows:

```
[ 3  7 11 15]
```

A summary of available aggregation functions

Function Name	NaN-safe Version	Description
<code>np.sum</code>	<code>np.nansum</code>	Compute sum of elements
<code>np.prod</code>	<code>np.nanprod</code>	Compute product of elements
<code>np.mean</code>	<code>np.nanmean</code>	Compute mean of elements
<code>np.std</code>	<code>np.nanstd</code>	Compute standard deviation
<code>np.var</code>	<code>np.nanvar</code>	Compute variance
<code>np.min</code>	<code>np.nanmin</code>	Find minimum value
<code>np.max</code>	<code>np.nanmax</code>	Find maximum value
<code>np.argmin</code>	<code>np.nanargmin</code>	Find index of minimum value
<code>np.argmax</code>	<code>np.nanargmax</code>	Find index of maximum value
<code>np.median</code>	<code>np.nanmedian</code>	Compute median of elements
<code>np.percentile</code>	<code>np.nanpercentile</code>	Compute rank-based statistics of elements
<code>np.any</code>	N/A	Evaluate whether any elements are true
<code>np.all</code>	N/A	Evaluate whether all elements are true

Boolean operations

```
In [72]: x = np.array([1,2,3,4,5,6])  
x>3
```

```
Out[72]: array([False, False, False,  True,  True,  True])
```

Operator	Equivalent function
==	np.equal
<	np.less
>	np.greater
!=	np.not_equal
<=	np.less_equal
>=	np.greater_equal

Masks

A boolean array can be used to index which element to extract from a second array:

In [73]:

```
print(x)
print(x>3)
x[x>3]
```

```
[1 2 3 4 5 6]
[False False False  True  True  True]
```

Out[73]: array([4, 5, 6])

Fancy indexing

```
In [74]: l = np.array([1,2,3,4,5])  
         l[[0,2]]
```

```
Out[74]: array([1, 3])
```

```
In [75]: l[[-1,0,-2]]
```

```
Out[75]: array([5, 1, 4])
```

Moreover:

```
In [76]: ind = np.array([[3, 0],  
                          [4, 1]])  
         l[ind]
```

```
Out[76]: array([[4, 1],  
                [5, 2]])
```

When using fancy indexing, the output has the same shape as the index.

Broadcasting

Broadcasting is a feature allowing for binary operations to be performed on arrays with different shapes.

```
In [77]: x = np.array([0,1,2])  
         print(x+3)  
         print(x+np.array([3,3,3]))
```

```
[3 4 5]  
[3 4 5]
```

```
In [78]: M = np.ones((3, 3))  
         M+x
```

```
Out[78]: array([[1., 2., 3.],  
                [1., 2., 3.],  
                [1., 2., 3.]])
```



```
In [79]: y = x.reshape((3,1))
print('y=\n',y)
print('-'*10)
print('x+y=\n',x+y)
```

