1 Bonds

1.1 Price

$$P = \frac{(1-p)\times 10}{1+r_D} + \frac{(1-p)^2 \times 10}{(1+r_D)^2} + \frac{(1-p)^3 \times 110}{(1+r_D)^3}$$

Where p **is the annual probability of default and** r_D **is the required return on the bond.**

$$r_D = r_f + \beta_0 \left(r_m - r_f\right)$$

In case $p = 5\%, r_f = 10\%, \beta_D = 0.2$ and $r_m - r_f = 7.5\%$, we obtain $r_0 = 11.5\%$ **and** $P = 83.81$

Discount Serves to compensate investors for bearing risk of default and the associated losses

Negative risk-free Negative risk-free rate indicated investors pay the government for keeping their money safe. It's like a fee in a bank

Clean/dirty price Bond prices drop after coupon payments. To avoid "see-saw" pattern, they are **quoted net of accrued interest = clean price**

Clean price = Dirty price − next coupon $\times \frac{d}{\Delta}$, where d is the number of days since the last coupon payment and Δ is the number of days between the last payment and the next.

Dirty price evolves as

$$PV\left(1 + r\frac{d}{\Delta}\right)$$

Amplitude of **seesaw pattern is decreasing** because of lower present value closer to maturity, thus smaller increase in price over the period

1.2 Duration

The **longer the maturity** of the bond, the **more sensitive to interest rates**. If interest changes, the bond with longer maturity is **locked in longer**

Duration defintion It's a measure of the average life of the cash flow that make up the bond. Formula:

$$\frac{\frac{\text{coupon}}{1+r} + \frac{2\times\ \text{coupon}}{(1+r)^2} + \ldots + \frac{N\times\ \text{coupon}}{(1+r)^N} + \frac{N\times face}{(1+r)^N}}{P}$$

or:

$$\frac{\sum_{n=1}^{N} \frac{n\times\text{coupon}}{(1+r)^n} + \frac{N\times\ \text{face}}{(1+r)^N}}{P}$$

Each cash flow is **weighted by the year in which it is received**

- **Zero-coupon bonds** have **duration=maturity**
- a **higher coupon rate decreases duration** as we get more return earlier
- **higher discount rate decreaes duration**

Duration definition 2 duration is **negative, normalised first derivative of the P w.r.t. DR**

$$\text{Duration } = -\frac{1+r}{P}\frac{dP}{dr}$$

The preceding relation can be rewritten as:

$$\frac{dP}{P} = -\frac{\text{Duration}}{1+r} \times dr \equiv -\text{ Modified Duration } \times dr$$

Modified duration Modified duration **relates change in the DR to the resulting** *percent* **change in the price of the bond**

Absolute change in price for 1bp change in yield Val01

$$Val01 = dP(1bp) = modDur \times 0.0001 \times Price$$

Delta Heding to **avoid exposure to identical changes in yield**, position is such that

$$P_{bondA} = \frac{Val01_{bondA}}{Val01_{bondB}} P_{bondB}$$

1.3 Convexity

Convexity is **measure of non-linearity of a bond**

$$\text{Convexity } = \frac{\sum_{n=1}^{N} \frac{n\times(n+1)\times\ \text{coupon}}{(1+r)^{n+2}} + \frac{N\times(N+1)}{(1+r)^{N+2}}}{P}$$

Convexity **decreases with coupan rate** and thus **increases with duration**

Convexity definition Convexity is **normalised second derivative of the price w.r.p. to the discount rate**

$$\text{Convexity } = \frac{1}{\text{P}}\frac{\text{d}^2\text{P}}{\text{dr}^2}$$

We have the relation:

$$\frac{\text{dP}}{\text{P}} \approx -\text{ Modified Duration } \times \text{dr} + \frac{1}{2}\text{ Convexity } \times (\text{dr})^2$$

1.4 Immunisation

Techniques used to shield an asset or liability **from changes in interest rates**

$$D_A = \frac{E}{E+L}D_E + \frac{L}{E+L}D_L$$
$$\Leftrightarrow D_E = \frac{A}{E}\left[D_A - \frac{L}{A}D_L\right]$$
$$\Leftrightarrow D_E = D_A + \frac{L}{E}\left[D_A - D_L\right]$$

D_E increases in leverage (L/E) where $\text{D}_\text{A} > \text{D}_\text{L}$; it decreases otherwise.

