1 1. Intro

1.1 FOC

$$\max_{(c_1,c_2)\geq 0,s} u\left(c_1,c_2\right) \text{ s.t. } c_2 = e_2 + \left(e_1 - c_1\right)(1+r)$$
$$\frac{\partial u(c_1,c_2)}{\partial c_1} - \lambda(1+r) = 0 \tag{1}$$
$$\frac{\partial u(c_1,c_2)}{\partial c_2} - \lambda = 0$$

1.2 Useful assumptions on utility

$$u\left(c_1, c_2\right) = v\left(c_1\right) + \beta v\left(c_2\right) \tag{2}$$

Examples

$$\begin{aligned} &\text{CARA:} \quad v(c) = -\exp(-c) \\ &\text{CRRA:} \quad v(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \square \neq 1 \\ &\log: \quad v(c) = \log(c) \end{aligned} \tag{3}$$

1.3 Inter-temporal equilibrium model

- H households and two periods, *t=1,2*
- **single, perishable good** and *no production*
- each household has some utility over consumption and exogenous endowments. They can write debt-contracts, i.e. they can borrow and save at a market prices (1+r)

Market clearing $\Rightarrow$ borrowing = saving

1.4 Competitive equilibrium - implicit assumptions

- price taking
- market-clearing
- rationality (Super strong assumption)
- rational expectations (super strong assumption)

Some formal results In equilibrium we must have that for each agent h

$$(1+r)\beta = \frac{v_h'\left(c_1^h\right)}{v_h'\left(c_2^h\right)}$$

Since v(.) is strictly concave this implies

$$c_2^1 > c_1^1 \Rightarrow c_2^h > c_1^h \forall h$$

if one agent consumes more than the other, then all agents must consume more Market clearing gives the results

Main take-away from this: We can get rid of individual consumption, be left with aggregate consumption only and read off asset prices from that