y=

```
[[0]
 [1]
 [2]]
```

x+y=

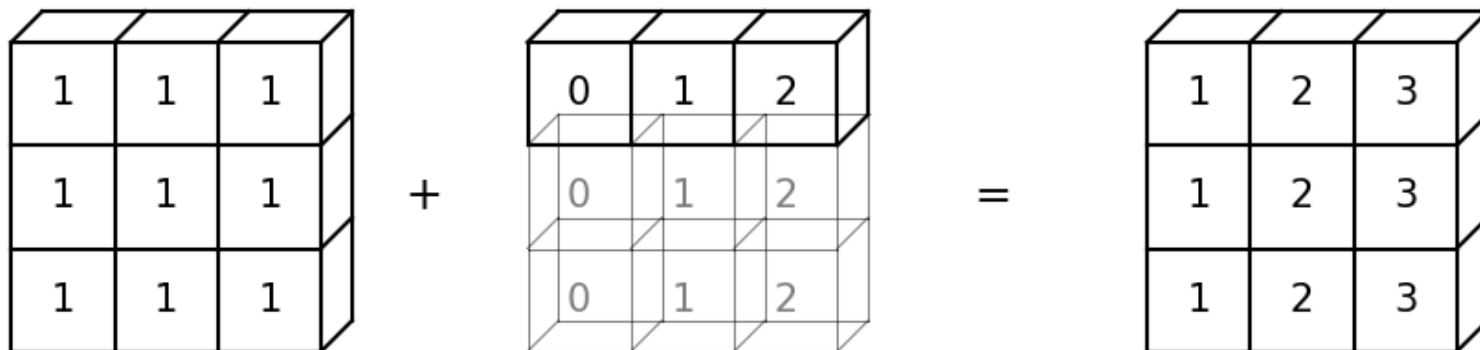
```
[[0 1 2]
 [1 2 3]
 [2 3 4]]
```

Rules of Broadcasting:

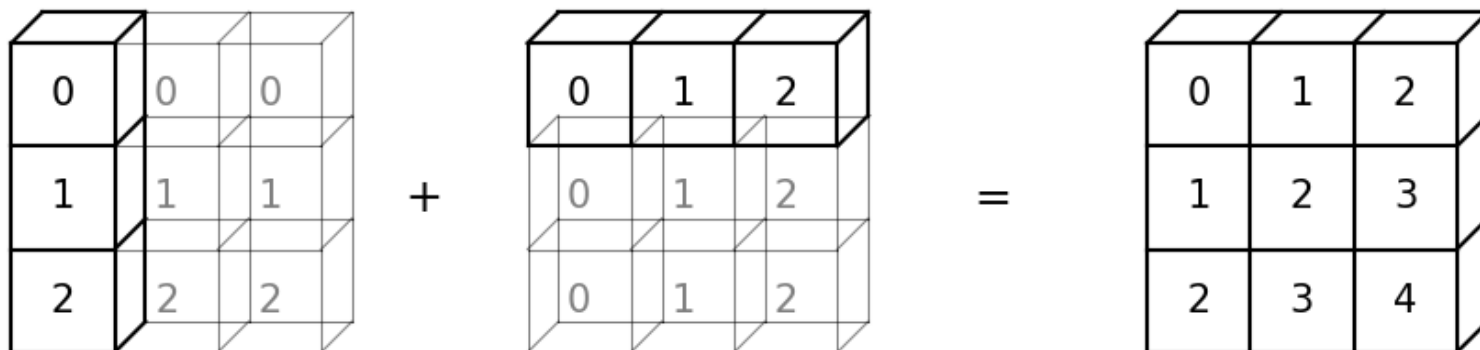
`np.arange(3) + 5`



`np.ones((3, 3)) + np.arange(3)`



`np.arange(3).reshape((3, 1)) + np.arange(3)`



Copy NumPy arrays (Deep-copy)

```
In [80]: A = np.arange(10)
         B = A
         B[0]= 100
         A
```

```
Out[80]: array([100,  1,  2,  3,  4,  5,  6,  7,  8,  9])
```

```
In [81]: A = np.arange(10)
         B = A.copy()
         B[0]=100
         A
```

```
Out[81]: array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
```

Pandas

Main data structures in Pandas:

- Series
- DataFrames

Series

```
In [82]: import pandas as pd  
         #from pandas import Series  
         pd.Series([0.1, 0.2, 0.3, 0.4, 0.5])
```

```
Out[82]: 0    0.1  
         1    0.2  
         2    0.3  
         3    0.4  
         4    0.5  
         dtype: float64
```

```
In [83]: s = pd.Series([0.1, 0.2, 0.3, 0.4, 0.5],  
                      index = ['a', 'b', 'c', 'd', 'e'])  
s #s[0] s['a'] s[s>2] s[:2] s[[3,4]]  
##i.e. ca be treated as nparrays
```

```
Out[83]: a      0.1  
        b      0.2  
        c      0.3  
        d      0.4  
        e      0.5  
dtype: float64
```