A bank with **negative**

$$D_E$$

increaes equity with increase in interest rates

Immuisation of equity = structure its assets and liabilities such that

$$D_A = \frac{L}{A}D_L \Leftrightarrow D_E = 0$$

Alternatively, the bank can enter into a **futures position F** such:

$$FD_F = -ED_E = -(AD_A - LD_L)$$

1.5 Information in Bond Prices and YTM

Bond prices are determined by discounting promised cash flows by **opportunity cost of capital**. Thus, observing the prices, we can calculate the **opportunity cost of capital or YTM**

Yield to Maturity and Current Yield Yield to maturity **is measure of the return one obtains when holding to maturity**

YTM is measure of annual return **IF** we can reinvest at the same yield after first period bonds carry **reinvestment risk** that *coupon payments cannot be reinviested at the initial YTM*

$$\text{current/initial yield} = \frac{coupon}{price}$$

1.6 Risks

- liquidity risk (**"of the run"** bonds are super liquid, purchasing without changing price)
- default risk
- reinvestment risk

2 Kentish town capital

2.1 Set up:

- two bonds with different YTM, different maturity
- Δ YTM is not due to diff. maturity, nor liquidity (both off-the-run, buying and selling quickly doesn't affect the price)
- might be due to reinvestment risk
- they have difference in convexity

2.2 Arbitrage position

Go long the cheaper (higher yield) and short the dearer (lower yield) bond. **Bet on converging yields**.\

Size Size of the position is determined by delta hedging

$$P_{10.625,\text{ long}} \quad -\frac{\text{Val01 }_{10.625}}{\text{Val 01}_{4.25}}P_{4.25,short}$$

Profit If yields converge fully, i.e. move by $\frac{\Delta}{2} = \frac{35bp}{2}$, the profit is=

Long leg profit − Short leg profit = $Val01_long + Val01_short =$
$modDur_long \times$ move in bp $\times P_long - modDur_short \times$ move in bp $\times P_short$

Epilogue *"Convexity is useful when there is more volatility or people being scared"*

3 Bank One

3.1 Asset-Liability management

Banks might be **asset sensitive** ⇒ interest rates on assets reset more quickly than those on liabilities; increase (decrease) in interest rates increases (decreases) earnings. Banks might:

- bear that risk
- hedge it out (swaps, off-balance sheet)
- transform it (balancing assets)

Swap hedge If a bank has too many floating rate assets and too many long term fixed liabilities, it receives (pays) too little (much) because of upward yield curve and , it goes opposite by a swap →pays float short term and receives long term higher

Balance sheet transformation Raise new floating-rate funds to be invested in fixed rate (LT) securities. Requires additional capital ⇒ badly influences ROA, ROE. Also might not be possible market-wise.

Swaps are better in the regard they are off-balance sheet, thus only interest is concerned, no principal payments take place.

To calculate how much to transform, we perform immunitization.

4 Rose Tree

Shock to assets ⇒ lowers equity (absorber) ⇒ increase in leverage

In **absolute terms**, decline in assets = decline in equity.

In **relative terms**, it is bigger in equity:

$$\delta_E = \frac{\Delta A}{E} = \frac{\Delta A}{eA} = \frac{\Delta}{e} > \Delta = \delta_A$$

Where **e** is fraction of assets.

If the firm wants to reverse the increase in leverage byselling assets, it needs to sell total amount is equal to:

$$\text{e} = \frac{\text{E}-\Delta\text{A}}{\text{A}-\Delta\text{A}-\Sigma\text{A}} = \frac{\text{eA}-\Delta\text{A}}{\text{A}-\Delta\text{A}-\Sigma\text{A}}$$
$$\Rightarrow \Sigma = \left(\frac{1}{\text{e}} - 1\right)\Delta$$

Partial loan forgivness

Restructuring mortgage means forgiving a part to get at least something than nothing at all.

Moral hazard! Other borrowers might demand the same favorable treatment.

Discount rate obtained by CAPM. Asset beta of mortgage portfolio is averaging REIT equity betas like so:

$$\beta_A = \frac{E}{E+D}\beta_E$$

Which is the same as duration.

5 Securitisation

Packaging asset to **issue securities that are secured by the package of assets**

Individual secutiries are generally **more liquid and creditworthy** than individual assets that secure them. (Typically mortgages, credit card receivables, auto loans)

5.1 What people did not realize

Securitisation **reduces risk** between the individual assets, but **increases risk** between two securities, because they are perfectly correlated.

$$\left.\begin{array}{l}\text{Mortage 1}\\ \text{Mortage 2}\end{array}\right\} \text{ Securitisation } \Rightarrow \left\{\begin{array}{l}\frac{M_1+M_2}{2} = S_1\\ \frac{M_1+M_2}{2} = S_2\end{array}\right.$$

$$Corr(S_1, S_2) = 1$$