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Time delay estimation in unresolved lensed quasars

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ABSTRACT

Context. Early universe (EU) measurements and late universe (LU) observations have resulted in a tension on the estimated value of the Hubble parameter H_0. Time-delay cosmography offers an alternative method to measure H_0. In this respect, the H0LiCoW collaboration has reported a 2.4% measurement of H_0 compatible with LU observations, increasing the tension at the 5.3σ level. Whereas, TDCOSMO+SLACS has reported a 5% measurement of H_0 in agreement with both EU and LU estimates, showing the need to collect more data in order to reduce the error in the H_0 estimation.
Aims. In time-delay cosmography, the fractional error on H_0 is directly related to the error of relative time delays measurements and it linearly decreases with the number of lensed systems considered. Therefore, in order to reduce it, more lensed systems should be analysed and, possibly, with a regular and long-term monitoring, of the order of years. This cannot be achieved with big telescopes, due to the huge amount of observational requests they have to fulfill. On the other hand, small/medium-sized telescopes are present in a much larger number and are often characterized by more versatile observational programs. However, the limited resolution capabilities of such instruments and their often not privileged geographical location prevent them from providing well-separated images of the same lensed source.
In this work, we present a novel approach to estimate the time-delay in unresolved lensed quasar systems. Our proposal is further motivated by recent developments in discovering more unresolved strongly-lensed QSO systems.
Methods. Our method uses ...
Results. ...

Key words. giant planet formation – κ-mechanism – stability of gas spheres

1. Introduction

The Hubble parameter H_0 quantifies the expansion rate of the Universe and can be measured in different ways. The Planck collaboration et al (2020) has found $H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1} from the Cosmic Microwave Background anisotropies spectrum [1], whereas, cosmic distance ladder observations Riess et al. (2019) give $H_0 = 74.03 \pm 1.42$ km s^{-1} Mpc^{-1}, resulting in a tension of about 4.4σ.
As first pointed out in Refsdal (1964), gravitational lensing offers an alternative way of determining the Hubble parameter. The light rays coming from a distant source, e.g. a quasar, can be deflected by the gravitational field of an intervening massive object, like a galaxy. If the field is strong enough, multiple images of the same source will be observed and, by tracking the light intensity of each image over time, one can obtain a so-called *light curve*. The light curves associated with different images will exhibit a mutual time-delay, due to the different paths that the photons have travelled. As shown in Refsdal (1964), this time-delay can be directly related to H_0. The H0LiCOW collaboration Wong et al. (2020) found $H_0 = 73.3^{+1.7}_{-1.8}$ km s^{-1} Mpc^{-1} from a sample of six lensed quasars monitored by the COSMOGRAIL Millon et al. (2020) project, enhancing the tension up to 5.3σ, and thus motivating further studies aimed at improving the precision in the estimate of H_0.
Time-delays between light curves of Gravitationally Lensed Quasars (GLQs) can be estimated with various methods Tewes et al. (2013), such as free-knot spline interpolation Jupp (1978) or via Gaussian Process (GP) regression Rasmussen & Williams (2006). Such methods work under the assumption that multiple images of the same quasar are fully resolved so that a light curve can be extracted for each of them. However, most known GLQs qua (????) have small angular separations $\Delta\theta <\sim 3arcsec$. This would make big telescopes the ideal instruments to perform lensed quasars monitoring, both in light of their high angular resolution and the geographical areas they are placed in, where the effects of atmospheric turbulence are less prominent. However, because of the time scales of the intrinsic variations of the source, such observation campaigns should last years Millon et al. (2020). Therefore, due to the amount of observational requests that big telescopes have to fulfill, they can not be employed for these purposes. On the other hand, small/medium sized telescopes (1-2m) Borgeest et al. (1996) can be used. Unfortunately their reduced angular resolution, together with their often less privileged geographical positions in terms of atmospheric seeing, may worsen the effective angular resolution up to $3arcsec$ Karttunen (2016). In addition, small telescopes cannot observe faint distant sources. These shortcomings prevent the acquisition of a large number of GLQ datasets suitable for the application of the aforementioned techniques Jupp (1978); Rasmussen & Williams (2006). Here, we propose a novel ap-

⋆ Just to show the usage of the elements in the author field
⋆⋆ The university of heaven temporarily does not accept e-mails
[1] Assuming the ΛCDM cosmological model.

proach based on Machine Learning (ML) algorithms to estimate the time-delay from non-resolved lensed quasar light curves. In particular, we train a one-dimensional Convolutional Neural Network (CNN) to map non-resolved double-lensed quasar images into the corresponding time-delay. Starting from open-source real data Cos (????) of resolved quasar light curves, we combine them into a single time series to mimic the real situation in which the quasar images are not resolved. Then, we feed the so-obtained time series into a GP-based algorithm, which artificially generates new realistic non-resolved quasar light curves along with their corresponding time-delays. Preliminary experiments based on data of quasars HE0435 and HS2209 demonstrate the effectiveness of the proposed method.