Be careful however about operations between different Series

```
In [84]: s1 = pd.Series({'a': 0.1, 'b': 1.2, 'c': 2.3})  
s2 = pd.Series({'a': 1.0, 'b': 2.0, 'c': 3.0})  
s3 = pd.Series({'c': 0.1, 'd': 1.2, 'e': 2.3})
```

```
In [85]: s1 + s2
```

```
Out[85]: a    1.1  
b    3.2  
c    5.3  
dtype: float64
```

```
In [86]: s1 + s3
```

```
Out[86]: a    NaN  
b    NaN  
c    2.4  
d    NaN  
e    NaN  
dtype: float64
```

```
In [87]: s1 = pd.Series([1,2,3],index=['a'] * 3)
s2 = pd.Series([4,5],index=['a'] * 2)
s1 + s2 #for non-unique indices: broadcasting to all common indices.
```

```
Out[87]: a      5
a      6
a      6
a      7
a      7
a      8
dtype: int64
```


It is possible to access the underlying arrays through the attributes `values` and `index`

```
In [88]: print(type(s3.values))  
s3.values
```

```
<class 'numpy.ndarray'>
```

```
Out[88]: array([0.1, 1.2, 2.3])
```

```
In [89]: s3.index = ['First', 'Second', 'Third']  
print(s3)  
s3.index[1]
```

```
First      0.1  
Second     1.2  
Third      2.3  
dtype: float64
```

```
Out[89]: 'Second'
```

```
In [90]: s = pd.Series([10,20,30],  
                        index=[13,2,89])  
## Now indexing is ambiguous!  
s[2]  
# s[0] # Error
```

```
Out[90]: 20
```

```
In [91]: s.iloc[0:2] ## s.iloc[0:2] ##i.e. slicing works
```

```
Out[91]: 13    10  
         2    20  
         dtype: int64
```

```
In [92]: s.loc[89] # s.loc[[13,89]]  
         ##i.e. fancy indexing works
```

```
Out[92]: 30
```

Notable Methods of the `Series` data structure

Accessed as `my_series.method()`

Name	Description
<code>head()</code> and <code>tail()</code>	Display the first five and the last five rows respectively (first/last n rows if n is given as an argument)
<code>isnull()</code>	Returns a Series with same indices and boolean values indicating where the values are NaNs or Nulls
<code>notnull()</code>	Negation of <code>isnull()</code>
<code>iloc()</code>	Access integer location of a Series
<code>loc()</code>	Access location according to indexing of the Series
<code>describe()</code>	Returns summary and statistics of the Series
<code>unique()</code>	Returns the unique elements of a Series
<code>drop(index)</code>	Drops elements with the selected index
<code>dropna()</code>	Drops all NaNs and Nulls elements
<code>fillna(value)</code>	Fills all NaNs and Nulls with value
<code>append(series)</code>	Appends a Series to another Series

DataFrame

Dataframes are a collection of Series .

```
In [93]: df = pd.DataFrame(np.array([[1,2],[3,4]]))  
df
```

```
Out[93]:
```

	0	1
0	1	2
1	3	4

```
In [94]: df.columns = ['col1', 'col2']  
df.index = ['row1', 'row2']  
df
```

```
Out[94]:
```

	col1	col2
row1	1	2
row2	3	4

```
In [95]: pd.DataFrame(np.array([[1,2],[3,4]]), columns=['col1', 'col2'], index = ['row1', 'row2'])
```

```
Out[95]:
```

	col1	col2
row1	1	2
row2	3	4

```
In [96]: s1 = pd.Series(np.arange(0,5))
s2 = pd.Series(np.arange(1,4))
s3 = pd.Series(np.arange(2,3))
pd.DataFrame({'col1': s1, 'col2': s2, 'col3': s3})
```

Out[96]:

	col1	col2	col3
0	0	1.0	2.0
1	1	2.0	NaN
2	2	3.0	NaN
3	3	NaN	NaN
4	4	NaN	NaN

```
In [97]: df = pd.DataFrame({'col'+str(1+i):pd.Series(np.arange(i,5.0-i)) for i in range(3)})#np.random.randint(0,3,3)
```

```
In [98]: df.describe()
```

Out[98]:

	col1	col2	col3
count	5.000000	3.0	1.0
mean	2.000000	2.0	2.0
std	1.581139	1.0	NaN
min	0.000000	1.0	2.0
25%	1.000000	1.5	2.0
50%	2.000000	2.0	2.0
75%	3.000000	2.5	2.0
max	4.000000	3.0	2.0

```
In [99]: df.sum() ### NaN automatically diregarded!
```

Out[99]:

col1	10.0
col2	6.0
col3	2.0
dtype:	float64

Selecting columns ...

```
In [100]: print(df['col1'])  
          print(type(df['col1']))
```

```
0    0.0  
1    1.0  
2    2.0  
3    3.0  
4    4.0  
Name: col1, dtype: float64  
<class 'pandas.core.series.Series'>
```

```
In [101]: print(df[['col1', 'col3']])  
          print(type(df[['col1', 'col3']]))
```

```
   col1  col3  
0    0.0    2.0  
1    1.0   NaN  
2    2.0   NaN  
3    3.0   NaN  
4    4.0   NaN  
<class 'pandas.core.frame.DataFrame'>
```

... selecting rows...

In [102]:

```
df[2:4]
```

Out[102]:

	col1	col2	col3
2	2.0	3.0	NaN
3	3.0	NaN	NaN

...and of course: selecting rows and columns...

In [103]:

```
df[2:4][['col2']]
```

Out[103]:

	col2
2	3.0
3	NaN

...deleting columns...

```
In [104]: df2 = df.copy() #Recall the `issue` in numpy?  
del df2['col2']  
df2
```

```
Out[104]:
```

	col1	col3
0	0.0	2.0
1	1.0	NaN
2	2.0	NaN
3	3.0	NaN
4	4.0	NaN

```
In [105]: df2.pop('col1')
```

```
Out[105]:
```

0	0.0
1	1.0
2	2.0
3	3.0
4	4.0

Name: col1, dtype: float64

```
In [106]: df2
```

```
Out[106]:
```

	col3
0	2.0
1	NaN
2	NaN
3	NaN
4	NaN

```
In [107]: df2 = df.drop(['col1', 'col3'], axis = 1)
df2
```

```
Out[107]:
```

	col2
0	1.0
1	2.0
2	3.0
3	NaN
4	NaN

```
In [108]: df
```

```
Out[108]:
```

	col1	col2	col3
0	0.0	1.0	2.0
1	1.0	2.0	NaN
2	2.0	3.0	NaN
3	3.0	NaN	NaN
4	4.0	NaN	NaN

Data import with Pandas

CSV files ([pandas.read_csv](#))

Comma-separated value files can be easily read using `pandas.read_csv`:

```
csv_data = pd.read_csv('file.csv')
```

Excel files (https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.read_excel.html)

```
csv_data = pd.read_excel('file.xlsx')
```

`pandas.read_excel` requires two arguments: the name of the file and the name of the sheet.

Moreover, more optional arguments can be parsed to these functions to specify where to start reading from, how many rows to read, etc.

Additionally, `pd.read_stata`, `pd.read_sql`, `pd.read_json`, [and more](#) (<https://pandas.pydata.org/pandas-docs/stable/reference/io.html>).

What to do with missing data?

- None Missing data inside of dataframe of type object
- NaN Missing numerical data

```
In [109]: # None + 1  
np.nan +1
```

```
Out[109]: nan
```

```
In [110]: pd.Series([1, np.nan, 2, None])  
## Notice both the mapping None -> NaN  
## as well as int -> float
```

```
Out[110]: 0    1.0  
1    NaN  
2    2.0  
3    NaN  
dtype: float64
```

Detection of missing data

```
In [111]: df.count() #count non-missing elements
```

```
Out[111]: col1      5  
col2      3  
col3      1  
dtype: int64
```

```
In [112]: df.notnull() # opposite: df.isnull()
```

```
Out[112]:
```

	col1	col2	col3
0	True	True	True
1	True	True	False
2	True	True	False
3	True	False	False
4	True	False	False

```
In [113]: df['col2'][df['col2'].notnull()]
```

```
Out[113]: 0      1.0  
1      2.0  
2      3.0  
Name: col2, dtype: float64
```