2. Light Curves Simulation

In this section the one-zone model of **?**, originally used to study the Cepheïd pulsation mechanism, will be briefly reviewed. The resulting stability criteria will be rewritten in terms of local state variables, local timescales and constitutive relations.

? investigates the stability of thin layers in self-gravitating, spherical gas clouds with the following properties:

- hydrostatic equilibrium,
- thermal equilibrium,
- energy transport by grey radiation diffusion.

For the one-zone-model Baker obtains necessary conditions for dynamical, secular and vibrational (or pulsational) stability (Eqs. (34a, b, c) in Baker **?**). Using Baker's notation:

M_r	mass internal to the radius r
m	mass of the zone
r_0	unperturbed zone radius
ρ_0	unperturbed density in the zone
T_0	unperturbed temperature in the zone
L_{r0}	unperturbed luminosity
E_{th}	thermal energy of the zone

and with the definitions of the *local cooling time* (see Fig. 1)

$$\tau_{\mathrm{co}} = \frac{E_{\mathrm{th}}}{L_{r0}}\,, \tag{1}$$

and the *local free-fall time*

$$\tau_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G}\frac{4\pi r_0^3}{3M_{\mathrm{r}}}}\,, \tag{2}$$

Baker's K and σ_0 have the following form:

$$\sigma_0 = \frac{\pi}{\sqrt{8}}\frac{1}{\tau_{\mathrm{ff}}} \tag{3}$$

$$K = \frac{\sqrt{32}}{\pi}\frac{1}{\delta}\frac{\tau_{\mathrm{ff}}}{\tau_{\mathrm{co}}}\,; \tag{4}$$

where $E_{\mathrm{th}} \approx m(P_0/\rho_0)$ has been used and

$$\begin{aligned}\delta &= -\left(\frac{\partial\ln\rho}{\partial\ln T}\right)_P \\ e &= mc^2\end{aligned} \tag{5}$$

is a thermodynamical quantity which is of order 1 and equal to 1 for nonreacting mixtures of classical perfect gases. The physical meaning of σ_0 and K is clearly visible in the equations above. σ_0 represents a frequency of the order one per free-fall time. K is proportional to the ratio of the free-fall time and the cooling time. Substituting into Baker's criteria, using thermodynamic identities and definitions of thermodynamic quantities,

Table 1. Opacity sources.

Source	$T/[\mathrm{K}]$
Yorke 1979, Yorke 1980a	$\leq 1700^{\mathrm{a}}$
Krügel 1971	$1700 \leq T \leq 5000$
Cox & Stewart 1969	$5000 \leq$

$$\Gamma_1 = \left(\frac{\partial\ln P}{\partial\ln\rho}\right)_S,\ \chi_\rho = \left(\frac{\partial\ln P}{\partial\ln\rho}\right)_T,\ \kappa_P = \left(\frac{\partial\ln\kappa}{\partial\ln P}\right)_T$$

$$\nabla_{\mathrm{ad}} = \left(\frac{\partial\ln T}{\partial\ln P}\right)_S,\ \chi_T = \left(\frac{\partial\ln P}{\partial\ln T}\right)_\rho,\ \kappa_T = \left(\frac{\partial\ln\kappa}{\partial\ln T}\right)_T$$

one obtains, after some pages of algebra, the conditions for *stability* given below:

$$\frac{\pi^2}{8}\frac{1}{\tau_{\mathrm{ff}}^2}(3\Gamma_1-4) > 0 \tag{6}$$

$$\frac{\pi^2}{\tau_{\mathrm{co}}\tau_{\mathrm{ff}}^2}\Gamma_1\nabla_{\mathrm{ad}}\left[\frac{1-3/4\chi_\rho}{\chi_T}(\kappa_T-4)+\kappa_P+1\right] > 0 \tag{7}$$

$$\frac{\pi^2}{4}\frac{3}{\tau_{\mathrm{co}}\tau_{\mathrm{ff}}^2}\Gamma_1^2\nabla_{\mathrm{ad}}\left[4\nabla_{\mathrm{ad}}-(\nabla_{\mathrm{ad}}\kappa_T+\kappa_P)-\frac{4}{3\Gamma_1}\right] > 0 \tag{8}$$

For a physical discussion of the stability criteria see **?** or **?**.

We observe that these criteria for dynamical, secular and vibrational stability, respectively, can be factorized into

1. a factor containing local timescales only,
2. a factor containing only constitutive relations and their derivatives.

The first factors, depending on only timescales, are positive by definition. The signs of the left hand sides of the inequalities (6), (7) and (8) therefore depend exclusively on the second factors containing the constitutive relations. Since they depend only on state variables, the stability criteria themselves are *functions of the thermodynamic state in the local zone*. The one-zone stability can therefore be determined from a simple equation of state, given for example, as a function of density and temperature. Once the microphysics, i.e. the thermodynamics and opacities (see Table 1), are specified (in practice by specifying a chemical composition) the one-zone stability can be inferred if the thermodynamic state is specified. The zone – or in other words the layer – will be stable or unstable in whatever object it is imbedded as long as it satisfies the one-zone-model assumptions. Only the specific growth rates (depending upon the time scales) will be different for layers in different objects.