Dropping missing values

```
In [114]: df.dropna()  
## drops all rows  
## with at least one missing value
```

```
Out[114]:
```

	col1	col2	col3
0	0.0	1.0	2.0

```
In [115]: df.dropna(axis='columns')
```

```
Out[115]:
```

	col1
0	0.0
1	1.0
2	2.0
3	3.0
4	4.0

Filling missing values

```
In [116]: df.fillna(0)
```

Out[116]:

	col1	col2	col3
0	0.0	1.0	2.0
1	1.0	2.0	0.0
2	2.0	3.0	0.0
3	3.0	0.0	0.0
4	4.0	0.0	0.0

```
In [117]: # forward-fill  
df.fillna(method='ffill') #bfill for back-fill
```

Out[117]:

	col1	col2	col3
0	0.0	1.0	2.0
1	1.0	2.0	2.0
2	2.0	3.0	2.0
3	3.0	3.0	2.0
4	4.0	3.0	2.0

```
In [118]: # change axis  
df.fillna(method='ffill',axis = 1)
```

Out[118]:

	col1	col2	col3
0	0.0	1.0	2.0
1	1.0	2.0	2.0
2	2.0	3.0	3.0
3	3.0	3.0	3.0
4	4.0	4.0	4.0

```
In [119]: df = pd.DataFrame({"A": [12, 4, 5, None, 1],  
                             "B": [None, 2, 54, 3, None],  
                             "C": [20, 16, None, 3, 8],  
                             "D": [14, 3, None, None, 6]})  
  
df
```

Out[119]:

	A	B	C	D
0	12.0	NaN	20.0	14.0
1	4.0	2.0	16.0	3.0
2	5.0	54.0	NaN	NaN
3	NaN	3.0	3.0	NaN
4	1.0	NaN	8.0	6.0


```
In [120]: # to interpolate the missing values  
df.interpolate(method='linear', limit_direction='forward', axis=1)
```

Out[120]:

	A	B	C	D
0	12.0	16.0	20.0	14.0
1	4.0	2.0	16.0	3.0
2	5.0	54.0	54.0	54.0
3	NaN	3.0	3.0	3.0
4	1.0	4.5	8.0	6.0

Alternatively:

- `linear`: Ignore the index and treat the values as equally spaced.
- `time`: Works on daily and higher resolution data to interpolate given length of interval.
- `index, values`: use the actual numerical values of the index.
- `pad`: Fill in NaNs using existing values.
- `nearest, zero, slinear, quadratic, cubic, spline, barycentric, polynomial`
- `krogh, piecewise_polynomial, spline, pchip, akima`

[More here \(https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.DataFrame.interpolate.html\)](https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.DataFrame.interpolate.html).

Probability and Statistics

Random generators

```
In [121]: import random as rnd  
rnd.random() ## Uniform in [0,1)
```

```
Out[121]: 0.42632677014265585
```

```
In [122]: # uniform in range  
rnd.uniform(1,10)
```

```
Out[122]: 5.054553636111223
```

```
In [123]: #simulate die  
rnd.randint(1,6)
```

```
Out[123]: 6
```

```
In [124]: greetings = ['Hi', 'Hello', 'Welcome!', 'Hola']  
rnd.choice(greetings)
```

```
Out[124]: 'Hola'
```

```
In [125]: #Simulate wheel spins  
colors = ['R', 'B', 'G'] # Red, Black and Green  
rnd.choices(colors, weights=[18,18,2] ,k =10)
```

```
Out[125]: ['B', 'B', 'B', 'R', 'B', 'B', 'R', 'G', 'R', 'G']
```

```
In [126]: # Shuffle cards
deck = list(range(1,53)) ## 52 cards
rnd.shuffle(deck)
print(deck)
```

```
[28, 23, 50, 8, 32, 47, 5, 51, 10, 52, 16, 43, 9, 49, 14, 29, 27, 39, 34, 40,
33, 22, 11, 12, 38, 41, 35, 30, 24, 31, 21, 42, 37, 6, 44, 26, 48, 19, 36, 2,
3, 20, 45, 1, 18, 25, 4, 15, 13, 7, 46, 17]
```

```
In [127]: #Sample a hand from the deck
hand = rnd.sample(deck,k=5)
print(hand)## only unique values
```

```
[9, 49, 16, 36, 48]
```