We will now write down the sign (and therefore stability) determining parts of the left-hand sides of the inequalities (6), (7) and (8) and thereby obtain *stability equations of state*.

The sign determining part of inequality (6) is $3\Gamma_1 - 4$ and it reduces to the criterion for dynamical stability

$$\Gamma_1 > \frac{4}{3}\,. \tag{9}$$

Fig. 1. Adiabatic exponent Γ_1. Γ_1 is plotted as a function of lg internal energy [erg g^{-1}] and lg density [g cm^{-3}].

Fig. 2. Vibrational stability equation of state $S_{\text{vib}}(\lg e, \lg \rho)$. > 0 means vibrational stability.

Stability of the thermodynamical equilibrium demands

$$\chi_\rho > 0, \quad c_v > 0\,, \tag{10}$$

and

$$\chi_T > 0 \tag{11}$$

holds for a wide range of physical situations. With

$$\Gamma_3 - 1 = \frac{P}{\rho T}\frac{\chi_T}{c_v} \quad > \quad 0 \tag{12}$$

$$\Gamma_1 = \chi_\rho + \chi_T(\Gamma_3 - 1) \quad > \quad 0 \tag{13}$$

$$\nabla_{\text{ad}} = \frac{\Gamma_3 - 1}{\Gamma_1} \quad > \quad 0 \tag{14}$$

we find the sign determining terms in inequalities (7) and (8) respectively and obtain the following form of the criteria for dynamical, secular and vibrational *stability*, respectively:

$$3\Gamma_1 - 4 =: S_{\text{dyn}} > \quad 0 \tag{15}$$

$$\frac{1 - 3/4\chi_\rho}{\chi_T}(\kappa_T - 4) + \kappa_P + 1 =: S_{\text{sec}} > \quad 0 \tag{16}$$

$$4\nabla_{\text{ad}} - (\nabla_{\text{ad}}\kappa_T + \kappa_P) - \frac{4}{3\Gamma_1} =: S_{\text{vib}} > \quad 0\,. \tag{17}$$

The constitutive relations are to be evaluated for the unperturbed thermodynamic state (say (ρ_0, T_0)) of the zone. We see that the one-zone stability of the layer depends only on the constitutive relations Γ_1, ∇_{ad}, χ_T, χ_ρ, κ_P, κ_T. These depend only on the unperturbed thermodynamical state of the layer. Therefore the above relations define the one-zone-stability equations of state S_{dyn}, S_{sec} and S_{vib}. See Fig. 2 for a picture of S_{vib}. Regions of secular instability are listed in Table 1.

3. Conclusions

1. The conditions for the stability of static, radiative layers in gas spheres, as described by Baker's (**?**) standard one-zone model, can be expressed as stability equations of state. These stability equations of state depend only on the local thermodynamic state of the layer.
2. If the constitutive relations – equations of state and Rosseland mean opacities – are specified, the stability equations of state can be evaluated without specifying properties of the layer.
3. For solar composition gas the κ-mechanism is working in the regions of the ice and dust features in the opacities, the H_2 dissociation and the combined H, first He ionization zone, as indicated by vibrational instability. These regions of instability are much larger in extent and degree of instability than the second He ionization zone that drives the Cepheïd pulsations.

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References

????, GLQ Database: https://research.ast.cam.ac.uk/lensedquasars/index.html
????, Released data – LASTRO - EPFL
Borgeest, U., Schramm, K.-J., & Von Linde, J. 1996, in Examining Big Bang Diffus. Backgr. Radiations (Springer Netherlands), 527–528
et al, P. 2020, A&A, 641, A6
Jupp, D. L. 1978, SIAM Journal on Numerical Analysis, 15, 328
Karttunen, H. 2016, Fundamental astronomy (Heidelberg: Springer)
Millon, M., Courbin, F., Bonvin, V., et al. 2020, Astron. Astrophys. [arXiv:2002.05736]
Rasmussen, C. & Williams, C. 2006, Gaussian Processes for Machine Learning, Adaptive Computation and Machine Learning (Cambridge, MA, USA: MIT Press), 248
Refsdal, S. 1964, Monthly Notices of the Royal Astronomical Society, 128, 307
Riess, A. G., Casertano, S., Yuan, W., Macri, L. M., & Scolnic, D. 2019, The Astrophysical Journal, 876, 85
Tewes, M., Courbin, F., & Meylan, G. 2013, Astron. Astrophys., 553 [arXiv:1208.5598]
Wong, K. C., Suyu, S. H., Chen, G. C.-F., et al. 2020, Mon. Not. R. Astron. Soc. [arXiv:arXiv:1907.04869v2]