NumPy random generators

```
In [128]: import numpy.random as rnd
```

```
In [129]: ## UNIFORM
print(rnd.rand(3,4))
```

```
[[0.93695749 0.274056    0.53011925 0.97679767]
 [0.7436092  0.03851414 0.27102461 0.41150143]
 [0.27627046 0.35089043 0.99386763 0.03871757]]
```

```
In [130]: ## STANDARD NORMAL
print(rnd.randn(3,4))
```

```
[[ -1.52295668 -0.13276398 -0.5029103   1.83349    ]
 [ -1.37253978  0.41098531  0.92069655 -0.33936542]
 [ -0.04344644  2.43514737  1.31225644  1.47672998]]
```

```
In [131]: ## UNIFORM INTEGERS
print(rnd.randint(0,100,(3,4)))
```

```
[[16 17 36 22]
 [61 66 21 13]
 [ 9 91  2 71]]
```

```
In [132]: rnd.shuffle(deck)
print(deck)
```

```
[19, 13, 23, 36, 22, 7, 35, 43, 32, 30, 27, 34, 52, 51, 40, 12, 2, 20, 50, 38,
17, 29, 24, 8, 25, 45, 11, 31, 15, 41, 18, 28, 1, 37, 49, 9, 14, 42, 46, 16, 4
4, 47, 3, 5, 39, 48, 21, 26, 10, 4, 33, 6]
```

Function	Description
<code>uniform(a, b, k)</code>	Returns k draws from $U(a, b)$.
<code>normal(μ, σ, k)</code>	Returns k draws from $\mathcal{N}(\mu, \sigma)$.
<code>multivariate_normal(μ, Σ, k)</code>	Returns k draws from $\mathcal{N}(\vec{\mu}, \Sigma)$.
<code>lognormal(μ, σ, k)</code>	Returns k draws from $\text{LogNormal}(\mu, \sigma)$.
<code>standard_t(ν, k)</code>	Returns k draws from Student-t(ν).
<code>chisquare(ν, k)</code>	Returns k draws from χ^2_ν .
<code>poisson(λ, k)</code>	Returns k draws from $\text{Poisson}(\lambda)$.
<code>binomial(n, p, k)</code>	Returns k draws from $B(n, p)$.
<code>binomial(1, p, k)</code>	Returns k draws from $\text{Bernoulli}(p)$.
<code>multinomial(n, p, k)</code>	Returns k draws from $\text{Multinomial}(n, \vec{p})$ (n trials, and a list of probabilities p).
<code>exponential(λ, k)</code>	Returns k draws from $\text{Exponential}(\lambda)$.
<code>f(ν_1, ν_2, k)</code>	Returns k draws from F_{ν_1, ν_2} .
<code>gamma(α, θ, k)</code>	Returns k draws from $\Gamma(\alpha, \theta)$ (α and θ the shape and scale parameters).
and more...	...

Note 1: call as `rnd.function_name(...)`.

Note 2: the argument `k` is optional.

Note 3: replace `k` with `(k, 1)` to obtain a $k \times l$ matrix instead.

More advanced statistical analysis packages

- [statsmodels](http://www.statsmodels.org/stable/index.html) (<http://www.statsmodels.org/stable/index.html>): mainly to estimate statistical models, and perform statistical tests. Includes: Linear Regression, Generalized Linear Models, Generalized Estimating Equations, Robust Linear Models, Linear Mixed Effects Models, Regression with Discrete Dependent Variables, ANOVA, Time Series analysis, Models for Survival and Duration Analysis, Statistics (e.g. Multiple Tests, Sample Size Calculations etc.), Nonparametric Methods, Generalized Method of Moments, Empirical Likelihood, ...
- [PyMC](http://pymc-devs.github.io/pymc/) (<http://pymc-devs.github.io/pymc/>): for Bayesian statistical models and fitting algorithms, including MCMC and Gaussian Processes.
- [scikit-learn](https://scikit-learn.org/stable/) (<https://scikit-learn.org/stable/>): for machine learning, data mining, and data analysis, including supervised and unsupervised learning. Includes tools for: Classification , Regression , Clustering , Dimensionality reduction , Model selection.

End of Lecture 2.1